

## Letter

# Sliding Mode Control for Recurrent Neural Networks With Time-Varying Delays and Impulsive Effects

Lanfeng Hua, Kaibo Shi, *Member, IEEE*, Zheng-Guang Wu,  
*Member, IEEE*, Soohee Han, *Senior Member, IEEE*, and  
Shouming Zhong

Dear Editor,

This letter studies the problem of sliding mode control (SMC) design for recurrent neural networks (RNNs) with impulsive disturbances and time-varying transmission delays. To this end, an appropriate integral sliding surface function and SMC law are adopted for use under impulsive disturbances and time-varying delays. Then, based on the finite-time stability and the discontinuous Lyapunov analysis method, the finite-time reachability of the designed sliding surface and the exponential stability of the resulting sliding mode dynamic are analyzed respectively. Finally, a numerical example is presented to illustrate the effectiveness of the proposed results.

**Introduction:** In the past decades, RNNs have attracted considerable attentions for their advantages of memory and self-adaptability [1]–[3]. Further, they have successful applications in many different fields [4] and [5]. It has been generally acknowledged that in the electronic implementation of RNNs, time delays are likely to occur due to sudden noise or signal propagation speed limitations. Besides delayed features of RNNs, there might also be impulsive disturbances, which could potentially affect the performance of the systems [6]–[8]. Therefore, the dynamic behaviors of the RNNs with time-varying delays and impulsive effects have been increasingly studied.

SMC is known as an effective approach to solve the control problem of complex dynamical systems due to its excellent properties [9]–[11]. In the literature, SMC law has been widely used in practical applications, such as robotics, power systems, etc. [12]–[14]. Furthermore, the SMC design for RNNs also has drawn widespread attention [15] and [16]. However, it is worth noting that the existing works on SMC mainly focus on continuous-time systems. Actually, many practical systems are inevitably subject to impulsive disturbances. This implies that the dynamic process of the systems is not continuous but may change suddenly at certain instants, which might lead to the unavailability of the existing widely used.

This letter addresses how to design the SMC law for the systems with impulsive disturbances and time-varying delays. The main contributions of this letter are that: an appropriate integral sliding sur-

face function and an SMC law are introduced for use under impulsive disturbances and time-varying delays. Then, based on the theories of finite-time stability and impulsive control, some sufficient conditions are deduced to ensure that the considered system states can be driven to the designed sliding surface  $s(t) = 0$  within a finite time. Meanwhile, the reaching-time up to the designed sliding surface can be estimated, which is dependent on the impulses. Moreover, by employing the average dwell-time and the Lyapunov analysis method, the exponential stability of the resulting sliding mode dynamic, which is modeled by a delayed impulsive nonlinear system, is guaranteed.

**Problem formulation:** Consider the RNN with time-varying delays and impulsive effects described by the following model:

$$\begin{cases} \dot{x}(t) = -Cx(t) + \mathcal{A}f(x(t)) \\ \quad + \mathcal{B}f(x(t - \tau(t))), \quad t \notin \mathcal{K} \\ \Delta x(t) = Gx(t^-), \quad t \in \mathcal{K} \end{cases} \quad (1)$$

where  $x(t) \in \mathcal{U} \subseteq \mathbb{R}^n$  is the state vector of the RNN, the function  $f: \mathcal{U} \rightarrow \mathbb{R}^n$  is the neuron activation function,  $\tau(t)$  is the time-varying transmission delay and satisfies  $0 \leq \tau(t) \leq \tau$ ,  $\dot{\tau}(t) \leq \mu < 1$ ,  $\Delta x(t) = x(t^+) - x(t^-)$  is the impulsive disturbance, the set  $\mathcal{K} = \{t_i, i \in \mathbb{Z}^+\}$  is a strictly increasing impulses sequence on  $(t_0, \infty)$ , the matrix  $C \in \mathbb{R}^{n \times n}$  is a positive definite diagonal matrix, the matrices  $\mathcal{B}, \mathcal{A} \in \mathbb{R}^{n \times n}$  are the connection weight with and without delays, and the matrix  $G \in \mathbb{R}^{n \times n}$  means the impulsive strength. The following assumptions are needed.

