Secrecy Capacity in Large Cooperative Networks in Presence of Eavesdroppers with Unknown Locations

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Abstract

In this paper, an extended large wireless network under the secrecy constraint is considered. In contrast to works which use idealized assumptions, a more realistic network situation with unknown eavesdroppers locations is investigated: the legitimate users only know their own Channel State Information (CSI), not the eavesdroppers CSI. Also, the network is analyzed by taking in to account the effects of both fading and path loss. Under these assumptions, a power efficient cooperative scheme, named *stochastic virtual beamforming*, is proposed. Applying this scheme, an unbounded secure rate with any desired outage level is achieved, provided that the density of the legitimate users tends to infinity. In addition, by tending the legitimate users density to the infinity, the tolerable density of eavesdroppers will become unbounded too.

I. INTRODUCTION

Nowadays, secrecy is an essential quality of service which is harder to meet in wireless networks, because their broadcast nature increases the possibility of eavesdropping. Common methods rely on using algorithms with high computational complexity that are hard to break for an adversary [1]. Another field which focuses on the attackers with unlimited computational power is information-theoretic physical layer secrecy. Wiretap channel, the basic model for information-theoretic secrecy, was introduced by Wyner in [2] through which reliable and secure transmission is possible if the channel between the transmitter and the eavesdropper is the degraded version of the direct channel, i.e., between the transmitter and the receiver.

There are many research works on wireless networks with few nodes [3], but wireless systems are getting larger and larger and their exact performance analysis is getting complex, actually impossible. This leads the research community to turn into the scaling laws and analyzing the asymptotic behavior. Large wireless networks was first investigated in [4] by Gupta and Kumar from the scaling laws point of view. They considered an ad hoc large network with n randomly located nodes and the total rate that they achieved is $O(\sqrt{n})$. Effects of secrecy on large wireless networks was investigated in [5] for the first time, where a large wireless network has been investigated that the distributions of the legitimate and eavesdropper nodes are according to the Poisson point processes with densities λ_l and λ_e , respectively. The result of [5] is that the secure communication with total rate of $O(\sqrt{n})$ is possible, as long as $\lambda_e/\lambda_l = O((\log n)^2)$, where n is the number of the legitimate nodes. These works showed that it is possible to achieve the total rate that scales like \sqrt{n} under per node power constraint, with and without secrecy. However, their main limiting assumption was considering a point-to-point multihopping communication which excludes the possibility of cooperation using relays.

Authors in [6] proposed a cooperative scheme to achieve a total rate with near linear scaling under per node power constraint in a large wireless network without secrecy constraints. In addition, they showed the possibility of zero cost communication, i.e., unbounded total rate for fixed total power constraint. In [7] using active cooperative relaying based schemes and with a bound on the number of the eavesdroppers, the authors showed that zero cost secure communication is also possible. Recent developments in wireless technology (e.g., self interference cancellation, power allocation scheme at the PHY layer, proper MAC protocol for the efficient implementation of the full-duplex transmission mode [8]) support the relaying based cooperative models, in contrast to the traditional multi-hop interference limited networks.

In the model of [5], [7], the Channel State Information (CSI) is known to the legitimate transmitter. However, knowing CSI leads to the knowledge of the location of the passive eavesdroppers; that is not reasonable in many practical cases. So the natural questions here are that if zero cost secure communication is possible under unknown CSI. And, how should the cooperative strategies change to achieve this result? In addition, another important aspect of wireless network, ignored in many works, is fading. How fading affects the secrecy rate in wireless systems is a challenging question.

The secrecy rate in large networks with *unknown CSI* is investigated in some recent works. In [9], [10], the total rate of order 1 was achieved in a large wireless network with fading when CSI is not known. The authors in [11] took the advantage of path diversity to achieve the total rate of order $\sqrt{\frac{n}{\ln(n)}}$ in the case of unknown CSI, by limiting the number of the eavesdroppers that can be tolerated. Adding network coding has improved this result in [12] to a scheme in which any number of eavesdroppers could be tolerated without any change in the total achievable rate. The unknown CSI assumption is also taken into account in other works such as [13], [14]. However, to the best of our knowledge, none of the existing works uses relaying to achieve zero cost secure communication with unknown CSI and/or fading.

In this paper, we answer the above question affirmatively by proposing a scheme that achieves zero cost secure communication in a fading network and in the case of unknown CSI (including the eavesdroppers location). We

consider a network with n_l legitimate nodes and n_e eavesdroppers that are distributed according to the Poisson point processes with densities λ_l and λ_e . In contrary to the existing works, we achieve zero cost secure communication, i.e., unbounded total secrecy rate, by using cooperation and distributed beamforming. In order to overcome the lack of CSI knowledge, we propose a new scheme called, *stochastic virtual beamforming*. In this 2-stage scheme, we benefit from the fading diversity by exploiting some relaying nodes near the transmitter. Actually, we design a decode and forward scenario to direct the majority of the power toward the receiver location. To make this possible, at the first step the transmitter sends the secure message to all the relaying nodes by using wiretap coding. The security of this transmission step is provided by using the distance advantage of the relaying nodes in comparison with the eavesdroppers. So we leverage the path-loss effect in a positive way. Then, at the second step, the relaying nodes accomplish a distributed beamforming by setting their transmission coefficients proportional to the complex conjugate of their channel gains to the receiver.

II. NETWORK MODEL AND PRELIMINARIES

Throughout the paper, use upper case letters are used for denoting the random variables and lower-case letters for their realizations. Also, superscripts l and e are used for denoting legitimate users and eavesdroppers, respectively. We note the desired secure rate and outage level by R_S and ϵ , respectively. Also, we define ϵ' to be equal to $\frac{\epsilon}{7}$. Considering both path loss and fading effects, we use a common model for characterizing the power attenuation in wireless mediums as [15]: $\frac{P_R}{P_T} = C\alpha^2 10^{\frac{\pi}{10}} d^{-\gamma}$, in which, P_R is the received power; P_T is the transmitted power; Cis a constant; α is the fading coefficient; $10^{\frac{\pi}{10}}$ denotes the shadow fading where $X \sim N(0, \sigma^2)$; d is the distance between the transmitter and the receiver; and γ is the path loss exponent which depends on the environment and normally $\gamma \ge 2$. α is assumed to have Rayleigh distribution with parameter μ . For simplicity, we ignore the effect of shadow fading comparing with path loss effect (we remark that the shadowing effect is a random variable varying with location not with time). Also, because of different and stochastic paths between the transmitter and the receiver, the phase of the received signal (shown by θ) is modeled by a uniform distribution on $[0, 2\pi]$. The letters h and dwith appropriate subscripts and superscripts are used for indicating fading coefficients and distances, respectively. So, the channel gain from the *i*th legitimate user to the *j*th legitimate user and also, to the *k*th eavesdropper can be characterized by:

$$G_{i,j}^{l} = h_{i,j}^{l} (d_{i,j}^{l})^{-\gamma/2} e^{j\theta_{i,j}^{l}}$$
(1)

$$G_{i,k}^{e} = h_{i,k}^{e} (d_{i,k}^{e})^{-\gamma/2} e^{j\theta_{ik}^{e}}.$$
(2)

We assume that the environment is isotropic. Hence the fading statistics is the same between every two nodes. We consider a network with n_l legitimate nodes and n_e eavesdroppers that are distributed according to the Poisson point processes with densities λ_l and λ_e . We consider the eavesdroppers as passive attackers with no collusion between them. In addition, we assume that neither the location nor the fading coefficient of any eavesdropper channel is not

known to the legitimate users. We consider an extended wireless network. In order to establish consistency between the density of legitimate users (λ_l) and their total number (n_l) , we consider the network as a square with the side equal to $\sqrt{\frac{n_l}{\lambda_l}}$. Also, for the sake of simplicity we let the transmitter to be located at the center of the square. The Rayleigh assumption for fading results in $\mathbb{E}[H^2] = 2\mu$. Also, for simplicity we assume that the noise variances of all the channels, either legitimate or non-legitimate, are the same and equal to unity.

