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Assessment of Robustness of Power Systems from a Network Perspective

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***Abstract*—In this paper, we study the robustness assessment of power systems from a network perspective. Based on Kirchhoff's laws and the properties of network elements, and combining with a complex network structure, we propose a model that generates power flow information given the electricity consumption and generation information. It has been widely known that large scale blackouts are the result of a series of cascading failures triggered by the malfunctioning of specific critical components. Power systems could be more robust if there were fewer such critical components or the network configuration was suitably designed. The percentage of unserved nodes (PUN) caused by a failed component and the percentage of non-critical links (PNL) that will not cause severe damage are used to provide quantitative indication of a power system's robustness. We assess robustness of the IEEE 118 Bus, Northern European Grid and some synthesized networks. The influence of network structure and location of generators are explored. Simulation results show that the connection with short average shortest path length can significantly reduce a power system's robustness, and that the system with lower generator resistance has better robustness with a given network structure. We also propose a new metric based on node-generator distance (DG) for measuring the accessibility of generators in a power network which is shown to affect robustness significantly.**

***Index Terms*—Power system, complex networks, cascading failure, robustness.**

I. Introduction

POWER systems, comprising connected electrical components, have become a critical type of infrastructures in modern society as they generate and transmit power to support virtually all residential, industrial, public service, commercial, business activities. Power blackouts inevitably caused inconvenience to millions of users as well as incurred huge economic loss. Enhancing the robustness of a power system has always been a priority for electrical engineers.

Since the scale-free and small-world properties have been revealed and defined in networked systems [1], [2], the research of complex systems in terms of their network properties has made rapid and fruitful progress. By abstracting power stations in the power grid as nodes and transmission lines as edges, the power grid is amenable to complex network analysis [3]. Many researchers have tried to apply complex network theory to power systems, aiming at gaining new insights into the power grid operation that would help enhance the reliability and performance of power systems.

In early studies [4], [5], real data from power grid in different regions were analyzed, with the objective of extracting structural characteristics of this man-made infrastructure. Average degree, degree distribution and betweenness distribution are three important parameters to reveal the power grids' structural properties. Results have shown that the average degree of most of power grids is between 2 and 3 while in terms of degree distribution and betweenness distribution, no uniform conclusions are drawn. Cotilla-Sanchez *et al.* [6] compared the structural and electrical properties using the concept of "resistance distance" which is an important parameter for measuring accessibility of nodes.

In addition, the functional properties of power grids, e.g., robustness, synchronization and efficiency [7], were explored in later studies, among which robustness has always drawn much attention. Static models were first used to study the grid's resilience to the failure of some specific nodes or lines. Rosas-Casals *et al.* [8] found that the power grid in Europe were more likely to disconnect when the high-degree nodes were removed compared to the removal of the same number of randomly chosen nodes.

Since many severe blackouts were caused by a series of complex dynamic processes which were in turn triggered by some specific component's failure, many researchers began to use dynamic models to study cascading failures. In previous studies [9]–[11], each component in the system carries its load as well as its rated capacity. When some of the components break down, the power flow will redistribute in the power system, and the components whose loads exceed their capacities will fail in succession. Such cascading failure continues until all the remaining components can work properly.

In dynamic models, deriving the load distribution in the network is the key issue. In the work by Motter and Lai [12], the total number of shortest paths passing through a node is used to represent the node's load, and this definition has been adopted in several papers. The cascading failure processes in the Italian grid [13] and the North American power system [4] have been simulated in terms of this topological parameter.

Power flow distribution in a power system is governed by electrical laws and components' properties. Analysis is either inadequate or inaccurate if it is based only on network topology. In order to exploit complex network methods for producing practically relevant results, better methods are needed [14]. The DC power model [15] has been used to calculate the power flow in a power grid [11], [14]. However, the DC power flow model falls short of providing critical

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information about voltage values [16], let alone giving a complete solution for voltages and currents in the network upon re-balancing of power generated and consumed after a fault (component's failure) occurs.

In this paper, we first introduce a model that uses the concepts of complex networks and electrical laws to obtain the power flow information in the system in Section II. Then, the cascading failure process is described in Section III. In order to quantitatively describe a system's robustness, two robustness parameters are proposed in Section IV, i.e., the percentage of unserved nodes (PUN) caused by a component's failure and the percentage of non-critical links (PNL) that will not cause severe damage. Section V shows robustness assessment results of some real power systems with the method proposed. Many factors can influence a power system's robustness, and Section VI specifically explores the influence of network structure, the locations of generators. Simulation results show that, for a given set of numbers of generators, consumers, and transmission lines, connections having short average shortest path length can significantly reduce a power system's robustness. To explore the effects of generators' distribution in the grid, we propose, in Section VI, a new metric based on node-generator resistance distance [17] (DG) for measuring the degree of accessibility to generators of all consumers in a power network which is shown to affect robustness significantly.

