Letter

A Linear Algorithm for Quantized Event-Triggered Optimization Over Directed Networks

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Dear Editor,

This letter investigates a class of distributed optimization problems with constrained communication. A quantized discrete-time event-triggered zero-gradient-sum algorithm (QDE-ZGS) is developed to optimize the sum of local functions over weight-balanced directed networks. Based on an encoder-decoder scheme and a zooming-in technique, an event-triggered quantization communication is designed. Theoretical analysis shows that the exact convergence to the global optimal solution is guaranteed when the triggering threshold is bounded and the scaled sequence introduced by the zooming-in technique is quadratic summable. When the scaled sequence is bounded by an exponential decay function, QDE-ZGS converges linearly to the unique global optimal solution. Numerical simulations are conducted to demonstrate the theoretical results.

In this letter, a class of distributed optimization problems, minimizing the sum of local objective functions $\sum_{i=1}^{n} f_i(x)$ via local communication over a network with n agents, is considered, which has attracted considerable attention with wide applications in large-scale machine learning, cognitive networks, power systems, etc. [1]. Due to the sublinear convergence of distributed subgradient decent algorithm (DGD) and its extensions [2], [3], auxiliary-variable algorithms are motivated with fast convergence but high communication and computation load [4]. Lately, a well-designed linear ZGS algorithm was proposed without any extra variable [5]–[7].

Most of literature, including aforementioned ones, heavily relies on the accurate state information, which inevitably brings great challenges for the communication capacity due to bandwidth constraints. To address this problem, the quantized communication was deployed widely in the network environment [8]. On this front, the zooming-in based quantized communication was introduced for average consensus subject to constrained data rate [9]. Leveraging the same mechanism, a series of subgradient based distributed optimization algorithms have been designed under constrained communication over undirected [10] and directed networks [11], [12], where only sublinear convergence was guaranteed. For the linear convergence, auxiliary-variable quantized algorithm was developed [13], resulting in the inevitable increase of computation cost. For a communication-efficient algorithm, it is expected to achieve fast convergence with less computation cost.

Besides, based on the fact that communication is contributed to more energy consumption compared with computation [14], the data transmission at every iteration puts strict requirements on the communication capacity, which is beyond the ability of agents with limited energy. Considering the communication bandwidth limitation, the event-triggered communication mechanism has been introduced from distributed control [15], [16] into distributed optimization [17], [18]. Based on the zooming-in technique, an event-triggered quantized communication mechanism was developed in [19] over time-varying communication networks, which is not applicable over fixed networks. A distributed constrained optimization problem was solved through the zooming-in technique based event-triggered quantized communication in [20]. The authors in [21] focused on the same problem in the continuous-time setting and inexact convergence was achieved. It should be noted that only the sublinear or asymptotic convergence was guaranteed in [19]–[21]. It is of practical significance to design a linear convergence algorithm with constrained communication.

Motivated by above discussions, this letter incorporates the event-triggered quantized communication mechanisms into the discrete-time ZGS algorithm and the linear convergence is achieved over directed networks. The main contributions are summarized as follows:

1) A zooming-in based event-triggered quantized communication mechanism is designed, where only the bounded triggering threshold is required.

2) The exact convergence is achieved under event-triggered quantized communication on the condition that the scaled sequence introduced by the zooming-in technique is quadratic summable.

3) When the scaled sequence is bounded by an exponential decay function, the linear convergence is established, which is faster than other quantized algorithms [10], [11], [13] and event-triggered based algorithms [19]–[21] without any auxiliary variables.

Notations: Given n-dimensional real vector space $\mathbb{R}^n$ and $n\times m$ real matrix space $\mathbb{R}^{n\times m}$, we denote identity matrix and the set of natural number by $I_n \in \mathbb{R}^{n\times n}$ and $\mathbb{N}$, respectively. $\mathbf{1}_n = [1,1,\ldots,1] \in \mathbb{R}^n$. $\otimes$ denotes the Kronecker product, and $\|\cdot\|$ presents the l_2-norm for a vector or matrix.

