Letter

Fixed-Time Cluster Optimization for Multi-Agent Systems Based on Piecewise Power-Law Design

Suna Duan, Xinchun Jia, and Xiaobo Chi

Dear Editor,

This letter focuses on the fixed-time (FXT) cluster optimization problem of first-order multi-agent systems (FOMASs) in an undirected network, in which the optimization objective is the sum of the objective functions of all clusters. A novel piecewise power-law control protocol with cooperative-competition relations is proposed. Furthermore, a sufficient condition is obtained to ensure that the FOMASs achieve the cluster consensus within an FXT. Then, on the basis of maintaining the cluster consensus, the agents in each cluster converge to the optimal solution of their objective functions in another FXT.

Cooperative control of multi-agent systems (MASs) has become a hot issue in recent years due to its wide applications in many fields, such as the ground vehicle systems [1] and the circuit systems [2]. The consensus is a fundamental problem of the cooperative control. So far, the asymptotic consensus of FOMASs was widely studied [3]. In practical applications, a faster settling time can improve the robustness of MASs. In this view, the finite-time consensus of FOMASs was considered in [4]. However, the settling time of the finite-time consensus depends on the initial values of the considered systems and it is uncertain if the initial state cannot be obtained in advance. Subsequently, the FXT consensus of FOMASs was studied in [5] and [6], the settling time does not depend on the initial values of the considered systems. It is noted that these previous works [3]–[6] mainly consider the complete consensus (namely, all the agents in an MAS attain a same object) of FOMASs. In practice, the agents may be divided into several clusters with cooperative or competitive relations to perform different tasks, such as the foraging cluster of mixed species and the multi-target round-up of unmanned aerial vehicles. In recent years, the FXT cluster consensus of FOMASs with cooperative-competition relations was investigated in [7].

Besides, the optimization is an inevitable issue when considering the control cost of MASs. In [8] and [9], the distributed asymptotic optimization of FOMASs was considered. In [10], the FXT optimization problem of FOMASs was studied. It can be observed that the optimization without clusters was considered in [8]–[10]. The asymptotic cluster optimization problem of FOMASs with cooperative-competition relations was analyzed in [11]. Up to now, there are few results on the FXT cluster optimization of FOMASs.

In summary, this letter is devoted to designing a control protocol with cooperative-competition relations to solve the FXT cluster optimization of FOMASs. The contributions of this letter are listed as follows. 1) The FXT cluster optimization problem of FOMASs is proposed for the first time. 2) In comparison with [10], the proposed control protocol in this letter only requires the gradient information of each agent's cost function, and is independent of the information of its Hessian matrix. 3) Unlike the asymptotic cluster optimization in [11], the piecewise power-law control protocol developed in this letter enables the FOMASs to achieve the FXT cluster optimization.

Notations: Let $\mathrm{sig}^{\mu}(z)=(\mathrm{sign}(z_1)|z_1|^{\mu},\ldots,\mathrm{sign}(z_n)|z_n|^{\mu})^T$. $\nabla F(z)$ and $\nabla^2 F(z)$ denote the gradient and the Hessian matrix of $F(z)$, respectively.

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Problem statement: Consider the following FOMAS:

$$\dot{x}_i(t)=u_i(t),\ \ i\in\mathcal{V} \tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ and $u_i(t)\in\mathbb{R}^n$ are the state and control input of agent i, respectively. $\mathcal{V}=\{1,2,\ldots,N\}$ is the agent set. In order to explore the FXT cluster optimization of an MAS, it is assumed that the agent set $\mathcal{V}$ can be divided into m nonempty and disjoint clusters. i.e., $\mathcal{V}=\cup_{k=1}^{m}\mathcal{V}_k$, where $\mathcal{V}_k=\{r_{k-1}+1,r_{k-1}+2,\ldots,r_k\}$, $r_0=0$ and $r_m=N$. Based on this agent division, the Laplacian matrix $\mathcal{L}$ of the cluster-based communication topology can be expressed as

$$\mathcal{L}=\begin{pmatrix}\mathcal{L}_{11} & \mathcal{L}_{12} & \cdots & \mathcal{L}_{1m}\\ \mathcal{L}_{21} & \mathcal{L}_{22} & \cdots & \mathcal{L}_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ \mathcal{L}_{m1} & \mathcal{L}_{m2} & \cdots & \mathcal{L}_{mm}\end{pmatrix}$$

where $\mathcal{L}_{pk}\in\mathcal{R}^{(r_p-r_{p-1})\times(r_k-r_{k-1})}$, and $p,k\in\{1,2,\ldots,m\}$.