II. Basic Model

Our model for the power system is based on the admittance model proposed by Grainger and Stevenson [15]. For a power system with n buses, the admittance model is written as

$$\begin{bmatrix} Y_{11} & -Y_{12} & \cdots & -Y_{1n} \\ -Y_{21} & Y_{22} & \cdots & -Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -Y_{n1} & -Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \tag{1}$$

which is composed of Kirchhoff's law equations for all nodes. Here, v_n and I_n are the voltage and external injected current at node n, respectively, Y_{ij} is the admittance of the transmission line connecting nodes i and j, and $Y_{ii} = -\sum_{j\neq i} Y_{ij}$. If there is no transmission line between nodes i and j, $Y_{ij} = 0$. The values of v_n and I_n are given in the time domain and can change with time, satisfying the constraints described by (1) at any instant of time. The time series of v_n and I_n describe the dynamic behavior of a power system. Equation (1) can be used to analyze the operation of a power system both in AC and DC. If the power system operates in AC and contains nonlinear components, harmonics will be included in (1).

Compared to models based on topological loads, where the loads carried by the components in the grid are represented with topological parameters, like the betweenness of the nodes and edges [10], [13], the above Grainger and Stevenson model provides real power information of the grid. However, since this model cannot perform load balance analysis and includes only a limited choice of types of nodes, it cannot provide a realistic analysis of the grid. In this paper we introduce a more comprehensive model. Four kinds of nodes are considered in our model, namely, the generation node, the consumer node, the distribution node and the transformer node.

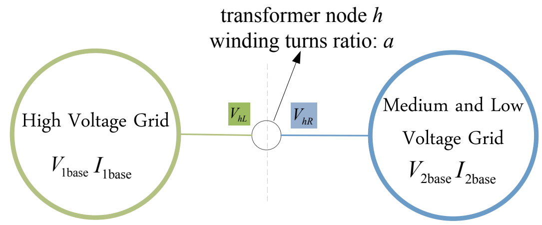


Fig. 1. Transformer h connecting grids of varying voltages.

(i) Consumer Nodes (Loads)

A consumer node i dissipates power, and at the circuit level, it sinks current I_i. The current value is negative as the node consumes power, i.e.,

$$\begin{bmatrix} -Y_{i1} & \cdots & Y_{ii} & \cdots & -Y_{in} \end{bmatrix} * V = I_i \tag{2}$$

where $V = \begin{bmatrix} \cdots & v_i & v_j & v_k & v_h & \cdots \end{bmatrix}^T$.

(ii) Distribution Nodes

A distribution node j is a connecting node that nether produces nor consumes power. Thus, we set $I_j = 0$, i.e.,

$$\begin{bmatrix} -Y_{i1} & \cdots & Y_{ii} & \cdots & -Y_{in} \end{bmatrix} * V = 0 \tag{3}$$

(iii) Generation Nodes

A generation node k is a fixed voltage source. The current emerging from this node depends on its own voltage, the power consumption of other nodes and the network topology. The nodal equation is

$$\begin{bmatrix} 0 & \cdots & y_k & \cdots & 0 \end{bmatrix} * V = v_k \tag{4}$$

where $y_k = 1$, and v_k is the voltage of node k.

(iv) Transformer Nodes

Transformer nodes connect the high-voltage grids with mid-voltage or low-voltage grids, as shown in Fig. 1. Here, a is the winding turns ratio; v_{hL} and v_{hR} are the voltages at node h's input side and output side. In this study, we perform our analysis in per unit (p.u.), and the base values at the two sides of h are set according to $V_{2\text{base}} = V_{\text{base}}/a$ and $I_{2\text{base}} = aI_{\text{base}}$. Thus, the p.u. voltage values of node h can be represented as $v_{hL} = v_{hR} = v_h$.