**Assumption 1:**  $x_i(t)$  have left and right limits, and  $x_i(t) = x_i(t^+)$  at all times.

**Assumption 2:**  $f_i(\cdot)$  satisfies Lipschitz condition, that is, there exists  $\lambda_i \in \mathbb{R}^+$  such that  $|f_i(y_1) - f_i(y_2)| \leq \lambda_i |y_1 - y_2|$  for  $\forall y_1, y_2 \in \mathbb{R}$ .

Letting the RNN (1) as the drive system, the corresponding response system are introduced as follows:

$$\begin{cases} \dot{y}(t) = -Cy(t) + \mathcal{A}f(y(t)) \\ \quad + \mathcal{B}f(y(t - \tau(t))) + \mathcal{D}U(t), \quad t \notin \mathcal{K} \\ \Delta y(t) = Gy(t^-), \quad t \in \mathcal{K} \end{cases} \quad (2)$$

where  $y(t) \in \mathcal{U} \subseteq \mathbb{R}^n$  is the state vector of the response system,  $U(t) \in \mathbb{R}^m$  is the control input to be designed later on, and the matrix  $\mathcal{D} \in \mathbb{R}^{n \times m}$  satisfies  $\text{rank}(\mathcal{D}) = m$ .

Define the synchronization error  $e(t) = y(t) - x(t)$ . Then, the synchronization error dynamic is deduced as follows:

$$\begin{cases} \dot{e}(t) = -Ce(t) + \mathcal{A}\tilde{f}(e(t)) \\ \quad + \mathcal{B}\tilde{f}(e(t - \tau(t))) + \mathcal{D}U(t), \quad t \notin \mathcal{K} \\ e(t) = \tilde{G}e(t^-), \quad t \in \mathcal{K} \end{cases} \quad (3)$$

where  $\tilde{f}(e(\cdot)) = f(y(\cdot)) - f(x(\cdot))$ , and  $\tilde{G} = I + G$ .

**Definition 1** [6]: The response RNN (2) is said to be exponentially synchronized onto the drive RNN (1) over the impulse sequences  $\mathcal{K}$ , if there exist constants  $\vartheta, \varepsilon > 0$  such that for any initial condition, the solution  $e(t; t_0, e_0)$  of the synchronization error system (3) satisfies

$$\|e(t; t_0, e_0)\| \leq \vartheta e^{-\varepsilon(t-t_0)} \|e_0\|, \quad \forall t \geq t_0.$$

**Main results:** In this section, we will study the SMC design for the RNNs. We design the following sliding function:

$$s(t) = Pe(t) + \int_0^t P(C - \mathcal{D}H)e(\sigma) d\sigma \quad (4)$$

where  $P$  is chosen such that  $P\mathcal{D}$  is a nonsingular matrix, and  $H$  will be designed later. Then the following SMC law is considered:

$$U(t) = He(t) - \varphi(t) \text{sgn}(s(t)) \quad (5)$$

where  $\varphi(t) = \beta + \gamma \|e(t)\| + \delta \|e(t - \tau(t))\|$ , and  $\tilde{l} = \max_{1 \leq i \leq n} \{\lambda_i\}$ ,  $\gamma \triangleq \tilde{l} \|(P\mathcal{D})^{-1} P\mathcal{A}\|$ ,  $\delta \triangleq \tilde{l} \|(P\mathcal{D})^{-1} P\mathcal{B}\|$ . We consider the impulses sequence  $\mathcal{K}_1$  satisfying

$$\min \left\{ i \in \mathbb{Z}^+ : t_i \geq t_0 + \xi_1^{\frac{1}{\rho} - \alpha} \frac{V_0^{1-\alpha}}{\rho(1-\alpha)} \right\} = I_0 < +\infty \quad (6)$$

where the positive constants  $\alpha$  and  $\rho$  will be given later, and  $\xi_1$  is determined by impulsive strength.

**Theorem 1:** For the RNNs (1) and (2), if Assumptions 1 and 2 hold, the impulses sequence  $\mathcal{K}$  satisfies the condition (6) with  $\xi_1 > 1$ ,  $\alpha = \frac{1}{2}$  and  $\rho = \frac{\beta}{\lambda}$ , and the following condition holds:

Corresponding author: Kaibo Shi.