The goal of this letter is to design a FXT cluster optimization control protocol for the FOMAS (1), so that the following optimization problem is solved within an FXT.

$$\begin{cases}\min\ \displaystyle\sum_{k=1}^{m}F_k(\hat{x}_k(t))=\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}f_i(x_i(t))\\ \text{s.t. } (\mathcal{L}_{kk}\otimes I_n)\hat{x}_k(t)=0,\ k\in\{1,2,\ldots,m\}\end{cases} \tag{2}$$

where $\hat{x}_k(t)=(x^T_{r_{k-1}+1}(t),x^T_{r_{k-1}+2}(t),\ldots,x^T_{r_k}(t))^T$, $f_i(x_i(t)):\mathbb{R}^n\to\mathbb{R}$ represents the local objective function of agent i, and $F_k(\hat{x}_k(t))$ is the objective function of the kth cluster.

Definition 1 [11]: If there is a settling time $T>0$ such that

$$\begin{cases}\displaystyle\lim_{t\to T}\|x_i(t)-x_j(t)\|=0,\ \ x_i(t)=x_j(t),\ t\ge T,\ i,j\in\mathcal{V}_k\\ \displaystyle\sup\lim_{t\to T}\|x_i(t)-x_j(t)\|>0,\ \ x_i(t)\ne x_j(t),\ t\ge T,\ i\in\mathcal{V}_k\\ \qquad j\in\mathcal{V}\setminus\mathcal{V}_k,k\in\{1,2,\ldots,m\}\end{cases}$$

then, the FOMAS (1) is said to reach the FXT cluster consensus.

Next, some Assumptions are given for the proof of our main result.

Assumption 1: The local objective function $f_i(x_i(t))$ $(i\in\mathcal{V})$ is twice continuously differentiable with respect to $x_i(t)$ and satisfies $\nabla^2 f_i(x_i(t))\ge\epsilon_i I_n$ with $\epsilon_i>0$.

Remark 1: In [11], $\nabla f_i(x_i(t))=\epsilon_i x_i(t)+\psi_i(x_i(t))$ needs to be satisfied, where $\psi_i(x_i(t))$ is bounded. However, only $\nabla^2 f_i(x_i(t))\ge\epsilon_i I_n$ is required here. Therefore, this assumption is more conducive to practical application.

Assumption 2: The cluster-based communication topology is connected and the balance of information communication among clusters is satisfied. (i.e., $\mathcal{L}_{pk}1_{r_k-r_{k-1}}=0_{r_p-r_{p-1}}$, $p\ne k$.)

Remark 2: In this letter, $a_{ij}>0$ indicates that agents i and j are cooperative relationship, while $a_{ij}<0$ represents that they are competitive relationship. In addition, suppose $a_{ii}=0$ $(i\in\mathcal{V})$.

Main result: In order to analyze the optimization problem (2) with cooperative-competition relations, the piecewise power-law control protocol is designed as follows:

$$u_i(t)=\begin{cases}\displaystyle\sum_{j\in\mathcal{V}_k}a_{ij}(\alpha_1\mathrm{sig}^{\mu}(\chi_{ji}(t))+\beta_1\mathrm{sig}^{\nu}(\chi_{ji}(t)))\\ \displaystyle\quad+\sum_{k'\ne k}(\alpha_2\mathrm{sig}^{\mu}(y_i(t))+\beta_2\mathrm{sig}^{\nu}(y_i(t))),\ \ t\in[0,T_1]\\ \displaystyle-\varsigma_1\mathrm{sig}^{\mu}(\sum_{j\in\mathcal{V}_k}\nabla f_j(x_j(t)))\\ \displaystyle-\tau_1\mathrm{sig}^{\nu}(\sum_{j\in\mathcal{V}_k}\nabla f_j(x_j(t))),\ \ t\in(T_1,T_2]\end{cases} \tag{3}$$

