Letter

A Novel Competition-Based Coordination Model With Dynamic Feedback for Multi-Robot Systems

Bo Peng, Xuerui Zhang, and Mingsheng Shang

Dear Editor,

Task allocation strategies are important in multi-robot systems and have been intensely investigated by researchers because they are critical in determining the performance of the system. In this letter, a novel competition-based coordination model is proposed to solve the multi-robot task allocation problem and applied to a multi-robot object tracking scenario. Both local and global stability of the proposed model are theoretically analyzed, and simulations are implemented to demonstrate the effectiveness of the proposed model.

Introduction: With the advance in technology, robots are deployed in more and more scenarios in place of human beings as they can provide more accurate and consistent working performance than human. Different from a traditional single robot system, a multi-robot system consists of a number of robots that act cooperatively with each other to achieve a set of given tasks [1], [2]. Due to the facts that multi-robot systems are robust and can accomplish tasks that are inefficient or impossible for single-robot systems to perform, multi-robot systems have been a hot studied field since the early days of robotic researches.

How to allocate tasks among a group of robots, which is also termed multi-robot task allocation (MRTA) problem, is crucial in designing an effective and efficient multi-robot system [3]. Among all other methodologies, nature-inspired design algorithms are actively applied in the research field of task allocation in multi-robot systems. In [4], an ant colony optimization scheme is developed to distribute a set of robot among different geographical points to perform various tasks. Wu *et al*. [5] propose a hybrid method of market-based allocation mechanism and Gini coefficient-based scheme to deal with the tasks assignment problem in a multi-robot system, obtaining the goal of maximizing the number of tasks completed and minimizing the energy resource consumed at the same time. Wu *et al.* propose an algorithm combined with particle swarm optimization and reinforcement learning in [6] to determine the real-time rescue assignment strategy for multiple underwater autonomous vehicle systems in complex environments. It is worth noting that these nature-inspired heuristic methods often involve the shortcoming of time consumption due to the fact a lot rounds of iteration are needed before the solution converges. When dealing with time-critical application scenarios such as moving object tracking, more efficient algorithms are demanded.

Winner-take-all (WTA) competition refers to a phenomenon that members of a group compete with one another for activation, and only the one with the most prominent input wins to be activated while the rest ones lose and stay deactivated. Inspired by this phenomenon, an algorithm named WTA is proposed to pick the maxima from all the inputs. A more general form of WTA algorithm is called k-winner-tall-all (k-WTA) [7], [8], which selects k largest values from a group of inputs. As the WTA model is computationally powerful and can generate useful functions needed in many applications, many models have been proposed by researchers to produce the WTA competition. A WTA network named Maxnet is constructed in [9] to deal with pattern classification problems. An alternative neural network based on Hopfield network topology is proposed as WTA functions in [10]. Yu *et al*. [11] propose a WTA circuit to model the inference process of hidden Markov models (HMMs) with time-invariant variables and suggest that the logarithm of posterior probability of the hidden variable could be encoded by the membrane potential of each neuron in the WTA circuit, and the posterior probability of HMM is proportional to the neural firing rate. In [12], a Lotka-Volterra type network is utilized to implement WTA competition. Recurrent neural networks, inspired by their successful applications in various fields, are used to investigate the WTA competition in [13]–[16]. Liu and Wang [17] prove that the WTA problem can be formulated as an optimization problem, and the result could be deduced by solving such optimization problems. It is worth noting that all these models mentioned above are continuous models, but discrete-time models are preferred when implemented on digital platforms.

In this letter, a novel discrete competition-based coordination model is proposed to generate the WTA behaviour and applied to handle the task allocation problem in a multi-robot system, which is engaged in moving object capturing activity. During the process, all robots compete with each other, and only the most suitable one gets activated to chase the target while all the other non-selected robots stay still for vigilance. Merits of the proposed model are simplicity and can be readily implemented. This letter's contributions are listed as follows:

1) A competition-based discrete task allocation model is proposed and implemented in behaviour coordination scheme among a set of robots to track a moving target. The model enjoys the merit of structure simplicity and is fit for real-time applications.

