Secure physical layer network coding versus secure network coding

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Abstract—Secure network coding realizes the secrecy of the message when the message is transmitted via noiseless network and a part of edges or a part of intermediate nodes are eavesdropped. In this framework, if the channels of the network has noise, we apply the error correction to noisy channel before applying the secure network coding. In contrast, secure physical layer network coding is a method to securely transmit a message by a combination of coding operation on nodes when the network is given as a set of noisy channels. In this paper, we give several examples of network, in which, secure physical layer network coding has advantage over secure network coding.

Index Terms—secrecy analysis, secure communication, noisy channel, network coding, computation and forward, physical layer security

I. INTRODUCTION

arXiv:1812.00117v1 [cs.CR] 1 Dec 2018

Secure network coding is a method to securely transmit the message via noiseless network when a part of edges or a part of intermediate nodes are eavesdropped [1], [2], [3], [4], [5], [6]. Since the real channel has noise, we apply the error correction to the real channel. Then, we apply secure network coding to the noiseless channel realized by error correction. That is, in the above scenario, we separately apply the error correction and secure network coding. Therefore, there is a possibility that we have an advantage by jointly applying the error correction and secure network coding. This idea is called physical network coding [7], [8], [9]. That is, to consider this improvement for the security, we discuss the secure version of physical layer network coding, i.e., secure physical layer network coding, which is a method to securely transmit a message by a combination of coding operation on nodes when the network is given as a set of noisy channels. There are two kinds of codes in secure physical layer network coding. Once we have secure network coding, we can attach physical layer network coding. This method can be considered as a simple combination of secure network coding and physical layer network coding. The other type of codes in secure physical layer network coding are codes that cannot be made by such a simple combination. Unfortunately, there are almost no studies for secure physical layer network coding of the latter type. That is, existing studies addressed

Manuscript submitted 1st December 2018; revised xxx, 2018.

only secure computation-and-forward, which is a method to securely transmit the modulo sum of two input message when noisy multiple access channel is given [10], [11], [12], [13], [14], [15]. The motivation of these studies is the realization of secure two way-relay channel with untrusted relay. To seek the further possibility of secure physical layer network coding, we need to find more examples of concrete coding schemes of secure physical layer network coding.

In fact, secure network code mainly focuses on the secrecy for the attack to channels. Some typical secure network codes cannot realize the secrecy when one of intermediate nodes is eavesdropped. In contrast, secure physical layer network coding is advantageous for attacks on intermediate nodes. In this paper, we give two examples of network, in which, secure physical layer network coding realizes a performance that cannot be realized by secure network coding. One is butterfly network [16] and the other is a network with three source nodes.

The remaining parts of this paper are organized as follows. Section II reviews the results of secure computation-andforward, which is a typical example of secure physical layer network coding. Section III discusses secure communication over butterfly network by using secure physical layer network coding. Section IV addresses secure communication over a network with three source nodes by using secure physical layer network coding.

II. SECURE COMPUTATION-AND-FORWARD

First, we review secure computation-and-forward. We consider secure computation-and-forward in a typical setting. Consider two senders V_1 , V_2 and one receiver *R*. Assume that

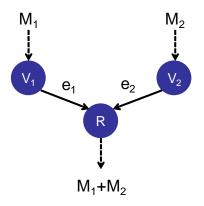


Fig. 1. Computation-and-forward.

The material in this paper was presented in part at the 2018 IEEE Information Theory Workshop (ITW), Guangzhou, China, 25-29 November 2018. [25]

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the sender V_i has message $M_i \in \mathbb{F}_q$, and the receiver R are linked by a (noisy) multiple access channel that has two inputs from the two senders V_1 and V_2 . Computation-and-forward is the task for the receiver R to obtain the modulo sum $M_1 + M_2$ via the (noisy) multiple access channel as Fig. 1.

To discuss computation-and-forward, many papers focused on a multiple access Gaussian channel. When the sender V_i sends the complex-valued signal X_i for i = 1, 2, the receiver *R* receives the complex-valued signal *Y* as

$$Y = h_1 X_1 + h_2 X_2 + Z, (1)$$

where $h_1, h_2 \in \mathbb{C}$ are the channel fading coefficients, and Z is a Gaussian complex random variable with average 0 and variance 1. In the following of this section, we assume multiple use of the above multiple access Gaussian channel.