Preliminaries and algorithm description: Consider a distributed unconstrained optimization problem with n agents, where all agents collaboratively minimize an objective function, that is

$$\min \ F(x) = \sum_{i=1}^{n} f_i(x) \tag{1}$$

where $f_i : \mathbb{R}^m \to \mathbb{R}$ is a private information, accessed only by agent i. The basic assumption is made as follows.

Assumption 1: For optimization problem (1), 1) the set of global minimizer $\mathcal{X}^*$ is nonempty and 2) twice continuously differentiable f_i, $\forall i \in \mathcal{V}$, is θ_i-strongly convex with $\theta_i > 0$.

In this letter, the communication among n agents is over an one-way communication network, which can be depicted as a diagraph $\mathcal{G} = \{\mathcal{V},\mathcal{E},\mathcal{A}\}$ where the collection of nodes is $\mathcal{V} = \{v_1,\ldots,v_n\}$ and edges set is $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$. Let $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ denote the weighted adjacency matrix, with $a_{ij} > 0$ if $(i,j) \in \mathcal{E}$, i.e., v_i can receive information from v_j, $a_{ij} = 0$ otherwise, and $a_{ii} = 0$. The in-neighbors and out-neighbors set of agent i are denoted as $\mathcal{N}_i^{in} = \{j|(i,j) \in \mathcal{E}\}$ and $\mathcal{N}_i^{out} = \{j|(j,i) \in \mathcal{E}\}$ separately. In graph $\mathcal{G}$, the degree matrix is $\mathcal{D} = \mathrm{diag}\{deg^1,\ldots,deg^n\}$ with agent i 's in-degree $deg^i = \sum_{j=1}^{n} a_{ij}$, while the Laplacian matrix is $L = \mathcal{D} - \mathcal{A}$. A graph is weight-balanced if $\sum_{i=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ij}$.

Assumption 2: The network in this letter is weight-balanced and strongly connected.

For the data rate constraints in the communication network, the event-triggered quantized communication is adopted and an encoding-decoding scheme is designed based on event-triggered mechanism to transmit quantized states to their own neighboring agents, when the triggering condition is satisfied.

For a vector $Z = (z_1,\ldots,z_n)^T \in \mathbb{R}^m$, a uniform quantizer $Q(Z) = (q(z_1), q(z_2),\ldots,q(z_n))^T$ is selected with $q(z_i) = l$ for $(2l-1)/2 \le z_i < (2l+1)/2$ for nonnegative integers l and $q(z_i) = -q(-z_i)$ when $z_i < -1/2$. Obviously, $|q(z_i) - z_i| \le 1/2$. For agent i, the encoder ϕ_i at time k executes as

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$$\begin{cases} z_i(k) = Q\left(\dfrac{x_i(k)-\xi_i(k-1)}{s(k-1)}\right) \\ \xi_i(k) = s(k-1)\tilde{z}_i(k)+\xi_i(k-1),\ k>0 \end{cases} \tag{2}$$

with the input $x_i(k)$ and the output $z_i(k)$. The quantization error is

$$\begin{aligned} \Delta_i(k) &= z_i(k) - \frac{x_i(k)-\xi_i(k-1)}{s(k-1)} \\ &= e_i(k) + \frac{\xi_i(k)-x_i(k)}{s(k-1)} \end{aligned} \tag{3}$$

with $\Delta_i^T(k)\Delta_i(k) \le m/4$.