Citation: L. F. Hua, K. B. Shi, Z.-G. Wu, S. Han, and S. M. Zhong, "Sliding mode control for recurrent neural networks with time-varying delays and impulsive effects," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 5, pp. 1319–1321, May 2023.

L. F. Hua is with the School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China (e-mail: hualanf@163.com).

K. B. Shi is with the School of Electronic Information and Electrical Engineering, Chengdu University, Chengdu 610106, China (e-mail: skbs111@163.com).

Z.-G. Wu is with the Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China (e-mail: nashwzhg@zju.edu.cn).

S. Han is with the Department of Electrical Engineering and Convergence IT Engineering, Pohang University of Science and Technology, Pohang 37673, South Korea (e-mail: soohee.han@postech.ac.kr).

S. M. Zhong is with the School of Mathematics Sciences, University of Electronic Science and Technology of China, Chengdu 611731, China (e-mail: zhongsm@uestc.edu.cn).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/JAS.2023.123372

$$\tilde{G}^T(P\mathcal{D})^{-1}\tilde{G} \leq \xi_1(P\mathcal{D})^{-1} \quad (7)$$

where  $\tilde{\lambda} = (\frac{1}{2}\lambda_{\max}((P\mathcal{D})^{-1}))^{\frac{1}{2}}$ , then the synchronization error system (3) can be driven onto the designed sliding surface (4) within a finite time under the SMC law (5) and maintain there thereafter. Furthermore, the reaching-time is upper bounded by

$$T(e_0, \mathcal{K}) \leq \tilde{T}_0 \triangleq t_0 + \xi_1^{\frac{1}{2}} \frac{2\tilde{\lambda}V_0^{\frac{1}{2}}}{\beta}. \quad (8)$$

Proof: Choose the Lyapunov function

$$V(t) = \frac{1}{2}s^T(t)(P\mathcal{D})^{-1}s(t). \quad (9)$$

Based on the sliding function (4), one has

$$\dot{s}(t) = -P[\mathcal{D}He(t) + \mathcal{A}\tilde{f}(e(t)) + \mathcal{B}\tilde{f}(e(t-\tau(t))) + \mathcal{D}U(t)]. \quad (10)$$

Then, the time derivative of the Lyapunov function (9) is given as

$$\begin{aligned} \dot{V}(t) &\leq \|s^T(t)\| \times \|(P\mathcal{D})^{-1}P\mathcal{A}\| \times \|\tilde{f}(e(t))\| \\ &\quad + \|s^T(t)\| \times \|(P\mathcal{D})^{-1}P\mathcal{B}\| \times \|\tilde{f}(e(t-\tau(t)))\| \\ &\quad - s^T(t)He(t) + s^T(t)U(t), \quad t \notin \mathcal{K}. \end{aligned} \quad (11)$$

Under Assumption 2, substituting the SMC law (5) into (11) and employing the inequality  $\|s(t)\| \leq \|s(t)\|_1$ , yield

$$\dot{V}(t) \leq -\beta\|s^T(t)\| \leq -\frac{\beta}{\tilde{\lambda}}V(t)^{\frac{1}{2}}, \quad t \notin \mathcal{K}. \quad (12)$$

From the condition (7), it turns out that

$$V(t) \leq \frac{1}{2}s^T(t^-)\tilde{G}^T(P\mathcal{D})^{-1}\tilde{G}s(t^-) \leq \xi_1 V(t^-), \quad t \in \mathcal{K}. \quad (13)$$

Denote  $T_0 = \frac{2\tilde{\lambda}V_0^{\frac{1}{2}}}{\beta}$ . Multiplying both sides of (12) with  $V(t)^{-\frac{1}{2}}$  and integrating them over the interval  $[t_0, t_1]$ , it follows that:

$$V(t)^{\frac{1}{2}} \leq V_0^{\frac{1}{2}} - \frac{\beta}{2\tilde{\lambda}}(t - t_0), \quad t \in [t_0, t_1]. \quad (14)$$