The nodal equation of node h is

$$\begin{bmatrix} Y_{h1} & \cdots & Y_{hh} & \cdots & Y_{hn} \end{bmatrix} * V = 0 \tag{5}$$

Combining equations (2)–(5), we get the following power system equation:

$$A * V = B \tag{6}$$

where

$$A = \begin{bmatrix} \ddots & & & & & & \cdots & \\ Y_{i1} & \cdots & Y_{ii} & Y_{ij} & Y_{ik} & Y_{ih} & \cdots & Y_{in} \\ Y_{j1} & \cdots & Y_{ji} & Y_{jj} & Y_{jk} & Y_{jh} & \cdots & Y_{jn} \\ 0 & \cdots & 0 & 0 & y_k & 0 & \cdots & 0 \\ Y_{h1} & \cdots & Y_{hi} & Y_{hj} & Y_{hk} & Y_{hh} & \cdots & Y_{in} \\ & & & & & & \cdots & \ddots \end{bmatrix},$$

$$B = \left[\begin{array}{cccccc} \cdots & I_i & 0 & v_k & 0 & \cdots \end{array} \right]^T,$$

and subscript i denotes a consumer node (load); j denotes a distribution node; k denotes a generation node; h denotes a transformer node. Given the power consumption, the generation information and the topology, the voltage of each node can be found using (6). Then, the currents flowing in the transmission lines can be calculated as

$$I_{ij} = (v_i - v_j) * Y_{ij} \tag{7}$$

Remarks: Equation (6) is derived from consideration of circuit laws and hence realistically describes the behavior of the power network. Furthermore, with the help of computation softwares, this model offers a convenient means for studying the power grid from a complex network perspective, producing results that are not obtainable from conventional circuit analysis. It should be noted that, in a connected system, the power provided by the generators should always be equal to the power consumed. When changes occur in a power system, the loads should be balanced manually or automatically. The DC model [18], [19] computes the power flow information with a given external injected power of each node. When some nodes fail and get disconnected from the network in a cascading failure process, their externally injected power becomes 0. This causes the loads of the remaining system to become unbalanced. Thus, before using the DC model to derive the updated power flow information, the loads of the remaining nodes should be balanced. The DC model cannot balance the loads automatically, and an algorithm or control method for balancing the loads should be used when analyzing the cascading failure process. The balancing algorithm can affect the results significantly. In our model, the generators are treated as voltage sources. The power emerging from generator nodes depends on their own voltages, the power consumption of other nodes and the network topology. Thus, the loads are balanced according to (6).

III. Cascading Failure Mechanism

When a link or node in the network breaks down, the structure of the power system will change, causing power flow to redistribute in the system according to (6). The nodes or links whose current loads exceed their capacities will fail successively. Thus, cascading failure continues until all the remaining components of the network can sustain their normal operation. Referring to Fig. 2, the cascading failure process can be described as follows.

1) *Initialization Settings:* At the start of the simulation, the voltages at the generation stations, the currents sunk at the consumer nodes, the winding turns ratios of the transformers and the admittances of the transmission lines need to be set. In order to reduce the effects of other factors on robustness and for simplicity, we set the voltages of generators at 1 p.u., nodes except generators each sinking 1 p.u. of current, and the admittance of each transmission line at 11 p.u. Then, with these initial values, we use (6) to obtain the initial power flow information in the system, i.e., the voltage at each nodes, the currents flowing through each link, and the load of each component. The node or link whose load exceeds its capacity will be removed. A transmission line's *current loading* is defined as the current through it, and its *capacity* is $1+\alpha$ times of its initial value $I_{ij}(0)$. A node's *power loading* is defined as $v_i(0) * I_{oi}(0)$, where $I_{oi}(0)$ is the sum of currents flowing out of node i, and its *capacity* is $1+\beta$ times of its initial value $v_i(0) * I_{0i}(0)$. Here, α and β denote the safety margins of the lines and nodes in the power grid, respectively. In reality, due to economic considerations, the safety margins limited and will not be very high. In this simulation the safety margins are set as $\alpha = 0.2$ and $\beta = 0.5$.

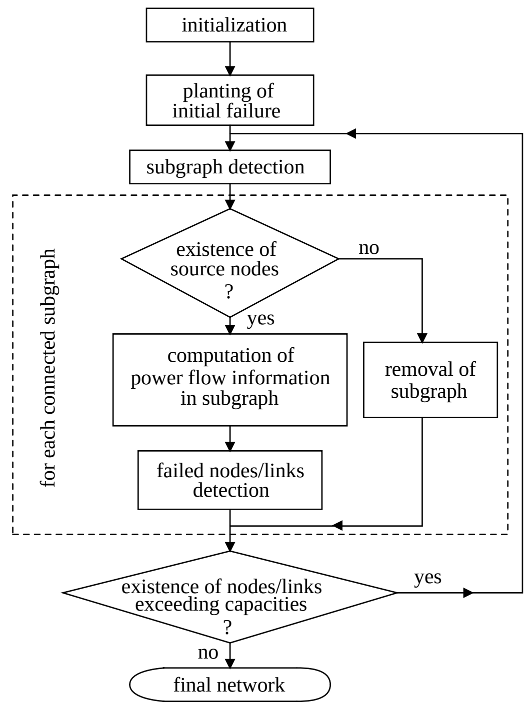


Fig. 2. Flow chart of cascading failure.

