Letter

Recursive Filtering for Nonlinear Systems With Self-Interferences Over Full-Duplex Relay Networks

Hailong Tan, Bo Shen, *Senior Member, IEEE*, Qi Li, and Wei Qian

Dear Editor,

In this paper, a recursive filtering problem (RRP) is addressed for nonlinear systems over full-duplex relay (FDR) networks. A FDR is adopted to forward measurements of the sensor to the filter. Because of concurrently transmitting and receiving, the FDR is interfered by the signals from itself, thereby exhibiting self-interference (SI). The motivation of this letter is to design a recursive filter (RR) for the nonlinear system subject to the SI. To alleviate the SI, a SI cancellation (SIC) method is first proposed for the FDR. By analyzing the dynamics of the SI and the filtering error, an upper bound (UB) is provided for the filtering error covariance (FEC). Then, the filter gain is parameterized to minimize the UB. Finally, the performance of the proposed filtering scheme is evaluated by a numerical example.

Filtering/State estimation has long been one of the most appealing research topics in control communities. So far, scores of filtering schemes with omnifarious performances have been available in existing literatures [1]–[5]. Notably, in the filtering problem of stochastic systems, the FEC is usually the most concerned performance that should be minimized as much as possible. However, in many scenarios, especially when the concerned stochastic system is subject to nonlinearities and uncertain parameters, it is usually theoretically impracticable to attain the precise FEC. In such a less-than-ideal situation, an alternative method is to search an UB for the real FEC and further minimize it by adequately designing the filter gain. Based on this method, plentiful and elegant research results have been achieved on the filtering issue of stochastic nonlinear/uncertain systems, see, e.g., [6]–[9].

It should be pointed out that, in existing literatures discussing the filtering problem, it has been always assumed that measurements received by sensors can be transmitted infinitely far. Unfortunately, such an assumption is in conflict with real scenes where the transmission distance of sensors is essentially limited because of the nonnegligible path loss in signal transmission. Owing to the distance-limited communication, filters may fail to receive the signal spread by sensors. To prevent this situation, relays have been widely employed to forward signals as far as possible. Recently, the filtering problem with relays has begun to attract the ever-growing research attention, see, e.g., [10].

Corresponding author: Bo Shen.

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H. L. Tan is with the School of Mathematics-Physics and Finance, Anhui Polytechnic University, Wuhu 241000, and also with the Key Laboratory of Advanced Perception and Intelligent Control of High-End Equipment, Ministry of Education, Anhui Polytechnic University, Wuhu 241000, China (e-mail: hl.tan@ahpu.edu.cn).

B. Shen is with the College of Information Science and Technology, Donghua University, Shanghai 201620, and also with the Engineering Research Center of Digitalized Textile and Fashion Technology, Ministry of Education, Shanghai 201620, China (e-mail: bo.shen@dhu.edu.cn).

Q. Li is with the School of Information Science and Engineering, Hangzhou Normal University, Hangzhou 311121, China (e-mail: liqimicky@gmail.com).

W. Qian is with the School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, China (e-mail: qweihpu@163.com).

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To further improve the spectral efficiency of relay networks, the full-duplex technique that supports synchronous transmission and reception has been deeply integrated with relays. Although the FDR has performed tremendous advantages on communication capacity, it brings a lot of intrinsical challenges as well. For example, due mainly to the simultaneously transmitting and receiving, the FDR will receive the signal broadcasted by itself, which gives rise to the so-called SI, see, e.g., [11]. The SI will lead to the distortion of the measurement and even cause instability of the relay system. Despite the fact that many SIC techniques have been available in communication communities, it is still impossible to completely suppress the SI in reality. Therefore, it is of both practical and theoretical significance to study the RRP subject to the SI of FDR networks. However, such an important issue has been largely overlooked, and this situation stimulates the current study on the RRP with FDRs.

In this letter, the RRP is studied for nonlinear systems with SI over FDR networks. The essential challenges can be sorted out as follows: 1) How can we describe the SI of the FDR? and 2) How can we develop a recursive filtering scheme to accommodate SI of FDR? The main contributions of this work are highlighted as follows: 1) A novel RRP is studied where the SI frequently appearing in FDR networks is taken into consideration; 2) A RR is proposed where an UB is guaranteed on the FEC; and 3) The filter gain matrix is parameterized through minimizing the obtained UB.

Problem formulation: The considered filtering problem is presented in Fig. 1. A FDR is located between the sensor and the filter to forward the measurement broadcasted by the sensor. The FDR can receive and transmit the signal at the same time and therefore it will receive the signal spread by itself, which gives rise to the SI in the signal received by the relay. In order to suppress the SI, a SIC method is first adopted in the FDR. Then, the FDR forwards signals through SIC to the filter with certain transmission power. Finally, the filter generates the desired state estimates.

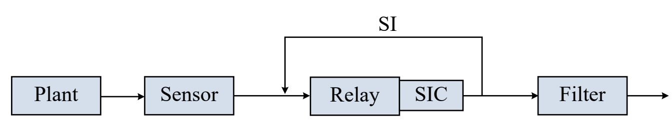

Fig. 1. Diagram for the filtering problem.