Moreover, $\tilde{z}_i(k)$ stands for the information from agent i at the latest triggering instant, i.e., $\tilde{z}_i(k) = z_i(k)$ when $k \in \kappa_i$, and $\tilde{z}_i(k) = \tilde{z}_i(k-1)$ otherwise, where $\kappa_i = \left\{k_i^l | l \in \mathbb{N}\right\}$ and k_i^l is the l-th triggering instant of agent i, determined by

$$k_i^{l+1} = \min_{k\in\mathbb{N}}\left\{k > k_i^l \big| \|e_i(k)\|^2 \ge E_i(k)\right\} \tag{4}$$

with the measurement error $e_i(k) = z_i(k) - \tilde{z}_i(k)$. Without loss of generality, for $i \in \mathcal{V}$, set $k_i^0 = 0$. Agent i receives $\tilde{z}_j(k)$ from ϕ_j, $j \in \mathcal{N}_i^{in}$ and decodes them to estimate the state of agent j, $\hat{x}_{ji}(k)$, by the decoder φ_{ji}

$$\begin{cases} \hat{x}_{ji}(0) = 0 \\ \hat{x}_{ji}(k) = s(k-1)\tilde{z}_j(k) + \hat{x}_{ji}(k-1),\ k>0 \end{cases} \tag{5}$$

invoking (2) yields that

$$\hat{x}_{ji}(k) = \hat{x}_j(k) = \xi_j(k). \tag{6}$$

Assumption 3: For sequences $\{s(k)\}$ and $E_i(k)$, the following statements are in force: 1) $\sum_{k=0}^{\infty} s^2(k) < \infty$; 2) $E(k) = \max_{i\in\mathcal{V}} E_i(k)$, and $\max_{k\in\mathbb{N}} E(k) = E_m$, which means sequence $\{E(k)\}$ is bounded.

To solve problem (1) under event-triggered quantized communication, the QDE-ZGS is proposed that

$$\begin{cases} x_i(k+1) = (\nabla f_i)^{-1}\left(c\displaystyle\sum_{j=1}^n a_{ij}\left(\hat{x}_j(k)-\xi_i(k)\right) + \nabla f_i(k)\right) \\ x_i(0) = x_i^* \end{cases} \tag{7}$$

where state x_i represents the estimate of agent i to the global optimal solution $x^* \in \mathcal{X}$ and $x_i(0) = x_i^*$ with the minimizer x_i^* of f_i. We use $\nabla f_i(k)$ to denote the gradient of $f_i(x)$ at $x = x_i(k)$. The inverse function of $g(\cdot)$ is g^{-1}, such that $g^{-1}(y) = x$ if $g(x) = y$. Since QDE-ZGS algorithm (7) is in the discrete-time setting, the Zeno behavior can be directly excluded.

Remark 1: From (7), one can easily have that $\sum_{i=1}^n \nabla f_i(k+1) - \sum_{i=1}^n \nabla f_i(k) = c\sum_{i=1}^n\sum_{j=1}^n a_{ij}(\hat{x}_j(k)-\xi_i(k)) = c\sum_{i=1}^n\sum_{j=1}^n a_{ij}(\hat{x}_j(k)-\hat{x}_i(k))$ = 0. Thus, $\sum_{i=1}^n \nabla f_i(0) = 0$ implies that $\sum_{i=1}^n \nabla f_i(k) = 0$ for $k \in \mathbb{N}$. In other words, every instant of this algorithm holds the optimal condition. Only if states among agents achieve consensus, the optimization problem (1) is well solved [22].

Noting (2) and (5), substituting (3) into (7) yields that

$$\begin{aligned} \nabla f_i(k+1) &= \nabla f_i(k) + c\sum_{j=1}^n a_{ij}(\hat{x}_j(k)-\xi_i(k)) \\ &= \nabla f_i(k) + c\sum_{j=1}^n a_{ij}(s(k-1)(\Delta_j(k)-e_j(k)) \\ &\quad + x_j(k) - (s(k-1)(\Delta_i(k)-e_i(k)) + x_i(k))). \end{aligned}$$

For the sake of convenience and simplicity, QDE-ZGS can be equivalently rewritten as

$$\begin{cases} \nabla F(k+1) = -c\mathcal{L}(s(k-1)(\Delta(k)-e(k)) + x(k)) \\ \qquad\qquad\qquad + \nabla F(k) \\ x(0) = [x_1^{*T}, \ldots, x_n^{*T}]^T \end{cases} \tag{8}$$

where $x(k) = \left[x_1(k)^T, \ldots, x_n(k)^T\right]^T$, $\mathcal{L} = L \otimes I_m$ and $\nabla F(k)$, $\Delta(k)$ and $e(k)$ are defined in the similar way.

