

Design of Robust Higher Order Sliding Mode Control for Microgrids

Michele Cucuzzella, Gian Paolo Incremona, and Antonella Ferrara

Abstract—This paper deals with the design of advanced control strategies of sliding mode type for microgrids. Each distributed generation unit (DGU), constituting the considered microgrid, can work in both grid-connected operation mode (GCOM) and islanded operation mode (IOM). The DGU is affected by load variations, nonlinearities and unavoidable modelling uncertainties. This makes sliding mode control particularly suitable as a solution methodology for the considered problem. In particular, a second order sliding mode (SOSM) control algorithm, belonging to the class of Suboptimal SOSM control, is proposed for both GCOM and IOM, while a third-order sliding mode (3-SM) algorithm is designed only for IOM, in order to achieve, also in this case, satisfactory chattering alleviation. The microgrid system controlled via the proposed sliding mode control laws exhibits appreciable stability properties, which are formally analyzed in the paper. Simulation results also confirm that the obtained closed-loop performances comply with the IEEE recommendations for power systems.

Index Terms—Sliding modes, power systems, uncertain systems.

I. INTRODUCTION

In recent years, the increasing of renewable energy sources has given rise to a new paradigm in power generation. There is a clear trend towards the realization of smaller DGUs [1], which enables to achieve economical and environmental benefits, in terms of energy efficiency and reduced carbon emissions [2]. DGUs also improve the service continuity [3], by supplying a portion of the load, even after being disconnected from the main grid [4].

In the literature, a set of interconnected DGUs, which are usually strictly close to the energy consumers, is identified as a “microgrid” [5]–[7]. The latter, characterized by some intelligent computation and metering capability, can be considered as the basic unit of the so-called “smart grid” [8]. Because of the intermittence and the uncertainty caused by meteorological factors, it is difficult to integrate renewable energy sources directly into the main grid. This is the reason why voltage control, power control, fault detection, reliability enforcement, and power losses minimization are among the issues to solve in order to integrate DGUs into the distribution network [9], [10]. In recent years, several control strategies have been proposed to deal with DGUs. Many of them are based on PI controllers

and consider the microgrid in IOM (see [11]–[15]). Others adopt more advanced control methodologies such as droop mode control [16], [17], predictive control [18], [19], adaptive control [20], [21], H^∞ control [22], and Plug-and-Play (PnP) decentralized algorithms [23].

One of the crucial problems in microgrids is the presence of the voltage-sourced-converter (VSC) as interface element with the main grid. The VSC can be viewed as a source of modelling uncertainty and disturbances. This fact makes the adoption of a robust control design methodology mandatory. Sliding mode (SM) control [24], [25] is a well-known control approach particularly appreciated for its robustness properties. Specifically, it is able to reject the so-called matched uncertainties, i.e., unknown terms which act on the same channel of the control variable, and not to amplify unmatched disturbances [26]. SM control is easy to implement, yet, it requires the use of discontinuous control laws, which can enforce the chattering effect, i.e., high frequency oscillations of the controlled variable due to the discontinuities of the control law, [27]–[29]. In the literature, several methods to alleviate chattering, such as boundary layer control or filtered control, have been proposed. Yet, in these cases the robustness properties typical of SM control could be lost. An effective way to perform chattering alleviation is instead to increase the order of the sliding mode. For this reason Higher Order Sliding Mode (HOSM) control laws [30], in particular of the second order, have been studied.

In this paper, a master-slave scheme with advanced control strategies, which belong to the class of Suboptimal SOSM algorithms [31]–[34] and of min–max Time-optimal third-order SM (3-SM) algorithms [30], is proposed. First, the use of SOSM control is investigated, observing how this approach can provide satisfactory chattering alleviation only in case of GCOM, since in that case the controlled system relative degree is unitary, while in IOM it is equal to 2. Then, to attain a chattering attenuation effect also in IOM, a 3-SM control law is designed for that case. So, on the whole, it is possible to devise a control policy which switches from a SOSM control law to a 3-SM control law, i.e., changes the order of the sliding modes which are generated, whenever a transition from GCOM to IOM occurs.

Note that, preliminary and partial versions of this work, not reporting the proofs of stability and robustness, have been published in [35], [36].

II. PRELIMINARY ISSUES

Consider the schematic electric single-line diagram of two interconnected DGUs in Fig. 1. The basic element of a DGU is usually an energy source of renewable type, which can

This is the final version of the accepted paper submitted to IEEE Journal on Emerging and Selected Topics in Circuits and Systems. Copyright (c) 2015 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending an email to pubs-permissions@ieee.org. M. Cucuzzella, G. P. Incremona and A. Ferrara are with the Dipartimento di Ingegneria Industriale e dell’Informazione, University of Pavia, via Ferrara 1, 27100 Pavia, Italy (e-mail: michele.cucuzzella@gmail.com, gp.incremona@gmail.com, a.ferrara@unipv.it).

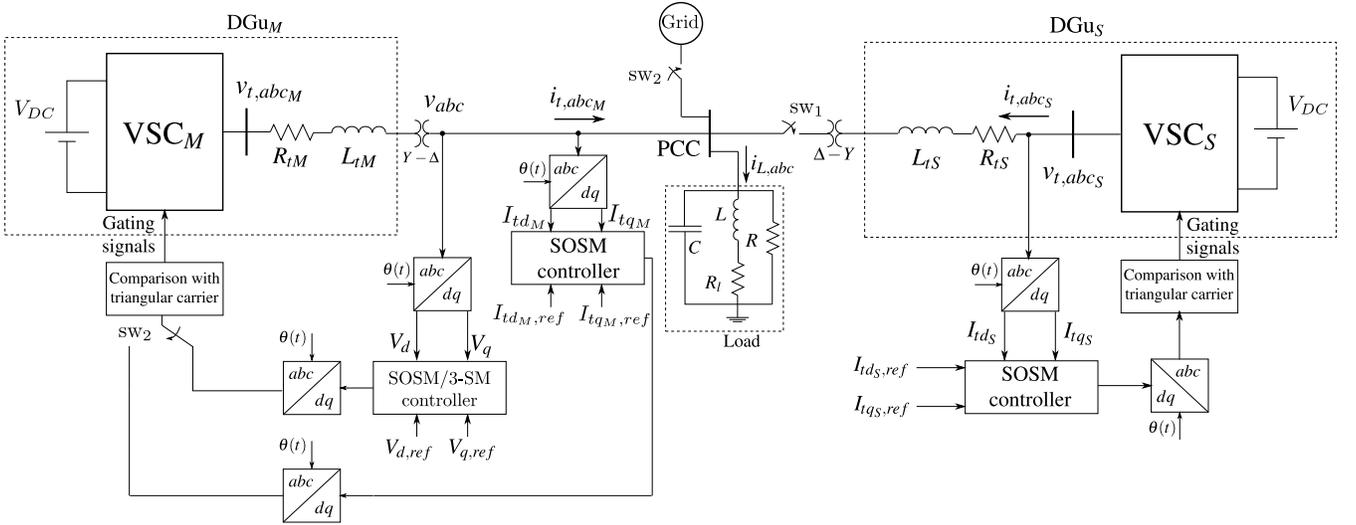


Figure 1. Representation of the master-slave system with HOSM controllers.

be represented by a direct current (DC) voltage source. This is interfaced with the main grid through two components: a voltage-sourced-converter (VSC) and a filter. In our case the first component is a pulse width modulation (PWM) inverter, while the second component is a resistive-inductive filter. The electric connection point of the DGU to the main grid is the so-called point of common coupling (PCC) where a local three-phase parallel resistive-inductive-capacitive load is connected.

In GCOM, the PCC voltage magnitude and frequency are dictated by the main grid. Thus, the system is forced to operate in stiff synchronization with the grid by using the so-called phase-locked-loop (PLL), which provides the reference angle θ for the Park's transformation [37]. Let V_d , V_q , I_d and I_q denote the direct and quadrature components of the load voltage v_{abc} and of the delivered current $i_{t,abc}$, respectively. In order to achieve the lock with the main grid, a proportional-integral (PI) controller is used to keep the PCC quadrature voltage component V_q as close as possible to zero. In such a case, the active and reactive power are equal to $P = 3/2 V_d I_d$ and $Q = -3/2 V_d I_q$. Hence, the DGU works in the so-called dq current control mode in order to supply the desired active and reactive power.

When an islanding event occurs, i.e., when the circuit breaker SW_2 in Fig. 1 opens, the PCC voltage and frequency could deviate significantly from the nominal values, due to the power mismatch between the DGU and the load. Therefore, in IOM the DGU has to provide the voltage control in order to keep the load voltage magnitude and frequency constant with respect to the reference values. In IOM, the Park's transformation angle θ is provided by an internal oscillator set to the nominal angular frequency, namely $\omega_0 = 2\pi f_0$.