III. MAIN RESULTS

The main result of this paper is summarized in the following theorem. This theorem states that the zero-cost secure communication is possible by using our proposed scheme when eavesdroppers CSI is not known to the legitimate users. The rest of this section is devoted to the proof of this result, where we analyze the scheme in detail and derive six constraints for different parameters of the network. These constraints are consistent and can be selected step by step.

Theorem 1: In the extended network with fading and unknown eavesdroppers CSI (defined in Section II), under the constant power constraint and by letting the legitimate users density to be sufficiently large, any desired pair of secure rate and outage level denoted by (R_S, ϵ) is achievable.

Proof: We propose a scheme which achieves the desired result. Our proof has two steps: (i) In the first step, we consider the transmission from the transmitter (source) to the relaying nodes and guarantee a specific secure rate R_S with high probability for this transmission. Our technique is based on defining two circles, denoted by B_l and B_e , centered at the transmitter and radii a_l and a_e , while $a_l < a_e$ (see Fig.1). Then λ_l , λ_e , a_l , and a_e are chosen such that the following three requirements are provided. First, with the probability greater than $1 - \epsilon'$, no eavesdropper lies in B_e . Second, with the probability greater than $1 - \epsilon'$ at least n_r legitimate users lie in B_l . Third, the difference of the worst legitimate channel and the best eavesdropper channel be greater than R_S with a probability greater than $1 - \epsilon'$. (ii) In the second step, we analyze the rate from the relay nodes to the receiver and guarantee the second rate using the cooperation of n_r relaying nodes. Actually, we make this distributed Multiple-Input Single-Output Single-antenna Eavesdropper (MISOSE) situation to concentrate the most of the transmitted power in a neighboring region of the receiver. It can be deduced from our following calculations that by increasing n_r , both R_S and ϵ can be improved, i.e., increased and decreased, respectively.

A. Step 1: First rate analysis

In this step, we guarantee a secure rate R_S for the transmission from the transmitter to the relaying nodes with an outage level of $2\epsilon'$. To make this possible, we choose the radius of the circles B_l and B_e in a way that even with considering possible exacerbating effects of the fading, the difference between the capacities of the worst legitimate channel and the best eavesdropper channel be greater than R_S which is done by obtaining proper upper and lower bounds on a_l and a_e , respectively. Hence, the following constraint must hold:

$$\min_{1 \le i \le n_r, 1 \le j \le n_e} C_i^l - C_j^e > R_S \tag{3}$$

where C_i^l is the rate of the link from the transmitter to the *i*-th legitimate user and C_j^e is the rate of the link from the transmitter to the *j*-th eavesdropper. To simplify the analysis, we work with a suboptimum problem and we guarantee the following two inequalities:

$$\min_{1 \le i \le n_r} C_i^l > (1+\rho)R_S,\tag{4}$$

$$\max_{1 < j < n_e} C_j^e < \rho R_S.$$
⁽⁵⁾

in which, ρ is an arbitrary positive constant and the problem can be optimized over ρ . Now, to establish (4) and (5), we present appropriate upper and lower bounds on a_l and a_e , respectively, where each bound holds with an outage level of ϵ' .

1) The legitimate rate analysis: In the following theorem, considering the constraint on C_i^l , we derive an appropriate upper bound on a_l .

Theorem 2 (Upper bound on a_l): A sufficient condition for having (4), with an outage level of ϵ' , is:

$$a_l < \left(\frac{-P_T \mu \ln\left(1 - \frac{\epsilon'}{n_r}\right)}{2^{(1+\rho)R_S} - 1}\right)^{\frac{1}{\gamma}}.$$
(6)

Proof: Using the union bound we guarantee the outage level of $\frac{\epsilon'}{n_r}$ for the rate of each of the n_r relaying nodes in B_l , to guarantee the outage level of ϵ' for the minimum of these rates. We write for one of them, chosen arbitrarily:

$$\log (1 + P_T h^2 d_l^{-\gamma}) > \log (1 + P_T h^2 a_l^{-\gamma}) > (1 + \rho) R_S \Rightarrow$$

$$a_l^{\gamma} < \frac{P h^2}{2^{(1+\rho)R_S} - 1}.$$
(7)

where d_l and h are the distance and the channel gain (respectively) between the transmitter and the chosen relay, so $d_l < a_l$. We require the validity of (7) with a probability more than $1 - \frac{\epsilon'}{n_r}$. With respect to the Rayleigh distribution assumption for h, the distribution of h^2 is exponential with parameter $\frac{1}{\mu}$. Hence, with the probability of $1 - \frac{\epsilon'}{n_r}$, we have: $h^2 > -\mu \ln (1 - \frac{\epsilon'}{n_r})$. Thus, if a_l satisfies the bound in (6), the inequality (7) holds with a probability more than $1 - \frac{\epsilon'}{n_r}$.

2) Network Layering scheme and eavesdropper rate analysis: To analyze the eavesdroppers rates, one can follow a similar approach to what presented in the previous part. However, the eavesdroppers are distributed in all around the network and their distances from the transmitter vary from a_e to the radius of the network. Hence, following the same approach would yield a loose bound on a_e . For deriving a tighter bound we propose a network layering scheme. In this scheme, as shown in Fig. 1, the network is divided to a number of layers and the eavesdroppers rates in each layer is analyzed separately. To be precise, the k-th layer is defined as the region of the network between the radii $2^{k-1}a_e$ and $2^k a_e$. We repeat this procedure till the boundary of the network. In the following, we propose

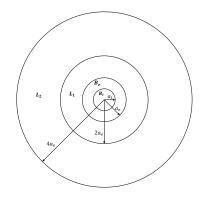


Fig. 1. The inner and outer circles and the two first layers in the network layering scheme

a lower bound on a_e using this idea. We denote the number of layers by K_L and the k-th layer by L_k . The area of L_k , denoted by S_k , is equal to $S_k = \pi \left(2^{2k} - 2^{2(k-1)}\right) a_e^2 = 3\pi 2^{2(k-1)}a_e^2$. To bound the number of eavesdroppers in each layer we present two following lemmas.

Lemma 1 (Number of eavesdroppers in each layer): For any positive constant β_k , define t_k as:

$$t_k \triangleq \left(\frac{\beta_k}{\epsilon'}, \frac{1}{\lambda_e S_k}\right)^{\frac{1}{2}}.$$
(8)

Then, with a probability larger than $1 - \frac{\epsilon'}{\beta_k}$, the number of eavesdroppers in L_k (denoted as $n_{e,k}$) satisfies:

$$n_{e,k} < (1+t_k)\lambda_e S_k. \tag{9}$$

Proof: Considering Poisson distribution of eavedroppers locations, we use Chebyshev's inequality for $n_{e,k}$ to write:

$$\Pr\left\{n_{e,k} > \lambda_e S_k + t_k \lambda_e S_k\right\}$$

$$< \Pr\left\{|n_{e,k} - \lambda_e S_k| > t_k \sqrt{\lambda_e S_k} \sqrt{\lambda_e S_k}\right\}$$

$$< \frac{1}{t_k^2 \lambda_e S_k}.$$

Lemma 2: If in Lemma 1, we set β_k s such that $\sum_{k=1}^{K_L} \frac{1}{\beta_k} < 1$ holds, then, the inequality (9) will be valid for all the layers, with a probability larger than $1 - \epsilon'$.