where α_1, α_2, β_1, β_2, ς_1 and τ_1 are positive constants. $i\in\mathcal{V}_k$, $\mu>1$, $0<\nu<1$, $\chi_{ji}(t)=x_j(t)-x_i(t)$, $y_i(t)=\sum_{j\in\mathcal{V}_{k'}}a_{ij}(\chi_{ji}(t))$, and $k,k'\in\{1,2,\ldots,m\}$. T_1 and T_2 are given later.

Remark 3: It is not difficult to find that if the FOMAS (1) achieves the FXT cluster consensus within $[0,T_1]$, the agents in each cluster converge to the optimal solution of their objective functions in $(T_1,T_2]$ while maintaining the cluster consensus.

The following notations are introduced for convenience:

$$\overline{N}=\max_{k=1,2,\ldots,m}\{r_k-r_{k-1}\},\ \underline{N}=\min_{k=1,2,\ldots,m}\{r_k-r_{k-1}\}$$

$$\hat{a}=\max_{i\in\mathcal{V}_k, j\in\mathcal{V}_{k'}, k'\neq k}|a_{ij}|,\ \xi_2=\beta_1\underline{\lambda}_2 2^{\nu}-\xi_4$$

$$\mathcal{L}_{kk}^{\frac{2}{\nu+1}}=\begin{cases}-a_{ij}^{\frac{2}{\nu+1}},\ i\neq j,\\ \sum\limits_{k=r_{k-1+1},\ j\neq i}^{r_k} a_{ij}^{\frac{2}{\nu+1}},\end{cases}\ \mathcal{L}_{kk}^{\frac{2}{\mu+1}}\text{ is similar to }\mathcal{L}_{kk}^{\frac{2}{\nu+1}}$$

$$\underline{\lambda}_2=\min_{k=1,2,\ldots,m}\left\{\lambda_2^{\frac{\nu+1}{2}}\left(\mathcal{L}_{kk}^{\frac{2}{\nu+1}}\right)\right\},\ \xi_3=\alpha_2N_1\hat{a}^{\mu}\overline{N}^{\mu-1}2^{\frac{1+\mu}{2}}$$

$$\check{\lambda}_2=\min_{k=1,2,\ldots,m}\left\{\lambda_2^{\frac{1+\mu}{2}}\left(\mathcal{L}_{kk}^{\frac{2}{1+\mu}}\right)\right\},\ \xi_4=\beta_2N_1\hat{a}^{\nu}(Nn)^{\frac{1-\nu}{2}}2^{\frac{1+\nu}{2}}$$

$$\xi_1=\alpha_1\check{\lambda}_2 2^{\mu}(n\overline{N}(\overline{N}-1)m)^{\frac{1-\mu}{2}}-\xi_3,\ N_1=N-\underline{N}.$$

Theorem 1: Suppose that Assumptions 1 and 2 hold. Under the control protocol (3), the FOMAS (1) subject to the optimization problem (2) achieves the cluster consensus in an FXT T_1 and converges to the optimal solution of the problem (2) in an FXT T_2, if the power-law parameters α_1 and β_1 satisfy the following conditions:

$$\alpha_1>\frac{\alpha_2N_1\hat{a}^{\mu}\overline{N}^{\frac{3\mu-3}{2}}(nm(\overline{N}-1))^{\frac{\mu-1}{2}}}{\check{\lambda}_2 2^{\frac{\mu-1}{2}}},\ \beta_1>\frac{\beta_2\hat{a}^{\nu}N_1(2nN)^{\frac{1-\nu}{2}}}{\underline{\lambda}_2}.$$

Proof: The proof can be divided into three parts.

Part I: Construct the following Lyapunov function:

$$V_1(t)=\frac{1}{2}\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}e_i^T(t)e_i(t)$$

where $e_i(t)=x_i(t)-\overline{x}_k(t)\ (i\in\mathcal{V}_k)$ and $\overline{x}_k(t)=\frac{\sum_{j\in\mathcal{V}_k}x_j(t)}{r_k-r_{k-1}}$.