Using lattice codes, the papers [17], [18], [19] derived an achievable rate under the energy constraint, which is called the computation rate. Also, based on the BPSK scheme, in which X_i is coded to $(-1)^{A_i}$ with $A_i \in \mathbb{F}_2$, the paper [20] derived an achievable rate $I(Y; A_1 + A_2)_{\text{Eq.}(1)}$, where the mutual information is given with the independent and uniform random numbers A_1 and A_2 . In this paper, we choose the base of logarithm to be *e*. Then, the papers [21], [22] proposed to use LDPC codes (regular LDPC codes and spatial coupling LDPC codes) with the BPSK scheme. The method proposed by [21], [22] can be efficiently realized, and realizes a rate close to $I(Y; A_1 + A_2)_{\text{Eq.}(1)}$.

When we additionally impose the secrecy for each message to the receiver R, this task is called secure computationand-forward. In this case, it is required that the receiver Robtains the modulo sum $M_1 + M_2$, but the variable in R's hand is independent of M_1 and M_2 . The papers [10], [11], [12], [13], [14] proposed a code for this task by using lattice code. Using an arbitrary algebraic code for computationand-forward given in [21], [22], the paper [15] proposed an efficiently realizable code. The paper [15] also derived an upper bound for the leaked information of the constructed finite length code. Also, the paper [15] showed that the rate $2I(Y; A_1 + A_2)_{Eq.(1)} - I(Y; A_1, A_2)_{Eq.(1)}$ is achievable in the BPSK scheme [15, (29)], where the mutual information is given with the independent and uniform random numbers A_1 and A_2 . That is, when the channel (1) is prepared and the receiver colludes with no sender, secure computation-andforward guarantees no information leakage of each message to the receiver while the receiver can recover the sum $M_1 + M_2$. In fact, all these papers [10], [11], [12], [13], [14] for secure computation-and-forward addressed only the case when the number of senders is 2. The paper [23] will address secure computation-and-forward when the number of senders is more than 2.

Unfortunately, we have no good application for secure computation-and-forward except for secure two way-relay channel with untrusted relay. The remaining part of this paper discusses its further application.

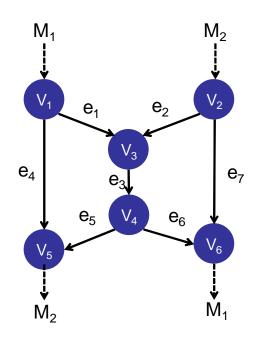


Fig. 2. Butterfly network coding.

III. BUTTERFLY NETWORK

A. conventional protocol

We consider butterfly network coding [16], which is a coding method that efficiently transmits the information in the crossing way as Fig. 2. The aim of this network is the following two tasks. The transmission of the message M_1 from V_1 to V_6 and the transmission of the message M_2 from V_2 to V_5 . When each channel can transmit only one element of \mathbb{F}_q , the channel e_3 from V_3 to V_4 is the bottleneck. In this network, only the node V_3 has the choice because other node has no other choice except for transmitting the received information. To resolve this bottleneck, the node V_3 transmits the modulo sum to the node V_4 via channel e_3 . Then, the sink node V_5 can recover the message M_2 from the received information M_1 and $M_1 + M_2$. Similarly, the other sink node V_6 can recover the message M_1 from the received information M_2 and $M_1 + M_2$.

B. Secure network coding

However, under this network code, the node V_3 obtains both messages M_1 and M_2 . The sink node V_5 obtains the intended message M_1 , and the other sink node V_6 has the same problem. Now, we consider impose the secrecy for the attack to one of intermediate nodes. That is, the information of all intermediate nodes needs to be independent of M_1 and M_2 , and the information of source node V_5 (V_6) needs to be independent of the unintended message M_1 (M_2). This kind of secrecy can be realized by employing a shared secret number L between V_1 and V_2 when messages M_1 and M_2 are elements of \mathbb{F}_q and q is not a power of 2 in the following way [24, Figure 2]. When the information transmitted Z_i on the edge e_i is given D as