Proof: Using the union bound for the undesired event in each layer, we can bound the global undesired event. Therefore, the probability that the inequality (9) does not hold in at least one layer is bounded by:

$$\sum_{k=1}^{K_L} \frac{\epsilon'}{\beta_k} = \epsilon' \sum_{k=1}^{K_L} \frac{1}{\beta_k}.$$

Assuming the condition introduced in the lemma on β_k s, this quantity will be less than ϵ' .

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Now we derive a proper bound on a_e using the above lemmas. In fact, each layer imposes a lower bound on a_e and the largest lower bound is the main constraint on a_e . Our technique of deriving these bounds is summarized in the following. We divide the tolerable error ϵ' between all the layers, dedicating the tolerable error $\epsilon_k = \frac{\epsilon'}{2^k}$ to the L_k , and we find a proper lower bound to guarantee the outage probability of ϵ_k for this layer. In the analysis of each layer we apply the union bound for the eavesdroppers in that layer. Finally, we apply the union bound on the outage events of these layers to find a bound on the probability of the total outage event. This total outage probability is less than ϵ' , because of how ϵ_k s are allocated.

Using Lemmas 1 and 2, we work with $(1+t_k)\lambda_e S_k$ as the maximum number of eavesdroppers in L_k . The following lemma gives the constraint on a_e concluded from L_k .

Lemma 3: Given the inequality (9), a sufficient condition to have,

$$\max_{j \in L_k} C_j^e < \rho R_S \tag{10}$$

with a probability greater than $1 - \epsilon_k$, is the following constraint on a_e :

$$a_e > a_e^{(k)} \triangleq 2^{-(k-1)} \left(\frac{-P_T \mu}{2^{\rho R_S} - 1} \ln\left(\frac{\epsilon'}{2^k \lambda_e (1 + t_k) S_k}\right) \right)^{\frac{1}{\gamma}}.$$
(11)

Proof: To guarantee the outage level of $\frac{\epsilon'}{2^k}$ for the validity of (10), relying on the union bound, we guarantee the outage level of $\frac{\epsilon'}{2^k n_{e,k}}$ for the validity of the following inequality:

$$C_i^e < \rho R_S \tag{12}$$

for each of the eavesdroppers, e.g., the *j*-th eavesdropper, in this layer. Considering the condition (9) on $n_{e,k}$, it suffices to guarantee the outage level of $\frac{\epsilon'}{2^k \lambda_e (1+t_k) S_k}$ for each of the eavesdroppers. For one of the eavesdroppers, arbitrarily chosen, we write:

$$\log\left(1 + Ph^2 d_e^{-\gamma}\right) < \log\left(1 + Ph^2 (2^{-k-1} a_e)^{-\gamma}\right) < \rho R_S.$$

The first inequality is deduced from $d_e < 2^{k-1}a_e$, in which d_e is the distance between the transmitter and the chosen eavesdropper. For simplicity, the other indices are eliminated. We want the second inequality to be valid with a probability greater than $1 - \frac{\epsilon'}{2^k \lambda_e (1+t_k) S_k}$. With the same probability, for the coefficient h^2 , considering its exponential distribution, the following inequality holds:

$$h^2 < h_{\max}^2 \triangleq -\mu \ln\left(\frac{\epsilon'}{2^k \lambda_e (1+t_k) S_k}\right).$$

Hence, to provide the desired outage level, it suffices to guarantee the inequality

$$\log\left(1 + Ph_{\max}^2(2^{-(k-1)}a_e)^{-\gamma}\right) < \rho R_S,$$

by proper choice of a_e . With a little algebraic efforts and displacing the variables, the recent inequality can be converted to (11).

Now, we put all the results together in the following theorem. Then, from this theorem and by some substitutions and calculations, we conclude the Corollary 1 in which an appropriate lower bound is finalized for a_e .

Theorem 3: Given the coefficients t_k and β_k consistent with the assumptions presented in Lemmas 1 and 2 and by choosing

$$a_e > \max_{1 \le k \le K_L} a_e^{(k)},\tag{13}$$

the rate inequality (5) holds with an outage probability less than $2\epsilon'$.

Proof: Note that the inequality (5) is valid if and only if the one in (10) is valid. We define the binary random variable O_k as

$$O_k = \begin{cases} 0 & \text{if } \max_{j \in L_k} C_j^e < \rho R_S, \\ 1 & \text{if } \max_{j \in L_k} C_j^e > \rho R_S. \end{cases}$$
(14)

In addition, we define the binary random variable Q_k as

$$Q_{k} = \begin{cases} 0 & \text{if } n_{e,k} < (1+t_{k})\lambda_{e}S_{k}, \\ 1 & \text{if } n_{e,k} > (1+t_{k})\lambda_{e}S_{k}. \end{cases}$$
(15)

Now, using the union bound, we write:

$$\Pr\left\{\max_{1 < j < n_e} C_j^e > \rho R_S\right\} < \sum_{k=1}^{L_K} \Pr\left\{O_k = 1\right\}.$$
(16)

Also, we expand the occurrence probability of O_k in Q_k as the following:

$$\Pr \{O_k = 1\} = \sum_{i=0,1} \Pr \{Q_k = i\} \Pr \{O_k = 1 | Q_k = i\}$$

$$\stackrel{(a)}{\leq} \Pr \{O_k = 1 | Q_k = 0\} + \Pr \{Q_k = 1\}.$$
(17)

Considering that the value of the probability function is not never more than the unity, the terms $Pr \{Q_k = 0\}$ and $Pr \{O_k = 1 | Q_k = 1\}$ in the inequality (a) are replaced by 1. First, we investigate the first term of (17). Given (13), the offered sufficient condition presented in the Lemma 3 ((11)) holds for all the layers. So, considering this lemma, we have:

$$\Pr\left\{O_k|Q_k=0\right\} < \frac{\epsilon'}{2^k}.\tag{18}$$

By summing up in all the layers, we write:

$$\sum_{k=1}^{L_K} \Pr\{O_k | Q_k = 0\} < \sum_{k=1}^{L_k} \frac{\epsilon'}{2^k} < \epsilon'.$$
(19)

Now we consider the second term in (17). For the summation of these terms, according to the Lemma 2, we have:

$$\sum_{k=1}^{K_L} \Pr\{Q_k = 1\} < \epsilon'.$$
(20)

Now the proof is completed by applying (17) in (16) and then using the inequalities (19) and (20).

Corollary 1: By choosing

$$a_e = \frac{(P_T \mu)^{\frac{1}{\gamma}}}{(2^{\rho R_S} - 1)^{\frac{1}{\gamma}}} \left(\ln \frac{-\ln (1 - \epsilon')^6}{\epsilon'} + \sqrt{\frac{2\epsilon'}{(-\ln (1 - \epsilon')^3)^3}} \right), \tag{21}$$

the outage probability of (5) is less than $2\epsilon'$.