Consider the following nonlinear systems:

$$\vartheta_{i+1}=f_i(\vartheta_i)+w_i \tag{1}$$

where i $(i=0,1,2,\ldots)$ is the sampling time instant, $\vartheta_i\in\mathbb{R}^n$ and $w_i\in\mathbb{R}^n$ represent the system state and the stochastic noise, $f_i(\vartheta_i)$ satisfies the following condition [12]:

$$\begin{aligned}&f_i(0)=0\\&\|f_i(\zeta_i)-f_i(\eta_i)-A_i(\zeta_i-\eta_i)\|_2^2\le a_i\|\zeta_i-\eta_i\|_2^2\end{aligned} \tag{2}$$

for all ζ_i, $\eta_i\in\mathbb{R}^n$, A_i is a known matrix with appropriate dimensions, and a_i is a positive scalar. In addition, the initial state ϑ_0 is assumed to follow the Gaussian distribution.

Denoting by $z_i\in\mathbb{R}^m$ the measurement of the sensor, then we describe z_i as follows:

$$z_i=C_i\vartheta_i+v_i \tag{3}$$

where C_i is a known matrix and v_i denotes the measurement noise.

The sensor transmits the measurement to the relay with certain transmission power. The signal received by the FDR is denoted as follows:

$$d_i=\sqrt{t_{s,i}}o_{sr,i}z_i+\sqrt{t_{r,i-1}}o_{rr,i}s_i+s_i^c+\upsilon_{s,i} \tag{4}$$

where s_i and s_i^c are the SI and the SIC, $t_{r,i}$ and $t_{s,i}$ are the transmission powers of the FDR and the sensor, $\upsilon_{s,i}$ is a white Gaussian noise, and $o_{sr,i}$ and $o_{rr,i}$ are the stochastic channel coefficients of sensor-to-relay and relay-to-relay channel with $\mathbb{E}o_{sr,i}=\bar{o}_{sr,i}$, $\mathbb{E}o_{rr,i}=\bar{o}_{rr,i}$ $\mathbb{E}(o_{sr,i}-\bar{o}_{sr,i})^2=\sigma_{sr,i}$ and $\mathbb{E}(o_{rr,i}-\bar{o}_{rr,i})^2=\sigma_{rr,i}$ ($\bar{o}_{rr,i}$, $\bar{o}_{sr,i}$, $\sigma_{sr,k}$ and $\sigma_{rr,k}$ are known positive scalars).

According to [13], the SI s_i is described as follows:

$$s_i = \begin{cases} 0, & i=0 \\ \alpha_{i-1} d_{i-1}, & i>0 \end{cases} \tag{5}$$

where $\alpha_i > 0$ is the given amplification factor of the relay.

In order to suppress the SI given in (5), the SIC s_i^c is designed as

$$s_k^c = \begin{cases} 0, & i=0 \\ -\sqrt{t_{r,i-1}}\bar{o}_{rr,i} s_i, & i>0. \end{cases} \tag{6}$$

Furthermore, the signal d_i is amplified and forwarded to the filter by the FDR. Then, signal g_i obtained by the filter is denoted as follows:

$$g_i = \alpha_i \sqrt{t_{r,i}} o_{rf,i} d_i + \upsilon_{f,i} \tag{7}$$

where $\upsilon_{f,i}$ is Gaussian distributed noise and $o_{rf,i}$ is the coefficient of relay-to-filter channel satisfying $\mathbb{E}o_{rf,i} = \bar{o}_{rf,i}$ and $\mathbb{E}(o_{rf,i} - \bar{o}_{rf,i})^2 = \sigma_{rf,i}$ ($\bar{o}_{rf,i} > 0$ and $\sigma_{rf,i} > 0$ are known parameters).

Remark 1: It is worth mentioning that, because of the effects of SI, the measurement d_i received by the FDR is related to d_{i-1}. Therefore, the measurement d_i performs dynamic features, which renders the existing filtering algorithm invalid. In this paper, we try our best to develop a RR for nonlinear systems with the SI of FDR.

Remark 2: In order to suppress the SI, the SIC (6) which eliminates the SI in the mean sense is proposed by utilizing the information of the stochastic channel coefficient. Such a SIC has advantages over low computational burden and easy-to-implement.

Assumption 1: The random variables ϑ_0, w_i, v_i, $\upsilon_{s,i}$ and $\upsilon_{f,i}$ are mutually uncorrelated and auto-uncorrelated and obey the following probability distributions:

$$\begin{aligned} &\vartheta_0 \sim \mathcal{N}(\bar{\vartheta}_0, P_0),\ \ w_i \sim \mathcal{N}(0, W_i),\ \ v_i \sim \mathcal{N}(0, V_i) \\ &\upsilon_{s,i} \sim \mathcal{N}(0, R_{s,i}),\ \ \upsilon_{f,i} \sim \mathcal{N}(0, R_{f,i}) \end{aligned} \tag{8}$$

where $\bar{\vartheta}_0$ is a given vector and P_0, W_i, V_i, $R_{s,i}$ and $R_{f,i}$ are given positive definite matrix parameters with suitable dimensions.