$$Z_{1} = 2M_{1} + L, Z_{4} = -(M_{1} + L),$$

$$Z_{2} = 2M_{2} + L, Z_{7} = -(M_{2} + L),$$

$$Z_{3} = Z_{1} + Z_{2} = 2M_{1} + 2M_{2} + 2L,$$

$$Z_{5} = Z_{6} = Z_{3}/2,$$

$$\hat{M}_{2} = Z_{5} + Z_{4} = M_{2}, \hat{M}_{1} = Z_{6} + Z_{7} = M_{1}.$$

where \hat{M}_2 (\hat{M}_1) is the recovered message by V_5 (V_6). Any intermediate edge and any intermediate node obtain no information for the messages M_1 and M_2 . Also, the sink node V_5 (V_6) obtains no information for the message M_1 (M_2) while it obtains the message M_2 (M_1). Hence, this code guarantees the following types of security; (B1) When the eavesdropper attacks only one of edges, she obtains no information for the messages M_1 and M_2 . (B2) When no node colludes with another node, each node obtains no information for the unintended messages.

When $q \ge 4$ is a power of 2, the above code can be modified as follows. We choose an element $e \in \mathbb{F}_q$ such that $e^2 + e \ne 0$, i.e., $e \ne 1, 0$. Then, we arrange our code as

$$Z_{1} = (1 + e)M_{1} + L, \quad Z_{4} = -(M_{1} + L),$$

$$Z_{2} = (1 + e)M_{2} + eL, \quad Z_{7} = -(M_{2} + L),$$

$$Z_{3} = Z_{1} + Z_{2} = (1 + e)(M_{1} + M_{2} + L),$$

$$Z_{5} = Z_{6} = Z_{3}/(1 + e),$$

$$\hat{M}_{2} = Z_{5} + Z_{4} = M_{2}, \quad \hat{M}_{1} = Z_{6} + Z_{7} = M_{1}.$$

This modification realizes the required security in this case.

C. Secure physical layer network coding

But, if there is no shared secret number between V_1 and V_2 , it is not so easy to realize this kind of secrecy for the butterfly network under the framework of secure network coding. Now, we assume the assumption; (B3) The pairs (e_1, e_2) , (e_4, e_5) , and (e_6, e_7) are given as multiple access Gaussian channels like (1). Only the channel e_3 is a single input Gaussian channel. In this case, in the multiple access Gaussian channel (e_1, e_2) at V_3 , we employ secure computation-and-forward so that the node V_3 obtains the information $M_1 + M_2$. Then, the node V_3 forwards the obtained information to the node V_4 , and the node V_4 receives the information $M_4 := M_1 + M_2$. In the multiple access Gaussian channel (e_4, e_5) at V_5 , we again employ secure computation-and-forward so that the node V_5 obtains the information $M_4 - M_1 = M_2$. In the same way, the node V_6 obtains the information $M_4 - M_2 = M_1$. That is, this code guarantees the following types of security; (B4) When no node colludes with another node and (B3) is satisfied, each node obtains no information for the unintended messages.

As another kind of secure physical layer network coding, we attach the computation-and-forward to the communications to nodes V_3 , V_5 , and V_6 in the protocol with q = 4 given in Section III-B. In this protocol, an element of \mathbb{F}_4 is regarded as a vector on \mathbb{F}_2 . While this protocol saves the time, it still requires the secure shared randomness *L*. This protocol can be regarded as a simple combination of secure network coding and physical layer network coding.

D. Comparison

To implement these protocols as wireless communication network, we compare the transmission speeds of the protocols given in Sections III-B and III-C when each edge is given as the BPSK scheme of a two-input Gaussian channel as (1) or a single-input Gaussian channel

$$Y = hX + Z, (2)$$

where $h \in \mathbb{C}$ are the channel fading coefficients, Z is a Gaussian complex random variable with average 0 and variance 1, and X is coded as $(-1)^A$ with $A \in \mathbb{F}_2$. In this comparison, for simplicity, we assume that $h_1 = h_2 = h$. Here, we assume that T is the time period to transmitting one Gaussian signal in each edge. Additionally, we assume that ideal codes are available as follows. The mutual information rate $I(Y; A)_{Eq.(2)}$ is available in the channel (2), the rate $I(Y; A_1 + A_2)_{Eq.(1)}$ is available for computation-and-forward in the channel (1), and the rate $2I(Y; A_1 + A_2)_{Eq.(1)} - I(Y; A_1, A_2)_{Eq.(1)}$ is available for secure computation-and-forward in the channel (1). Notice that the relation $I(Y; A_2|A_1)_{\text{Eq.}(1)} = I(Y; A_1|A_2)_{\text{Eq.}(1)}$ holds in this case. Also, the mutual information rate pair $(I(Y; A_1A_2)_{\text{Eq.}(1)}/2, I(Y; A_1A_2)_{\text{Eq.}(1)}/2)$ is available in the MAC channel (1) when both senders intend to send their own message to the receiver. In the above discussion, the random variables A_1 , A_2 , and A are subject to the uniform distribution independently.