Proof: We start from the right side of the constraint (13) in the recent theorem and insert the value of $a_e^{(k)}$ from (11). Also, we replace t_k by its calculated value from (8) and for any $1 \le k \le K_L$, we set:

$$\beta_k = 2^k. \tag{22}$$

It's clear that by this choice, the required condition in the Lemma 2 for β_k s is established. Furthermore, we replace λ_e by its value from (48). Now, we can write:

$$\begin{split} a_{e} &= \max_{1 \leq k \leq K_{L}} a_{e}^{(k)} \\ &= \left(\frac{P\mu}{2^{\rho R_{s}} - 1}\right)^{\frac{1}{\gamma}} \max_{1 \leq k \leq K_{L}} -2^{-(k-1)} \ln \frac{\epsilon'}{2^{k} \lambda_{e} S_{k}(1 + t_{k})} \\ &= \left(\frac{P\mu}{2^{\rho R_{s}} - 1}\right)^{\frac{1}{\gamma}} \max_{1 \leq k \leq K_{L}} 2^{-(k-1)} \times \ln \frac{-3 \times 2^{3k-2} \ln \left(1 - \epsilon'\right) + \left(\frac{4}{-3\epsilon' \ln \left(1 - \epsilon'\right)}\right)^{\frac{1}{2}} \beta_{k}^{\frac{1}{2}}}{\epsilon'} \\ &\stackrel{(a)}{=} \left(\frac{P\mu}{2^{\rho R_{s}} - 1}\right)^{\frac{1}{\gamma}} \max_{1 \leq k \leq K_{L}} 2^{-(k-1)} \ln \left(c_{1} 2^{3k} + c_{2} 2^{\frac{k}{2}}\right) \\ &= \left(\frac{P\mu}{2^{\rho R_{s}} - 1}\right)^{\frac{1}{\gamma}} \max_{1 \leq k \leq K_{L}} 2^{-(k-1)} \ln c_{1} 2^{3k} \left(1 + \frac{c_{2}}{c_{1}} 2^{\frac{-5}{2}k}\right) \\ &\stackrel{(b)}{\leq} \left(\frac{P\mu}{2^{\rho R_{s}} - 1}\right)^{\frac{1}{\gamma}} \max_{1 \leq k \leq K_{L}} 2^{-(k-1)} (3k \ln 2 + \ln c_{1} + \frac{c_{2}}{c_{1}} 2^{-\frac{5}{2}k}) \\ &\stackrel{(c)}{=} \left(\frac{P\mu}{2^{\rho R_{s}} - 1}\right)^{\frac{1}{\gamma}} \left(3 \ln 2 + \ln c_{1} + \frac{c_{2}}{4\sqrt{2}c_{1}}\right) \end{split}$$

In the equation (a), the following definitions are used:

$$c_1 \triangleq \frac{-3\ln\left(1-\epsilon'\right)}{4\epsilon'},\tag{23}$$

$$c_2 \triangleq \sqrt{\frac{4}{-3\epsilon' \ln\left(1-\epsilon'\right)}}.$$
(24)

Inequality (b) is deduced from $\ln x < x - 1$ for x > 0 and $x \neq 1$. The inequality (c) is obtained by considering that the argument of maximization is a decreasing function in k, so it takes its maximum at k = 1. Substituting this value for k, the last line is obtained.

By replacing c_1 and c_2 by their values we reach to the same value in the relation (21). Now we set a_e equal to this value. Hence, the desired condition in the theorem 3 will be valid, too.

Remark 1: We remark that the proposed idea for layering the network is really effective. In fact, as it's clear from (21), the final constraint on a_e is independent of the number of the eavesdroppers (and so from the size of

the network). Therefore, by extending the network and so increasing the total number of the eavesdroppers, it's not necessary to limit the eavesdropper-free region anymore.

B. Step 2: Second rate analysis

In this subsection we analyze the second rate, i.e., forward rate, and from it we derive the proper constraint on n_r .

1) Fading calculations: In this part we pick arbitrarily one of the eavesdroppers and do the calculations for it. We denote the channel gain vector between the source and the legitimate and the non-legitimate users, respectively, as:

$$\underline{g}^{l} = \left((d_{1}^{l})^{-\gamma/2} h_{1}^{l} e^{j\theta_{1}^{l}}, \dots, (d_{n_{r}}^{l})^{-\gamma/2} l h_{n_{r}}^{l} e^{j\theta_{n_{r}}^{l}} \right),$$
(25)

$$\underline{g}^{e} = ((d_{1}^{e})^{-\gamma/2}h_{1}^{e}e^{j\theta_{1}^{e}}, \dots, (d_{n_{r}}^{e})^{-\gamma/2}h_{n_{r}}^{e}e^{j\theta_{n_{r}}^{e}}).$$
(26)

We assume that the distance between every two users is greater than half of the wavelength. This assumption yields the uncorrelation of different fading gains and phases [16]. We establish a virtual and distributed MISOSE situation using adequate relaying nodes. This scheme has two advantage comparing with conventional Multiple-Input Multiple-Output (MIMO) schemes. First, it does not need the devices to be equipped with multiple antennas. Second, as noted in [1], the maximum number of antennas in practical MIMO systems has physical limitation. But, in this scheme we can exploit more relaying nodes and benefit more from channel diversity. In [17], for MISOSE situation with ergodic capacity criterion and known CSI and only for legitimate users, it has been proved that the efficient strategy for the beamforming of the transmission vector is to align it in the direction of the fading vector. Thus, by a similar technique, we align the beamforming vector in the direction of complex conjugate of channel gain vector, i.e., $(\underline{g}^l)^*$, to maximize the correlation between these two vectors. In order to control the total consumed power, we set the beamforming vector equal to $\frac{(g^l)^*}{n_r}$. Given only legitimate users CSI, it is a reasonable strategy.

Message transmission scheme: For the message set $M = [1 : 2^{nR}]$ and for any $m \in M$, a proper codeword X^n generated from Wyner wiretap coding is chosen and transmitted by the transmitter. We denote the average power of the transmitter by P_T . The relaying nodes decode their received sequence to obtain the transmitted message m. In the next step, the *i*-th relaying node uses the same codebook to send the sequence $F_i = \frac{1}{\sqrt{n_r}} (d_i^l)^{-\gamma/2} h_i X^n e^{-j\theta_i^l}$ in n transmission intervals. So the power consumed by the *i*-th relaying node and the total consumed power equal to:

$$P_i = \frac{1}{n_t n_r} \sum_{t=1}^{n_t} (d_i^l)^{-\gamma} h_i^2 |X(t)|^2 = \frac{(d_i^l)^{-\gamma} h_i^2}{n_r} P_T,$$
(27)

$$P_T^{(\text{tot})} = \sum_{i=1}^{n_r} P_i = \left(\frac{1}{n_r} \sum_{i=1}^{n_r} (d_i^l)^{-\gamma} h_i^2\right) P_T.$$
(28)

Furthermore, the received signals at the end of the *t*-th transmission interval are:

$$Y(t) = \left(\frac{1}{\sqrt{n_r}} \sum_{i=1}^{n_r} (d_i^l)^{-\gamma} (h_i^l)^2 \right) X(t),$$

$$Z(t) = \left(\frac{1}{\sqrt{n_r}} \sum_{i=1}^{n_r} (d_i^l)^{-\gamma/2} (d_i^e)^{-\gamma/2} h_i^l h_i^e e^{j(\theta_i^e - \theta_i^l)} \right) X(t).$$

Finally, the received powers at the legitimate user and the eavesdroppers are:

$$P_l = \left(\frac{1}{\sqrt{n_r}} \sum_{i=1}^{n_r} (d_i^l)^{-\gamma} h_i^2\right)^2 P_T,$$
(29)

$$P_e = \left| \frac{1}{\sqrt{n_r}} \sum_{i=1}^{n_r} (d_i^l)^{-\gamma/2} (d_i^e)^{-\gamma/2} h_i^l h_i^e e^{j(\theta_i^e - \theta_i^l)} \right|^2 P_T.$$
(30)

Now we consider these two recent random variables (i.e., P_l, P_e) and give bounds on their expected values and variances. Having these in hand, we can use a bounding inequality, like Chebyshev's inequality, to predict the behavior of these two quantities with high probability.

Probabilistic results: Based on the assumptions we noted previously about the fading coefficients, we proved the following bounds for the expected value and variances of P_l and P_e . The proof is provided in appendix A.