The transition from GCOM to IOM has to be smooth to avoid system performance degradation. Thus, when the voltage control is activated, the phase angle, provided by the PLL, must be used as the initial condition for the internal oscillator. To avoid hard transients, also before the reconnection to the main grid, the PCC voltage must be resynchronized with the

grid voltage, for instance as proposed in [38], [39].

In Fig. 1, the proposed control scheme with two DGUs is illustrated. It has a master-slave structure, which can be extended to the case with several DGUs. In GCOM, all the DGUs regulate their own active and reactive power. When an islanding event occurs, the Master DGu_M switches to the voltage control mode and becomes responsible to keep the voltage amplitude and frequency constant with respect to their references for the Slave DGUs of the microgrid.

III. PROBLEM FORMULATION

Consider the scheme of a single DGU and assume the system to be symmetric and balanced. According to the stationary abc -frame, the governing equations for the DGU in IOM, are

$$\begin{cases} i_{t,abc} = \frac{1}{R} v_{abc} + i_{L,abc} + C \frac{dv_{abc}}{dt} \\ v_{t,abc} = L_t \frac{di_{t,abc}}{dt} + R_t i_{t,abc} + v_{abc} \\ v_{abc} = L \frac{di_{L,abc}}{dt} + R_l i_{L,abc} \end{cases} \quad (1)$$

where $i_{t,abc}$, v_{abc} , $i_{L,abc}$ and $v_{t,abc}$ represent the currents delivered by the DGU, the load voltages, the currents fed into the inductance load (L) and the VSC output voltages, respectively. Each three-phase variable of (1) can be transferred to the rotating dq -frame by applying the Clarke's and Park's transformations. Then, the so-called state-space representation of (1) results in being

$$\begin{cases} \dot{x}_1(t) = -\frac{1}{RC} x_1(t) + \omega_0 x_2(t) + \frac{1}{C} x_3(t) - \frac{1}{C} x_5(t) \\ \dot{x}_2(t) = -\omega_0 x_1(t) - \frac{1}{RC} x_2(t) + \frac{1}{C} x_4(t) - \frac{1}{C} x_6(t) \\ \dot{x}_3(t) = -\frac{1}{L_t} x_1(t) - \frac{R_t}{L_t} x_3(t) + \omega_0 x_4(t) + \frac{1}{L_t} u_1(t) \\ \dot{x}_4(t) = -\frac{1}{L_t} x_2(t) - \omega_0 x_3(t) - \frac{R_t}{L_t} x_4(t) + \frac{1}{L_t} u_2(t) \\ \dot{x}_5(t) = \frac{1}{L} x_1(t) - \frac{R_l}{L} x_5(t) + \omega_0 x_6(t) \\ \dot{x}_6(t) = \frac{1}{L} x_2(t) - \omega_0 x_5(t) - \frac{R_l}{L} x_6(t) \\ y_{dIOM}(t) = x_1(t) \\ y_{qIOM}(t) = x_2(t) \end{cases} \quad (2)$$

where $x = [V_d \ V_q \ I_d \ I_q \ I_{Ld} \ I_{Lq}]^T \in \mathcal{X} \subset \mathbb{R}^6$ is the state variables vector, $u = [V_{d,ref} \ V_{q,ref}]^T \in \mathcal{U} \subset \mathbb{R}^2$ is the input vector, and $y_{IOM} =$

$[V_d V_q]^T \in \mathbb{R}^2$ is the output vector. Analogously, the state-space model of the DGU in GCOM is

$$\begin{cases} \dot{x}_1(t) = -\frac{1}{RC}x_1(t) + \omega x_2(t) + \frac{1}{C}x_3(t) - \frac{1}{C}x_5(t) - \frac{1}{C}x_7(t) \\ \dot{x}_2(t) = -\omega x_1(t) - \frac{1}{RC}x_2(t) + \frac{1}{C}x_4(t) - \frac{1}{C}x_6(t) - \frac{1}{C}x_8(t) \\ \dot{x}_3(t) = -\frac{1}{L}x_1(t) - \frac{R}{L}x_3(t) + \omega x_4(t) + \frac{1}{L}u_1(t) \\ \dot{x}_4(t) = -\frac{1}{L}x_2(t) - \omega x_3(t) - \frac{R}{L}x_4(t) + \frac{1}{L}u_2(t) \\ \dot{x}_5(t) = \frac{1}{L}x_1(t) - \frac{R}{L}x_5(t) + \omega x_6(t) \\ \dot{x}_6(t) = \frac{1}{L}x_2(t) - \omega x_5(t) - \frac{R}{L}x_6(t) \\ \dot{x}_7(t) = \frac{1}{L_s}x_1(t) - \frac{R_s}{L_s}x_7(t) + \omega x_8(t) - \frac{1}{L_s}u_3(t) \\ \dot{x}_8(t) = \frac{1}{L_s}x_2(t) - \omega x_7(t) - \frac{R_s}{L_s}x_8(t) - \frac{1}{L_s}u_4(t) \\ y_{dGCOM}(t) = x_3(t) \\ y_{qGCOM}(t) = x_4(t) \end{cases} \quad (3)$$

where $x = [V_d V_q I_{id} I_{iq} I_{Ld} I_{Lq} I_{gd} I_{gq}]^T \in \mathcal{X} \subset \mathbb{R}^8$ is the state variables vector, $u = [V_{id} V_{iq} V_{gd} V_{gq}]^T \in \mathcal{U} \subset \mathbb{R}^4$ is the input vector, and $y_{GCOM} = [I_{id} I_{iq}]^T \in \mathbb{R}^2$ is the output vector, $I_{gd}, I_{gq}, V_{gd}, V_{gq}$ being the dq -components of the currents exchanged with the grid and the grid voltages.

The aim of this paper consists in designing a control scheme capable of guaranteeing that the tracking error between any controlled variable and the corresponding reference is steered to zero in a finite time in spite of the uncertainties.

IV. THE PROPOSED SOLUTION: HIGHER ORDER SLIDING MODE CONTROL SCHEME

In this section, the use of HOSM control to solve the aforementioned control problem is discussed.

A. Suboptimal SOSM Controller

Consider the IOM state-space model (2) and select the so-called ‘‘sliding variables’’ as

$$\sigma_{dIOM}(t) = y_{dIOM,ref} - y_{dIOM}(t) \quad (4)$$

$$\sigma_{qIOM}(t) = y_{qIOM,ref} - y_{qIOM}(t) \quad (5)$$

where $y_{iIOM,ref}$, $i = d, q$ are assumed to be of class C^2 and with second time derivative Lipschitz continuous. Denote with r the relative degree of the system, i.e., the minimum order r of the time derivative $\sigma^{(r)}$ of the sliding variable in which the control u explicitly appears. With reference to (4)-(5), it appears that r is equal to 2. This implies that a SOSM control naturally applies [31], [32]. According to the SOSM control theory, we need to define the so-called auxiliary variables $\xi_{d,1IOM} = \sigma_{dIOM}$ and $\xi_{q,1IOM} = \sigma_{qIOM}$ such that the corresponding auxiliary systems can be expressed as

$$\begin{cases} \dot{\xi}_{i,1IOM}(t) = \xi_{i,2IOM}(t) \\ \dot{\xi}_{i,2IOM}(t) = f_{iIOM}(x(t)) + g_{iIOM}u_{iIOM}(t) \end{cases} \quad i = d, q \quad (6)$$

where u_{iIOM} are the dq -components of the VSC output voltages, $\xi_{i,2IOM}$ are assumed to be unmeasurable, and

$$\begin{aligned} f_{dIOM}(x(t)) &= (\omega_0^2 - \frac{1}{(RC)^2} + \frac{1}{L_s C} + \frac{1}{L C})x_1(t) + \frac{2\omega_0}{RC}x_2(t) \\ &\quad + (\frac{1}{RC^2} + \frac{R}{L C})x_3(t) - \frac{2\omega_0}{C}x_4(t) \\ &\quad - (\frac{1}{RC^2} + \frac{R}{L C})x_5(t) + \frac{2\omega_0}{C}x_6(t) + \ddot{x}_{1,ref}(t) \\ f_{qIOM}(x(t)) &= -\frac{2\omega_0}{RC}x_1(t) + (\omega_0^2 - \frac{1}{(RC)^2} + \frac{1}{L_s C} + \frac{1}{L C})x_2(t) \\ &\quad + \frac{2\omega_0}{C}x_3(t) + (\frac{1}{RC^2} + \frac{R}{L C})x_4(t) \\ &\quad - \frac{2\omega_0}{C}x_5(t) - (\frac{1}{RC^2} + \frac{R}{L C})x_6(t) + \ddot{x}_{2,ref}(t) \\ g_{iIOM} &= -\frac{1}{L C}, \quad i = d, q \end{aligned} \quad (7)$$

are allowed to be uncertain with known bounds

$$|f_{iIOM}(\cdot)| \leq F_{iIOM}, \quad G_{i,mIOM} \leq |g_{iIOM}| \leq G_{i,MIOM} \quad (8)$$