2) *Planting of Initial Failure:* With a set of initial settings, one component is randomly chosen as the first failed component, and it will be removed from the network.
3) *Cascading Iteration:* The removal of a component changes the structure and the operation of the power system. When an initial failure is planted, a series of cascading iterations begins. First, connected subgraphs will be identified. For a subgraph containing no generators, all the nodes in it are *unserved nodes*. For a subgraph containing at least one generator, (6) is used to compute the actual power flow distribution. The node or link that exceeds its capacity will be removed. This procedure repeats until all existing nodes and links can sustain their respective loadings. Then, we get the final balanced condition of the system.

IV. Robustness Parameters

Robustness refers to the ability of a system to tolerate faults. For a power system, robustness can be defined in terms of a measure that describes the ability of the system in providing

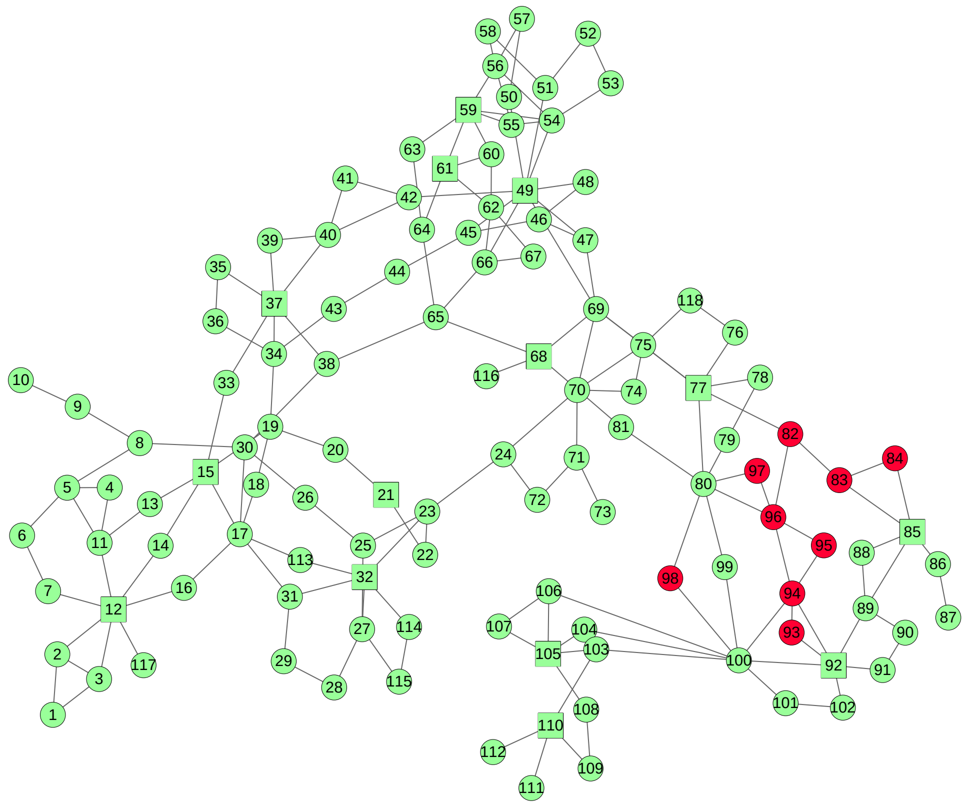


Fig. 3. Simulation of cascading failure triggered by breakdown of transmission line (77, 82) of IEEE 118 Bus. Squares are generators. Red nodes are unserved nodes.

normal service to a critical percentage of clients under the condition that some components of the system fail. It is important to define appropriate metrics that can quantitatively indicate a power system's robustness. In our study, a power system is represented as an undirected graph W with n nodes and m links. Formally, a graph W is $\{N, M\}$, where N is the set of all nodes and M is the set of all links. Also, G represents the set of generators in W, and $G \subseteq N$.