If  $t_1 \geq t_0 + T_0$ , then we have  $V(t) \leq V_0, \forall t \in [t_0, t_0 + T_0]$  and  $V(t) \equiv 0, \forall t \geq t_0 + T_0$ , that is, the synchronization error system  $e(t)$  can reach the designed sliding surface  $s(t) = 0$  within the finite time  $T_0$ . If  $t_1 \leq t_0 + T_0$ , it implies that  $t_0 \geq 2$ . Since  $\xi_1 > 1$ , with a similar method as above, it can be deduced from (12)–(14) that

$$\begin{aligned} V(t)^{\frac{1}{2}} &\leq \xi_1^{\frac{1}{2}} \left[ V_0^{\frac{1}{2}} - \frac{\beta}{2\tilde{\lambda}}(t_1 - t_0) \right] - \frac{\beta}{2\tilde{\lambda}}(t - t_1) \\ &\leq \xi_1^{\frac{1}{2}} V_0^{\frac{1}{2}} - \frac{\beta}{2\tilde{\lambda}}(t - t_0), \quad t \in [t_1, t_2]. \end{aligned} \quad (15)$$

Finally, we have

$$\begin{aligned} V(t)^{\frac{1}{2}} &\leq \xi_1^{\frac{1}{2}} \left[ V(t_{l_0-2})^{\frac{1}{2}} - \frac{\beta}{2\tilde{\lambda}}(t_{l_0-1} - t_{l_0-2}) \right] - \frac{\beta}{2\tilde{\lambda}}(t - t_{l_0-1}) \\ &\leq \xi_1^{\frac{1}{2}} V_0^{\frac{1}{2}} - \frac{\beta}{2\tilde{\lambda}}(t - t_0), \quad t \in [t_{l_0-1}, t_{l_0}]. \end{aligned} \quad (16)$$

It is obvious that  $V(t) \leq \xi_1^{l_0-1} V_0, \forall t \in [t_0, \tilde{T}_0]$  and  $V(t) \equiv 0, \forall t \geq \tilde{T}_0$ , that is, the system  $e(t)$  can reach the designed sliding surface within the finite time  $\tilde{T}_0$  under the SMC law (5). Then, it is not hard to deduce from (13) that the synchronization error system  $e(t)$  will still maintain on the surface  $s(t) = 0$  under the impulsive disturbance. Therefore, it turns out that the synchronization error system (3) can reach the designed sliding surface (4) within a finite time and stay on the surface, and the reaching-time is upper bounded by (8). ■

According to Theorem 1, when the states of synchronization error system (3) slide along the designed sliding surface, from  $\dot{s}(t) = 0$ , we obtain the equivalent controller  $U_e(t)$  as follows:

$$U_e(t) = He(t) - (P\mathcal{D})^{-1}P\mathcal{A}\tilde{f}(e(t)) - (P\mathcal{D})^{-1}P\mathcal{B}\tilde{f}(e(t-\tau(t))). \quad (17)$$

By substituting the equivalent controller (17) into the synchronization error system (3), the following sliding mode dynamic can be acquired:

$$\begin{cases} \dot{e}(t) = -\tilde{C}e(t) + \tilde{\mathcal{A}}\tilde{f}(e(t)) + \tilde{\mathcal{B}}\tilde{f}(e(t-\tau(t))), & t \notin \mathcal{K} \\ e(t) = \tilde{G}e(t^-), & t \in \mathcal{K} \end{cases} \quad (18)$$

where  $\tilde{C} = C - \mathcal{D}H$ ,  $\tilde{\mathcal{A}} = [I - \mathcal{D}(P\mathcal{D})^{-1}P]\mathcal{A}$  and  $\tilde{\mathcal{B}} = [I - \mathcal{D}(P\mathcal{D})^{-1}P]\mathcal{B}$ .

Next, we present a criteria to ensure the stability of the synchronization error system (3). For this purpose, we consider the impulse

sequence  $\mathcal{K}_2$  that satisfies the following condition:

$$N(t, \tilde{T}_0) \leq N_0 + \frac{t - \tilde{T}_0}{\ell}, \quad \forall t \geq \tilde{T}_0 \geq 0 \quad (19)$$

where  $N(t, \tilde{T}_0)$  is the number of impulsive points on the interval  $[\tilde{T}_0, t]$ ,  $N_0$  is a positive constant, and  $\ell$  is average dwell-time.