Main results: First, we show an another iteration form of (7)

$$x_i(k+1) = x_i(k) + d_i(k)(\nabla f_i(k+1) - \nabla f_i(k)) \tag{9}$$

with

$$d_i(k) = \frac{\|x_i(k+1)-x_i(k)\|^2}{(\nabla f_i(k+1)-\nabla f_i(k))^T(x_i(k+1)-x_i(k))}.$$

Let $D(k) = \mathrm{diag}\{d_1(k), \ldots, d_n(k)\}$. From Assumption 1, $D(k) \le (1/\underline{\theta})I_n$ with $\underline{\theta} = \min_{i\in\mathcal{V}} \theta_i$. Combining (8), rewriting (9) in the matrix form yields

$$x(k+1) = x(k) + (D(k)\otimes I_m)(\nabla F(k+1) - \nabla F(k)). \tag{10}$$

Theorem 1: Under Assumptions 1−3, QDE-ZGS (7) under event-triggered quantized communication (2) and (5) with (4) solves problem (1) if the gain c satisfies that $0 < c < (a-2)\varepsilon\underline{\theta}/6a$ with constants $a > 2$.

Proof: Define a Lyapunov candidate

$$V(k) = \sum_{i=1}^n \left(f_i(x^*) - f_i(k) + \nabla f_i(k)^T(x_i(k) - x^*)\right).$$

By Assumption 1, it is obviously that

$$V(k) \ge \sum_{i=1}^n \frac{\theta_i}{2}\|x_i(k) - x^*\|^2 \tag{11}$$

where $V(k) = 0$ only when $x_i(k) = x^*$. The difference of $V(k)$ can be calculated as

$$\begin{aligned} \triangle V(k) &= V(k+1) - V(k) \\ &= \sum_{i=1}^n \Big(-f_i(x_i(k+1)) + f_i(x_i(k)) \\ &\quad - \nabla f_i(k)^T(x_i(k) - x_i(k+1)) \\ &\quad + (\nabla f_i(k+1) - \nabla f_i(k))^T x_i(k+1)\Big) \end{aligned}$$

where the last equality comes from Remark 1.

Since θ_i-strong convexity of $f_i(x)$, one obtains that $f_i(x_i(k+1)) - f_i(x_i(k)) \ge \nabla f_i(k)^T(x_i(k+1) - x_i(k)) + (\theta_i/2)\|x_i(k+1) - x_i(k)\|^2 \ge 0$. Based on (8), it yields that

$$\begin{aligned} \triangle V(k) \le& \sum_{i=1}^n -\frac{\theta_i}{2}\|x_i(k+1) - x_i(k)\|^2 \\ &+ (\nabla F(k+1) - \nabla F(k))^T x(k+1). \end{aligned} \tag{12}$$

In view of Lemma 1 and A.2 in [7], there exists a constant ε satisfying $\varepsilon = \sup\left\{\epsilon | \epsilon LL^T \le L + L^T\right\}$. From Young's inequality, for a positive constant $a > 2$, it can be conducted that

$$\begin{aligned} &-s(k-1)(\Delta(k) - e(k))^T \mathcal{L} x(k) \\ &\le \frac{as^2(k-1)}{2\varepsilon}\Delta(k)^T\Delta(k) + \frac{as^2(k-1)}{2\varepsilon}e(k)^T e(k) + \frac{1}{a}y(k) \end{aligned} \tag{13}$$

where $y(k) = x(k)^T\left(\left(L + L^T\right)\otimes I_m\right)x(k) \in \mathbb{R}$ is used to simplify the proof.