Secure network coding protocol given in Section III-B needs to avoid a crossed line when we do not use multiple access Gaussian channel. Hence, its whole network needs four time slots at least as Table I. Therefore, to transmit message with size e^R , the required time in this case is calculated to be $\frac{4RT}{I(Y;A)_{Eq.}(2)}$.

 TABLE I

 Secure network coding without multiple access Gaussian

 channel

Time slot	Time 1	Time 2	Time 3	Time 4
Channel	e_1, e_4	e_2, e_7	e_3	e_5, e_6

When we use multiple access Gaussian channel, secure network coding protocol given in Section III-B can be realized with three time slots as Table II. In this case, to transmit message with size e^R , the required time is calculated to be $\frac{2RT}{I(Y;A)_{Eq.(2)}} + \frac{2RT}{I(Y;A_{1,A_2})_{Eq.(1)}}$. Although we can design the whole process as Table III, this design requires the time length $\frac{RT}{I(Y;A)_{Eq.(2)}} + \frac{4RT}{I(Y;A_{1,A_2})_{Eq.(1)}}$, which is larger than $\frac{2RT}{I(Y;A)_{Eq.(2)}} + \frac{2RT}{I(Y;A_{1,A_2})_{Eq.(1)}}$ because $\frac{I(Y;A_{1,A_2})_{Eq.(1)}}{2} \leq I(Y;A)_{Eq.(2)}$.

Secure physical layer network coding protocol given in the 1st paragraph of Section III-C can be realized only with three time slots as in Table III, where the pairs (e_1, e_2) , (e_4, e_5) , and (e_6, e_7) are realized by secure computation-and-forward based on the Gaussian MAC channel (1). Therefore, to transmit message with size e^R , the required time in this case is calculated to be $\frac{2RT}{2I(Y;A_1+A_2)_{\text{Eq.}(1)}-I(Y;A_1,A_2)_{\text{Eq.}(1)}} + \frac{RT}{I(Y;A)_{\text{Eq.}(2)}}$.

 TABLE II

 Secure Network coding with multiple access Gaussian channel

Time slot	Time 1	Time 2	Time 3
Channel	(e_1, e_2)	e_3, e_4, e_7	e_5, e_6

 (e_i, e_j) expresses a multiple access Gaussian channel of the joint transmission on the edges e_i and e_j .

Secure physical layer network coding protocol given in the 2nd paragraph of Section III-C also can be realized only with three time slots as in Table III. Therefore, to transmit message with size e^R , the required time in this case is calculated to be $\frac{2RT}{I(Y;A_1+A_2)_{Eq.(1)}} + \frac{RT}{I(Y;A)_{Eq.(2)}}$.

TABLE III Secure physical layer network coding with multiple access Gaussian channel

Time slot	Time 1	Time 2	Time 3
Channel	(e_1, e_2)	e_3	$(e_4, e_5), (e_6, e_7)$

Fig. 3 gives the numerical comparison among $\frac{4RT}{I(Y;A)_{Eq.(2)}}$ 2RT2RT+ $\frac{I(Y;A_1,A_2)_{\text{Eq.}(1)}}{2RT}$, $\frac{I(Y;A)_{\text{Eq.}(2)}}{RT}$ $\frac{I(Y;A_{\text{Eq.}(2)})}{RT} + \frac{1}{I(Y;A_1,A_2)_{\text{Eq.}(1)}}, \quad \frac{2I(Y;A_1+A_2)_{\text{Eq.}(1)} - I(Y;A_1,A_2)_{\text{Eq.}(1)}}{2I(Y;A_1+A_2)_{\text{Eq.}(1)}} + \frac{RT}{I(Y;A)_{\text{Eq.}(2)}}, \text{ and } \frac{2RT}{I(Y;A_1+A_2)_{\text{Eq.}(1)}} + \frac{RT}{I(Y;A)_{\text{Eq.}(2)}}. \text{ Secure network}$ coding protocol given in Section III-B requires shorter time length for the transmission than secure physical layer network coding protocol given in Section III-C in this comparison. Since the difference is not so extensive, secure physical layer network coding protocol given in the first paragraph of Section III-C is useful when it is not easy to prepare secure shared randomness between two source nodes. In fact, when we use the butterfly network, it is usual that the direct communication between two source nodes is not easy. In this case, such a secure shared randomness requires an additional cost. However, secure physical layer network coding protocol given in the second paragraph of Section III-C has no advantage over the secure network coding protocol with MAC channel. That is, a simple combination of secure network coding and physical layer network coding is not so useful in this case.