Theorem 2: For the RNNs (1) and (2), if Assumptions 1 and 2 hold, the impulses sequence  $\mathcal{K}$  satisfies the condition (19) with the average dwell-time  $\ell > \frac{\ln(2\theta\xi_2)}{\varepsilon}$ , the impulsive strengths matrices  $\tilde{G}$  satisfy  $\tilde{G}^T\tilde{G} \leq \xi_2 I$  with  $\xi_2 > 1$ , and the matrices  $H$  satisfy

$$\Omega = I - \tilde{C} + \frac{1}{2}\tilde{\mathcal{A}}\tilde{\mathcal{A}}^T + \frac{\tilde{I}^2}{4(1-\mu)}\tilde{\mathcal{B}}\tilde{\mathcal{B}}^T + \frac{1}{2}L^2 < 0 \quad (20)$$

then, the response RNN (2) is exponentially synchronized onto the drive RNN (1) under the SMC law (5).

Proof: Choose the Lyapunov function

$$V(t) = \frac{1}{2}e^T(t)e(t) + \int_{t-\tau(t)}^t e^T(\sigma)e(\sigma)d\sigma. \quad (21)$$

Then, it can be easily seen that for  $t \notin \mathcal{K}$ ,

$$\begin{aligned} \dot{V}(t) &= e^T(t)\dot{e}(t) + e^T(t)e(t) - (1 - \dot{\tau}(t))e^T(t-\tau(t))e(t-\tau(t)) \\ &\leq e^T(t)(I - \tilde{C})e(t) + e^T(t)\tilde{\mathcal{A}}\tilde{f}(e(t)) + e^T(t)\tilde{\mathcal{B}} \\ &\quad \times \tilde{f}(e(t-\tau(t))) - (1 - \mu)e^T(t-\tau(t))e(t-\tau(t)). \end{aligned} \quad (22)$$

Based on inequality technique and Assumption 2, we have

$$\dot{V}(t) \leq e^T(t)\Omega e(t) \leq -\zeta e^T(t)e(t), \quad t \notin \mathcal{K}. \quad (23)$$

where  $\zeta = \lambda_{\min}(-\Omega)$ . Letting  $\varpi(t) = e^{\varepsilon(t-\tilde{T}_0)}V(t)$ , we obtains

$$\begin{aligned} \dot{\varpi}(t) &= \varepsilon e^{\varepsilon(t-\tilde{T}_0)}V(t) + e^{\varepsilon(t-\tilde{T}_0)}\dot{V}(t) \\ &= \varepsilon e^{\varepsilon(t-\tilde{T}_0)}\left(\frac{1}{2}e^T(t)e(t) + \int_{t-\tau(t)}^t e^T(\sigma)e(\sigma)d\sigma\right) \\ &\quad - \zeta e^{\varepsilon(t-\tilde{T}_0)}e^T(t)e(t), \quad t \notin \mathcal{K}. \end{aligned} \quad (24)$$

By integrating (24) over the interval  $[\tilde{T}_0, t]$ , we have

$$\begin{aligned} \varpi(t) - \varpi(\tilde{T}_0) &\leq \frac{1}{2}\varepsilon \int_{\tilde{T}_0}^t e^{\varepsilon(s-\tilde{T}_0)}e^T(s)e(s)ds \\ &\quad + \varepsilon \int_{\tilde{T}_0}^t \int_{s-\tau}^s e^{\varepsilon(s-\tilde{T}_0)}e^T(\sigma)e(\sigma)d\sigma ds \\ &\quad - \zeta \int_{\tilde{T}_0}^t e^{\varepsilon(s-\tilde{T}_0)}e^T(s)e(s)ds, \quad t \in [\tilde{T}_0, t_k] \end{aligned} \quad (25)$$

where  $t_k > \tilde{T}_0$  is the first impulse point after the system  $e(t)$  reaches the designed sliding surface. Changing the order of integration yields

$$\begin{aligned} \int_{\tilde{T}_0}^t \int_{s-\tau}^s e^{\varepsilon(s-\tilde{T}_0)}e^T(\sigma)e(\sigma)d\sigma ds &\leq \tau e^{\varepsilon\tau} \int_{\tilde{T}_0-\tau}^{\tilde{T}_0} e^T(\sigma)e(\sigma)d\sigma \\ &\quad + \tau e^{\varepsilon\tau} \int_{\tilde{T}_0}^t e^{\varepsilon(\sigma-\tilde{T}_0)}e^T(\sigma)e(\sigma)d\sigma. \end{aligned} \quad (26)$$