Theorem 4: By appropriate choices for η and ν , the following bounds hold:

$$\frac{\mathbb{E}\left[P_l\right]}{P_T} > \eta n_r (d_{TR} + a_l)^{-2\gamma},\tag{31}$$

$$\frac{\mathbb{E}[P_e]}{P_T} < \eta (a_e - a_l)^{-\gamma} (d_{TR} - a_l)^{-\gamma},$$
(32)

$$\frac{\sigma^2(P_l)}{P_T^2} < \nu^2 n_r (d_{TR} - a_l)^{-4\gamma},\tag{33}$$

$$\frac{\sigma^2(P_e)}{P_T^2} < \nu^2 (a_e - a_l)^{-2\gamma} (d_{TR} - a_l)^{-2\gamma}.$$
(34)

where, d_{TR} is the distance between the transmitter and the receiver. It is assumed that the receiver is out of the inner circle (B_l) .

To continue, we look for the sufficient number of relaying nodes in order to attain the secure rate R_S with outage probability less than $2\epsilon'$. We use Chebyshev's inequality to establish proper bounds on probability of the undesired events defined on the amount of P_l and P_e . We wish to have $C_l - \max_{i \in \mathcal{E}} C_i^e > R_S$ with a probability greater than $1 - \epsilon'$. But, for the sake of simplicity, we guarantee the following bounds, each with the probability of $1 - \epsilon'$:

$$C_l > (1+\kappa)R_S,\tag{35}$$

$$\max_{i \in \mathcal{E}} C_i^e < \kappa R_S. \tag{36}$$

where, κ is an arbitrary positive constant which can be optimized if necessary. Now, we derive the proper bounds on the network parameters by analyzing the above limitations. Instead of (36), using a union bound approach, we consider the following constraint for each eavesdropper:

$$C_i^e < \kappa R_S \tag{37}$$

with the probability of $1 - \frac{\epsilon'}{n_e}$.

2) Legitimate rate analysis: The constraint in (35) implies $\Pr \{C_l = \log (1 + P_l) < (1 + \kappa)R_S\} < \epsilon'$, or equivalently:

$$\Pr\left\{P_l < 2^{(1+\kappa)R_S} - 1\right\} < \epsilon'.$$
(38)

Now, we apply Chebyshev's inequality and drive a lower bound on n_r which guarantees (38). Noting the expected value and the variance of P_l by η_l and ν_l^2 , respectively, we apply the inequalities of Theorem 4 for these two values. First, we write:

$$\Pr\left\{P_l < 2^{(1+\kappa)R_S} - 1\right\} \stackrel{(a)}{<} \Pr\left\{P_l < \eta_l - \alpha\nu_l\right\}$$
$$< \Pr\left\{|P_l - \eta_l| > \alpha\nu_l\right\} \stackrel{(b)}{<} \frac{1}{\alpha^2} \stackrel{(c)}{\leq} \epsilon'.$$

where, (b) follows from Chebyshev's inequality, for (c) we set: $\alpha = \sqrt{\frac{1}{\epsilon'}}$, and for (a), it's sufficient to have: $2^{(1+\kappa)R_S} - 1 < \eta_l - \alpha \nu_l$, or equivalently: $\nu_l < \frac{1}{\alpha}(\eta_l - 2^{(1+\kappa)R_S} + 1)$. Considering (31) and (33), it's sufficient to establish the following chain:

$$\nu_l \stackrel{\text{(d)}}{<} \nu_{\sqrt{n_r}} (d_{TR} - a_l)^{-2\gamma} P_T$$

$$\stackrel{\text{(e)}}{<} \frac{1}{\alpha} (\eta n_r (d_{TR} + a_l)^{-2\gamma} P_T - 2^{(1+\kappa)R_S} + 1)$$

$$\stackrel{\text{(f)}}{<} \frac{1}{\alpha} (\eta_l - 2^{(1+\kappa)R_S} + 1).$$

where, (d) and (f) are deduced from Theorem 4. We establish (e) by choosing n_r sufficiently large. After some algebraic calculations, (e) can be written as the following quadratic inequality in $\sqrt{n_r}$:

$$n_r(\eta (d_{TR} + a_l)^{-2\gamma} P_T) - \sqrt{n_r} (\alpha \nu (d_{TR} - a_l)^{-2\gamma} P_T) - 2^{(1+\kappa)R_s} + 1 > 0.$$

in which only one of the two roots is positive and so acceptable. By choosing n_r greater than the square of this root, we reach a constraint on n_r presented in the following theorem.

Theorem 5 (Lower bound for n_r): A sufficient condition on n_r for guaranteeing (35) with an outage level of ϵ' is to have:

$$n_{r} > \frac{(d_{TR} - a_{l})^{-4\gamma}}{4\eta^{2}(d_{TR} + a_{l})^{-4\gamma}} (\frac{\nu}{\sqrt{\epsilon'}} + \sqrt{\zeta})^{2},$$

$$\zeta = \frac{\nu^{2}}{\epsilon'} + 4\eta \frac{(d_{TR} + a_{l})^{-2\gamma}}{P_{T}(d_{TR} - a_{l})^{-4\gamma}} (2^{(1+\kappa)R_{S}} - 1).$$
(39)

In order to get an intuition from the behavior of this constraint, we put a simplifying assumption on a_l , which makes this constraint independent of a_l . For this, we assume:

$$a_l < d_{TR}/2. \tag{40}$$

We justify this assumption by noting that if the receiver lies in B_l , it is not necessary to exploit the *stochastic virtual beamforming* scheme. Actually in this situation, based on the calculations for the first rate, the message can be delivered securely to the receiver by direct transmission. Here, for the sake of simplicity, after choosing a valid value for a_l , i.e., a value which satisfies the constraint in (6), we divide it by two. By this choice, we can send the secure message directly to the receiver whenever the receiver is in the distance of at most $2a_l$. Therefore, we use our proposed scheme, i.e., the *stochastic virtual beamforming*, only when (40) holds. Using this, constraint (39) is turned to the following simplified version. The proof is simple and is completed by bounding $d_{TR} \pm a_l$ properly.

Corollary 2: A simplified sufficient condition on n_r to guarantee (35) with the outage level of ϵ' is to have:

$$n_r > \frac{81}{4\eta^2} \left(\frac{\nu}{\sqrt{\epsilon'}} + \sqrt{\frac{\nu^2}{\epsilon'}} + \frac{4\eta}{P_T} d_{TR}^{2\gamma} (2^{(1+\kappa)R_S} - 1) \right)^2.$$
(41)

3) Eavesdropper rate analysis: Now, we proceed in a similar way to obtain another constraint to guarantee (37) for the eavesdropper rate with high probability. As noted previously, for the arbitrarily chosen eavesdropper, we wish to have: $\Pr \{\log(1 + P_e) > \kappa R_S\} = \Pr \{P_e > 2^{\kappa R_s} - 1\} < \frac{\epsilon'}{n_e}$. Similar to the previous part, we use η_e and ν_e^2 to denote the expected value and the variance of P_e . We start with:

$$\Pr\left\{P_e > 2^{\kappa R_S} - 1\right\} \stackrel{\text{(a)}}{<} \Pr\left\{P_e > \eta_e + \alpha \nu_e\right\}$$
$$< \Pr\left\{|P_e - \eta_e| > \alpha \nu_e\right\} \stackrel{\text{(b)}}{<} \frac{1}{\alpha^2} \stackrel{\text{(c)}}{\leq} \frac{\epsilon'}{n_e},$$

where (b) follows from Chebyshev's inequality and for (c) we set $\alpha = \sqrt{\frac{n_e}{\epsilon'}}$. Similar to the previous part, for establishing (a), it's sufficient to have $\nu_e < \frac{1}{\alpha}(2^{\kappa R_s} - \eta_e - 1)$. Now, considering (32) and (34), it is sufficient to establish the following chain,

$$\nu_{e} \stackrel{\text{(d)}}{<} \nu(a_{e} - a_{l})^{-\gamma} (d_{TR} - a_{l})^{-\gamma} P_{T}$$

$$\stackrel{\text{(e)}}{<} \frac{1}{\alpha} \left(2^{\kappa R_{S}} - \eta(a_{e} - a_{l})^{-\gamma} (d_{TR} - a_{l})^{-\gamma} P_{T} - 1 \right)$$

$$\stackrel{\text{(f)}}{<} \frac{1}{\alpha} \left(2^{\kappa R_{S}} - \eta_{e} - 1 \right).$$