IV. NETWORK WITH THREE SOURCES

Next, we consider the network given in Fig 4. This network has three source nodes S_1 , S_2 , and S_3 , three intermediate nodes I_1 , I_2 , and I_3 , and one terminal node T. The aim of this network is secure transmission from the three source nodes to the terminal node T. The source node S_i intends to transmit an element $M_i \in \mathbb{F}_q$ to the terminal node T.

A. Secure network coding

First, we consider this network with the framework of secure network coding. Each edge expresses a channel to transmit one element of \mathbb{F}_q without error. We consider two settings.

(1) Eve can eavesdrop one edge among three edges between the intermediate nodes and the terminal node.

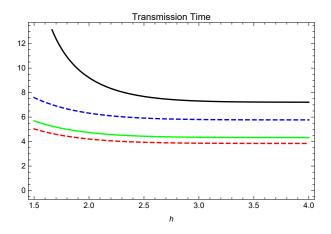


Fig. 3. Transmission Time for four schemes when RT = 1. Upper solid line (Black) expresses the time $\frac{2RT}{2I(Y;A_1+A_2)\text{Eq.}(1)} - I(Y;A_1,A_2)\text{Eq.}(1)} + \frac{RT}{I(Y;A)\text{Eq.}(2)}$ of secure physical layer network coding protocol given in the 1st paragraph of Section III-C. Upper dashed line (Blue) expresses the time $\frac{4RT}{I(Y;A)\text{Eq.}(2)}$ of secure network coding protocol given in Section III-B without MAC channel. Lower dashed line (Red) expresses the time $\frac{2RT}{I(Y;A)\text{Eq.}(2)} + \frac{2RT}{I(Y;A)\text{Lq.}(3)}$ of secure network coding protocol given in Section III-B with MAC channel. Lower solid line (Green) expresses the time $\frac{2RT}{I(Y;A)\text{Eq.}(2)} + \frac{RT}{I(Y;A)\text{Eq.}(2)}$ of secure physical layer network coding protocol given in the 2nd paragraph of Section III-C.

(2) Eve can eavesdrop one intermediate node among three intermediate nodes.

1) Case (1): The following code is secure in the case (1) when q is not a power of 2. Notice that the matrix 0 1 1 -1/21/21/20 1 1/2-1/21/21 is invertible because is 1 0 1/21/21 -1/2the inverse matrix. Source node S_i sends M_i in each edge. Each intermediate node sends the sum of the received information. -1/21/21/2Finally, applying the inverse matrix 1/2-1/21/21/21/2-1/2to the received information, the node T recovers all messages. In this code, each of the messages $M_1 + M_2$, $M_2 + M_3$, and $M_3 + M_1$ is independent of anyone of M_1 , M_2 , and M_3 . Hence, the security in the case (1) is satisfied. The rate of this protocol is the optimal even without the secrecy condition.

Next, we consider the case when $q \ge 4$ is a power of 2. We choose an element $e \in \mathbb{F}_q$ such that $e^2 + e \neq 0$, $0 \ 1 \ 1$ 0 is invertible because its е which implies that e e 0 determinant is $e^2 + e \neq 0$. For example, when q = 4, since 1+e e e $e^2 = e + 1$, the inverse matrix is 1 + ee 1 Then, е 1 the following code is secure; Source node S_i sends M_i in each edge. The intermediate nodes I_1 , I_2 , and I_3 send the received information $Z_1 := M_2 + M_3$, $Z_2 := M_1 + eM_3$, and $Z_3 := eM_1 + eM_2$, respectively. Finally, applying the inverse 0 1 1 Z_1

matrix of $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & e \\ e & e & 0 \end{pmatrix}$ to the received information $\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix}$, the node *T* recovers all messages. In this code, each of the

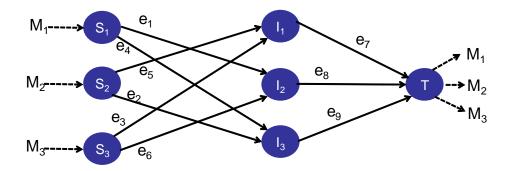


Fig. 4. Network with three sources.

informations $eM_1 + eM_2$, $M_2 + M_3$, and $eM_3 + M_1$ is independent of anyone of M_1 , M_2 , and M_3 . Hence, the security in the case (1) is satisfied. That is, this code guarantees the following type of security; (T1) When the eavesdropper attacks only one of edges, she obtains no information for anyone of the messages M_1 , M_2 and M_3 .