The derivative of $V_1(t)$ is

$$\begin{aligned}\dot{V}_1(t)=&\sum_{k=1}^{m}\sum_{i,j\in\mathcal{V}_k}a_{ij}e_i^T(t)(\alpha_1\mathrm{sig}^{\mu}(\chi_{ji}(t))+\beta_1\mathrm{sig}^{\nu}(\chi_{ji}(t)))\\&+\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\sum_{k'\neq k}e_i^T(t)(\alpha_2\mathrm{sig}^{\mu}(y_i(t))+\beta_2\mathrm{sig}^{\nu}(y_i(t))).\end{aligned}\tag{4}$$

Next, each item in (4) will be handled.

By using the symmetry of an undirected network, Lemma 5 in [10] and Lemma 2.2 in [11], one has

$$\begin{aligned}&\alpha_1\sum_{k=1}^{m}\sum_{i,j\in\mathcal{V}_k}a_{ij}e_i^T(t)\mathrm{sig}^{\mu}(\chi_{ji}(t))\\&\leq-\frac{\alpha_1}{2}(n\overline{N}(\overline{N}-1))^{\frac{1-\mu}{2}}\sum_{k=1}^{m}\left[2\lambda_2\left(\mathcal{L}_{kk}^{\frac{2}{1+\mu}}\right)\hat{e}_k^T(t)\hat{e}_k(t)\right]^{\frac{1+\mu}{2}}\\&\leq-\alpha_1\check{\lambda}_2 2^{\mu}(n\overline{N}(\overline{N}-1)m)^{\frac{1-\mu}{2}}V_1^{\frac{1+\mu}{2}}(t)\end{aligned}\tag{5}$$

where $\hat{e}_k(t)=(e_{r_{k-1}+1}^T(t),e_{r_{k-1}+2}^T(t),\ldots,e_{r_k}^T(t))^T$.

Similarly, we get

$$\beta_1\sum_{k=1}^{m}\sum_{i,j\in\mathcal{V}_k}a_{ij}e_i^T(t)\mathrm{sig}^{\nu}(\chi_{ji}(t))\leq-\beta_1\underline{\lambda}_2 2^{\nu}V_1^{\frac{\nu+1}{2}}(t).\tag{6}$$

According to Lemmas 2.2−2.3 in [11], it yields

$$\begin{aligned}&\alpha_2\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\sum_{k'\neq k}e_i^T(t)\mathrm{sig}^{\mu}(y_i(t))\\&\leq\alpha_2\hat{a}^{\mu}\overline{N}^{\mu-1}\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\sum_{l=1}^{n}|e_i^l(t)|^{1+\mu}(N-(r_k-r_{k-1}))\\&\leq\alpha_2N_1\hat{a}^{\mu}\overline{N}^{\mu-1}2^{\frac{1+\mu}{2}}V_1^{\frac{1+\mu}{2}}(t).\end{aligned}\tag{7}$$

Meanwhile, by applying Lemmas 2.2−2.3 in [11], one gets

$$\beta_2\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\sum_{k'\neq k}e_i^T(t)\mathrm{sig}^{\nu}(y_i(t))\leq\xi_4V_1^{\frac{1+\nu}{2}}(t).\tag{8}$$

Submitting the above (5)−(8) into (4), we have

$$\dot{V}_1(t)\leq-\xi_1V_1^{\frac{1+\mu}{2}}(t)-\xi_2V_1^{\frac{1+\nu}{2}}(t).$$

Based on Lemma 2.1 in [11], one can conclude that $V_1(t)$ converges to zero in an FXT T_1 and T_1 is estimated by

$$T_1\leq\frac{2}{\xi_2}\left(\frac{\xi_2}{\xi_1}\right)^{\frac{1-\nu}{\mu-1}}\left(\frac{1}{1-\nu}+\frac{1}{\mu-1}\right).\tag{9}$$

Furthermore, when $t\geq T_1$, $x_i(t)=\overline{x}_k(t)$ for $i\in\mathcal{V}_k$.