From (10) and (8), we have

$$\begin{aligned} &(s(k-1)(\Delta(k) - e(k)) + x(k))^T\left(L^T D(k) L \otimes I_m\right) \\ &\quad \times (s(k-1)(\Delta(k) - e(k)) + x(k)) \\ &\le \frac{3}{\varepsilon\underline{\theta}}y(k) + \frac{3\|L\|^2 s^2(k-1)}{\underline{\theta}}\left(e(k)^T e(k) + \Delta(k)^T\Delta(k)\right). \end{aligned} \tag{14}$$

By substituting (13) and (14) into (12), for $k > 0$, one obtains that

$$\begin{aligned} \triangle V(k) \le& s^2(k-1)\left(\frac{ac}{2\varepsilon} + \frac{3c^2\|L\|^2}{\underline{\theta}}\right)e(k)^T e(k) \\ &+ s^2(k-1)\left(\frac{ac}{2\varepsilon} + \frac{3c^2\|L\|^2}{\underline{\theta}}\right)\Delta(k)^T\Delta(k) \\ &+ \left(\frac{2-a}{2a}c + \frac{3c^2}{\varepsilon\underline{\theta}}\right)y(k) \end{aligned}$$

invoking (4) and the boundness of quantization error yields that

$$\begin{aligned}\triangle V(k) \le & -\frac{a-2}{2a}c\left(1-\frac{6ac}{\varepsilon\underline{\theta}(a-2)}\right)y(k)\\ & + s^2(k-1)\left(\frac{ac}{2\varepsilon}+\frac{3c^2\|L\|^2}{\underline{\theta}}\right)\left(nE(k)+\frac{mn}{4}\right)\end{aligned} \tag{15}$$

summing up which from 1 to r–1 with respect to k yields

$$\begin{aligned}V(r) \le & V(1)-\sum_{k=1}^{r-1}\frac{a-2}{2a}c\left(1-\frac{6ac}{\varepsilon\underline{\theta}(a-2)}\right)y(k)\\ & +\left(\frac{anc}{8\varepsilon}+\frac{3nc^2\|L\|^2}{4\underline{\theta}}\right)\sum_{k=1}^{r-1}s^2(k-1)(4E(k)+m).\end{aligned} \tag{16}$$

From Assumption 3, when $r\to\infty$, the third term of (16) exists. Considering $1-6ac/\varepsilon\underline{\theta}(a-2)>0$ when $c<(a-2)\varepsilon\underline{\theta}/6a$ holds, one obtains that

$$\lim_{r\to\infty}\frac{a-2}{2a}c\left(1-\frac{6ac}{\varepsilon\underline{\theta}(a-2)}\right)y(k)=0$$

indicating $\lim_{k\to\infty}\mathcal{L}x(k)=0$. Recalling Remark 1, the problem (1) is well solved through the algorithm (7). ■

Next, the linear convergence rate is discussed.

Theorem 2: If conditions given in Theorem 1 are satisfied, when the gain c satisfies that

$$0<c<\tilde{c}\triangleq\min\left\{\frac{(a-2)\varepsilon\underline{\theta}}{6a},\frac{2a}{\rho(a-2)}\right\} \tag{17}$$

and there exist positive constants $C_s>0$ and $0<\beta<1$, $\beta\ne1+\zeta$ that $s(k)\le C_s\beta^{\frac{k}{2}}$ with $\zeta\triangleq-(\rho(a-2)/2a)c\left(1-(6ac/\varepsilon\underline{\theta}(a-2))\right)$, QDE-ZGS linearly solves problem (1) such that

$$\sum_{i=1}^{n}\|x_i(k)-x^*\|^2\le\frac{2}{\underline{\theta}}\left(\left(V(1)-\frac{\Xi}{\beta-\zeta-1}\right)(1+\zeta)^k+\frac{\Xi}{\beta-\zeta-1}\beta^k\right) \tag{18}$$

where Ξ is defined in (19).