2) Case (2): In the case (2), the following code is secure. We use the channels between the intermediate nodes and the terminal node twice, but, we use the channels between the source nodes and the intermediate nodes only once. Source node S_i prepares scramble variable L_i . Source node S_i sends the scramble variable L_i to the intermediate node I_{i+1} via the edge e_i . Source node S_i sends the variable $M_i - L_i$ to Intermediate node I_{i-1} via the edge e_{3+i} . Here i + 1 and i - 1are regarded as elements of \mathbb{Z}_3 . Each intermediate node sends both received variables to the terminal node by using the channel twice. Since the terminal node T obtains information $L_1, L_2, L_3, M_1 - L_1, M_2 - L_2$, and $M_3 - L_3$, it can recover the messages $M_1 = (M_1 - L_1) + L_1$, $M_2 = (M_2 - L_2) + L_2$, and $M_3 = (M_3 - L_3) + L_3$. The information on the intermediate node I_i is the pair of L_{i+1} and $M_{i-1} - L_{i-1}$, which is independent of anyone of M_1 , M_2 , and M_3 . Hence, this code guarantees the following type of security; (T2) When no intermediate node colludes with another node, each intermediate node obtains no information for the messages.

B. Secure physical layer network coding

1) Use of secure computation-and-forward: Now, we assume the assumption; (T3) The pairs (e_1, e_6) , (e_2, e_4) , and (e_3, e_5) are given as multiple access Gaussian channels like (1). We assume that the eavesdropper can access one of the information on the intermediate nodes, which corresponds to Case 2 of Section IV-A. Then, using secure computation-and-forward, we construct a required protocol.

First, we consider the case when q is not a power of 2. In the multiple access Gaussian channel (e_1, e_6) , we employ secure computation-and-forward so that the node I_2 obtains the information $M_1 + M_3$. Similarly, I_1 and I_3 obtain the information $M_2 + M_3$ and $M_1 + M_2$, respectively. Therefore, the information on each intermediate node is independent of the messages M_1 , M_2 , and M_3 . In the next step, the intermediates nodes I_1 , I_2 , and I_3 transmit their obtained information M'_1 , M'_2 , and M'_3 to the terminal node T via the multiple access Gaussian channels with three input signals. Then, applying separate decoding, the terminal node T recovered the information M'_1, M'_2 , and M'_3 . Using the method given in Section IV-A1, the terminal node T obtains the original information M_1, M_2 , and M_3 .

When $q \ge 4$ is a power of 2, to apply the method given in Section IV-A1, the node I_2 needs to obtain the information $M_1 + eM_3$. It can be realized by secure computation-andforward with a 2-dimensional vector over the finite field \mathbb{F}_2 by the prior conversion from M_3 to eM_3 in the node S_3 before use of the multiple access Gaussian channel (e_1, e_6) . The same method is applied to the multiple access Gaussian channels (e_2, e_4) and (e_3, e_5) . Then, the remaining part can be done in the same way as the above.

Therefore, under the framework of secure physical layer network coding, we can realize the secure code for the attack to an intermediate node by using secure computation-andforward. That is, this code guarantees the following types of security; (T4) When no node colludes with another node and (T3) is satisfied, each intermediate node obtains no information for the messages. This code does not require additional random number like the code given in Section IV-A2.

2) Use of computation-and-forward: Next, using computation-and-forward, we construct a required protocol. For this aim, we employ the protocol given in Section IV-A2. In this protocol, at the node T, to recover M_1 we employ computation-and-forward of two edges e_8 and e_9 . Similarly, to recover M_2 (M_3), we employ computation-and-forward of two edges e_7 and e_9 (e_7 and e_8).