2) The proposed model's stability and convergence property are theoretically proved.

3) Simulations of task allocation in a multi-robot system are conducted to demonstrate the feasibility and effectiveness of the proposed model.

Problem formulation: Firstly, we formulate the task allocation problem of a multi-robot system in the moving-target tracking scenario as follows. A competition-based behaviour coordination algorithm is designed for a group of n robots, such that only the most appropriate robot is selected and entrusted with the task to capture the target capturing and all the other robots stay put on guard.

Suppose at the time instance k, the state of a group of n robots is $S^k=[s_1^k,s_2^k,\ldots,s_n^k]^T\in\mathbb{R}^n$, the input of the WTA network is $U^k=[u_1^k, u_2^k,\ldots,u_n^k]^T\in\mathbb{R}^n$ with $u_i^k\neq u_j^k$ for $i\neq j$, the output of the WTA network is $O^k=[o_1^k,o_2^k,\ldots,o_n^k]^T\in\mathbb{R}^n$, then the dynamics of competition-based coordination model for the multi-robot system can be formulated as follows:

$$
\begin{aligned}
\begin{bmatrix} s_1^{k+1}\\ s_2^{k+1}\\ \vdots\\ s_n^{k+1}\end{bmatrix} &= \begin{bmatrix} u_1^k & & & \\ & u_2^k & & \\ & & \ddots & \\ & & & u_n^k\end{bmatrix}\begin{bmatrix} o_1^k\\ o_2^k\\ \vdots\\ o_n^k\end{bmatrix}\\
\begin{bmatrix} o_1^{k+1}\\ o_2^{k+1}\\ \vdots\\ o_n^{k+1}\end{bmatrix} &= \begin{bmatrix} s_1^k\\ s_2^k\\ \vdots\\ s_n^k\end{bmatrix} \Big/ \sqrt{(s_1^k)^2+(s_2^k)^2+\cdots+(s_n^k)^2}.
\end{aligned}
\tag{1}
$$

Corresponding author: Mingsheng Shang.

Citation: B. Peng, X. R. Zhang, and M. S. Shang, "A novel competition-based coordination model with dynamic feedback for multi-robot systems," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 10, pp. 2029–2031, Oct. 2023.

B. Peng is with the Chongqing Key Laboratory of Big Data and Intelligent Computing, Chongqing Institute of Green and Intelligent Technology, Chinese Academy of Sciences, Chongqing 400714, and also with Chongqing School, University of Chinese Academy of Sciences, Chongqing 400714, China (e-mail: msshang@cigit.ac.cn).

X. R. Zhang and M. S. Shang are with the Chongqing Key Laboratory of Big Data and Intelligent Computing, Chongqing Institute of Green and Intelligent Technology, Chinese Academy of Sciences, Chongqing 400714, China (e-mail: zxr@cigit.ac.cn; msshang@cigit.ac.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JAS.2023.123267

The above states equation (1) can be concisely transformed into the following:

$$\begin{aligned} S^{k+1} &= \mathrm{Diag}(U^k)O^k \\ O^{k+1} &= S^k/\|S^k\|_2 \end{aligned} \tag{2}$$

where $\mathrm{Diag}(U^k)$ is a matrix transformation with $\mathrm{Diag}(U^k) = I \odot (U^k 1^T)$; I is the identity matrix; 1^T is an all-ones vector with $1^T = [1,1,...,1]^T \in \mathbb{R}^n$; $\|S^k\|$ is the Euclidean norm of vector S^k.

Local and global stability analyses: In this section, the local and global stability of the competition-based coordination model for multi-robot system (1) are theoretically analysed and with regard to the local stability of system (1), the following theorems can be given.