By some substitution and assuming the other parameters to be constant, the inequality (e) can be converted to a constraint on n_e , as follows:

$$n_e < \epsilon' \left(\frac{2^{\kappa R_s} - \eta (a_e - a_l)^{-\gamma} (d_{TR} - a_l)^{-\gamma} - 1}{\nu (a_e - a_l)^{-\gamma} (d_{TR} - a_l)^{-\gamma} P_T}\right)^2.$$
(42)

C. Poisson calculations

1) Constraint related to the inner circle: We must have at least n_r legitimate relaying nodes in the circle B_l , where n_r is chosen appropriately regarding the former constraint in (39). In the following, we start by bounding the probability of undesirable event, i.e., having less than n_r nodes in B_l , using Chebyshev's inequality. Then, using this

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bound, we derive a sufficient condition for λ_l , in order to keep the probability of undesirable event less than ϵ' . By defining the k_l as the number of legitimate nodes in B_l , we write:

$$\Pr\{k_l < n_r\} < \Pr\{|k_l - \lambda_l \pi a_l^2| > \lambda_l \pi a_l^2 - n_r\}$$

$$= \Pr\left\{|k_l - \lambda_l \pi a_l^2| > \sqrt{\lambda_l \pi a_l^2} \frac{\lambda_l \pi a_l^2 - n_r}{\sqrt{\lambda_l \pi a_l^2}}\right\}$$

$$< \frac{\lambda_l \pi a_l^2}{(\lambda_l \pi a_l^2 - n_r)^2}.$$
(43)

To satisfy the outage probability constraint, it suffices to set (43) less than or equal to ϵ' . In the equality case, we reach the following equation from which a lower bound on λ_l is deduced. By satisfying this constraint, we have at least n_r legitimate nodes in the inner circle, B_l , with probability larger than $1 - \epsilon'$.

$$\lambda_{l}^{2}(\pi^{2}a_{l}^{4}) - \lambda_{l}(2n_{r} + \frac{1}{\epsilon'})\pi a_{l}^{2} + n_{r}^{2} = 0 \Rightarrow$$

$$\lambda_{l} = \frac{n_{r} + \frac{1}{2\epsilon'} \pm \sqrt{\left(n_{r} + \frac{1}{2\epsilon'}\right)^{2} - n_{r}^{2}}}{\pi a_{l}^{2}}$$
(44)

Note that the smaller solution in (44) is not acceptable, because it yields values for λ_l which are lower than $\frac{n_r}{\pi a_l^2}$. So we work with the greater solution. As a sufficient condition, we can choose λ_l to be greater than this solution. Therefore,

$$\lambda_{l} > \frac{n_{r} + \frac{1}{2\epsilon'} + \sqrt{(n_{r} + \frac{1}{2\epsilon'})^{2} - n_{r}^{2}}}{\pi a_{l}^{2}} = \frac{n_{r}}{\pi a_{l}^{2}} \Big(\underbrace{1 + \frac{1}{2\epsilon' n_{r}} + \sqrt{(1 + \frac{1}{2\epsilon' n_{r}})^{2} - 1}}_{\beta_{l}(\epsilon)} \Big).$$
(45)

By the above definition of $\beta_l(\epsilon)$, we summarize the constraint as:

$$\lambda_l > \beta_l(\epsilon) . \frac{n_r}{\pi a_l^2}.$$
(46)

2) Constraint related to the outer circle: As mentioned previously we need the circle C^e to be free of eavesdroppers, with probability larger than $1 - \epsilon'$. By defining k_e as the number of eavesdroppers in B_e , we want to have:

$$\Pr\left\{k_e = 0\right\} > 1 - \epsilon' \Rightarrow e^{-\lambda_e \pi a_e^2} > 1 - \epsilon' \tag{47}$$

which results in the constraint:

$$\lambda_e < \frac{-\ln(1-\epsilon')}{\pi a_e^2} \tag{48}$$

The main six constraints for a_l , a_e , n_r , n_e , λ_l and λ_e are given in (6), (21), (41), (42), (46) and (48), respectively. To achieve any desired pair of (R_S, ϵ) , we proceed as follows. First, we set n_r satisfying (41), which just depends on R_S and ϵ and not the other five parameters. Knowing n_r , we choose a_l properly from its constraint in (6). Then, the required λ_l is calculated by inserting the value of n_r and a_l in (46). In addition, minimum of a_e is computed by knowing R_S and ϵ from (21). Then, the maximum tolerable amount of λ_e is derived from (48). It is seen that by tending λ_l to infinity, the maximum tolerable density of eavesdroppers tends to infinity, too. This completes the proof.

APPENDIX

PROOF OF THEOREM 4

First, we consider the problem without the path loss effect and analyze the four desired quantities for this case. Then, we add the effect of path loss and update the previous bounds for this case.

The following fading vectors are in fact the simplified versions of channel gain vectors, \underline{g}^l and \underline{g}^e , when all the coefficients related to path loss effect are substituted by unity:

$$\underline{h}^{l} = (h_{1}^{l} e^{j\theta_{1}^{l}}, \dots, h_{n_{r}}^{l} e^{j\theta_{n_{r}}^{l}}),$$
(49)

$$\underline{h}^e = (h_1^e e^{j\theta_1^e}, \dots, h_{n_r}^e e^{j\theta_{n_r}^e}).$$
(50)

In the following lemma, the values of the four desired quantities are given when the path loss effect is eliminated.

Lemma 4: Under the assumptions noted in the paper for the fading coefficients, the mean and the variance of P_l and P_e satisfy the following constraints, when the path loss effect is eliminated:

$$\frac{\mathbb{E}\left[P_{l}\right]}{P_{T}} = (n_{r} - 1)\mathbb{E}^{2}\left[H^{2}\right] + \mathbb{E}\left[H^{4}\right]$$

$$> (n_{r} - 1)\mathbb{E}^{2}\left[H^{2}\right] + \mathbb{E}^{2}\left[H^{2}\right] = n_{r}\mathbb{E}^{2}\left[H^{2}\right],$$
(51)

$$\frac{\mathbb{E}\left[P_e\right]}{P_T} = \mathbb{E}^2\left[H^2\right],\tag{52}$$

$$\frac{\sigma^2(P_l)}{P_T^2} = 4n_r \left(\mathbb{E} \left[H^4 \right] \mathbb{E}^2 \left[H^2 \right] - \mathbb{E}^4 \left[H^2 \right] \right) + \dots = \mathcal{O} \left(n_r \right),$$
(53)

$$\frac{\sigma^2(P_e)}{P_T^2} = \frac{1}{n_r} \mathbb{E}^4 \left[H^2 \right] + \dots = \mathbf{O}(1) \,.$$
(54)