Part II: According to the above analysis, for $t>T_1$, $u_i(t)$ can be written as

$$u_i(t)=-\varsigma_1\mathrm{sig}^{\mu}\Big(\sum_{j\in\mathcal{V}_k}\nabla f_j(\overline{x}_k(t))\Big)-\tau_1\mathrm{sig}^{\nu}\Big(\sum_{j\in\mathcal{V}_k}\nabla f_j(\overline{x}_k(t))\Big).\tag{10}$$

According to (10), the agents of each intra-cluster have the same dynamics. Therefore, for $t>T_1$, the optimization problem (2) can be written as

$$\min\ \sum_{k=1}^{m}\widetilde{F}_k(\overline{x}_k(t))=\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}f_i(\overline{x}_k(t)).\tag{11}$$

Constructing the following Lyapunov function:

$$V_{2k}(t)=\frac{1}{2}(\widetilde{F}_k(\overline{x}_k(t))-\widetilde{F}_k(x_k^*))^2$$

where x_k^* is the optimal solution of the problem (11).

Based on (11), the derivative of $V_{2k}(t)$ is as follows:

$$\begin{aligned}\dot{V}_{2k}(t)=&(\widetilde{F}_k(\overline{x}_k(t))-\widetilde{F}_k(x_k^*))\nabla^T\widetilde{F}_k(\overline{x}_k(t))\dot{\overline{x}}_k(t)\\\leq&-(\widetilde{F}_k(\overline{x}_k(t))-\widetilde{F}_k(x_k^*))[\varsigma_1n^{\frac{1-\mu}{2}}\|\nabla\widetilde{F}_k(\overline{x}_k(t))\|_2^{\mu+1}\\&+\tau_1\|\nabla\widetilde{F}_k(\overline{x}_k(t))\|_2^{\nu+1}].\end{aligned}\tag{12}$$

Because x_k^* is the optimal solution of the problem (11), there is $\widetilde{F}_k(\overline{x}_k(t))-\widetilde{F}_k(x_k^*)>0$ and $\nabla\widetilde{F}_k(x_k^*)=0$. Therefore,

$$\begin{aligned}\widetilde{F}_k(\overline{x}_k(t))-\widetilde{F}_k(x_k^*)&=\frac{1}{2}(\overline{x}_k(t)-x_k^*)^T\nabla^2\widetilde{F}_k(\eta_1)(\overline{x}_k(t)-x_k^*)\\&\geq\frac{\underline{\epsilon}_k}{2}(\overline{x}_k(t)-x_k^*)^T(\overline{x}_k(t)-x_k^*)\end{aligned}\tag{13}$$

where $\underline{\epsilon}_k=\min_{i\in\mathcal{V}_k}\{\epsilon_i\}$, $\eta_1=x_k^*+\vartheta_1(\overline{x}_k(t)-x_k^*)$ with $0<\vartheta_1<1$. Furthermore, $\widetilde{F}_k(x_k^*)-\widetilde{F}_k(\overline{x}_k(t))\geq\nabla^T\widetilde{F}_k(\overline{x}_k(t))(x_k^*-\overline{x}_k(t))$, where $\eta_2=\overline{x}_k(t)+\vartheta_2(x_k^*-\overline{x}_k(t))$ with $0<\vartheta_2<1$. Therefore,

$$\begin{aligned}&\widetilde{F}_k(\overline{x}_k(t))-\widetilde{F}_k(x_k^*)\\&\leq\frac{1}{\underline{\epsilon}_k}\|\nabla\widetilde{F}_k(\overline{x}_k(t))\|_2^2+\frac{1}{2}(\widetilde{F}_k(\overline{x}_k(t))-\widetilde{F}_k(x_k^*)).\end{aligned}\tag{14}$$

Combined with (13) and (14), one has

$$\|\nabla\widetilde{F}_k(\overline{x}_k(t))\|_2^2\geq\frac{\underline{\epsilon}_k}{2}(\widetilde{F}_k(\overline{x}_k(t))-\widetilde{F}_k(x_k^*)).\tag{15}$$