For all $i > 0$, it can be obtained from (3) and (7) that

$$\begin{aligned} g_i = &\alpha_i \sqrt{t_{r,i} t_{s,i}} o_{rf,i} o_{sr,i} C_i \vartheta_i + \alpha_i \sqrt{t_{r,i}} o_{rf,i} \upsilon_{s,i} \\ &+ \alpha_i \sqrt{t_{r,i} t_{s,i}} o_{rf,i} o_{sr,i} v_i + \varepsilon_i s_i + \upsilon_{f,i} \end{aligned} \tag{9}$$

where $\varepsilon_i = \alpha_i \sqrt{t_{r,i} t_{r,i-1}} o_{rf,i}(o_{rr,i} - \bar{o}_{rr,i})$.

Based on the measurement g_i, the following filter is constructed for the concerned nonlinear system (1):

$$\begin{cases} \hat{\vartheta}_{i+1} = f_i(\hat{\vartheta}_i) + K_i \chi_i \\ \hat{\vartheta}_0 = \bar{\vartheta}_0 \end{cases} \tag{10}$$

where $\hat{\vartheta}_i$ depicts the estimate for ϑ_i, K_i is the gain matrix to be designed, and $\chi_i = g_i - \alpha_i \sqrt{t_{r,i} t_{s,i}} \bar{o}_{rf,i} \bar{o}_{sr,i} C_i \hat{\vartheta}_i$.

Letting the filtering error be $\tilde{\vartheta}_i = \vartheta_i - \hat{\vartheta}_i$, we define the FEC as $\Xi_i = \mathbb{E}\{\tilde{\vartheta}_i \tilde{\vartheta}_i^T\}$. Considering the effects of nonlinear parameters, the precise FEC Ξ_i is technically unavailable. In this paper, we aim to find a matrix sequence $\{\Sigma_i\}_{i=0}^{\infty}$ satisfying $\Xi_i \le \Sigma_i$ and further design the filter gain such that Σ_i is minimized at each time instant.

Main results: In this section, a RR is designed for the nonlinear system over FDR networks.

Lemma 1: For the nonlinear system (1), if there are a positive scalar μ_i ($i = 0,1,2,\ldots$) and a semi-positive definite matrix sequence $\{\tilde{M}_i\}_{i=0}^{\infty}$ satisfying

$$\begin{aligned} &\tilde{M}_{i+1} = (1+\mu_i) a_i \mathrm{tr}\{\tilde{M}_i\} I + (1+\mu_i^{-1}) A_i \tilde{M}_i A_i^T + W_i \\ &\tilde{M}_0 = P_0 + \bar{\vartheta}_0 \bar{\vartheta}_0^T \end{aligned} \tag{11}$$

then, we have $M_i \le \tilde{M}_i$ with $M_i = \mathbb{E}\{\vartheta_i \vartheta_i^T\}$.

Proof: The proof can be directly obtained from (1) and (2). The details are omitted here. ■

Lemma 2: Define $S_i = \mathbb{E}\{s_i s_i^T\}$. If the semi-definite matrix $\tilde{S}_i$ ($i = 0,1,\ldots$) is the solution to the following recursion:

$$\begin{aligned} \tilde{S}_{i+1} = &\alpha_i^2 t_{r,i-1} \sigma_{rr,i} \tilde{S}_i + \alpha_i^2 t_{s,i} \tilde{\sigma}_{sr,i} V_i \\ &+ \alpha_i^2 t_{s,i} \tilde{\sigma}_{sr,i} C_i \tilde{M}_i C_i^T + \alpha_i^2 R_{s,i} \\ \tilde{S}_0 = &\,0 \end{aligned} \tag{12}$$

where $\tilde{\sigma}_{sr,i} = \sigma_{sr,i} + \bar{o}_{sr,i}^2$, then $\tilde{S}_i$ is an UB for S_i, i.e.,

$$S_i \le \tilde{S}_i. \tag{13}$$

Proof: The mathematical induction method is adopted to prove this lemma. From the initial value, we easily know that $\tilde{S}_0 = S_0 = 0$. Then, assuming that $S_i \le \tilde{S}_i$, we intend to show $S_{i+1} \le \tilde{S}_{i+1}$. It can be obtained from (3)−(5) that

$$\begin{aligned} s_{i+1} = &\alpha_i d_i = \alpha_i \sqrt{t_{s,i}} o_{sr,i} C_i \vartheta_i + \alpha_i \sqrt{t_{s,i}} o_{sr,i} v_i \\ &+ \alpha_i \sqrt{t_{r,i-1}} (o_{rr,i} - \bar{o}_{rr,i}) s_i + \alpha_i \upsilon_{s,i}. \end{aligned} \tag{14}$$

Based on (14), one can easily obtain that

$$\begin{aligned} S_{i+1} = &\alpha_i^2 t_{r,i-1} \sigma_{rr,i} S_i + \alpha_i^2 t_{s,i} \tilde{\sigma}_{sr,i} V_i \\ &+ \alpha_i^2 t_{s,i} \tilde{\sigma}_{sr,i} C_i M_i C_i^T + \alpha_i^2 R_{s,i} \le \tilde{S}_{i+1}. \end{aligned} \tag{15}$$