Proof: Analysis for mean of P_l :

$$\frac{\mathbb{E}\left[P_{l}\right]}{P_{T}} = \mathbb{E}\left[\left(\frac{1}{\sqrt{n_{r}}}\sum_{i=1}^{n_{r}}(h_{i}^{l})^{2}\right)^{2}\right]$$
$$= \frac{1}{n_{r}}\mathbb{E}\left[\sum_{i=1}^{n_{r}}\sum_{k=1}^{n_{r}}(h_{i}^{l})^{2}(h_{k}^{l})^{2}\right]$$
$$= \frac{1}{n_{r}}\left(n_{r}\mathbb{E}\left[H^{4}\right] + n_{r}(n_{r}-1)\mathbb{E}^{2}\left[H^{2}\right]\right)$$
$$= (n_{r}-1)\mathbb{E}^{2}\left[H^{2}\right] + \mathbb{E}\left[H^{4}\right]$$
$$> (n_{r}-1)\mathbb{E}^{2}\left[H^{2}\right] + \mathbb{E}^{2}\left[H^{2}\right] = n_{r}\mathbb{E}^{2}\left[H^{2}\right]$$

Analysis for mean of P_e :

$$\begin{split} \frac{\mathbb{E}\left[P_{e}\right]}{P_{T}} &= \mathbb{E}\left[\left|\frac{1}{\sqrt{n_{r}}}\sum_{i=1}^{n_{r}}h_{i}^{l}h_{i}^{e}e^{j\left(\theta_{i}^{e}-\theta_{i}^{l}\right)}\right|^{2}\right] \\ &= \frac{1}{n_{r}}\mathbb{E}\left[\sum_{i=1}^{n_{r}}\sum_{k=1}^{n_{r}}h_{i}^{l}h_{i}^{e}h_{k}^{l}h_{k}^{e}e^{j\left(\theta_{i}^{e}-\theta_{i}^{l}-\theta_{k}^{e}+\theta_{k}^{l}\right)}\right] \\ &\stackrel{(a)}{=} \frac{1}{n_{r}}\sum_{i=1}^{n_{r}}\mathbb{E}\left[\left(h_{i}^{l}\right)^{2}\right]\mathbb{E}\left[\left(h_{i}^{e}\right)^{2}\right] + \frac{n_{r}(n_{r}-1)}{n_{r}}\sum_{i=1}^{n_{r}}\sum_{k\neq i}\mathbb{E}\left[h_{i}^{l}h_{i}^{e}h_{k}^{l}h_{k}^{e}\right]\underbrace{\mathbb{E}\left[e^{j\theta_{i}^{e}}\right]}_{0}\mathbb{E}\left[e^{j\left(-\theta_{i}^{l}-\theta_{k}^{e}+\theta_{k}^{l}\right)}\right] \\ &= \frac{n_{r}}{n_{r}}\mathbb{E}^{2}\left[H^{2}\right] = \mathbb{E}^{2}\left[H^{2}\right]. \end{split}$$

where (a) is deduced from the uniform distribution assumption for the random phases and the independence of the fading coefficients and also the phases of the legitimate and non-legitimate channels.

Analysis for variance of P_l :

$$\begin{aligned} \frac{\sigma^2(P_l)}{P_T^2} &= \mathbb{E}\left[\left(\frac{\left(\sum_{i=1}^{n_r} (h_i^l)^2\right)^2}{n_r} - \left(\mathbb{E}\left[H^4\right] + (n_r - 1)\mathbb{E}^2\left[H^2\right]\right) \right)^2 \right] \\ &= \frac{1}{n_r^2} \mathbb{E}\left[\left(\sum_{i=1}^{n_r} h_i^4 + \sum_{k=1}^{n_r} \sum_{q \neq k} h_k^2 h_q^2 - n_r \mathbb{E}\left[H^4\right] - n_r (n_r - 1)\mathbb{E}^2\left[H^2\right] \right)^2 \right] \\ &= \frac{1}{n_r^2} \mathbb{E}\left[\left(\sum_{i=1}^{n_r} h_i^4 - \mathbb{E}\left[H^4\right] + \sum_{k=1}^{n_r} \sum_{q \neq k} h_k^2 h_q^2 - \mathbb{E}^2\left[H^2\right] \right)^2 \right] \\ &= \frac{1}{n_r^2} \left(\mathbb{E}\left[S_1^2\right] + 2\mathbb{E}\left[S_1 S_2\right] + \mathbb{E}\left[S_2^2\right] \right), \end{aligned}$$

where

$$\begin{split} 2\mathbb{E}\left[S_{1}S_{2}\right] &= 4n_{r}(n_{r}-1)\mathbb{E}\left[(H_{1}^{4}-\mathbb{E}\left[H^{4}\right])(H_{1}^{2}H_{2}^{2}-\mathbb{E}^{2}\left[H^{2}\right])\right] \\ &= 4n_{r}(n_{r}-1)(\mathbb{E}\left[H^{6}\right]\mathbb{E}\left[H^{2}\right]-\mathbb{E}\left[H^{4}\right]\mathbb{E}^{2}\left[H^{2}\right]), \\ \mathbb{E}\left[S_{1}^{2}\right] &= n_{r}\mathbb{E}\left[(H^{4}-\mathbb{E}\left[H^{4}\right])^{2}\right] \\ &= n_{r}(\mathbb{E}\left[H^{8}\right]-\mathbb{E}^{2}\left[H^{4}\right]), \\ \mathbb{E}\left[S_{2}^{2}\right] &= n_{r}(n_{r}-1)(\mathbb{E}^{2}\left[H^{4}\right]-\mathbb{E}^{4}\left[H^{2}\right]) + 4n_{r}(n_{r}-1)(n_{r}-2)\mathbb{E}\left[(H_{1}^{2}H_{2}^{2}-\mathbb{E}^{2}\left[H^{2}\right])(H_{1}^{2}H_{3}^{2}-\mathbb{E}^{2}\left[H^{2}\right])\right] \\ &= n_{r}(n_{r}-1)(\mathbb{E}^{2}\left[H^{4}\right]-\mathbb{E}^{4}\left[H^{2}\right]) + 4n_{r}(n_{r}-1)(n_{r}-2)\left(\mathbb{E}\left[H^{4}\right]\mathbb{E}^{2}\left[H^{2}\right]-\mathbb{E}^{4}\left[H^{2}\right]\right). \end{split}$$

Hence:

$$\frac{\sigma^2(P_l)}{P_T^2} = \frac{1}{n_r^2} \Big(n_r (\mathbb{E} \left[H^8 \right] - \mathbb{E}^2 \left[H^4 \right]) + n_r (n_r - 1) (\mathbb{E}^2 \left[H^4 \right] - \mathbb{E}^4 \left[H^2 \right]) \\ + 4n_r (n_r - 1) (n_r - 2) \left(\mathbb{E} \left[H^4 \right] \mathbb{E}^2 \left[H^2 \right] - \mathbb{E}^4 \left[H^2 \right] \right) \\ + 4n_r (n_r - 1) (\mathbb{E} \left[H^6 \right] \mathbb{E} \left[H^2 \right] - \mathbb{E} \left[H^4 \right] \mathbb{E}^2 \left[H^2 \right]) \Big) \\ = \frac{4(n_r - 1)(n_r - 2)}{n_r} \Big(\mathbb{E} \left[H^4 \right] \mathbb{E}^2 \left[H^2 \right] - \mathbb{E}^4 \left[H^2 \right] \Big) + \cdots .$$