F_{iIOM} , $G_{i,mIOM}$ and $G_{i,MIOM}$ being positive constants. Note that, the existence of these bounds is true in practice due to the fact that $f_{iIOM}(\cdot)$, $i = d, q$, depend on electric signals related to the finite power of the system and g_{iIOM} , $i = d, q$, are constant values. The control laws, which are proposed to steer $\xi_{i,1IOM}(t)$ and $\xi_{i,2IOM}(t)$, $i = d, q$, to zero in a finite time in spite of the uncertainties, in analogy with [31], can be expressed as follows

$$u_{iIOM}(t) = -\alpha_{iIOM} U_{iIOM,max} \operatorname{sgn}\left(\xi_{i,1IOM}(t) - \frac{1}{2}\xi_{i,1IOM,max}\right) \quad (9)$$

with bounds

$$U_{iIOM,max} > \max\left(\frac{F_{iIOM}}{\alpha_{iIOM}^* G_{i,mIOM}}; \frac{4F_{iIOM}}{3G_{i,mIOM} - \alpha_{iIOM}^* G_{i,MIOM}}\right) \quad (10)$$

$$\alpha_{iIOM}^* \in (0, 1] \cap \left(0, \frac{3G_{i,mIOM}}{G_{i,MIOM}}\right) \quad (11)$$

Analogously, in GCOM, the sliding variables are selected as

$$\sigma_{dGCOM}(t) = y_{dGCOM,ref} - y_{dGCOM}(t) \quad (12)$$

$$\sigma_{qGCOM}(t) = y_{qGCOM,ref} - y_{qGCOM}(t) \quad (13)$$

where $y_{iGCOM,ref}$, $i = d, q$ are assumed to be of class C and with first time derivative Lipschitz continuous. In this second case, the natural relative degree of the system is equal to 1. So, a first order sliding mode controller would be adequate. Yet, in order to alleviate the chattering phenomenon [27]–[29], [40], [41], which can be dangerous in terms of harmonics affecting the electric signals, SOSM control is used also in this case, by artificially increasing the relative degree of the system. Specifically, by defining the auxiliary variables $\xi_{d,1GCOM} = \sigma_{dGCOM}$ and $\xi_{q,1GCOM} = \sigma_{qGCOM}$, one has

$$\begin{cases} \dot{\xi}_{i,1GCOM}(t) = \xi_{i,2GCOM}(t) \\ \dot{\xi}_{i,2GCOM}(t) = f_{iGCOM}(x(t), u(t)) + g_{iGCOM}w_{iGCOM}(t) \\ \dot{u}_{iGCOM}(t) = w_{iGCOM}(t) \end{cases} \quad i = d, q \quad (14)$$

where u_{iGCOM} , are the dq -components of the VSC output voltages, $\xi_{i,2GCOM}$ are assumed to be unmeasurable, and

$$\begin{aligned} f_{dGCOM}(x(t), u(t)) &= -(\frac{1}{RLC} + \frac{R}{L^2})x_1(t) + \frac{2\omega}{L}x_2(t) \\ &\quad + (\omega^2 + \frac{1}{LC} - \frac{R^2}{L^2})x_3(t) + \frac{2\omega R}{L}x_4(t) \\ &\quad - \frac{1}{L_s C}x_5(t) - \frac{1}{L_s C}x_7(t) + \frac{R}{L_s}u_{dGCOM}(t) \\ &\quad - \frac{\omega}{L_s}u_{qGCOM}(t) + \ddot{x}_{3,ref}(t) \\ f_{qGCOM}(x(t), u(t)) &= -\frac{2\omega}{L}x_1(t) - (\frac{1}{RLC} + \frac{R}{L^2})x_2(t) \\ &\quad - \frac{2\omega R}{L}x_3(t) + (\omega^2 + \frac{1}{LC} - \frac{R^2}{L^2})x_4(t) \\ &\quad - \frac{1}{L_s C}x_6(t) - \frac{1}{L_s C}x_8(t) + \frac{\omega}{L_s}u_{dGCOM}(t) \\ &\quad + \frac{R}{L_s}u_{qGCOM}(t) + \ddot{x}_{4,ref}(t) \\ g_{iGCOM} &= -\frac{1}{L_s}, \quad i = d, q \end{aligned} \quad (15)$$

are allowed to be uncertain with known bounds

$$|f_{iGCOM}(\cdot)| \leq F_{iGCOM}, \quad G_{i,mGCOM} \leq |g_{iGCOM}| \leq G_{i,MGCOM} \quad (16)$$

F_{iGCOM} , $G_{i,mGCOM}$ and $G_{i,MGCOM}$ being positive constants. The

control laws, which are proposed to steer $\xi_{i,1GCOM}(t)$ and $\xi_{i,2GCOM}(t)$, $i = d, q$, to zero in a finite time in spite of the uncertainties, in this second case, can be expressed as follows

$$w_{iGCOM} = -\alpha_{iGCOM} U_{iGCOM,max} \operatorname{sgn}\left(\xi_{i,1GCOM} - \frac{1}{2}\xi_{i,1GCOM,max}\right) \quad (17)$$

with bounds

$$U_{iGCOM,max} > \max\left(\frac{F_{iGCOM}}{\alpha_{iGCOM}^* G_{i,mGCOM}}; \frac{4F_{iGCOM}}{3G_{i,mGCOM} - \alpha_{iGCOM}^* G_{i,MGCOM}}\right) \quad (18)$$

$$\alpha_{iGCOM}^* \in (0, 1] \cap \left(0, \frac{3G_{i,mGCOM}}{G_{i,MGCOM}}\right) \quad (19)$$

Note that, the discontinuity of the SOSM control laws w_{iGCOM} , $i = d, q$ only affects $\ddot{\sigma}_{iGCOM}$. The actual control variables u_{iGCOM} , $i = d, q$, are continuous, so that the chattering is alleviated.

Moreover, in order to face some undesired overshoot on the currents, due to the reconnection to the main grid, as well as step variations of the current references, a constrained SOSM (SOSM_c) can be used in GCOM. According to [42], this is able to fulfil the constraints imposed on $\xi_{i,1GCOM}$ and $\xi_{i,2GCOM}$.

B. 3-SM Controller for chattering attenuation in IOM

To provide a chattering attenuation also in IOM, the procedure suggested in [31], consisting in artificially increasing the system relative degree, is applied. Inspired by [30], in this paper we propose a 3-SM control law to solve the microgrid voltage control problem in question with chattering attenuation. Since the 3-SM is applied only in IOM, the subscript *IOM* is omitted in this subsection.

By defining the auxiliary variables $\xi_{d,1} = \sigma_d$ and $\xi_{q,1} = \sigma_q$, one has

$$\begin{cases} \dot{\xi}_{i,1}(t) = \xi_{i,2}(t) \\ \dot{\xi}_{i,2}(t) = \xi_{i,3}(t) \\ \dot{\xi}_{i,3}(t) = \varphi_i(x(t), u(t)) + \gamma_i w_i(t) \\ \dot{u}_i(t) = w_i(t) \end{cases} \quad i = d, q \quad (20)$$

where $\xi_{i,2}$, $\xi_{i,3}$ are assumed to be unmeasurable, and

$$\begin{aligned} \varphi_d(x(t), u(t)) &= \left(\omega_0^2 - \frac{1}{(RC)^2} + \frac{1}{L_r C} + \frac{1}{LC}\right)\dot{x}_1(t) + \frac{2\omega_0}{RC}\dot{x}_2(t) \\ &\quad + \left(\frac{1}{RC^2} + \frac{R_l}{L_r C}\right)\dot{x}_3(t) - \frac{2\omega_0}{C}\dot{x}_4(t) \\ &\quad - \left(\frac{1}{RC^2} + \frac{R_l}{LC}\right)\dot{x}_5(t) + \frac{2\omega_0}{C}\dot{x}_6(t) + x_{1,ref}^{(3)}(t) \\ \varphi_q(x(t), u(t)) &= -\frac{2\omega_0}{RC}\dot{x}_1(t) + \left(\omega_0^2 - \frac{1}{(RC)^2} + \frac{1}{L_r C} \right. \\ &\quad \left. + \frac{1}{LC}\right)\dot{x}_2(t) + \frac{2\omega_0}{C}\dot{x}_3(t) + \left(\frac{1}{RC^2} + \frac{R_l}{L_r C}\right)\dot{x}_4(t) \\ &\quad - \frac{2\omega_0}{C}\dot{x}_5(t) - \left(\frac{1}{RC^2} + \frac{R_l}{LC}\right)\dot{x}_6(t) + x_{2,ref}^{(3)}(t) \\ \gamma_i &= -\frac{1}{L_r C}, \quad i = d, q \end{aligned} \quad (21)$$

are allowed to be uncertain with known bounds

$$|\varphi_i(\cdot)| \leq \Phi_i, \quad \Gamma_{i,m} \leq |\gamma_i| \leq \Gamma_{i,M} \quad (22)$$