Theorem 1: The dynamics of the competition-based coordination model for multi-robot system (1) is locally stable at point $(s^*, o^*) = \pm(u_{j^*} b_{j^*}, b_{j^*})$, where $j^* = \mathrm{argmax}_{i=1,2,\ldots,n}(u_i)$ with u_i being the ith input of the dynamic system, and b_{j^*} is a basis vector of the form $b_{j^*} = [0,\ldots,0,1,0,\ldots,0]^T \in \mathbb{R}^n$ with only the j^* the element being 1 and all other elements being 0. What is more, the dynamic system (1) is locally unstable at points $(s^*, o^*) = \pm(u_i b_i, b_i)$ with $i \neq j^*$.

Proof: We first begin with proving that the competition-based coordination model for multi-robot system (1) is in the equilibrium state at points $(s^*, o^*) = \pm(u_i b_i, b_i)$ where u_i is the ith input of the dynamic system, and b_i is a basis vector in space $\mathbb{R}^T$ with the ith equity being 1 and all others being 0.

As point (s^*, o^*) is an equilibrium point, from system (2), we may have

$$S^* = \mathrm{Diag}(U)O^* \tag{3}$$

$$O^* = S^*/\|S^*\|_2 \tag{4}$$

where $S^* = [s_1^*, s_2^*, \ldots, s_n^*]^T \in \mathbb{R}^n$, $O^* = [o_1^*, o_2^*, \ldots, o_n^*]^T \in \mathbb{R}^n$, and $U = [u_1, u_2, \ldots, u_n]^T \in \mathbb{R}^{\mathrm{n}}$ are the state vector, output vector and input vector of dynamic system (2), respectively. Substitute (4) into (3), we can have

$$S^* = \mathrm{Diag}(U)S^*/\|S^*\|_2 \tag{5}$$

which can be rearranged as

$$\mathrm{Diag}(U)S^* = \|S^*\|_2 S^*. \tag{6}$$

Expand (6) into matrix form, we have

$$\begin{bmatrix} u_1 & & & \\ & u_2 & & \\ & & \ddots & \\ & & & u_n \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} = \sqrt{(s_1)^2 + (s_2)^2 + \ldots + (s_n)^2} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}. \tag{7}$$

From the above equation, it can be easily deduced that $S^* = \pm[0,\ldots,0,u_i,0,\ldots,0]^T = \pm u_i b_i$ for $i = 1,2,\ldots,n$ are the solutions of (6), which means that points $(S^*, O^*) = \pm(u_i b_i, b_i)$ are the equilibrium points of dynamic system (1).

Then, we proceed with the local stability analysis around these equilibrium points. Substitute (3) into (4), we have the evolving state equation for system output O^{k+1} as

$$O^{k+1} = \mathrm{Diag}(U^k)O^k/\|\mathrm{Diag}(U^k)O^k\|_2. \tag{8}$$

Considering the case at equilibrium point $(S^*, O^*) = (u_i b_i, b_i)$, it can be deduced that $\|\mathrm{Diag}(U^k)O^*\|_2 = \|u_i b_i\|_2 = u_i$. So, the above (8) can be written as

$$\begin{bmatrix} o_1^{k+1} \\ o_2^{k+1} \\ \vdots \\ o_n^{k+1} \end{bmatrix} = \begin{bmatrix} u_1/u_i & & & \\ & u_2/u_i & & \\ & & \ddots & \\ & & & u_n/u_i \end{bmatrix} \begin{bmatrix} o_1^k \\ o_2^k \\ \vdots \\ o_n^k \end{bmatrix}. \tag{9}$$

Take the definition for $j^* = \mathrm{argmax}_{i=1,2,\ldots,n}(u_i)$ into account, then the j^*th diagonal entity of the above matrix is $u_{j^*}/u_i > 1$ for any $i \neq j^*$, which means that diagonal matrix have an eigenvalue greater than one. Thereby state evolving (9) is unstable, which means that the dynamic system (1) is locally unstable at point $(s^*, o^*) = \pm(u_i b_i, b_i)$ for any $i \neq j^*$. ■

The analysis of the global stability of competition-based coordination model for multi-robot system (1) proceeds as follows.