In the field of power system analysis, the extent of unserved area is usually used to measure the size of a blackout [20]. Here, we propose to use the *fraction of unserved area* caused by failure of a component to indicate the importance of that component. Specifically we define PUN(i) as the *percentage of unserved nodes* caused by failure of component i, i.e.,

$$\mathrm{PUN}(i) = \frac{n_{\mathrm{unserved}}(i)}{n} \tag{8}$$

where $n_{\mathrm{unserved}}(i)$ is the number of unserved nodes due to component i's malfunctioning. Unserved nodes are the nodes that are deprived of power in a blackout. As mentioned previously in Section III, unserved nodes are either nodes whose power loadings exceed their capacities or nodes that exist in a subgraph containing no generators. A component that has a large PUN, upon failure, can seriously damage the network. Conversely, a component with a small PUN will not have a significant influence when it fails. Thus, a power system is more resilient to faults that occur in components having small values of PUN, and we call this kind of components *non-critical components*. If a power system is resilient to faults that occur in most of the components, i.e., most of the components are non-critical, then we can say that the system is robust.

To measure the robustness of the whole system, we propose to use the *percentage of non-critical links* (PNL) whose PUNs are smaller than a *threshold* to indicate the ability of a network in tolerating faults. The PUN threshold is a specific percentage of nodes in the power grid. We define PNL(threshold) as the *percentage of non-critical links* for a given threshold, i.e.,

$$\mathrm{PNL}(\mathrm{threshold}) = \frac{1}{m} \sum_{i \in M} \delta(i) \tag{9}$$

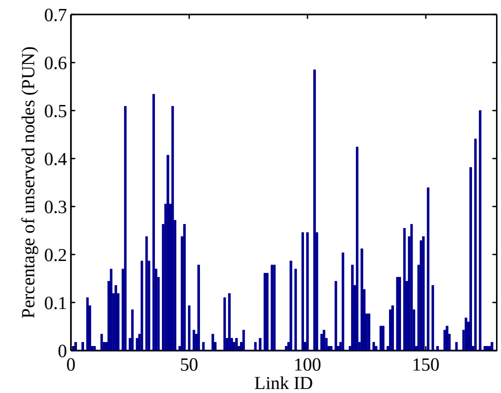
where

$$\delta(i) = \begin{cases} 1, & \mathrm{PUN}(i) < \mathrm{threshold} \\ 0, & \mathrm{otherwise.} \end{cases}$$

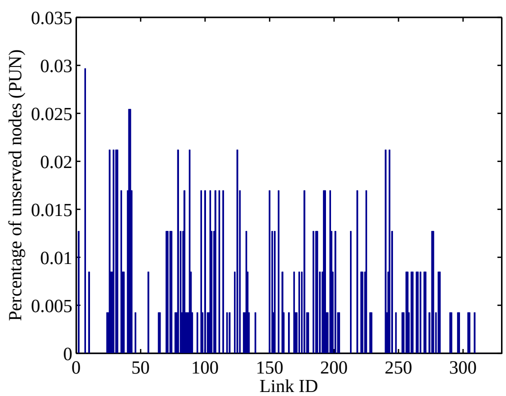
A large PNL means that the power system has a large portion of links whose failures will not lead to serious damages (i.e., the percentage of unserved nodes remains larger than the threshold) to the grid, in other words the system can tolerate faults occurred a large percentage of components of the system. The power system with a large PNL is robust.

V. Preliminary Study of Practical Systems

In this section, we present simulation results of robustness assessment of some real power systems. The IEEE 118 Bus is a power flow test case offered in [21] and the Northern European Grid (NEG) data is obtained from [22]. It should be noted that, in our study, we set the voltages of generators at 1 p.u., nodes except generators each sinking 1 p.u. of current;



(a)



(b)

Fig. 4. Simulation results of cascading failure and robustness assessment. (a) PUN of each link in IEEE 118 Bus; (b) PUN of each link in Northern European Grid.

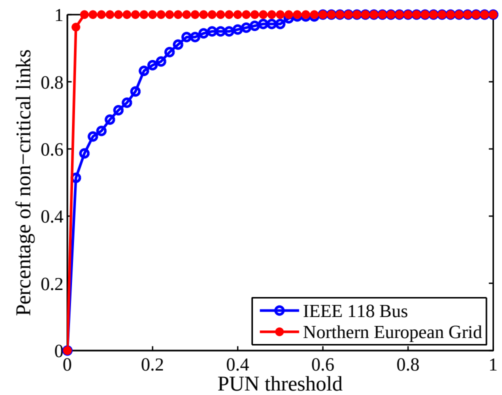


Fig. 5. Robustness assessment of IEEE 118 Bus and Northern European Grid.