Φ_i , $\Gamma_{i,m}$ and $\Gamma_{i,M}$ being positive known constants. The control laws, proposed to steer $\xi_{i,1}(t)$, $\xi_{i,2}(t)$ and $\xi_{i,3}(t)$, $i = d, q$, to zero in a finite time in spite of the uncertainties, can be

expressed as follows

$$w_i(\bar{\sigma}_i) = -\alpha_i \begin{cases} w_{i,1} = \operatorname{sgn}(\ddot{\sigma}_i), & \bar{\sigma}_i \in M_{i,1}/M_{i,0} \\ w_{i,2} = \operatorname{sgn}\left(\dot{\sigma}_i + \frac{\ddot{\sigma}_i^2 w_{i,1}}{2\alpha_{i,r}}\right), & \bar{\sigma}_i \in M_{i,2}/M_{i,1} \\ w_{i,3} = \operatorname{sgn}(s_i(\bar{\sigma}_i)), & \text{else} \end{cases} \quad (23)$$

where one has that $\bar{\sigma}_i = (\sigma_i, \dot{\sigma}_i, \ddot{\sigma}_i)^T$ and $s_i(\bar{\sigma}_i) = \sigma_i + \frac{\ddot{\sigma}_i^3}{3\alpha_{i,r}^2} + w_{i,2} \left[\frac{1}{\sqrt{\alpha_{i,r}}} \left(w_{i,2} \dot{\sigma}_i + \frac{\ddot{\sigma}_i^2}{2\alpha_{i,r}} \right)^{\frac{3}{2}} + \frac{\ddot{\sigma}_i \dot{\sigma}_i}{\alpha_{i,r}} \right]$, $\alpha_{i,r}$ being the reduced control amplitude, such that

$$\alpha_{i,r} = \alpha_i \Gamma_{i,m} - \Phi_i > 0 \quad (24)$$

In (23), (24) there are no parameters to be tuned, except for the control amplitudes α_i , $i = d, q$. In (23) the manifolds $M_{i,0}$, $M_{i,1}$, $M_{i,2}$ are defined as

$$\begin{aligned} M_{i,0} &= \{\bar{\sigma}_i \in \mathbb{R}^3 : \sigma_i = \dot{\sigma}_i = \ddot{\sigma}_i = 0\} \\ M_{i,1} &= \left\{ \bar{\sigma}_i \in \mathbb{R}^3 : \sigma_i - \frac{\ddot{\sigma}_i^3}{6\alpha_{i,r}^2} = 0, \dot{\sigma}_i + \frac{\ddot{\sigma}_i |\dot{\sigma}_i|}{2\alpha_{i,r}} = 0 \right\} \\ M_{i,2} &= \{\bar{\sigma}_i \in \mathbb{R}^3 : s_i(\bar{\sigma}_i) = 0\} \end{aligned} \quad (25)$$

Note that, in this case, the 3-SM algorithm requires that the discontinuous controls are $w_i(t)$, $i = d, q$, which only affect $\sigma_i^{(3)}$, but not $\ddot{\sigma}_i$, so that the controls actually fed into the plant are continuous and the chattering is alleviated.

V. STABILITY ANALYSIS

With reference to the proposed SOSM control approach, the following results can be proved.

Theorem 1: Given system (2) in IOM and system (3) in GCOM case, by applying the control laws (9)-(11) and (17)-(19), respectively, the sliding variables $\sigma_{d\nu}(t)$ and $\sigma_{q\nu}(t)$ in (4)-(5) and in (12)-(13), ν being the subscript *IOM* or *GCOM*, depending on the case, are steered to zero in a finite time.

Proof: This result directly follows from [31, Theorem 1] for the IOM and the GCOM case, by virtue of the choice of the control laws (9)-(11) and (17)-(19). In brief, it can be proved that, with the constraints (10)-(11) and (18)-(19), the control laws (9) and (17) establish the generation of a sequence of states with coordinates featuring a contraction of the extremal values, i.e., $|\xi_{i,1\nu,max,k+1}| < |\xi_{i,1\nu,max,k}|$, where $\xi_{i,1\nu,max,k}$ is the k -th extremal value of variable $\xi_{i,1\nu}(t)$. Moreover, it can be proved that $\lim_{k \rightarrow \infty} t_{i\nu,max,k} < \frac{\beta_{i\nu}}{1-\gamma_{i\nu}} + t_{i\nu,max,1}$ where $\{t_{i\nu,max,k}\}$, $i = d, q$, denote the sequences of the time instants when an extremal value of $\sigma_{d\nu}(t)$ and $\sigma_{q\nu}(t)$ occurs and $\gamma_{i\nu} < 1$, with

$$\beta_{i\nu} = \sqrt{|\xi_{i,1\nu,max,1}| \frac{(G_{i,m\nu} + \alpha_{i\nu}^* G_{i,M\nu}) U_{i\nu,max}}{(G_{i,m\nu} U_{i\nu,max} - F_{i\nu}) \sqrt{\alpha_{i\nu}^* G_{i,M\nu} U_{i\nu,max} + F_{i\nu}}}}$$

This allows one to conclude about the finite time convergence of the sliding variables in both the operation modes. ■

Remark 1: Note that, in GCOM, from [42, Lemma 3], it can also be proved that the convergence occurs while complying with state constraints.

Now, consider the IOM case. Let $e = [e_1, e_2, e_3, e_4, e_5, e_6]^T$ denote the state of the error system, with

$$e_j = x_{j,ref} - x_j \quad j = 1, \dots, 6 \quad (26)$$

x_j being the state variables of (2).

Theorem 2: Consider system (2) in IOM and the sliding variables (4)-(5), controlled via the SOSM algorithm in (9)-(11). $\forall t \geq t_r$, t_r being the time instant when $\sigma_{i_{IOM}}, \dot{\sigma}_{i_{IOM}}, i = d, q$, are identically zero, $\forall x(t_r) \in \mathcal{X}$, the origin of the error system state space is a finite time stable equilibrium point.

Proof: The auxiliary variables $\xi_{d,1_{IOM}}(t), \xi_{q,1_{IOM}}(t)$ are zero $\forall t \geq t_r$, since they coincide with the sliding variables $\sigma_{d_{IOM}}(t), \sigma_{q_{IOM}}(t)$, respectively. By comparing (26) with (4) and (5), taking into account system (2), one can conclude that also e_1 and e_2 are zero $\forall t \geq t_r$. Since, by virtue of the generation of a SOSM, also $\dot{\sigma}_{d_{IOM}}(t)$ and $\dot{\sigma}_{q_{IOM}}(t)$ are zero $\forall t \geq t_r$, it also follows that $e_1(t)$ and $e_2(t)$ are equal to zero $\forall t \geq t_r$. Then, inspecting (2), one has that $e_3 = e_5$ and $e_4 = e_6$, so that it yields

$$\begin{cases} -\frac{R_l}{L_l} e_3(t) + \frac{1}{L_l} u_{d_{IOM}}(t) = -\frac{R_l}{L_l} e_3(t) \\ -\frac{R_l}{L_l} e_4(t) + \frac{1}{L_l} u_{q_{IOM}}(t) = -\frac{R_l}{L_l} e_4(t) \end{cases} \quad (27)$$

According to [34], one can compute the equivalent control in case of SOSM, $\forall t \geq t_r$, by posing in (6) that $\xi_{i,2_{IOM}}(t), i = d, q$, are equal to zero, i.e.,

$$u_{eq,i_{IOM}} = -\frac{f_{i_{IOM}}(x(t))}{g_{i_{IOM}}} \quad i = d, q \quad (28)$$

By substituting (28) into (27), one has that $e_j, j = 3, \dots, 6$ are zero $\forall t \geq t_r$, which proves the theorem. ■

Theorem 3: Consider system (3) in GCOM and the sliding variables (12)-(13), controlled via the SOSM algorithm in (17)-(19). $\forall t \geq t_r$, t_r being the time instant when $\sigma_{i_{GCOM}}, \dot{\sigma}_{i_{GCOM}}, i = d, q$, are identically zero, $\forall x(t_r) \in \mathcal{X}$, the origin of the error system state space is a finite time stable equilibrium point.