Proof: It is clear that $V(k)\le x(k)^T(P\otimes I_m)x(k)$, where $P=[P_{ij}]\in\mathbb{R}^{n\times n}$ is well defined as follows:

$$P_{ij}=\begin{cases}\left(\frac{1}{2}-\frac{1}{n}\right)\Theta_i+\frac{1}{2n^2}\sum_{l=1}^{n}\Theta_l, & \text{if } i=j\\ -\frac{\Theta_i+\Theta_j}{2n}+\frac{1}{2n^2}\sum_{l=1}^{n}\Theta_l, & \text{otherwise}\end{cases}$$

which implies that $P\ge0$ and $P\mathbf{1}_n=\mathbf{0}$. By Lemma A.2 and Section 2.2 in [7], there is a positive constant ρ satisfying $\rho=\sup\{\varrho|\varrho P\le L+L^T\}$ such that for $k>0$, $V(k)\le(1/\rho)y(k)$. It follows from (15) and $1-6ac/\varepsilon\underline{\theta}(a-2)>0$ that:

$$\begin{aligned}\triangle V(k)\le & -\frac{\rho(a-2)}{2a}c\left(1-\frac{6ac}{\varepsilon\underline{\theta}(a-2)}\right)V(k)\\ & +s^2(k-1)\left(\frac{anc}{8\varepsilon}+\frac{3nc^2\|L\|^2}{4\underline{\theta}}\right)(4E(k)+m)\\ \le & \zeta V(k)+s^2(k-1)\left(\frac{anc}{8\varepsilon}+\frac{3nc^2\|L\|^2}{4\underline{\theta}}\right)(4E(k)+m)\end{aligned}$$

where ζ is defined in (18). Together with (17) and $a>2$, $-1<-\rho(a-2)/2ac<\zeta<0$. Then

$$\begin{aligned}V(k+1)\le & (1+\zeta)V(k)\\ & +s^2(k-1)\left(\frac{anc}{8\varepsilon}+\frac{3nc^2\|L\|^2}{4\underline{\theta}}\right)(4E(k)+m)\\ & \cdots\\ \le & (1+\zeta)^kV(1)+\left(\frac{anc}{8\varepsilon}+\frac{3nc^2\|L\|^2}{4\underline{\theta}}\right)\\ & \times\sum_{r=0}^{k-1}(1+\zeta)^rs^2(k-r-1)(4E(k-r)+m).\end{aligned}$$

Considering $s(k)\le C_s\beta^{\frac{k}{2}}$ and $E(k)<E_m$, then

$$\begin{aligned}V(k+1)\le & (1+\zeta)^kV(1)+\Xi\sum_{r=0}^{k-1}(1+\zeta)^r\beta^{k-r-1}\\ = & \left(V(1)-\frac{\Xi}{\beta-\zeta-1}\right)(1+\zeta)^k+\frac{\Xi}{\beta-\zeta-1}\beta^k\end{aligned} \tag{19}$$

with $\Xi\triangleq C_snc\left(a\underline{\theta}+6c\varepsilon\|L\|^2\right)(4E_m+m)/8\varepsilon\underline{\theta}$. The result (18) follows readily from (11) and (19). ■

Simulation: Theoretical findings are illustrated by the following numerical example. Consider a multi-agent system with 5 agents interconnected through the communication network depicted in Fig. 1, where the weights on (2,1), (2,5), (3,1) are randomly selected and the others are constructed following Assumption 2. The local objective functions of each agent is designed as $f_i(x)=(1/2)(x-y_i)^2$ with $y_i=[1.12,2.04,2.98,3.82,4.74]$, where the first three comes from [7]. In this case, the local optimal solution $x_i^*=y_i$, and $x(0)=[1.12,2.04,2.98,3.82,4.74]$; the global optimal solution $x^*=2.94$.

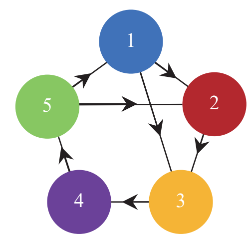

Fig. 1. Communication network.

We choose $s(k)=10/(k+1)$ and $s(k)=10\times0.98^k$ to show the effect of Theorems 1 and 2, respectively. Furthermore, $E(k)=10\times0.8^k$ is the maximal triggering threshold and $E_i(k)=E(k)$, $i\in\mathcal{V}$.