Theorem 2: Given the input and output vector of the competition-based coordination model for multi-robot system (1) be $U = [u_1, u_2, ...u_n]^T \in \mathbb{R}^n$ and $O = [o_1, o_2, \ldots, o_n]^T \in \mathbb{R}^n$, respectively, suppose $j^* = \mathrm{argmax}_{i=1,2,\ldots,n}(u_i)$, then the output o_i for system (1) converges to ± 1 for the case $i = j^*$, and converges to 0 for the case $i \neq j^*$.

Proof: From (2), the evolving dynamic equation for output vector O_{k+1} could be written as

$$O_{k+1} = \mathrm{Diag}(U)O_k/\|\mathrm{Diag}(U)O_k\|_2. \tag{10}$$

By expanding (10), we get

$$\begin{aligned} O_{k+1} &= \mathrm{Diag}(U)O_k/\|\mathrm{Diag}(U)O_k\|_2 \\ &= \mathrm{Diag}^2(U)O_{k-1}/\|\mathrm{Diag}^2(U)O_{k-1}\|_2 \\ &= \cdots \\ &= \mathrm{Diag}^k(U)O_1/\|\mathrm{Diag}^k(U)O_1\|_2. \end{aligned} \tag{11}$$

Transform (10) into matrix form,

$$\begin{bmatrix} o_1^{k+1} \\ o_2^{k+1} \\ \vdots \\ o_n^{k+1} \end{bmatrix} = \begin{bmatrix} (\frac{u_1}{u_j^*})^k & & & \\ & (\frac{u_2}{u_j^*})^k & & \\ & & \ddots & \\ & & & (\frac{u_n}{u_j^*})^k \end{bmatrix} \times \begin{bmatrix} o_1^1 \\ o_2^1 \\ \vdots \\ o_n^1 \end{bmatrix} / \sqrt{\sum_1^n ((\frac{u_i}{u_j^*})^k o_i^1)^2}. \tag{12}$$

Note that $j^* = \mathrm{argmax}_{i=1,2,\ldots,n}(u_i)$, then $u_i/u_{j^*} < 1$ for $i \neq j^*$, so for $\lim_{k\to\infty}(u_i/u_{j^*})^k$, we have

$$\lim_{k\to\infty} (\frac{u_i}{u_{j^*}})^k = \begin{cases} 0, & i \neq j^* \\ 1, & i = j^*. \end{cases} \tag{13}$$

From (12), we have

$$\begin{aligned} \lim_{k\to\infty} \begin{bmatrix} o_1^{k+1} \\ \vdots \\ o_n^{k+1} \end{bmatrix} &= \lim_{k\to\infty} \begin{bmatrix} (\frac{u_1}{u_j^*})^k & & \\ & \ddots & \\ & & (\frac{u_n}{u_j^*})^k \end{bmatrix} \times \begin{bmatrix} o_1^1 \\ \vdots \\ o_n^1 \end{bmatrix} / \sqrt{\sum_1^n \lim_{k\to\infty} ((\frac{u_i}{u_j^*})^k o_i^1)^2} \\ &= \begin{bmatrix} 0 & & & & \\ & \ddots & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ o_{j^*}^1 \\ \vdots \\ 0 \end{bmatrix} / \left\|o_{j^*}^1\right\|_2 = \begin{bmatrix} 0 \\ \vdots \\ \frac{o_{j^*}^1}{\left\|o_{j^*}^1\right\|} \\ \vdots \\ 0 \end{bmatrix}. \end{aligned} \tag{14}$$

And for $o_{j^*}^1/\|o_{j^*}^1\|_2$, we have

$$\frac{o^1_{j^*}}{\left\|o^1_{j^*}\right\|} = \begin{cases} 1, & o^1_{j^*} > 0 \\ -1, & o^1_{j^*} < 0. \end{cases} \tag{15}$$

■

Simulations: We first consider a multi-robot system of 20 robots ($n = 20$) to engage in the task to capture a moving target. The task allocation scheme is based on a WTA model, such that at every time instance, only the robot that is nearest to the moving target is active and entrusted with the task to capture the target. The update time interval for the discrete-time model is set as $\tau = 0.05$ s.