Proof: The proof is analogous to that of Theorem 2. ■

By virtue of the use of the Suboptimal SOSM control approach, the designed control system turns out to be naturally robust with respect to any uncertainty included in $f_{i_{IOM}}(\cdot), f_{i_{GCOM}}(\cdot), i = d, q$. It is furthermore interesting to analyze the robustness of the proposed control approach versus disturbances or uncertainties, gathered in a signal $u_{d_{VSC}}(t)$, due to the presence of the VSC. Consider the system in IOM and in GCOM expressed as

$$\dot{x}_v(t) = A_v x_v(t) + B_v u_v(t) + u_{d_{VSC_v}}(t) \quad (29)$$

where, we assume $u_{d_{VSC_v}}(t) = B_v h_{VSC}(t)$, v being the subscript *IOM* or *GCOM*, depending on the case, and the physical bound $\|h_{VSC}(t)\| \leq h_{VSC_{max}}$, $h_{VSC_{max}}$ being a positive constant. Note that, the associated auxiliary systems can be rewritten as in (6) and (14), replacing $f_{i_{IOM}}(\cdot), f_{i_{GCOM}}(\cdot)$ with $\bar{f}_{i_{IOM}}(\cdot), \bar{f}_{i_{GCOM}}(\cdot), i = d, q$, to include the additive term $u_{d_{VSC_v}}(t)$, with bounds $|\bar{f}_{i_{IOM}}(\cdot)| \leq \bar{F}_{i_{IOM}}$ and $|\bar{f}_{i_{GCOM}}(\cdot)| \leq \bar{F}_{i_{GCOM}}, \bar{F}_{i_{IOM}}, \bar{F}_{i_{GCOM}}, i = d, q$, being positive constants.

Theorem 4: System (29) in IOM, controlled by applying (9)-(11), with $U_{i_{IOM,max}}$ in (10) replaced by $\bar{U}_{i_{IOM,max}}, i = d, q$, and

$$\bar{U}_{i_{IOM,max}} > \max \left(\frac{\bar{F}_{i_{IOM}}}{\alpha_{i_{IOM}}^* G_{i,m_{IOM}}}; \frac{4\bar{F}_{i_{IOM}}}{3G_{i,m_{IOM}} - \alpha_{i_{IOM}}^* G_{i,M_{IOM}}} \right)$$

$\forall t \geq t_r$ and $\forall x(t_r) \in \mathcal{X}$, is robust with respect to the uncertain term h_{VSC} .

Proof: Consider the auxiliary systems (6) expressed as

$$\begin{cases} \dot{\xi}_{i,1_{IOM}}(t) = \xi_{i,2_{IOM}}(t) \\ \dot{\xi}_{i,2_{IOM}}(t) = \bar{f}_{i_{IOM}}(x(t)) + g_{i_{IOM}} u_{i_{IOM}}(t) \end{cases} \quad i = d, q \quad (30)$$

According to [34], one can compute the equivalent control in case of SOSM, $\forall t \geq t_r$, by posing in (30) that $\xi_{i,2_{IOM}}(t), i = d, q$, are equal to zero, i.e.,

$$u_{eq,i_{IOM}} = -\frac{\bar{f}_{i_{IOM}}(x(t))}{g_{i_{IOM}}} \quad i = d, q \quad (31)$$

Substituting (31) in (29), one can determine the equivalent dynamics in Filippov's sense [43] of the error system, which does not depend on the uncertain term $h_{VSC}(t)$. So, in spite of its presence, for Theorem 2, the origin of the error system state space results in being a finite time stable equilibrium point. ■

Theorem 5: System (29) in GCOM, controlled by applying (17)-(19), with $U_{i_{GCOM,max}}$ in (18) replaced by $\bar{U}_{i_{GCOM,max}}, i = d, q$, and

$$\bar{U}_{i_{GCOM,max}} > \max \left(\frac{\bar{F}_{i_{GCOM}}}{\alpha_{i_{GCOM}}^* G_{i,m_{GCOM}}}; \frac{4\bar{F}_{i_{GCOM}}}{3G_{i,m_{GCOM}} - \alpha_{i_{GCOM}}^* G_{i,M_{GCOM}}} \right)$$

$\forall t \geq t_r$ and $\forall x(t_r) \in \mathcal{X}$, is robust with respect to the uncertain term h_{VSC} .

Proof: The proof is analogous to that of Theorem 4. ■

Now, with reference to the proposed 3-SM control, the following results can be proved. Since the 3-SM is applied only in IOM, the subscript *IOM* is omitted in the following.

Theorem 6: Given system (2) in IOM, by applying the control law (23) with the constraint (24), the sliding variables $\sigma_d(t)$ and $\sigma_q(t)$ in (4)-(5), are steered to zero in a finite time.

Proof: In analogy with [30], the controlled system can be expressed as a differential inclusion [44]

$$\dot{\sigma}_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \bar{\sigma}_i + \begin{pmatrix} 0 \\ 0 \\ \varphi_i + \gamma_i w_i \end{pmatrix} \quad i = d, q \quad (32)$$

with $\varphi_i \in [-\Phi_i, \Phi_i]$ and $\gamma_i \in [\Gamma_{i,m}, \Gamma_{i,M}], i = d, q$. By applying Theorem 2 in [30], it can be proved that the origin is a uniformly global finite-time stable equilibrium point for (32), controlled through (23). This allows one to conclude about the finite time convergence of the sliding variables σ_d and σ_q . ■

Theorem 7: Consider system (2) in IOM and the sliding variables (4)-(5), controlled via the 3-SM algorithm in (23). $\forall t \geq t_r$, t_r being the time instant when $\sigma_i, \dot{\sigma}_i, \ddot{\sigma}_i, i = d, q$, are identically zero, $\forall x(t_r) \in \mathcal{X}$, the origin of the error system state space is a finite time stable equilibrium point.

Proof: As explained in the proof of Theorem 2, by virtue of the generation of a 3-SM, system (27) is obtained. According to the so-called ‘‘equivalent control’’ concept [24], [34], one can compute the continuous control equivalent in terms of effects to the discontinuous one, in case of 3-SM, $\forall t \geq t_r$, by posing in (20) that $\xi_{i,3}(t), i = d, q$, are equal to zero, i.e.,

$$w_{eq,i}(t) = -\frac{\varphi_i(x(t))}{\gamma_i} \quad i = d, q \quad (33)$$

Since the relative degree of the system is increased by virtue of the 3-SM algorithm, the control input fed into the plant is

u_i . Its equivalent version can be determined from (33) as

$$u_{eq,i}(t) = \int_{t_r}^t w_{eq,i}(\zeta) d\zeta = -\frac{f_i(x(t))}{\gamma_i} \quad i = d, q \quad (34)$$

By substituting (34) into (27), it yields that e_j , $j = 3, \dots, 6$ are zero $\forall t \geq t_r$, which proves the theorem. ■

By using the 3-SM control approach, the designed control system turns out to be naturally robust with respect to any uncertainty included in $\varphi_i(\cdot)$, $i = d, q$, while guaranteeing chattering alleviation. As for the SOSM case, it is worth analyzing the robustness of the 3-SM control approach with respect to matched disturbances or uncertainties captured by signal $u_{dVSC}(t)$ which acts on the same channel of the control variable. To this end, let us consider the perturbed system version (29) in IOM. Note that, the term $h_{VSC,i}(t)$, $h_{VSC,i}(t)$ being the i -th component of vector $h_{VSC}(t)$, can be included into φ_i in the auxiliary system (20). Then $\varphi_i(\cdot)$ is replaced with $\bar{\varphi}_i(\cdot)$, $i = d, q$, with known bounds $|\bar{\varphi}_i(\cdot)| \leq \bar{\Phi}_i$, $\bar{\Phi}_i$, $i = d, q$ being positive constants.

Theorem 8: System (29) in IOM, controlled by applying (23), with the reduced control amplitude $\bar{\alpha}_{i,r} = \alpha_i \Gamma_{i,m} - \bar{\Phi}_i > 0$, $\forall t \geq t_r$ and $\forall x(t_r) \in \mathcal{X}$, is robust with respect to the uncertain term h_{VSC} .