From the inductive method, (13) is directly obtained. ■

When $i = 0$, it can be easily obtained from the initial state that $\tilde{\vartheta}_0 = \vartheta_0 - \hat{\vartheta}_0 = \vartheta_0 - \bar{\vartheta}_0$ and $\Xi_0 = P_0$. As for $i = 1$, the filtering error $\tilde{\vartheta}_i$ can be deduced from (1), (3)−(7) and (10) as follows:

$$\tilde{\vartheta}_i = f_{i-1}(\vartheta_{i-1}) + w_{i-1} - f_{i-1}(\hat{\vartheta}_{i-1}) - K_{i-1}\chi_{i-1} = \Upsilon_{i-1} \tag{16}$$

where

$$\begin{aligned} \Upsilon_{i-1} = &f_{i-1}(\vartheta_{i-1}) - f_{i-1}(\hat{\vartheta}_{i-1}) - K_{i-1}\upsilon_{f,i-1} - \alpha_{i-1} \\ &\times \sqrt{t_{r,i-1} t_{s,i-1}} \bar{o}_{rf,i-1} \bar{o}_{sr,i-1} K_{i-1} C_{i-1} \tilde{\vartheta}_{i-1} \\ &- \alpha_{i-1} \sqrt{t_{r,i-1} t_{s,i-1}} o_{rf,i-1} o_{sr,i-1} K_{i-1} v_{i-1} \\ &- \alpha_{i-1} \sqrt{t_{r,i-1}} o_{rf,i-1} K_{i-1} \upsilon_{s,i-1} + w_{i-1} \\ &- \alpha_{i-1} \sqrt{t_{r,i-1} t_{s,i-1}} (o_{rf,i-1} o_{sr,i-1} - \bar{o}_{rf,i-1} \\ &\times \bar{o}_{sr,i-1}) K_{i-1} C_{i-1} \vartheta_{i-1}. \end{aligned} \tag{17}$$

By utilizing (16), the FEC at $i = 1$ is computed as $\Xi_i = \mathbb{E}\{\Upsilon_{i-1}\Upsilon_{i-1}^T\}$. For all $i > 1$, we know from (1), (4), (9) and (10) that

$$\tilde{\vartheta}_i = \Upsilon_{i-1} - \varepsilon_{i-1} K_{i-1} s_{i-1}. \tag{18}$$

In this case, the FEC is derived as

$$\Xi_i = \mathbb{E}\{(\Upsilon_{i-1} - \varepsilon_{i-1} K_{i-1} s_{i-1}) \times (\Upsilon_{i-1} - \varepsilon_{i-1} K_{i-1} s_{i-1})^T\}. \tag{19}$$

In the following lemma, an UB matrix Σ_i is provided for the FEC Ξ_i at each time instant.

Lemma 3: If there is a matrix $\Sigma_i > 0$ ($i = 0,1,2,\ldots$) satisfying

$$\Sigma_i = \begin{cases} P_0, & i=0 \\ \Psi_{i-1}, & i=1 \\ \Phi_{i-1}, & i>1 \end{cases} \tag{20}$$

where

$$\begin{aligned} \Psi_{i-1} = &(1+\epsilon_{i-1}) a_{i-1} \mathrm{tr}\{\Sigma_{i-1}\} I + (1+\epsilon_{i-1}^{-1}) \\ &\times (A_{i-1} - \alpha_{i-1} \sqrt{t_{r,i-1} t_{s,i-1}} \bar{o}_{rf,i-1} \bar{o}_{sr,i-1} \\ &\times K_{i-1} C_{i-1}) \Sigma_{i-1} (A_{i-1} - \alpha_{i-1} \bar{o}_{rf,i-1} \\ &\times \sqrt{t_{r,i-1} t_{s,i-1}} \bar{o}_{sr,i-1} K_{i-1} C_{i-1})^T \\ &+ K_{i-1} \Lambda(\tilde{M}_{i-1}) K_{i-1}^T + W_{i-1} \\ \Phi_{i-1} = &\Psi_{i-1} + \zeta_{i-1} K_{i-1} \tilde{S}_{i-1} K_{i-1}^T \\ \tilde{\sigma}_{rf,i} = &\sigma_{rf,i} + \bar{o}_{rf,i}^2,\ \zeta_i = \alpha_i^2 t_{r,i} t_{r,i-1} \tilde{\sigma}_{rf,i} \sigma_{rr,i} \\ \Lambda(\tilde{M}_{i-1}) = &\alpha_{i-1}^2 t_{r,i-1} t_{s,i-1} \tilde{\sigma}_{rf,i-1} \tilde{\sigma}_{sr,i-1} V_{i-1} \\ &+ \alpha_{i-1}^2 t_{r,i-1} \tilde{\sigma}_{rf,i-1} R_{s,i-1} + R_{f,i-1} \\ &+ \alpha_{i-1}^2 t_{r,i-1} t_{s,i-1} (\tilde{\sigma}_{rf,i-1} \tilde{\sigma}_{sr,i-1} \\ &- \bar{o}_{rf,i-1}^2 \bar{o}_{sr,i-1}^2) C_{i-1} \tilde{M}_{i-1} C_{i-1}^T \end{aligned}$$

and $\epsilon_i > 0$ is a given auxiliary parameter, then we have $\Xi_i \le \Sigma_i$ for all $i = 0,1,2,\ldots$