C. Comparison

To implement these protocols as wireless communication network, we compare the transmission speeds of the protocols given in Sections IV-A and IV-B when each edge is given as the BPSK scheme of a single-input Gaussian channel (2), a two-input Gaussian channel (1), or a three-input Gaussian channel (1)

$$Y = hX_1 + hX_2 + hX_3 + Z, (3)$$

where $h \in \mathbb{C}$ are the channel fading coefficients, Z is a Gaussian complex random variable with average 0 and variance 1, and X_i is coded as $(-1)^{A_i}$ with $A_i \in \mathbb{F}_2$. In this comparison, we make the same assumptions for h_1 , h_2 , and

T. Additionally, we assume that ideal codes given in Section III-D are available, and that the mutual information rate triple $(I(Y; A_1A_2A_3)_{\text{Eq.}(3)}/3, I(Y; A_1A_2A_3)_{\text{Eq.}(3)}/3, I(Y; A_1A_2A_3)_{\text{Eq.}(3)}/3, I(Y; A_1A_2A_3)_{\text{Eq.}(3)}/3, is available in the MAC channel (3) when three senders intend to send their own message to the receiver, where the random variables <math>A_1$, A_2 , and A_3 are subject to the uniform distribution independently. Under these assumptions, we compare secure network coding protocol given in Section IV-A2 and secure physical layer network coding protocols realize the secrecy for intermediate nods.

When we do not use multiple access Gaussian channel. secure network coding protocol given in Section IV-A2 needs five time slots at least as Table IV. In particular, the edges e_7 , e_8 , and e_9 need to send twice information as the remaining edges. Therefore, to transmit message with size e^R , the required time in this case is calculated to be $\frac{8RT}{I(Y;A)E_{G,(2)}}$. When we use multiple access Gaussian channel, secure network coding protocol given in Section IV-A2 can be realized with two time slots as Table V. In this case, to transmit message with size e^R , the required time in this case is calculated to be $\frac{6RT}{I(Y;A_1A_2A_3)E_{G,(3)}} + \frac{2RT}{I(Y;A_1A_2)E_{G,(1)}}$.

TABLE IV Secure network coding without multiple access Gaussian Channel

Time span	Time 1	Time 2	Time 3	Time 4	Time 5
Channel	e_1, e_2, e_3	e_4, e_5, e_6	e_7	e_8	e 9

 TABLE V

 Secure network coding with multiple access Gaussian channel

Time span	Time 1	Time 2
Channel	$(e_1, e_6), (e_2, e_4), (e_3, e_5)$	(e_7, e_8, e_9)

Secure physical layer network coding protocol given in Section IV-B1 can be realized only with two time slots as in Table VI, where the pairs (e_1, e_2) , (e_4, e_5) , and (e_6, e_7) are realized by secure computation-and-forward based on the Gaussian MAC channel (1). Therefore, to transmit message with size e^R , the required time in this case is calculated to be $\frac{3RT}{I(Y;A_1A_2A_3)_{\text{Eq.}(3)}} + \frac{RT}{2I(Y;A_1+A_2)_{\text{Eq.}(1)}-I(Y;A_1,A_2)_{\text{Eq.}(1)}}.$

TABLE VI Secure physical layer network coding with secure computation-and-forward

Time span	Time 1	Time 2	
Channel	$(e_1, e_6), (e_2, e_4), (e_3, e_5)$	(e_7, e_8, e_9)	

Another secure physical layer network coding protocol given in Section IV-B2 can be realized only with two time slots as in Table VII, where the pairs (e_1, e_2) , (e_4, e_5) , and (e_6, e_7)

T. Additionally, we assume that ideal codes given in Section III-D are available, and that the mutual information rate triple $(I(Y; A_1A_2A_3)_{\text{Eq.}(3)}/3, I(Y; A_1A_2A_3)_{\text{Eq.}(3)}/3, I(Y; A_1A_2A_3)_{\text{Eq.}(3)}/3, I(Y; A_1A_2A_3)_{\text{Eq.}(3)}/3)$ with size e^R , the required time in this case is calculated to be available in the MAC channel (3) when three senders $\frac{3RT}{I(Y;A_1+A_2)_{\text{Eq.}(1)}} + \frac{2RT}{I(Y;A_1A_2A_2)_{\text{Eq.}(1)}}$.