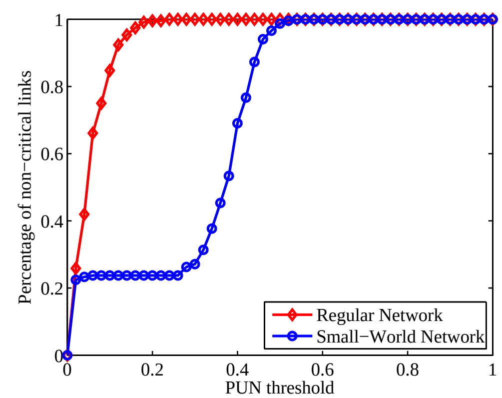


Fig. 6. Robustness assessment of small-world and regular networks.

TABLE I
AVERAGE SHORTEST PATH LENGTH (ASPL), AND PERCENTAGE OF GENERATORS (PG) OF NETWORKS

	ASPL	PG
IEEE 118 Bus	6.33	8%
Northern European Grid	8.99	50%

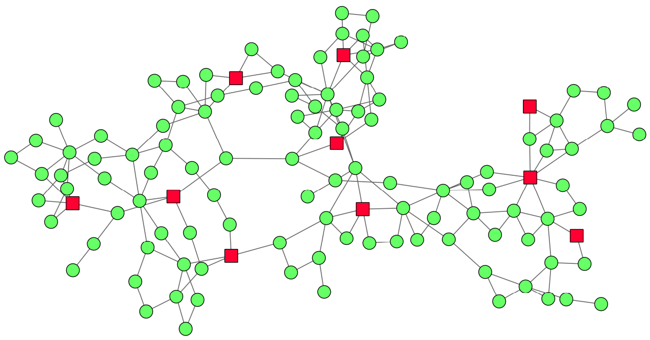
and the admittance of each transmission line to be 11 p.u. Also, the safety margins of nodes and links are set as $\alpha = 0.2$ and $\beta = 0.5$. The simulation software used here is Matlab, with the toolbox library [23] developed by Lev Muchnik which provides the basic functions for the computation of complex network parameters.

Fig. 3 shows a cascading failure result triggered by malfunctioning of line (77, 82). The rectangular nodes are generators, and the circle nodes are current sinks. The unserved nodes caused by the malfunctioning of this line are colored red. From Fig. 3, the PUN of this link is 7.6%, indicating that for the IEEE 118 Bus, the failure of line (77, 82) can deprive 7.6% of the network from power.

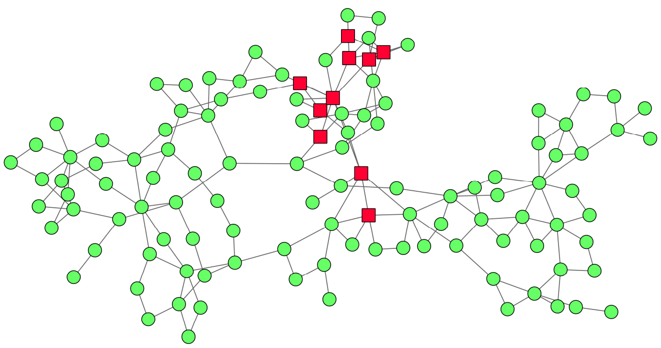
Fig. 4 shows the PUNs of all links in the IEEE 118 Bus and the Northern European Grid. It can be observed that the roles of different links in the same power system are prominently different, as they have different PUNs. From Fig. 4, the percentage of non-critical links of the Northern European Grid is larger than that of the IEEE 118 Bus. In order to distinguish the robustness of the two systems, we plot the PNLs of these two networks for different PUN thresholds. As shown in Fig. 5, the PNLs of the Northern European Grid are always larger than those of the IEEE 118 Bus, for the *threshold* ranging from 0 to 0.33. This means that the Northern European Grid is more robust than the IEEE 118 Bus.

The above result transpires a series of important questions. Why does the Northern European Grid have better robustness than the IEEE 118 Bus? What are the factors that affect a power system's robustness and in what way do these factors influence a power system's robustness? Is there a consolidated metric that can conveniently measure the robustness of a system? The answers to these questions will offer useful clues and design guidelines for power engineers to construct more reliable power transmission systems.

Table I lists the average shortest path length (ASPL), and the percentage of generators (PG) of the two networks. ASPL describes the structural characteristics of a network, whereas PG gives information about power availability. The Northern European Grid's ASPL is larger than the IEEE 118 Bus', indicating that the nodes of the IEEE 118 Bus are more closely connected. The network structure can play an important role in affecting the robustness of a power system. At the same



(a)



(b)

Fig. 7. Topologies of power networks. (a) IEEE 118 Bus A; and (b) IEEE 118 Bus B. Squares represent generators.