Proof: Since $\Xi_0 = \Sigma_0 = P_0$, it is obvious that

$$\Xi_0 \le \Sigma_0. \tag{21}$$

When $i = 1$, it is obtained from (16) and the statistical characteristics of $o_{sr,i-1}$ and $o_{rf,i-1}$ that

$$\Xi_i \le \mathbb{E}\{\vec{f}_{i-1}(\vartheta_{i-1}) \vec{f}_{i-1}^T(\vartheta_{i-1})\} + K_{i-1} \Lambda(\tilde{M}_{i-1}) K_{i-1}^T + W_{i-1} \tag{22}$$

where

$$
\begin{aligned}
\vec{f}_{i-1}(\vartheta_{i-1}) = {} & f_{i-1}(\vartheta_{i-1}) - f_{i-1}(\hat{\vartheta}_{i-1}) - A_{i-1}\tilde{\vartheta}_{i-1} \\
& + (A_{i-1} - \alpha_{i-1}\bar{o}_{rf,i-1}\bar{o}_{sr,i-1} \\
& \times \sqrt{t_{r,i-1}t_{s,i-1}}K_{i-1}C_{i-1})\tilde{\vartheta}_{i-1}.
\end{aligned}
$$

By means of the following inequality:

$$
mn^T + nm^T \le \epsilon mm^T + \epsilon^{-1}nn^T \tag{23}
$$

where m and n denote vectors with appropriate dimensions and $\epsilon > 0$ is a given scalar, we further have

$$
\begin{aligned}
& \mathbb{E}\{\vec{f}_{i-1}(\vartheta_{i-1})\vec{f}^T_{i-1}(\vartheta_{i-1})\} \\
& \le (1+\epsilon_{i-1})a_{i-1}\mathrm{tr}\{\Xi_{i-1}\}I + (1+\epsilon^{-1}_{i-1}) \\
& \quad \times (A_{i-1} - \alpha_{i-1}\sqrt{t_{r,i-1}t_{s,i-1}}\bar{o}_{rf,i-1}\bar{o}_{sr,i-1} \\
& \quad \times K_{i-1}C_{i-1})\Xi_{i-1}(A_{i-1} - \alpha_{i-1}\bar{o}_{rf,i-1} \\
& \quad \times \sqrt{t_{r,i-1}t_{s,i-1}}\bar{o}_{sr,i-1}K_{i-1}C_{i-1})^T.
\end{aligned} \tag{24}
$$

From (22), (24) and Lemma 1, it is easily obtained that

$$
\Xi_i \le \Psi_{i-1} = \Sigma_i. \tag{25}
$$

So, the conclusion holds at $i = 1$.

When $i > 1$, we first assume that $\Sigma_{i-1} \ge \Xi_{i-1}$. Then, it is obtained from (19) that

$$
\begin{aligned}
\Xi_i \le {} & \mathbb{E}\{\vec{f}_{i-1}(\vartheta_{i-1})\vec{f}^T_{i-1}(\vartheta_{i-1}) - \mathfrak{R}_i - \mathfrak{R}_i^T \\
& + \varepsilon^2_{i-1}K_{i-1}s_{i-1}s^T_{i-1}K^T_{i-1} - \mathfrak{A}_i - \mathfrak{A}_i^T\} \\
& + K_{i-1}\Lambda(\tilde{M}_{i-1})K^T_{i-1} + W_{i-1}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathfrak{R}_i = {} & \varepsilon_{i-1}K_{i-1}s_{i-1}\vec{f}^T_{i-1}(\vartheta_{i-1}), \\
\mathfrak{A}_i = {} & \varepsilon_{i-1}K_{i-1}s_{i-1}\vartheta^T_{i-1}C^T_{i-1}K^T_{i-1}\sqrt{t_{r,i-1}t_{s,i-1}} \\
& \times \alpha_{i-1}(o_{rf,i-1}o_{sr,i-1} - \bar{o}_{rf,i-1}\bar{o}_{sr,i-1}).
\end{aligned}
$$

Noting the statistical characteristics of $o_{rr,i}$ and $o_{rf,i}$, we have

$$
\begin{aligned}
& \mathbb{E}\{\varepsilon_{i-1}\} = 0,\ \mathbb{E}\{\varepsilon^2_{i-1}\} = \zeta_{i-1} \\
& \mathbb{E}\{\varepsilon_{i-1}(o_{rf,i-1}o_{sr,i-1} - \bar{o}_{rf,i-1}\bar{o}_{sr,i-1})\} = 0
\end{aligned} \tag{26}
$$

from which together with Lemma 2, we know that

$$
\begin{aligned}
& \mathbb{E}\{\varepsilon^2_{i-1}K_{i-1}s_{i-1}s^T_{i-1}K^T_{i-1}\} \\
& = \zeta_{i-1}K_{i-1}S_{i-1}K^T_{i-1} \le \zeta_{i-1}K_{i-1}\tilde{S}_{i-1}K^T_{i-1}
\end{aligned} \tag{27}
$$

and

$$
\mathbb{E}\{\mathfrak{R}_i\} = 0,\ \mathbb{E}\{\mathfrak{A}_i\} = 0. \tag{28}
$$

Substituting (24), (27) and (28) into (26), one has

$$
\Xi_i \le \Phi_{i-1} = \Sigma_i. \tag{29}
$$

From (21), (25) and (29), we obtain that $\Xi_i \le \Sigma_i$ at each time step. ■

In what follows, the filter gain K_i is designed by minimizing the UB matrix Σ_i.