According to (7) with (2)–(5), the simulation results are given as follows. The time evolutions of states x_i, $i\in\mathcal{V}$ using both $\{s(k)\}$ are depicted in Fig. 2. Moreover, the number of triggering instants for each agent is $[96,94,93,93,94]$ when $s(k)=10/(k+1)$ and is $[49,48,48,56,60]$ when $s(k)=10\times0.98^k$, which are shown in Fig. 3. We define the residual as $\sum_{i=1}^{n}\|x_i(k)-x^*\|$ to demonstrate the convergence rate, and Fig. 4 depicts the evolutions of residual for both choice of $\{s(k)\}$. Additionally, the residual evolutions of $s(k)=10\times0.98^k$ without event-triggered scheme, seen as the triggering threshold is 0, is also shown in Fig. 4, where the linear convergence rate is naturally obtained for $s(k)=10\times0.98^k$. It reveals that the usage of event-triggered scheme not only decrease the communication burden, but also have a more exact result with barely changed performance.

Conclusions: This letter has designed QDE-ZGS to solve the distributed optimization problems with constrained communication. An event-triggered quantized communication has been developed based on the encoder-decoder scheme and the zooming-in technique. Theoretical analysis has shown that exact convergence to the global optimal solution under a summable scaled sequence, and the linear convergence is achieved. Note that the unbalanced networks is still an open issue. Furthermore, how to analysis the convergence rate with the finite-level uniform quantizer or logarithmic quantizer is also an interesting issue, which deserves to be studied in the near future.

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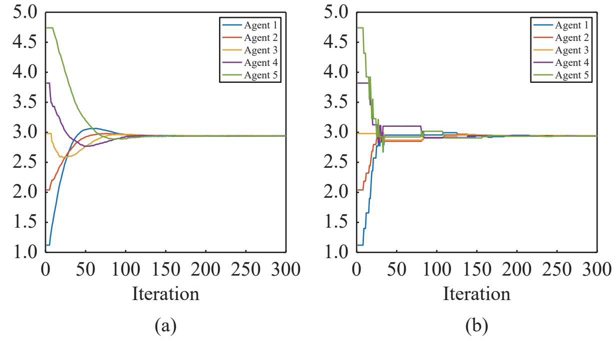

Fig. 2. Time evolutions of state $x_i, i \in \mathcal{V}$ for different $s(k)$. (a) $s(k) = 10/(k+1)$; (b) $s(k) = 10 \times 0.98^k$.

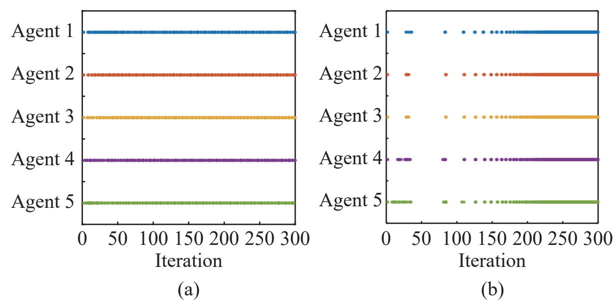

Fig. 3. Triggering instants for different $s(k)$. (a) $s(k) = 10/(k+1)$; (b) $s(k) = 10 \times 0.98^k$.

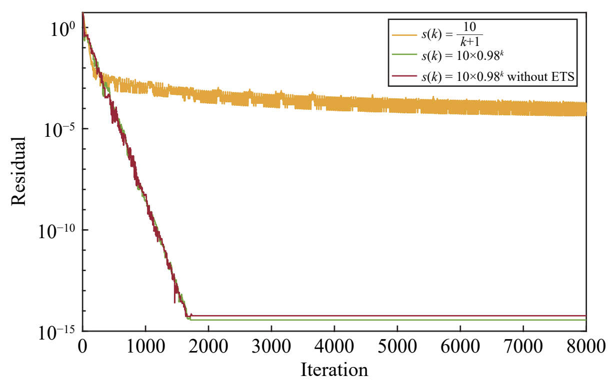

Fig. 4. Performance comparison.

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