Proof: Consider the auxiliary systems (20) expressed as

$$\begin{cases} \dot{\xi}_{i,1}(t) = \xi_{i,2}(t) \\ \dot{\xi}_{i,2}(t) = \xi_{i,3}(t) = f_i(x(t)) + g_i u_i(t) + g_i h_{VSC,i}(t) \\ \dot{\xi}_{i,3}(t) = \bar{\varphi}_i(x(t)) + \gamma_i w_i(t) \end{cases} \quad (35)$$

where $\gamma_i = g_i = 1/(L_i C)$ and $f_i(\cdot)$, $i = d, q$, as in (7). At this point, one can compute that

$$\bar{\varphi}_i(x(t)) = \varphi_i(x(t)) + g_i \dot{h}_{VSC,i}(t) = \dot{f}_i(x(t)) + g_i \dot{h}_{VSC,i}(t) \quad (36)$$

According to the so-called ‘‘equivalent control’’ concept [24], [34], one can compute the continuous equivalent control in case of 3-SM, $\forall t \geq t_r$, by posing in (35) that $\dot{\xi}_{i,3}(t)$, $i = d, q$, are equal to zero, i.e.,

$$w_{eq,i}(t) = -\frac{\varphi_i(x(t))}{g_i} - \dot{h}_{VSC,i}(t) \quad i = d, q \quad (37)$$

Since the relative degree of the system is increased by virtue of the 3-SM algorithm, the control input fed into the plant is u_i . Its equivalent version can be determined from (37) as

$$u_{eq,i}(t) = \int_{t_r}^t w_{eq,i}(\zeta) d\zeta = -\frac{f_i(x(t))}{g_i} - h_{VSC,i}(t) \quad (38)$$

Substituting (38) in (29), one can determine the equivalent dynamics in Filippov’s sense [43] of the error system, which does not depend on the uncertain term $h_{VSC}(t)$. So, in spite of its presence, for Theorem 7, the origin of the error system state space results in being a finite time stable equilibrium point. ■

VI. SIMULATION RESULTS

In this section the proposed HOSM control strategies, are verified in simulation by implementing the master-slave model of a microgrid composed of three DGUs. The electric parameters of the single DGu are reported in Table I. Note that, when three DGUs are considered, an additional load, which absorbs an active and reactive power equal to $P = 25$ kW and $Q = 1.5$ kvar,

Table I
ELECTRIC PARAMETERS OF THE SINGLE DGU

Quantity	Value	Description
V_{DC}	1000 V	DC voltage source
f_c	10 kHz	PWM carrier frequency
R_f	40 m Ω	VSC filter resistance
L_f	10 mH	VSC filter inductance
R	4.33 Ω	Load resistance
L	100 mH	Load inductance
C	1 pF	Load capacity
R_s	0.1 Ω	Grid resistance
f_0	60 Hz	Nominal grid frequency
V_n	120 V	Nominal grid phase-voltage (RMS)

respectively, is introduced. For the GCOM case, the SOSM control parameters $U_{i,max} = 5.0 \times 10^7$ and $\alpha_i^* = 0.9$, have been selected taking into account (18)-(19) and the upperbounds in (16), i.e., $\bar{F}_d = 4.5 \times 10^9$, $\bar{F}_q = 2.5 \times 10^7$ and $G_i = 1.0 \times 10^2$, $i = d, q$. For the IOM case, the 3-SM control parameters $\alpha_i = 5.0 \times 10^7$, $\alpha_{r,d} = 1.0 \times 10^{15}$ and $\alpha_{r,q} = 5.0 \times 10^{15}$ have been chosen taking into account (24) and the upperbounds in (22), i.e., $\bar{\Phi}_d = 4.0 \times 10^{15}$, $\bar{\Phi}_q = 5.0 \times 10^{13}$, and $\Gamma_i = 1.0 \times 10^8$, $i = d, q$. For all the simulation tests the sampling time is $T_s = 1 \times 10^{-6}$ s.

A. Transition To and From an Islanding Event

The transition time instants are imposed equal to $t_{isl} = 0.1$ s and $t_{grid} = 0.3$ s, in which the microgrid is islanded from and reconnected to the grid. Fig. 2 illustrates the currents exchanged with the main grid, by applying both SOSM and SOSM_c control, and it is apparent that the SOSM_c algorithm better tracks the reference. Fig. 3 shows the three-phase load voltage and the resynchronization of the PCC voltage to the grid voltage before reconnecting. Finally, the bottom of Fig. 3 shows the currents exchanged with the main grid when both the synchronization and resynchronization tools are inactive.

B. Unknown Load Dynamics

Consider the microgrid in IOM and in presence of balanced load condition. Then, from $t = 0.15$ s to $t = 0.25$ s a resistive load, which absorbs an active power of 3 kW, is equally added in the three phases, such that the resulting load is still balanced. Fig. 4 shows that during the load variation, the DGu_M increases the delivered currents to supply the added load, while keeping the load voltage equal to its reference value. Consider now that at $t = 0.15$ s the resulting load becomes unbalanced, i.e., $R_a = 5R$, $R_b = 4R$, $R_c = 2R$ and $L_c = L$ are added in phases a , b and c , respectively. In order to verify that the proposed controllers comply with the IEEE recommendations [45] for power systems, the voltage imbalance ratio V_N/V_P (where V_N and V_P are the magnitudes of negative and positive sequence components of load voltage) is calculated with the empirical formula proposed in [46]. Fig. 5 shows that, when the 3-SM is applied, the voltage imbalance ratio settles to a value approximately equal to 2.5%, which is less than the maximum

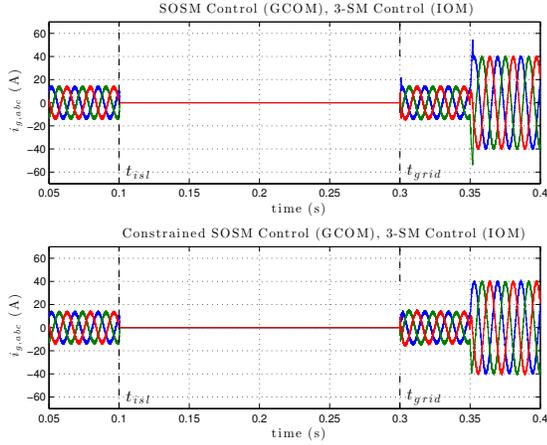


Figure 2. Performance evaluation of SOSM and $SOSM_c$ control algorithms, comparing the instantaneous currents exchanged with the main grid.

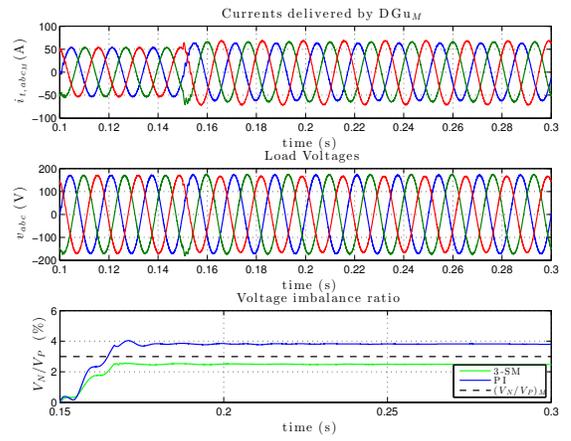


Figure 5. Unbalanced load condition. Instantaneous currents delivered by DGu_M , load voltages and V_N/V_P value by applying PI and 3-SM controllers.

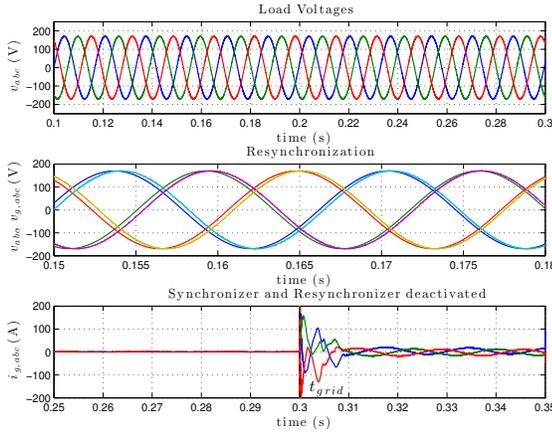


Figure 3. Load voltages in IOM, resynchronization of the load voltages to the grid voltage and currents exchanged with the grid when both synchronizer and resynchronizer are inactive.

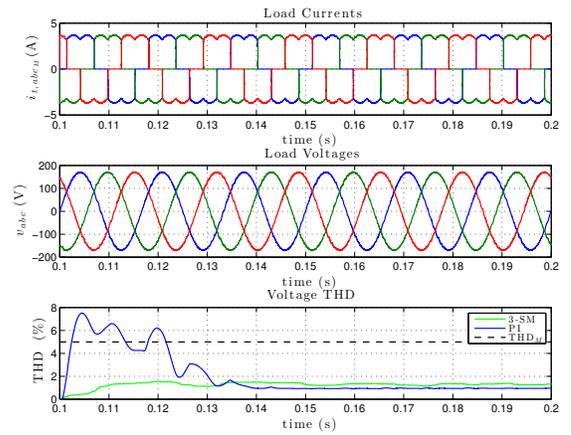


Figure 6. Nonlinear load conditions. Instantaneous load currents, load voltages and Total Harmonic Distortion (THD) value by applying PI and 3-SM controllers.

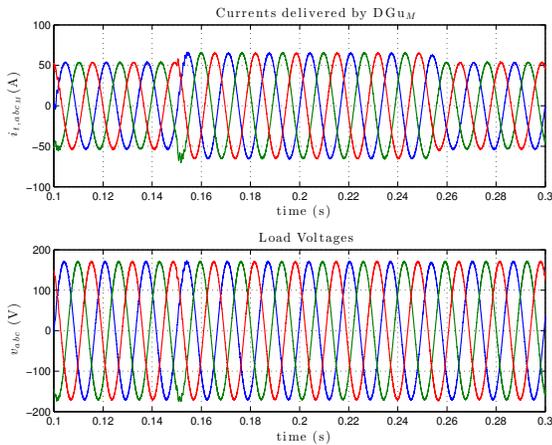


Figure 4. Instantaneous currents delivered by DGu_M and load voltages in presence of parameter uncertainties (balanced load conditions).