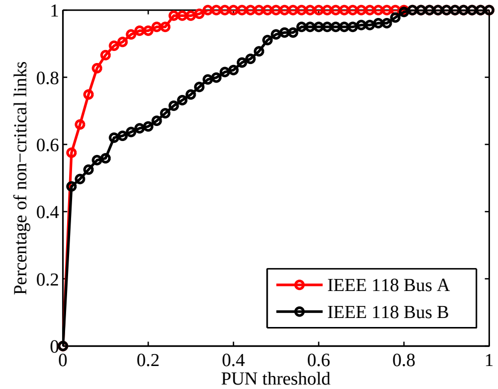


Fig. 8. Robustness assessment of IEEE 118 Bus. Buses A and B only differ in the locations of generators, with Bus A having more decentralized distribution of generators.

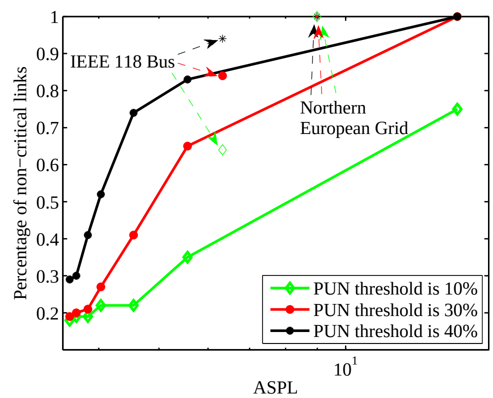


Fig. 9. Effects of small-world connectivity on robustness of power systems.

time, the Northern European Grid has a larger percentage of generators than the IEEE 118 Bus. The percentage of generators is also an important factor. Many other factors can influence the robustness of a power system as well, e.g., the locations of generators, the safety margins, and so on. It should be noted that the robustness of the two systems as inferred from Fig. 5 is the result of combined influence of these factors. In the next section, we will compare the effects of various parameters systematically, aiming to develop an effective metric that can be used to assess robustness.

VI. Network Properties and Robustness Assessment

In this paper, we focus on network properties that determine the robustness of a network. Specifically, we consider the network structure and the availability of generators in a network. Our purpose is to derive effective guidelines that can be used by electrical engineers to determine the network structure and generator distribution in order to optimize robustness. Note that we do not consider component parameters, e.g., ratings and safety margins, which can be considered as post-design parameters and be dealt with separately after the desired network is constructed.

A. Effect of Network Structure

Phadke *et al.* [24] pointed out that the graph of a power system is relevant to its efficiency and robustness. Here, we investigate the influence of a grid's topology on its robustness. To study the effect of network structure, we generate networks of specific structures for in-depth study. Small-world networks are one typical kind of networks whose ASPL is very small. Watts and Strogatz [1] showed that small-world connectivity could have significant effects on the dynamics of networked systems. To verify the effect of the connection with short ASPL, we first study the robustness of small-world networks. For instance, we construct regular and small-world networks of similar scale and identical percentage of generator nodes. Specifically, we generate a regular network of 118 nodes with an average degree of 4. The small-world network is generated by rewiring the links of the regular network with a probability $q = 0.3$. The percentage of generators is 8%. In order to scale the effects of other factors such as locations of the generators, we construct 100 realizations of the small-world network to get the average results. Fig. 6 shows that the PNLs of the regular network are much higher than those of the small-world network for PUN threshold ranging from 0.02 to 0.60.

In order to further explore the effect of the connection with short ASPL, we generate 7 groups of networks with the link-rewiring probability q ranging from 0 to 0.6. Each group contains 100 realizations, similar to the group with $q = 0.3$ mentioned above. The group with $q = 0$ is essentially the regular network group. Table II lists the averaged PNLs with

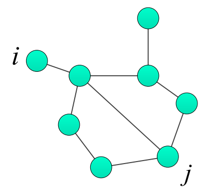


Fig. 10. An example of electrical network.