Substituting (26) into (25), we have that for sufficiently small  $\varepsilon$

$$\varpi(t) \leq \frac{1}{2}e^T(\tilde{T}_0)e(\tilde{T}_0) + (1 + \varepsilon\tau e^{\varepsilon\tau}) \int_{\tilde{T}_0-\tau}^{\tilde{T}_0} e^T(\sigma)e(\sigma)d\sigma, \quad t \in [\tilde{T}_0, t_k]. \quad (27)$$

As a result, there exists a positive constant  $\vartheta$  such that

$$\varpi(t) \leq \vartheta e^T(\tilde{T}_0)e(\tilde{T}_0), \quad t \in [\tilde{T}_0, t_k] \quad (28)$$

which leads to that

$$\|e(t)\| \leq \vartheta^{\frac{1}{2}} e^{-\frac{1}{2}\varepsilon(t-\tilde{T}_0)}\|e(\tilde{T}_0)\|, \quad t \in [\tilde{T}_0, t_k]. \quad (29)$$

With a similar method as above, it can be deduced from (23) that

$$\begin{aligned} V(t) &\leq \vartheta e^{-\varepsilon(t-t_{k+j-1})}e^T(t_{k+j-1})e(t_{k+j-1}) \\ &\leq \vartheta e^{-\varepsilon(t-t_{k+j-1})} \times 2\xi_2 \vartheta e^{-\varepsilon(t_{k+j-1}-t_{k+j-2})}e^T(t_{k+j-2})e(t_{k+j-2}) \\ &\leq \vartheta(2\theta\xi_2)^j e^{-\varepsilon(t-\tilde{T}_0)}e^T(\tilde{T}_0)e(\tilde{T}_0), \quad t \in [t_{k+j-1}, t_{k+j}] \end{aligned} \quad (30)$$

where  $j \in \mathbb{Z}^+$ . From (19) and (30), it holds that for  $\forall t > t_k$

$$\begin{aligned} V(t) &\leq \vartheta(2\theta\xi_2)^{N(t, \tilde{T}_0)} e^{-\varepsilon(t-\tilde{T}_0)}e^T(\tilde{T}_0)e(\tilde{T}_0) \\ &\leq \vartheta(2\theta\xi_2)^{(N_0 + \frac{t-\tilde{T}_0}{\ell})} e^{-\varepsilon(t-\tilde{T}_0)}e^T(\tilde{T}_0)e(\tilde{T}_0) \\ &= \vartheta e^{N_0 \ln(2\theta\xi_2) - (\varepsilon - \frac{\ln(2\theta\xi_2)}{\ell})(t-\tilde{T}_0)} e^T(\tilde{T}_0)e(\tilde{T}_0). \end{aligned} \quad (31)$$

It can be deduced from (31) that

$$\|e(t)\| \leq \tilde{\theta} e^{-\tilde{\varepsilon}(t-\tilde{T}_0)} \|e(\tilde{T}_0)\|, \quad \forall t > t_k \quad (32)$$

where  $\tilde{\theta} = \theta^{\frac{1}{2}} e^{\frac{1}{2} N_0 \ln(2\theta\xi_2)}$  and  $\tilde{\varepsilon} = \frac{1}{2}(\varepsilon - \frac{\ln(2\theta\xi_2)}{\ell})$ . Thus, we finally conclude that the response RNN (2) is exponentially synchronized onto the drive RNN (1). ■