TABLE VII Secure physical layer network coding with computation-and-forward

Time span	Time 1	Time 2	Time 3	Time 4
Channel	$(e_1, e_6), (e_2, e_4), (e_3, e_5)$	(e_8, e_9)	(e_7, e_9)	(e_7, e_8)

gives the numerical comparison among RT $\frac{\frac{3RT}{I(Y;A_1A_2A_3)_{\text{Eq.}(3)}} + \frac{RT}{2I(Y;A_1+A_2)_{\text{Eq.}(1)} - I(Y;A_1,A_2)_{\text{Eq.}(1)}}, \frac{3RT}{I(Y;A_1+A_2)_{\text{Eq.}(3)}} + \frac{2RT}{I(Y;A_1A_2)_{\text{Eq.}(2)}}, \text{ and } \frac{6RT}{I(Y;A_1A_2A_3)_{\text{Eq.}(3)}} + \frac{2RT}{I(Y;A_1A_2)_{\text{Eq.}(1)}}.$ Codes of secure physical layer network coding protocol given in Section IV-B require shorter time length for the transmission than secure network coding protocol given in Section IV-A2 in this comparison when the coefficient h is larger than about 1.7. This comparison shows the advantage of the secure physical layer network coding protocol given in Section IV-B1 over the secure network coding protocol given in Section IV-A2. Also, this comparison indicates the advantage of the simple combination of secure network coding and physical layer network coding given in Section IV-B2 over the secure network coding protocol given in Section IV-A2 with MAC channel.

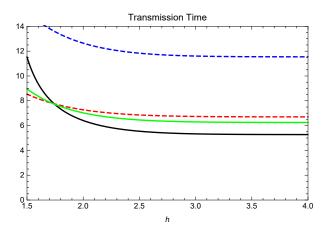


Fig. 5. Transmission Time for four schemes when RT = 1. Solid line (Black) expresses the time $\frac{3RT}{I(Y;A_1A_2A_3)_{Eq.(3)}} + \frac{RT}{2I(Y;A_1+A_2)_{Eq.(1)}-I(Y;A_1,A_2)_{Eq.(1)}}$ of secure physical layer network coding protocol given in Section IV-B1. Solid line (Green) expresses the time $\frac{3RT}{I(Y;A_1+A_2)_{Eq.(1)}} + \frac{2RT}{I(Y;A_1-A_2)_{Eq.(1)}}$ of secure physical layer network coding protocol given in Section IV-B2. Upper dashed line (Blue) expresses the time $\frac{8RT}{I(Y;A_1+A_2)_{Eq.(2)}}$ of secure network coding protocol given in Section IV-B2. Upper dashed line (Blue) expresses the time $\frac{8RT}{I(Y;A_1+A_2)_{Eq.(2)}}$ of secure network coding protocol given in Section IV-A2 without MAC channel. Lower dashed line (Red) expresses the time $\frac{6RT}{I(Y;A_1A_2A_3)_{Eq.(3)}} + \frac{2RT}{I(Y;A_1,A_2)_{Eq.(1)}}$ of secure network coding protocol given in Section IV-A2 with MAC channel. Solid line (Black), Solid line (Green), and Lower dashed line (Red) intersect around h = 1.7.

V. CONCLUSION

We have discussed the advantages of secure physical layer network coding over secure network coding. To clarify this advantage, we have addressed two typical networks. One is the butterfly network (Fig 2), and the other is a network with three source nodes (Fig. 4). That is, we have given a concrete protocol that efficiently works on these examples. We have also compared transmission times of proposed codes.

In these examples, secure physical layer network coding can realize the secrecy against intermediate nodes. Therefore, we can consider that secure physical layer network coding is useful when we realize the secrecy against intermediate nodes. In fact, when adversary attacks an intermediate node secure network coding requires more randomness than when adversary attacks an edge. Further, there are still a small number of applications of secure physical layer network coding. Hence, it is a future study to find more fruitful applications of secure physical layer network coding.

ACKNOWLEDGMENTS

The author is grateful to Prof. Ángeles Vazquez-Castro, Prof. Tadashi Wadayama, and Dr. Satoshi Takabe for discussions on secure physical layer network coding. The author is also grateful to Dr. Go Kato and Prof. Masaki Owari for discussions on secure network coding. The work reported here was supported in part by the JSPS Grant-in-Aid for Scientific Research (A) No.17H01280, (B) No. 16KT0017, (C) No. 16K00014, and Kayamori Foundation of Informational Science Advancement.

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