Theorem 1: If the filter gain K_i is chosen as

$$
K_i = \begin{cases} \Omega_i\Gamma^{-1}_i, & i = 0 \\ \Omega_i(\Gamma_i + \zeta_i\tilde{S}_i)^{-1}, & i > 0 \end{cases} \tag{30}
$$

where

$$
\begin{aligned}
\Omega_i = {} & (1+\epsilon^{-1}_i)\alpha_i\sqrt{t_{r,i}t_{s,i}}\bar{o}_{rf,i}\bar{o}_{sr,i}A_i\Sigma_iC^T_i \\
\Gamma_i = {} & (1+\epsilon^{-1}_i)\alpha^2_it_{r,i}t_{s,i}\bar{o}^2_{rf,i}\bar{o}^2_{sr,i}C_i\Sigma_iC^T_i + \Lambda(\tilde{M}_i)
\end{aligned} \tag{31}
$$

then, the UB Σ_i achieves the minimum value.

Proof: When $i = 1$, we can rewrite the UB matrix Σ_i as follows:

$$
\begin{aligned}
\Sigma_i = {} & (K_{i-1} - \Omega_{i-1}\Gamma^{-1}_{i-1})\Gamma_{i-1}(K_{i-1} - \Omega_{i-1}\Gamma^{-1}_{i-1})^T \\
& + W_{i-1} + (1+\epsilon_{i-1})a_{i-1}\mathrm{tr}\{\Sigma_{i-1}\}I \\
& + (1+\epsilon^{-1}_{i-1})A_{i-1}\Sigma_{i-1}A^T_{i-1} + W_{i-1} \\
& - \Omega_{i-1}\Gamma^{-1}_{i-1}\Omega^T_{i-1}.
\end{aligned} \tag{32}
$$

Obviously, when K_{i-1} is selected as (30), Σ_i is minimized at $i = 1$. As for all $i > 1$, the upper bound Σ_i is rewritten as

$$
\begin{aligned}
\Sigma_i = {} & (K_{i-1} - \Omega_{i-1}(\Gamma_{i-1} + \zeta_{i-1}\tilde{S}_{i-1})^{-1}) \\
& \times (\Gamma_{i-1} + \zeta_{i-1}\tilde{S}_{i-1})[K_{i-1} - \Omega_{i-1}(\Gamma_{i-1} \\
& + \zeta_{i-1}\tilde{S}_{i-1})^{-1}]^T + (1+\epsilon_{i-1})a_{i-1}\mathrm{tr}\{\Sigma_{i-1}\}I \\
& + (1+\epsilon^{-1}_{i-1})A_{i-1}\Sigma_{i-1}A^T_{i-1} + W_{i-1} \\
& - \Omega_{i-1}(\Gamma_{i-1} + \zeta_{i-1}\tilde{S}_{i-1})^{-1}\Omega^T_{i-1}.
\end{aligned} \tag{33}
$$

From (33), one can easily obtain that Σ_i ($i > 1$) is minimized when K_{i-1} is determined in the form of (30). ■

Remark 3: In fact, the K_i at $i = 0$ in (30) can be recognized as the filter gain minimizing the UB in the SI-free case. Meanwhile, the K_i for $i > 0$ is the desired filter gain when there exists SI in the measurements received by FDRs. Obviously, the effect from the SI to the filtering scheme is reflected by the term $\zeta_i\tilde{S}_i$.

Numerical example: In this part, an illustrative example is provided to display the effectiveness of the proposed recursive filtering method.

The parameters of the nonlinear system (1) with the measurement (3) are given as follows:

$$
\begin{aligned}
& f_i(\vartheta_i) = A_i\vartheta_i + \begin{bmatrix} -0.05\sin\vartheta_{1,i} \\ 0.02\sin\vartheta_{2,i} \end{bmatrix},\ a_i = 0.005 \\
& A_i = \begin{bmatrix} 0.63 & 0.52 \\ 0.5\sin i & 0.74 \end{bmatrix},\ C_i = \begin{bmatrix} 2 & 1.8 \end{bmatrix}.
\end{aligned}
$$

Moreover, the covariances of the noises are $W_i = 0.003I_2$, $v_i = 0.001$ and $R_{s,i} = R_{f,i} = 0.001$. The mean value and covariance of the initial value are given as $\bar{\vartheta}_0 = [-0.2\ \ 0.1]^T$ and $P_0 = 0.02I_2$.