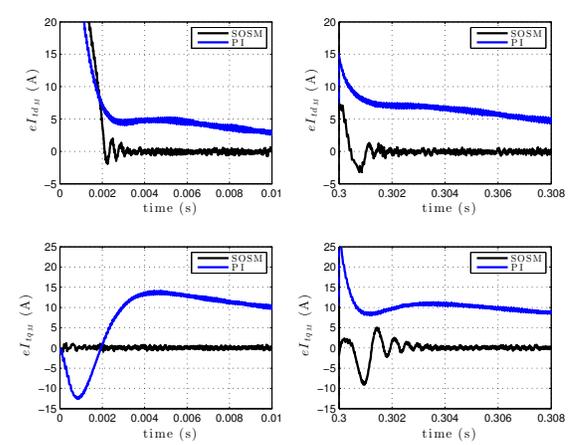


Figure 7. Tracking performance evaluation of SOSM and PI control: dq components error of the currents delivered by DGu_M at $t = t_0$ and at $t = t_{grid}$.

Table II
RMS VALUES OF CURRENTS AND VOLTAGES ERROR

Configuration	PI	SOSM	SOSM _c	3-SM
GCOM standard	100%	65%	59%	-
IOM balanced	100%	-	-	22%
IOM unbalanced	100%	-	-	23%
IOM nonlinear	100%	-	-	27%

admissible value (3%) indicated by IEEE, while, by using PI controllers the imbalance ratio reaches a value greater than the maximum admissible. However, the proposed control strategies cannot face arbitrarily large unbalanced load conditions, since the convergence properties depend on the choice of the control amplitude. Anyway, during all the tests that we run, they always result more performant than PI control. Note that the gains of PI controllers have been tuned relying on the standard Ziegler-Nichols method [47] to obtain a satisfactory behaviour of the controlled system, given the type of control law. This method, which is an heuristic, does not produce an optimal tuning of the control parameters but gives parameters which are normally highly satisfactory.

C. Nonlinear Load Conditions

Consider again the microgrid in IOM. Then, from $t = 0.1$ s to $t = 0.2$ s a three-phase six-pulse diode-bridge rectifier, feeding a purely resistive load with $R = 80 \Omega$, is connected to the PCC. In order to verify that the proposed controllers comply with the IEEE recommendations for power systems, the Total Harmonic Distortion (THD) has been calculated. Fig. 6 shows that, when the 3-SM is applied, the THD settles to a value approximately equal to 1%, which is less than the maximum admissible value (5%) recommended by IEEE, while, by using PI controllers, the THD reaches values greater than the maximum permissible during transients. As for the case in Subsection VI.B, also in this case the proposed control strategies cannot face arbitrarily large nonlinear load conditions, since the convergence properties depend on the choice of the control amplitude. Anyway, during all the tests that we run, they always result more performant than the PI control, especially during transients. Note that, the more conservative the choice of the control amplitude is, the more likely it is that the control system is able to counteract critical unbalanced and nonlinear load conditions. Reasonably, a compromise choice of the control amplitude has to be made, so as to avoid excessive control energy expenditure, though guaranteeing robustness to unforeseen load conditions. The same holds for the case of unbalanced load condition.

D. Comparative Analysis

In this subsection the proposed control strategies and the traditional PI control are compared in terms of tracking performance. Fig.7 shows the time evolution of the direct and quadrature components of the currents errors delivered by the DG_{uM}. Table II reports the root mean square error (expressed

in %) of the controlled variables I_{IdM} , I_{IqM} in GCOM, and V_d and V_q in IOM, with respect to their references. Finally, we compare the 3-SM with a filtered-SOSM (fSOSM), i.e., a SOSM control law filtered via a first order filter. The use of a 3-SM allows to obtain a reduction of about 11% of the sign changes of the sliding variables with respect to a fSOSM.

VII. CONCLUSIONS

In this paper, higher order sliding mode control strategies for microgrids have been studied. More precisely, a second order sliding mode control strategy, belonging to the class of Suboptimal algorithms, and a third order sliding mode control strategy have been proposed. The second order sliding mode has been used both in grid-connected and islanded operation mode. Then, by virtue of the fact that the natural relative degree of the system, in islanded mode, is equal to 2, it is observed how a significant beneficial effect can be obtained in terms of chattering alleviation by applying the third-order algorithm. The proposed controllers are analyzed theoretically in the paper. An extensive simulation analysis is also provided, considering a three degree-of-freedom microgrid with master-slave architecture. The proposed controllers show satisfactory closed-loop performance, complying with the IEEE recommendations for power systems and result in being more robust than traditional PI controllers.

REFERENCES

- [1] H. B. Puttgen, P. MacGregor, and F. Lambert, "Distributed generation: Semantic hype or the dawn of a new era?" *Power and Energy Magazine, IEEE*, vol. 1, no. 1, pp. 22–29, Jan 2003.
- [2] R. Lasseter and P. Paigi, "Microgrid: a conceptual solution," in *Proc. 35th IEEE Power Electron. Specialists Conf.*, vol. 6, June 2004, pp. 4285–4290 Vol.6.
- [3] J. Guerrero, P. C. Loh, T.-L. Lee, and M. Chandorkar, "Advanced control architectures for intelligent microgrids - part ii: Power quality, energy storage, and ac/dc microgrids," *IEEE Trans. Ind. Electron.*, vol. 60, no. 4, pp. 1263–1270, Apr. 2013.
- [4] F. Katiraei, M. Iravani, and P. Lehn, "Micro-grid autonomous operation during and subsequent to islanding process," *IEEE Trans. Power Del.*, vol. 20, no. 1, pp. 248–257, Jan. 2005.
- [5] R. Lasseter, "Microgrids," in *Power Engineering Society Winter Meeting, 2002. IEEE*, vol. 1, 2002, pp. 305–308 vol.1.
- [6] E. Planas, A. G. de Muro, J. Andreu, I. Kortabarria, and I. M. de Alegria, "General aspects, hierarchical controls and droop methods in microgrids: A review," *Renewable and Sustainable Energy Reviews*, vol. 17, no. 0, pp. 147 – 159, 2013.
- [7] O. Palizban, K. Kauhaniemi, and J. M. Guerrero, "Microgrids in active network management—part i: Hierarchical control, energy storage, virtual power plants, and market participation," *Renewable and Sustainable Energy Reviews*, vol. 36, no. 0, pp. 428 – 439, 2014.
- [8] S. Amin and B. Wollenberg, "Toward a smart grid: power delivery for the 21st century," *IEEE Power Energy Mag.*, vol. 3, no. 5, pp. 34–41, Sep. 2005.
- [9] P. Paigi and R. Lasseter, "Autonomous control of microgrids," in *Power Eng. Society General Meeting, 2006*, p. 8.
- [10] M. Zamani, G. Riahy, and A. Ardakani, "Modifying power curve of variable speed wind turbines by performance evaluation of pitch-angle and rotor speed controllers," in *Electrical Power Conference, 2007. EPC 2007. IEEE Canada*, Oct.s 2007, pp. 347–352.
- [11] M. Hamzeh, A. Ghazanfari, H. Mokhtari, and H. Karimi, "Integrating hybrid power source into an islanded mv microgrid using chb multilevel inverter under unbalanced and nonlinear load conditions," *Energy Conversion, IEEE Transactions on*, vol. 28, no. 3, pp. 643–651, Sep. 2013.
- [12] M. Hamzeh, H. Karimi, and H. Mokhtari, "A new control strategy for a multi-bus mv microgrid under unbalanced conditions," *Power Systems, IEEE Transactions on*, vol. 27, no. 4, pp. 2225–2232, Nov. 2012.