Introducing (15) into (12), it yields

$$\dot{V}_{2k}(t)\leq-\varsigma_1n^{\frac{1-\mu}{2}}\underline{\epsilon}_k^{\frac{1+\mu}{2}}2^{\frac{1-\mu}{4}}V_{2k}^{\frac{3+\mu}{4}}(t)-\tau_1\underline{\epsilon}_k^{\frac{1+\nu}{2}}2^{\frac{1-\nu}{4}}V_{2k}^{\frac{3+\nu}{4}}(t).$$

From Lemma 2.1 in [11], one can obtain that $\widetilde{F}_k(\overline{x}_k(t))$ converges to $\widetilde{F}_k(x_k^*)$ in an FXT T_{2k}, and an upper bound of T_{2k} is

$$T_{2k}\leq\frac{4}{\tau_1\underline{\epsilon}_k^{\frac{1+\nu}{2}}2^{\frac{1-\nu}{4}}}\left(\frac{\tau_12^{\frac{\mu-\nu}{4}}}{\varsigma_1n^{\frac{1-\mu}{2}}\underline{\epsilon}_k^{\frac{\mu-\nu}{2}}}\right)^{\frac{1-\nu}{\mu-\nu}}\left(\frac{1}{1-\nu}+\frac{1}{\mu-1}\right).\tag{16}$$

Part III: Let $\hat{T}_2=\max_{k=1,2,\ldots,m}\{T_{2k}\}$, $T_2=T_1+\hat{T}_2$. We get that

$$\lim_{t\to T_2}\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\nabla f_i(x_i(t))=\sum_{k=1}^{m}\nabla\widetilde{F}_k(x_k^*(t))=0_n.$$

Invoking Lemma 1 in [8], there exists an FXT T_2 such that the differentiable convex function $\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}f_i(x_i(t))$ can attain the minimum value as $t\to T_2$. According to Parts I−III and Definition 1, the agents in the FOMAS (1) converge to the optimal value of the problem (2) within an FXT T_2. ■

Remark 4: In the protocol (3), the consensus control protocol and the optimization control protocol are designed respectively according to a piecewise idea. This ensures that the FOMAS (1) in a cooperative-competitive network not only obtains the FXT consensus but also realizes the FXT optimization.

Numerical example: Consider the FOMAS (1) with 5 agents, its agents can be divided into two clusters, and the communication topology among the agents is shown in Fig. 1. The local cost func-

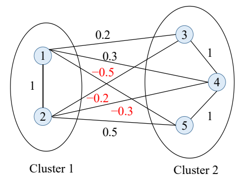

Fig. 1. Network topology $\mathcal{G}$.

tion of each agent is $f_i(x_i(t)) = 2/3(x_{i1}(t)-i)^2 + 1/3(x_{i2}(t)+i/2)^2$, where $x_i = (x_{i1}, x_{i2})$, $i = 1, 2, \ldots, 5$. The optimization problem is expressed as

$$\begin{cases} \min \displaystyle\sum_{k=1}^{2} F_k(\hat{x}_k(t)) = \sum_{k=1}^{2}\sum_{i\in\mathcal{V}_k} f_i(x_i(t)) \\ \text{s.t. } (\mathcal{L}_{kk}\otimes I_n)\hat{x}_k(t) = 0,\ k = 1, 2. \end{cases} \quad (17)$$

Case 1: Choose the initial values of the above system as $x_1(0) = (1, -1/2)$, $x_2(0) = (-1, 1)$, $x_3(0) = (3, 1)$, $x_4(0) = (-1, 3)$ and $x_5(0) = (0, 2)$. The parameters in the control protocol (3) are selected as $\mu = 1.2$, $\nu = 0.5$, $\alpha_2 = \beta_2 = 0.5$, $\varsigma_1 = 6$ and $\tau_1 = 5$. By calculation, one has $\tilde{\lambda}_2 = \lambda_2 = 1$, $\alpha_1 = 2.5428$, $\beta_1 = 3.243$, $\xi_1 = 2.5079$, $\xi_2 = 1.4142$, $T_1 = 2.3639$ and $T_2 = 11.5786$.