Target tracking process simulation: We randomly initiate the positions of all the robots of a multi-robot system and the target in the simulation, and the resulting process is shown in Fig. 1. From Fig. 1(a), it can be seen that at the beginning a robot (marked black) which is nearest to the target is selected as the winner to pursue the target, while all the other robots stay still. The distance between the target and all the robots changes as the capturing process proceeds as shown in Fig. 1(b). In one instance Fig. 1(c), a new winner is selected to continue with the capturing and the original winner stops. The output of the proposed competition-based coordination model is shown in Fig. 1(d), which shows the process of selecting new winners. A tracking process with a robot group size of 5 ($n = 5$) is also simulated on the CoppeliaSim platform (Fig. 2), which also demonstrates the task allocation process of the multi-robot system and thereby the effectiveness of the coordination mechanism.

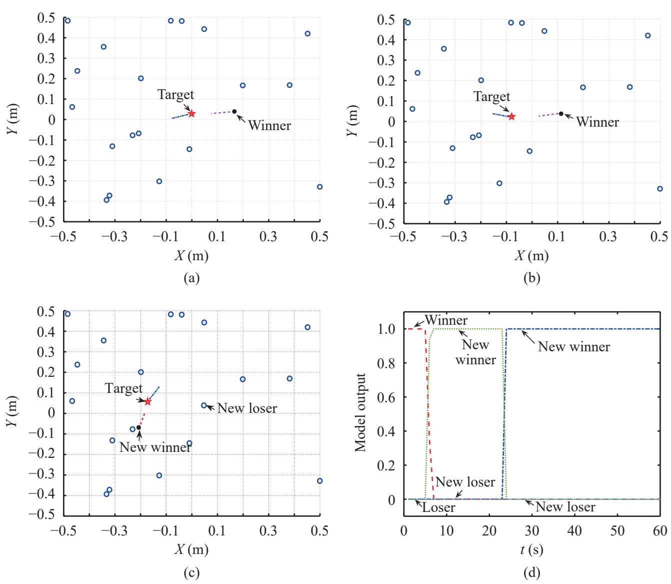

Fig. 1. Snapshots of the moving target tracking process, where initial positions of the moving target and robots are randomly generated. (a) Snapshot at $t = 0$ s; (b) Snapshot at $t = 6.2$ s; (c) Snapshot at $t = 24.3$ s; (d) Output of the WTA network.

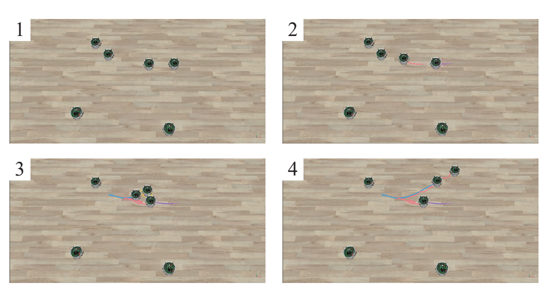

Fig. 2. Tracking process of a multi-robot system consisted of 5 Epuck robots on the CoppeliaSim platform.

Conclusion: In this letter, a novel competition-based discrete-time coordination model has been proposed to solve the multi-robot task allocation problem in a multi-robot system. The simplicity of the discrete-time model can be readily implemented on the digital platform. Local and global stability of the proposed model have been theoretically analyzed. For future study, we plan to expand the WTA model into a more general k-WTA model.

Acknowledgments: This work was supported by the Key Cooperation Project of Chongqing Municipal Education Commission (HZ2021008, HZ2021017) and the Project of "Fertilizer Robot" of Chongqing Committee on Agriculture and Rural Affairs.

References

[1] L. Ma, Y.-L. Wang, and Q.-L. Han, "Cooperative target tracking of multiple autonomous surface vehicles under switching interaction topologies," *IEEE/CAA J. Autom. Sinica.*, vol. 10, no. 3, pp. 673–684, 2023.