Analysis for variance of P_e :

$$\begin{split} \frac{\sigma^2(P_e)}{P_T^2} &= \mathbb{E}\left[\left(\left|\frac{1}{\sqrt{n_r}}\sum_{i=1}^{n_r}h_i^lh_i^e e^{j(\theta_i^e - \theta_i^l)}\right|^2 - \mathbb{E}^2\left[H^2\right]\right)^2\right] \\ &= \frac{1}{n_r^2}\mathbb{E}\left[\left(\sum_{\substack{i=1\\i=1\\A_1}}^{n_r}(h_i^l)^2(h_i^e)^2 - \mathbb{E}^2\left[H^2\right] + \sum_{\substack{k=1\\i=1\\A_2}}^{n_r}\sum_{\substack{i=1\\i=1\\A_2}}h_k^lh_k^eh_q^lh_q^e e^{j(\theta_k^l - \theta_k^e - \theta_q^l + \theta_q^e)}\right)^2\right] \\ &= \frac{1}{n_r^2}\left(\mathbb{E}\left[A_1^2\right] + 2\mathbb{E}\left[A_1A_2\right] + \mathbb{E}\left[A_2^2\right]\right), \end{split}$$

where

$$\mathbb{E}\left[A_{1}^{2}\right] = n_{r}\mathbb{E}\left[\left(H_{1}^{2}H_{2}^{2} - \mathbb{E}^{2}\left[H^{2}\right]\right)^{2}\right] = n_{r}\left(\mathbb{E}^{2}\left[H^{4}\right] - \mathbb{E}^{4}\left[H^{2}\right]\right),$$
$$\mathbb{E}\left[A_{2}^{2}\right] = n_{r}(n_{r}-1)\mathbb{E}^{4}\left[H^{2}\right],$$
$$2\mathbb{E}\left[A_{1}A_{2}\right] = 0.$$

results in:

$$\frac{\sigma^2(P_e)}{P_T^2} = \frac{n_r - 1}{n_r} \mathbb{E}^4 \left[H^2 \right] + \frac{1}{n_r} \left(\mathbb{E}^2 \left[H^4 \right] - \mathbb{E}^4 \left[H^2 \right] \right) = \mathbb{E}^4 \left[H^2 \right] + \dots$$

Corollary 3: There are positive coefficients η and ν , such that for the without path loss case, we have:

$$\frac{\mathbb{E}\left[P_l\right]}{P_T} > \eta n_r,\tag{55}$$

$$\frac{\mathbb{E}\left[P_e\right]}{P_T} = \eta,\tag{56}$$

$$\frac{\sigma^2(P_l)}{P_T^2} < \nu^2 n_r,\tag{57}$$

$$\frac{\sigma^2(P_e)}{P_T^2} < \nu^2. \tag{58}$$

Proof: For η we just set

$$\eta = \mathbb{E}^2 \left[H^2 \right] = 4\mu^2. \tag{59}$$

Now, we prove the main results proposed in the Theorem 4, i.e., the results for the complete model when the path loss effect is taken in to account. First, we prove (31) and (32). The proofs of (33) and (34) are more elaborate and needs two lemmas to be proved.

Proof of (31) *and* (32): Considering the geometry of the network, we have the following common bounds for all $d_i^l s$ and $d_i^e s$:

$$d_{TR} - a_l < d_i^l < d_{TR} + a_l, \tag{60}$$

$$a_e - a_l < d_i^e. agenum{61}{61}$$

By using these bounds, we extract the quantities related to the path loss effect from the summations in the expressions of (29) and (30), so that the remaining terms in the summations change to the same expressions related to the case without considering the path loss effect. Now, using the results stated in Corollary 3 and by considering the linearity and monotonicity of the expected value function, (31) and (32) are simply concluded.

Now we prove the two variance results ((33) and (34)). First, we present the following lemmas.

Lemma 5: For any non-negative random variable H, with positive mean, the following inequality is true:

$$\mathbb{E}\left[H^3\right] \ge \mathbb{E}\left[H^2\right] \mathbb{E}\left[H\right]. \tag{62}$$

Proof: Since $H \ge 0$, by using Cauchy-Schwartz inequality for the two random variables $H^{\frac{1}{2}}$ and $H^{\frac{3}{2}}$, we can write:

$$\mathbb{E} \left[H^3 \right] \mathbb{E} \left[H \right] = \mathbb{E} \left[\left(H^{3/2} \right)^2 \right] \mathbb{E} \left[\left(H^{1/2} \right)^2 \right]$$
$$\geq \mathbb{E}^2 \left[H^2 \right] \geq \mathbb{E} \left[H^2 \right] \mathbb{E}^2 \left[H \right].$$

Now, considering its positivity, we divide the above relations by $\mathbb{E}[H]$ to obtain the inequality (62).

Lemma 6: For every two i.i.d. random variables X and Y with positive mean and variance and for any two positive constants a and b such that a < b, the following inequality is true:

$$\operatorname{Var}[(aX + bY)^{2}] < b^{4}\operatorname{Var}[(X + Y)^{2}].$$
(63)

Proof: By expanding the left side, we show that substituting a by b will increase the variance.

$$\begin{split} \operatorname{Var}[(aX+bY)^2] =& \operatorname{Var}[a^2X^2 + b^2Y^2 + 2abXY] \\ =& \mathbb{E}\left[\left(a^2(X^2 - \mathbb{E}\left[X^2\right]) + b^2(Y^2 - \mathbb{E}\left[Y^2\right]) + 2ab(XY - \mathbb{E}\left[XY\right])\right)^2\right] \\ =& \mathbb{E}\left[a^4(X^2 - \mathbb{E}\left[X^2\right])^2\right] + \mathbb{E}\left[b^4(Y^2 - \mathbb{E}\left[Y^2\right])^2\right] + \mathbb{E}\left[4a^2b^2(XY - \mathbb{E}\left[XY\right])^2\right] \\ & + 2a^2b^2\mathbb{E}\left[(X^2 - \mathbb{E}\left[X^2\right])(Y^2 - \mathbb{E}\left[Y^2\right])\right] + 4a^3b\mathbb{E}\left[(X^2 - \mathbb{E}\left[X^2\right])(XY - \mathbb{E}\left[XY\right])\right] \\ & + 4ab^3\mathbb{E}\left[(Y^2 - \mathbb{E}\left[Y^2\right])(XY - \mathbb{E}\left[XY\right])\right]. \end{split}$$

In the last equality, the three first terms are clearly non-negative and increasing the coefficients, will increase the total result. So, substituting a by b increases the Variance. According to the independence assumption, the forth term can be decomposed to two expected value terms, which both of them are zero. The fifth and the sixth sentence have a similar form. We show below that the fifth term is always positive. A similar argument is true for the sixth term.

$$\mathbb{E}\left[(X^2 - \mathbb{E}\left[X^2\right])(XY - \mathbb{E}\left[XY\right])\right] \stackrel{\text{(a)}}{=} \mathbb{E}\left[X^3\right] \mathbb{E}\left[Y\right] - \mathbb{E}\left[X^2\right] \mathbb{E}\left[X\right] \mathbb{E}\left[Y\right]$$
$$= \mathbb{E}\left[Y\right] \left(\mathbb{E}\left[X^3\right] - \mathbb{E}\left[X^2\right] \mathbb{E}\left[X\right]\right) \stackrel{\text{(b)}}{>} 0.$$

where (a) is deduced from the independence assumption; (b) is concluded from Lemma 5 and the positivity of the mean of Y. So, the fifth and also the sixth sentence are positive and therefore all the six sentences have a non-negative value. Hence, replacing a by b will increase the amount of the variance and the validity of (63) is established.

proof of (33) *and* (34): Using induction, the recent lemma can be generalized to any number of random variables. Considering the independence of the fading coefficients and their Rayleigh distribution and using the inequalities (60) and (61), the generalization of Lemma 6 results in (33) and (34).

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