- [13] M. Babazadeh and H. Karimi, "A robust two-degree-of-freedom control strategy for an islanded microgrid," *IEEE Trans. Power Del.*, vol. 28, no. 3, pp. 1339–1347, Jul. 2013.
- [14] H. Karimi, H. Nikkhajoei, and R. Iravani, "Control of an electronically-coupled distributed resource unit subsequent to an islanding event," *IEEE Trans. Power Del.*, vol. 23, no. 1, pp. 493–501, Jan. 2008.
- [15] H. Karimi, E. Davison, and R. Iravani, "Multivariable servomechanism controller for autonomous operation of a distributed generation unit: Design and performance evaluation," *IEEE Trans. Power Syst.*, vol. 25, no. 2, pp. 853–865, May 2010.
- [16] K. De Brabandere, B. Bolsens, J. Van den Keybus, A. Woyte, J. Driesen, and R. Belmans, "A voltage and frequency droop control method for parallel inverters," *IEEE Trans. Power Electronics*, vol. 22, no. 4, pp. 1107–1115, July 2007.
- [17] C.-T. Lee, C.-C. Chu, and P.-T. Cheng, "A new droop control method for the autonomous operation of distributed energy resource interface converters," *IEEE Trans. Power Electronics*, vol. 28, no. 4, pp. 1980–1993, April 2013.
- [18] R. Palma-Behnke, C. Benavides, F. Lanas, B. Severino, L. Reyes, J. Llanos, and D. Saez, "A microgrid energy management system based on the rolling horizon strategy," *Smart Grid, IEEE Transactions on*, vol. 4, no. 2, pp. 996–1006, June 2013.
- [19] A. Parisio, E. Rikos, and L. Glielmo, "A Model Predictive Control Approach to Microgrid Operation Optimization," *IEEE Trans. Control Syst. Techn.*, vol. 22, no. 5, pp. 1813–1827, 2014.
- [20] D. Salomonsson, L. Soder, and A. Sannino, "An adaptive control system for a dc microgrid for data centers," *IEEE Trans. Industry Applications*, vol. 44, no. 6, pp. 1910–1917, Nov 2008.
- [21] A. Bidram, A. Davoudi, F. Lewis, and S. S. Ge, "Distributed adaptive voltage control of inverter-based microgrids," *IEEE Trans. Energy Conversion*, vol. 29, no. 4, pp. 862–872, Dec 2014.
- [22] T. Hornik and Q.-C. Zhong, "A current-control strategy for voltage-source inverters in microgrids based on h^∞ and repetitive control," *IEEE Trans. Power Electronics*, vol. 26, no. 3, pp. 943–952, March 2011.
- [23] S. Rivero, F. Sarzo, and G. Ferrari-Trecate, "Plug-and-play voltage and frequency control of islanded microgrids with meshed topology," *CoRR*, vol. abs/1405.2421, 2014.
- [24] V. I. Utkin, *Sliding Modes in Optimization and Control Problems*. New York: Springer Verlag, 1992.
- [25] C. Edwards and S. K. Spurgeon, *Sliding Mode Control: Theory and Applications*. London, UK: Taylor and Francis, 1998.
- [26] M. Rubagotti, A. Estrada, F. Castanos, A. Ferrara, and L. Fridman, "Integral sliding mode control for nonlinear systems with matched and unmatched perturbations," *Automatic Control, IEEE Transactions on*, vol. 56, no. 11, pp. 2699–2704, Nov 2011.
- [27] L. Fridman, "Singularly perturbed analysis of chattering in relay control systems," *IEEE Trans. Automat. Control*, vol. 47, no. 12, pp. 2079 – 2084, Dec. 2002.
- [28] I. Boiko, L. Fridman, A. Pisano, and E. Usai, "Analysis of chattering in systems with second-order sliding modes," *IEEE Trans. Automat. Control*, vol. 52, no. 11, pp. 2085 – 2102, Nov. 2007.
- [29] A. Levant, "Chattering analysis," *IEEE Trans. Automat. Control*, vol. 55, no. 6, pp. 1380 – 1389, Jun. 2010.
- [30] F. Dinuzzo and A. Ferrara, "Higher order sliding mode controllers with optimal reaching," *IEEE Trans. Automat. Control*, vol. 54, no. 9, pp. 2126 – 2136, Sep. 2009.
- [31] G. Bartolini, A. Ferrara, and E. Usai, "Chattering avoidance by second-order sliding mode control," *IEEE Trans. Automat. Control*, vol. 43, no. 2, pp. 241–246, Feb. 1998.
- [32] —, "Output tracking control of uncertain nonlinear second-order systems," *Automatica*, vol. 33, no. 12, pp. 2203 – 2212, Dec. 1997.
- [33] G. Bartolini, A. Ferrara, E. Usai, and V. Utkin, "On multi-input chattering-free second-order sliding mode control," *IEEE Trans. Automat. Control*, vol. 45, no. 9, pp. 1711–1717, Sep. 2000.
- [34] G. Bartolini, A. Ferrara, and E. Usai, "On boundary layer dimension reduction in sliding mode control of siso uncertain nonlinear systems," in *Proc. IEEE Int. Conf. Control Applications*, vol. 1, Trieste, Italy, Sep. 1998, pp. 242 – 247 vol.1.
- [35] M. Cucuzzella, G. P. Incremona, and A. Ferrara, "Master-slave second order sliding mode control for microgrids," in *Proc. IEEE American Control Conf. (ACC)*, Jul. 2015.
- [36] —, "Third order sliding mode voltage control in microgrids," in *Proc. IEEE European Control Conf. (ECC)*, Jul. 2015.
- [37] R. H. Park, "Two-reaction theory of synchronous machines - generalized method of analysis - part i," *Trans. American Instit. Electr. Eng.*, vol. 48, no. 3, pp. 716 – 727, 1929.
- [38] R. Vijayan, S. Ch, and R. Roy, "Dynamic modeling of microgrid for grid connected and intentional islanding operation," in *Proc. Advances in Power Conv. and Energy Techn. Int. Conf.*, Aug. 2012, pp. 1–6.
- [39] I. Balaguer, Q. Lei, S. Yang, U. Supatti, and F. Z. Peng, "Control for grid-connected and intentional islanding operations of distributed power generation," *IEEE Trans. Ind. Electron.*, vol. 58, no. 1, pp. 147–157, Jan. 2011.
- [40] I. Boiko, "Analysis of chattering in sliding mode control systems with continuous boundary layer approximation of discontinuous control," in *Proc. American Control Conf.*, San Francisco, CA, USA, Jul. 2011, pp. 757 – 762.
- [41] H. Lee and V. I. Utkin, "Chattering suppression methods in sliding mode control systems," *Annual Reviews in Control*, vol. 31, no. 2, pp. 179 – 188, Oct. 2007.
- [42] M. Rubagotti and A. Ferrara, "Second order sliding mode control of a perturbed double integrator with state constraints," in *Proc. American Control Conf.*, Baltimore, MD, USA, Jun. 2010, pp. 985–990.
- [43] A. F. Filippov and F. M. Arscott, *Differential Equations with Discontinuous Right-hand Sides: Control Systems*. Springer, 1988, vol. 18.
- [44] J. Aubin and A. Cellina, *Differential Inclusions*. Springer Verlag Berlin / Heidelberg, 1984.
- [45] IEEE, "Recommended practice for monitoring electric power quality," *IEEE Std 1159-2009 (Revision of IEEE Std 1159-1995)*, pp. c1–81, Jun. 2009.
- [46] P. Pillay and M. Manyase, "Definitions of voltage unbalance," *IEEE Power Engineering Review*, vol. 21, no. 5, pp. 50–51, 2001.
- [47] J. G. Ziegler and N. B. Nichols, "Optimum settings for automatic controllers," *ASME. J. Dyn. Sys.*, vol. 64, no. 759, 1942.



Triggered Control with application to the Microgrids.

Michele Cucuzzella received the Bachelor Degree (with honor) in Industrial Engineering and the Master Degree (with honor) in electric Engineering from the University of Pavia, Italy in 2012 and 2014, respectively. Currently he is a Ph.D. Student in Electronics, Computer Science and electric Engineering for the Identification and Control of Dynamic Systems Laboratory at the University of Pavia under the supervision of Professor Antonella Ferrara. His research activities are mainly in the area of Variable Structure and Sliding Mode Control and of Event-



real-time physical systems, optimal control and variable structure control methods of sliding mode type.

Gian Paolo Incremona received the Master Degree (with honor) in electric Engineering from the University of Pavia, Italy in 2012. He was a student of the *Almo Collegio Borromeo* of Pavia, and of the class of *Science and Technology* of the Institute for Advanced Studies *IUSS* of Pavia. He is now a Ph.D. Candidate in Electronics, electric and Computer Engineering at the Identification and Control of Dynamic Systems Laboratory of the University of Pavia under the supervision of Professor Antonella Ferrara. His research deals with industrial robotics,



Systems Society, and, among others, member of the *IEEE Technical Committee on Variable Structure and Sliding Mode Control*, and of the *IFAC Technical Committee on Transportation Systems*. Since July 2013 she is *Chair of the Women in Control Standing Committee of the Control Systems Society*.

Antonella Ferrara is Full Professor of Automatic Control at the University of Pavia. Her research deals with sliding mode and nonlinear control with application to traffic, automotive and robotics. She has authored/co-authored more than 300 papers, including more than 90 journal papers. She was *Associate Editor* of the *IEEE Transactions on Control Systems Technology* and of the *IEEE Transactions on Automatic Control*. Since January 2014, she is *Associate Editor* of the *IEEE Control Systems Magazine*. She is *Senior Member* of the *IEEE Control*