[2] X. Ge, Q.-L. Han, J. Wang, and X. M. Zhang, "A scalable adaptive approach to multi-vehicle formation control with obstacle avoidance," *IEEE/CAA J. Autom. Sinica.*, vol. 9, no. 6, pp. 990–1004, 2022.

[3] J. Wang, Y. Hong, J. Wang, J. Xu, Y. Tang, and Q.-L. Han, "Cooperative and competitive multi-agent systems: From optimization to games," *IEEE/CAA J. Autom. Sinica.*, vol. 9, pp. 763–783, 2022.

[4] G. Q. Gao, M. Yi, Y. H. Jia, W. N. Browne, and B. Xin, "Adaptive coordination ant colony optimization for multipoint dynamic aggregation," *IEEE Trans. Cybern.*, vol. 52, no. 8, pp. 7362–7376, 2021.

[5] D. F. Wu, G. P. Zeng, L. G. Meng, W. J. Zhou, and L. M. Li, "Gini coefficient-based task allocation for multi-robot systems with limited energy resources," *IEEE/CAA J. Autom. Sinica*, vol. 5, no. 1, pp. 155–168, 2017.

[6] J. H. Wu, C. X. Song, J. Ma, J. S. Wu, and G. J. Han, "Reinforcement learning and particle swarm optimization supporting real-time rescue assignments for multiple autonomous underwater vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 7, pp. 6807–6820, 2021.

[7] L. Jin, S. Q. Liang, X. Luo, and M. C. Zhou, "Distributed and time-delayed k-winner-take-all network for competitive coordination of multiple robots," *IEEE Trans. Cybern.*, vol. 53, no. 1, pp. 641–652, 2022.

[8] M. Liu and M. S. Shang, "On RNN-based k-WTA models with time-dependent inputs," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 11, pp. 2034–2036, 2022.

[9] R. Lippmann, "An introduction to computing with neural nets," *IEEE Trans. ASSP Mag.*, vol. 4, no. 2, pp. 4–22, Apr. 1987.

[10] G. Dempsey and E. McVey, "Circuit implementation of a peak detector neural network," *IEEE Trans. Circuit Syst. II*, *Analog Digit. Signal Process*, vol. 40, no. 9, pp. 585–591, 1993.

[11] Z. Yu, S. Guo, F. Deng, Q. Yan, K. Huang, J. K. Liu, and F. Chen, "Emergent inference of hidden Markov models in spiking neural networks through winner-take-all," *IEEE Trans. Cybern.*, vol. 50, no. 3, pp. 1347–1354, Mar. 2020.

[12] T. Asai, M. Ohtani, and H. Yonezu, "Analog integrated circuits for the Lotka-Volterra competitive neural networks," *IEEE Trans. Neural Netw.*, vol. 10, no. 5, pp. 1222–1231, 1999.

[13] Y. Fang, M. A. Cohen, and T. G. Kincaid, "Dynamic analysis of a general class of winner-take-all competitive neural networks," *IEEE Trans. Neural Netw.*, vol. 21, no. 5, pp. 771–783, 2010.

[14] S. Li and L. Jin, *Competition-Based Neural Networks With Robotic Applications*. Singapore: Springer, 2018.

[15] Y. M. Qi, L. Jin, X. Luo, Y. Shi, and M. Liu, "Robust k-WTA network generation, analysis, and applications to multiagent coordination," *IEEE Trans. Cybernt.*, vol. 52, no. 8, pp. 8515–8527, Aug. 2022.

[16] M. Liu, X. Y. Zhang, M. S. Shang, and L. Jin, "Gradient-based differential kWTA network with application to competitive coordination of multiple Robots," *IEEE/CAA J. Autom. Sinica.*, vol. 9, no. 8, pp. 1452–1463, 2022.

[17] S. Liu and J. Wang, "A simplified dual neural network for quadratic programming with its KWTA application," *IEEE Trans. Neural Netw.*, vol. 17, no. 6, pp. 1500–1510, 2006.