The transmission powers of the FDR and sensor are $t_{s,i} = t_{r,i} = 2$ and the amplification factor of the relay is $\alpha_i = 1$. As for the stochastic coefficients, we set $\bar{o}_{sr,i} = \bar{o}_{rf,i} = 0.5$, $\bar{o}_{rr,i} = 0.6$ and $\sigma_{sr,i} = \sigma_{rr,i} = \sigma_{rf,i} = 0.01$. In addition, μ_i and ϵ_i are chosen as $\mu_i = 10$ and $\epsilon_i = 0.08$.

The actual state and their estimates are presented in Figs. 2 and 3. In Fig. 2, the first element $\vartheta_{1,i}$ of the state ϑ_i and the corresponding estimates $\hat{\vartheta}_{1,i}$ are depicted simultaneously. Fig. 3 shows the real value and the estimates of the state $\vartheta_{2,i}$ which is the second element of ϑ_i.

In light of the unavailability of actual filtering error covariance, we use the mean square error MSE_i to approximate it. The MSE_i is defined as follows:

$$
\mathrm{MSE}_i = \frac{\sum^M_{l=1}(\vartheta^l_i - \hat{\vartheta}^l_i)(\vartheta^l_i - \hat{\vartheta}^l_i)^T}{M}
$$

where ϑ^l_i and $\hat{\vartheta}^l_i$ stand for the actual state and its estimate of the lth implementation of the developed filtering method, respectively, and M is a positive integer. By running the proposed filtering algorithm $M = 100$ times, the results of MSE_i and Σ_i are reflected in Figs. 4 and 5. Detailedly, Fig. 4 depicts the values of MSE^{11}_i and Σ^{11}_i which are the first elements in main diagonal of MSE_i and Σ_i. It can be easily seen that MSE^{11}_i is less than Σ^{11}_i, which conforms to the conclusion obtained in the paper. Moreover, same results can be obtained for MSE^{22}_i and Σ^{22}_i where MSE^{22}_i (respectively, Σ^{22}_i) is the second element in main diagonal of MSE_i (respectively, Σ_i). Based on the simulation results, we conclude that the proposed RR is competent to estimate the state of the concerned nonlinear system with FDRs.

Conclusion: This letter has been concerned with the RRP for nonlinear systems with SI over FDR networks. A FDR which endorses concurrent transmission and reception has been adopted to forward measurements transmitted by sensors. First, a RR has been constructed in the presence of SI of FDR. Moreover, an UB has been guaranteed on the real FEC. Then, the desired filter gain has been designed such that the UB is minimized at each time step. Finally, we have verified the usefulness of the RR by an illustrative example. Future research interests include the control issue over FDR networks [14], [15].

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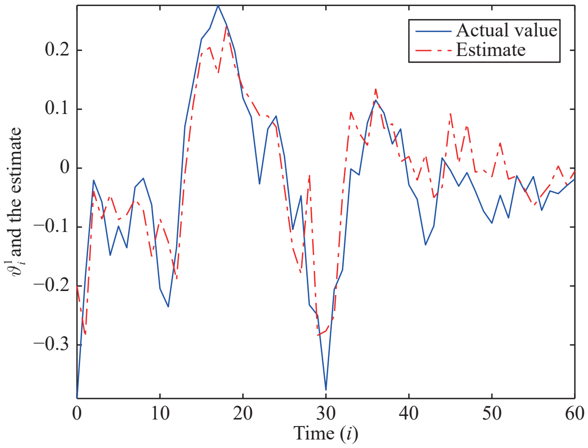

Fig. 2. $\vartheta_{1,i}$ and $\hat{\vartheta}_{1,i}$.

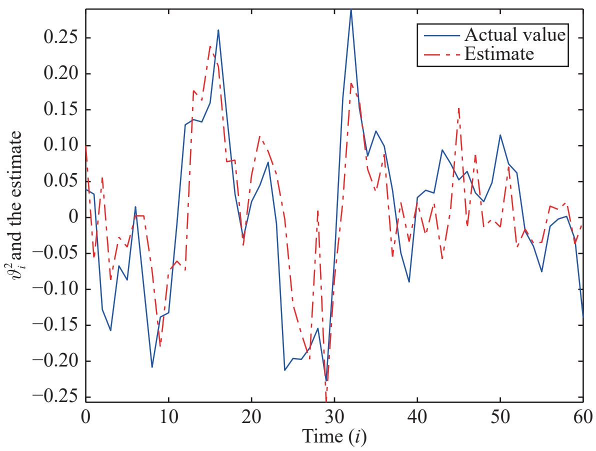

Fig. 3. $\vartheta_{2,i}$ and $\hat{\vartheta}_{2,i}$.

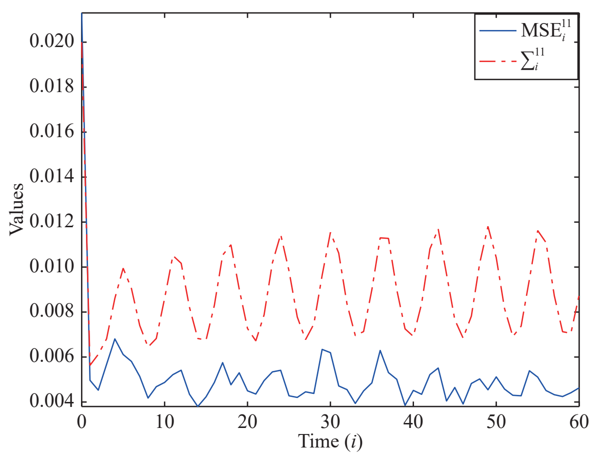

Fig. 4. MSE_i^{11} and Σ_i^{11}.