Case 2: Choose the initial values of the above system as $x_1(0) = (10, -5)$, $x_2(0) = (-10, 10)$, $x_3(0) = (20, 10)$, $x_4(0) = (-10, 30)$ and $x_5(0) = (0, 20)$. The parameters taken in the control protocol (3) are the same as those in Case1. Two settling times are computed as $T_1 = 2.3639$ and $T_2 = 11.5786$. The control protocol (3) is degraded to the finite-time control by removing the terms with the power of μ. For finite-time control, its settling time $\bar{T}_1$ is estimated as 6.5953. Obviously, $\bar{T}_1 > T_1$, which shows that our result is more effective.

Figs. 2 and 3 show the trajectories of the system states and the objective functions under the control protocol (3) for the above different initial value conditions. The optimization problem (17) is solved, and Fig. 4 shows the corresponding trajectories under the finite-time control. Clearly, the convergence speeds of the trajectories in Fig. 3 are faster than those in Fig. 4. In addition, combining with Figs. 2 and 3, the settling time of the FOMAS (2) does not change when the initial states change.

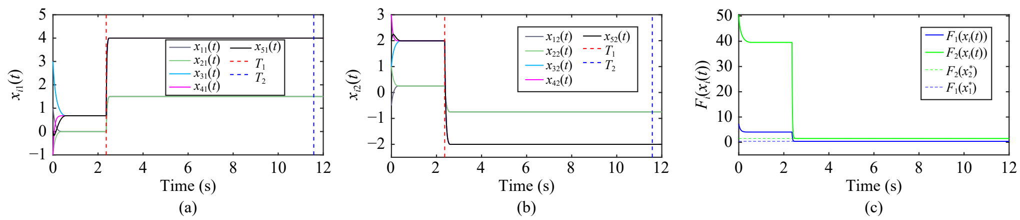

Fig. 2. Case 1 trajectory under the control protocol (4). (a) and (b) Trajectories of the agents; (c) Trajectories of the objective function.

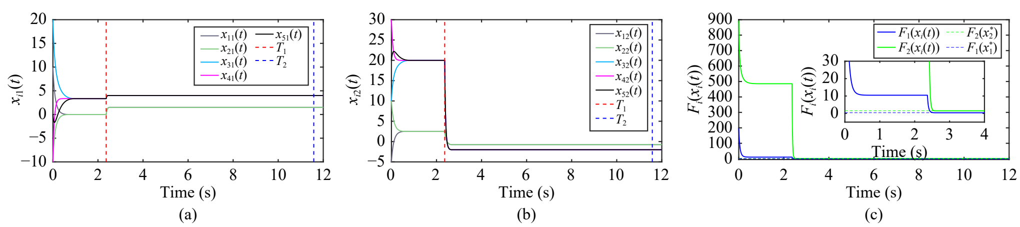

Fig. 3. Case 2 trajectory under the control protocol (4). (a) and (b) Trajectories of the agents; (c) Trajectories of the objective function.

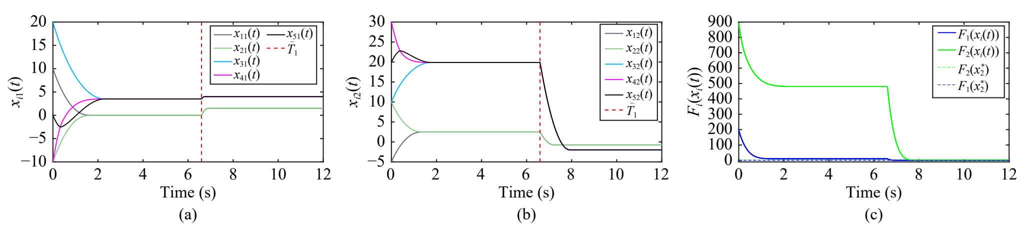

Fig. 4. Case 2 trajectory under the finite-time control protocol. (a) and (b) Trajectories of the agents; (c) Trajectories of the objective function.

Conclusion: In this letter, a piecewise power-law control protocol has been proposed for FOMASs. It has been proved by Lyapunov stability theory that the FXT cluster optimization problem of the FOMASs can be solved. Our future work will consider the FXT cluster optimization problem of higher-order MASs.

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