**Numerical example:** In this section, a numerical example is presented. Consider the drive-response RNNs (1) and (2) with the parameters given by [2]:  $\mathcal{A} = \begin{bmatrix} -2.3 & 1 \\ 1.1 & -1.8 \end{bmatrix}$ ,  $\mathcal{B} = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$ ,  $\mathcal{C} = \mathcal{D} = I$ , the neuron activation functions  $f_1(x) = f_2(x) = \frac{1}{2}(|x+1| - |x-1|)$ , and the delay satisfies  $0 \leq \tau(t) = 1$ . The systems are subject to the impulsive disturbances as  $\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix} = \begin{bmatrix} -2.5 \\ -2.5 \end{bmatrix} \begin{bmatrix} x_1(t^-) \\ x_2(t^-) \end{bmatrix}$ ,  $t \notin \mathcal{K}$  where  $\mathcal{K} = \{0.5i, i \in \mathbb{Z}^+\}$ . The initial values of the RNNs (1) and (2) satisfy  $x_0 = [-2.5, 3.2]^T$  and  $y_0 = [1.25, -2.7]^T$  for  $t \in [-1, 0]$ , respectively. Fig. 1 shows the simulation results for the synchronization error dynamics without control input. It is also observed that the response RNN (2) without control input can not be synchronized onto the drive RNN (1).

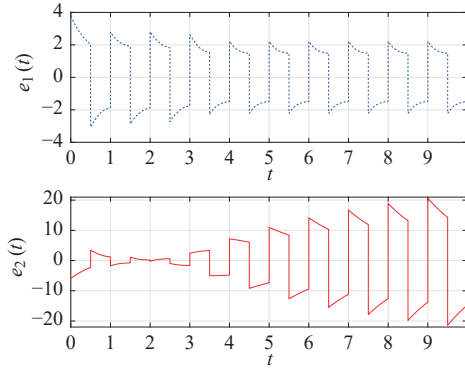


Fig. 1. Simulation results for the synchronization error dynamics of the drive-response RNNs (1) and (2) without control input.

Choose the parameter matrices  $P = \frac{1}{2}I$ ,  $H = -3I$  of the proposed sliding surface function (4), and the parameter  $\beta = 15$  of the proposed SMC law (5). From the parameters of the RNNs, it follows that  $L = I$ ,  $\gamma = -0.9718$  and  $\delta = 2$ . Then the impulses sequence  $\mathcal{K}$  satisfies the condition (6) with  $l_0 = 5$ , and when  $i \geq 5$  also satisfies the condition (19) with  $\ell = 0.5$ . In addition, the conditions (7) and (20) hold, and  $\xi_1 = \xi_2 = 2.25$ . Therefore, according to Theorems 1 and 2, the proposed SMC law can ensure that the response RNN (2) is synchronized onto the drive RNN (1) under impulsive disturbances and time-varying delays. The simulation results appear in Fig. 2.

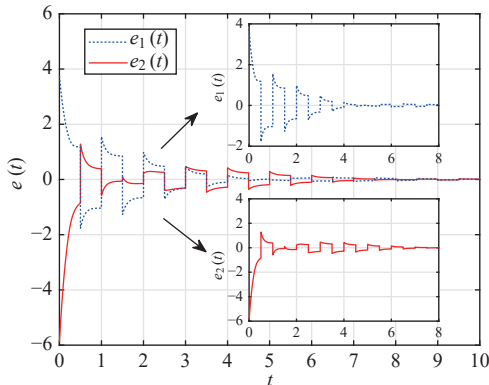


Fig. 2. Simulation results for the synchronization error dynamics of the drive-response RNNs (1) and (2) with the SMC law (5).

**Conclusion:** In this letter, we have shown how to design the SMC law to guarantee the exponential synchronization of the drive-response RNNs with impulsive disturbances and time-varying delays. Firstly, the finite-time reachability of the designed integral sliding surface has been guaranteed, where the reaching-time of the sliding surface is dependent on a class of impulse sequences, and the resulting sliding mode dynamic is modeled by a delayed impulsive nonlinear system. Then, some sufficient conditions dependent on impulse sequence have been deduced to ensure the exponential stability of the resulting sliding mode dynamic. Lastly, a numerical example has been presented to illustrate the effectiveness of the proposed results.