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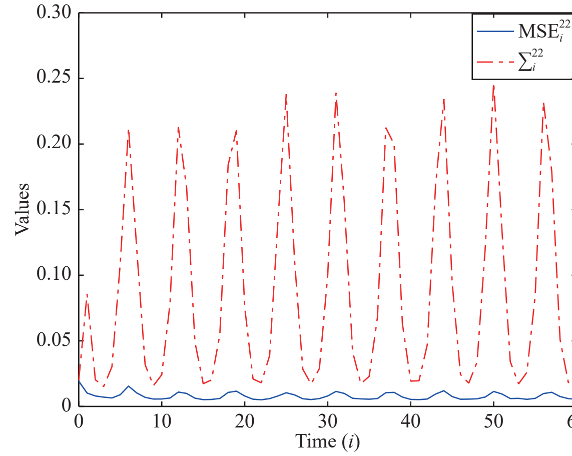

Fig. 5. MSE_i^{22} and Σ_i^{22}.

References

[1] D. Ding, Z. Wang, and Q.-L. Han, "A set-membership approach to event-triggered filtering for general nonlinear systems over sensor networks," *IEEE Trans. Automatic Control*, vol. 65, no. 4, pp. 1792–1799, Apr. 2020.

[2] H. Fang, N. Tian, Y. Wang, M. Zhou, and M. A. Haile, "Nonlinear Bayesian estimation: From Kalman filtering to a broader horizon," *IEEE/CAA J. Autom. Sinica*, vol. 5, no. 2, pp. 401–417, Mar. 2018.

[3] L. Pedroso, P. Batista, P. Oliveira, and C. Silvestre, "Discrete-time distributed Kalman filter design for networks of interconnected systems with linear time-varying dynamics," *Int. J. Systems Science*, 2021. DOI: 10.1080/00207721.2021.2002461.

[4] F. Zhao, C. Chen, W. He, and S. Ge, "A filtering approach based on MMAE for a SINS/CNS integrated navigation system," *IEEE/CAA J. Autom. Sinica*, vol. 5, no. 6, pp. 1113–1120, Nov. 2018.

[5] L. Zou, Z. Wang, and D. H. Zhou, "Moving horizon estimation with non-uniform sampling under component-based dynamic event-triggered transmission," *Automatica*, vol. 120, p. 109154, Oct. 2020.

[6] J. Hu, Z. Wang, G.-P. Liu, C. Jia, and J. Williams, "Event-triggered recursive state estimation for dynamical networks under randomly switching topologies and multiple missing measurements," *Automatica*, vol. 115, p. 108908, May 2020.

[7] W. Li, Y. Jia, and J. Du, "Resilient filtering for nonlinear complex networks with multiplicative noise," *IEEE Trans. Automatic Control*, vol. 64, no. 6, pp. 2522–2528, Jun. 2019.

[8] H. Tan, B. Shen, and H. Shu, "Robust recursive filtering for stochastic systems with time-correlated fading channels," *IEEE Trans. Systems, Man and Cybernetics: Systems*, vol. 52, no. 5, pp. 3102–3112, May 2022.

[9] Y. Theodor and U. Shaked, "Robust discrete-time minimum-variance filtering," *IEEE Trans. Signal Processing*, vol. 44, no. 2, pp. 181–189, Feb. 1996.

[10] A. S. Leong and D. E. Quevedo, "Kalman filtering with relays over wireless fading channels," *IEEE Trans. Automatic Control*, vol. 61, no. 6, pp. 1643–1648, Jan. 2016.

[11] S. Zhang, X. Chai, K. P. Long, A. V. Vasilakos, and L. Hanzo, "Full duplex techniques for 5G networks: Self-interference cancellation, protocol design, and relay selection," *IEEE Commun. Magazine*, vol. 53, no. 5, pp. 128–137, May 2015.

[12] B. Shen, Z. Wang, D. Wang, J. Luo, H. Pu, and Y. Peng, "Finite-horizon filtering for a class of nonlinear time-delayed systems with an energy harvesting sensor," *Automatica*, vol. 100, pp. 144–152, Feb. 2019.

[13] H. Cui, M. Ma, L. Song, and B. Jiao, "Relay selection for two-way full duplex relay networks with amplify-and-forward protocol," *IEEE Trans. Wireless Communications*, vol. 13, no. 7, pp. 3768–3777, Jul. 2014.

[14] X. Ge, Q.-L. Han, J. Wang, and X.-M. Zhang, "Scalable and resilient platooning control of cooperative automated vehicles," *IEEE Trans. Vehicular Technology*, 2022. DOI: 10.1109/TVT.2022.3147371.

[15] P. Gong, Q.-L. Han, and W. Lan, "Finite-time consensus tracking for incommensurate fractional-order nonlinear multiagent systems with directed switching topologies," *IEEE Trans. Cybernetics*, vol. 52, no. 1, pp. 65–76, Jan. 2022.