## References

- [1] H. Zhang, Z. Wang, and D. Liu, "A comprehensive review of stability analysis of continuous-time recurrent neural networks," *IEEE Trans. Neural Networks and Learning Systems*, vol. 25, no. 7, pp. 1229–1262, 2014.
- [2] J. Cao and J. Wang, "Global asymptotic and robust stability of recurrent neural networks with time delays," *IEEE Trans. Circuits and Systems I: Regular Papers*, vol. 52, no. 2, pp. 417–426, 2005.
- [3] B. Tao, M. Xiao, W. X. Zheng, J. Cao, and J. Tang, "Dynamics analysis and design for a bidirectional super-ring-shaped neural network with n neurons and multiple delays," *IEEE Trans. Neural Networks and Learning Systems*, vol. 32, no. 7, pp. 2978–2992, 2021.
- [4] J. Fei and L. Liu, "Real-time nonlinear model predictive control of active power filter using self-feedback recurrent fuzzy neural network estimator," *IEEE Trans. Industrial Electronics*, vol. 69, no. 8, pp. 8366–8376, 2022.
- [5] L. Hua, H. Zhu, S. Zhong, Y. Zhang, K. Shi, and O.-M. Kwon, "Fixed-time stability of nonlinear impulsive systems and its application to inertial neural networks," *IEEE Trans. Neural Networks and Learning Systems*, pp. 1–12, 2022. DOI: 10.1109/TNNLS.2022.3185664
- [6] B. Jiang, J. Lu, and Y. Liu, "Exponential stability of delayed systems with average-delay impulses," *SIAM J. Control Optimization*, vol. 58, no. 6, pp. 3763–3784, 2020.
- [7] P. Naghshtabrizi, J. P. Hespanha, and A. R. Teel, "Stability of delay impulsive systems with application to networked control systems," *Trans. Institute Measurement Control*, vol. 32, no. 5, pp. 511–528, 2010.
- [8] Y. Wang, X. Li, and S. Song, "Input-to-state stabilization of nonlinear impulsive delayed systems: An observer-based control approach," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 7, pp. 1273–1283, Jul. 2022.
- [9] J. Fei, Z. Wang, and Q. Pan, "Self-constructing fuzzy neural fractional-order sliding mode control of active power filter," *IEEE Trans. Neural Networks Learning Syst.*, pp. 1–12, 2022. DOI: 10.1109/TNNLS.2022.3169518
- [10] L. Wu, J. Liu, S. Vazquez, and S. K. Mazumder, "Sliding mode control in power converters and drives: A review," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 3, pp. 392–406, Mar. 2022.
- [11] W. H. Chen, X. Deng, and W. X. Zheng, "Sliding-mode control for linear uncertain systems with impulse effects via switching gains," *IEEE Trans. Automatic Control*, vol. 67, no. 4, pp. 2044–2051, 2022.
- [12] X. Li and Y. Zhao, "Sliding mode control for linear impulsive systems with matched disturbances," *IEEE Trans. Autom. Control*, vol. 67, no. 11, pp. 6203–6210, 2022.
- [13] J. Song, Y. Niu, and Y. Zou, "Finite-time stabilization via sliding mode control," *IEEE Trans. Automatic Control*, vol. 62, no. 3, pp. 1478–1483, 2017.
- [14] F. Zhang, "High-speed nonsingular terminal switched sliding mode control of robot manipulators," *IEEE/CAA J. Autom. Sinica*, vol. 4, no. 4, pp. 775–781, 2016.
- [15] J. J. Xiong, G. B. Zhang, J. X. Wang, and T. H. Yan, "Improved sliding mode control for finite-time synchronization of nonidentical delayed recurrent neural networks," *IEEE Trans. Neural Networks Learning Syst.*, vol. 31, no. 6, pp. 2209–2216, 2020.
- [16] H. Yan, H. Zhang, X. Zhan, Y. Wang, S. Chen, and F. Yang, "Event-triggered sliding mode control of switched neural networks with mode-dependent average dwell time," *IEEE Trans. Syst., Man, and Cybernetics: Syst.*, vol. 51, no. 2, pp. 1233–1243, 2021.