TABLE II
AVERAGE SHORTEST PATH LENGTH (ASPL), PERCENTAGE OF GENERATORS (PG) OF NETWORKS WITH DIFFERENT LEVELS OF SMALL-WORLD CONNECTIVITY CHARACTERIZED BY THE LINK-REWIRING PROBABILITY q. THEIR CORRESPONDING PNLS FOR THRESHOLD OF PUN SET AT 10%, 30% AND 40% ARE SHOWN.

q	ASPL	PG	PNL(10%)	PNL(30%)	PNL(40%)
0.0	15.13	8%	0.75	1.00	1.00
0.1	5.56	8%	0.35	0.65	0.83
0.2	4.55	8%	0.22	0.41	0.74
0.3	4.03	8%	0.22	0.27	0.52
0.4	3.84	8%	0.19	0.21	0.41
0.5	3.68	8%	0.19	0.20	0.30
0.6	3.59	8%	0.18	0.19	0.29

three thresholds, along with ASPL and PG of each group. We see that as q increases, ASPL decreases. In Fig. 9, we plot the relationship between PNL and ASPL. The lines are results derived from the 7 groups of synthesized networks listed in Table II, and the dots are robustness assessment results of IEEE 118 Bus and Northern European Grid. It is obvious that the value of PNL will be lower if the system has a smaller value of ASPL. In other words, short-ASPL connectivity deteriorates the robustness of a power system. Hence, we can conclude that with equal percentage of generator nodes, transmission lines, and same power consumption, the connection with short ASPL degrades a power system's robustness significantly when the safety margins are limited. This is consistent with the robustness assessment results for the IEEE 118 Bus and the Northern European Grid, i.e., the one with shorter average shortest path length is less robust.

Several prior studies have focused on the influence of small-world connectivity on the robustness of a power system. Mei *et al.* [25] drew a similar conclusion that small-world networks are prone to cascading failure, while Quattrociocchi *et al.* [26] reported that small-world networks were more readily recovered from failures, indicating that small-world networks are more robust. The main reason for the discrepancies in these studies is that their assumptions are different. In [26], no constraints are imposed on the amount of flow that can be transported by any link, i.e., the capacities of the components are infinite and the cascading processes are not considered. From a topological viewpoint, small-world networks have better connectivity than regular networks. Thus, a small-world network is more readily repaired by adding new links when the network decomposes. In reality, due to economic considerations, the safety margins cannot be infinite. It should be noted that the conclusion derived in our model is based on the condition that the capacities of the components of the power system are limited.

B. *Effect of Accessibility to Generators*

Power grids of the same structure can also display distinct robustness performances. We generate two power systems based on the IEEE 118 Bus, namely, IEEE 118 Bus A and IEEE 118 Bus B. Fig. 7 shows the graph layouts of these two systems, where the red rectangle nodes are generators and the green circle nodes are consumers. IEEE 118 Buses A and B share the same characteristics including network structure, percentage of generators, and safety margins, but the generators in the two networks are located differently. From Fig. 8, we see that the IEEE 118 Bus A is more robust than the IEEE 118 Bus B. Thus, the locations of the generators affect the robustness of the system.

In terms of generator distribution, the IEEE 118 Bus A is more decentralized than the IEEE 118 Bus B. Theoretically, for a given number (percentage) of available generators, a decentralized distribution of generators permits most of the consumers in the network to reach a power source within shorter distances. To transmit the same amount of power from generators to consumers, highly decentralized locations of generators can reduce the total "traffic" volume in the transmission lines as well as the distribution nodes.

It is desirable to find a variable that quantitatively describes the location information of the generators in a network. Here, we review the concept of *resistance distance* of a power system proposed by Klein and Randić [17]. Essentially, the *resistance distance* between two nodes refers to the effective resistance between them.

Referring to Fig. 10, when calculating the effective resistance between nodes i and j, we set node i as a voltage source with V_i, node j as a current sink with of I_j and all other nodes as distribution nodes with sink currents of 0. Using (6), V_j can be readily derived. The *effective resistance* between nodes i and j is defined by

$$R_{ij} = \frac{V_i - V_j}{I_j}. \tag{10}$$

When considering the electrical distance in the DC model [18], [19], the equivalent metric of the resistance distance between nodes i and j can be interpreted as reactance distance x_{ij}, with the resistances of transmission lines ignored. Similarly, when all other nodes are set as distribution nodes with zero external injected power, and the external powers of nodes i and j are balanced, x_{ij} can be computed using

$$x_{ij} = \frac{\theta_i - \theta_j}{P_j}. \tag{11}$$

where θ_i and θ_i are the voltage phase angles of nodes i and j, and P_j is the injected power of node j.

The minimum effective resistance of consumer node i to any nearest generator represents its shortest distance to a power source. This is a measure of the distance over which power is transmitted between the pair of nodes. Thus, the minimum effective resistance of consumer i represents the *accessibility* to power sources of this node. Specifically, we define the resistance distance of node i to its nearest generator, $d(i)$, as

$$d(i) = \min\{R_{is}, s \in G\} \tag{12}$$

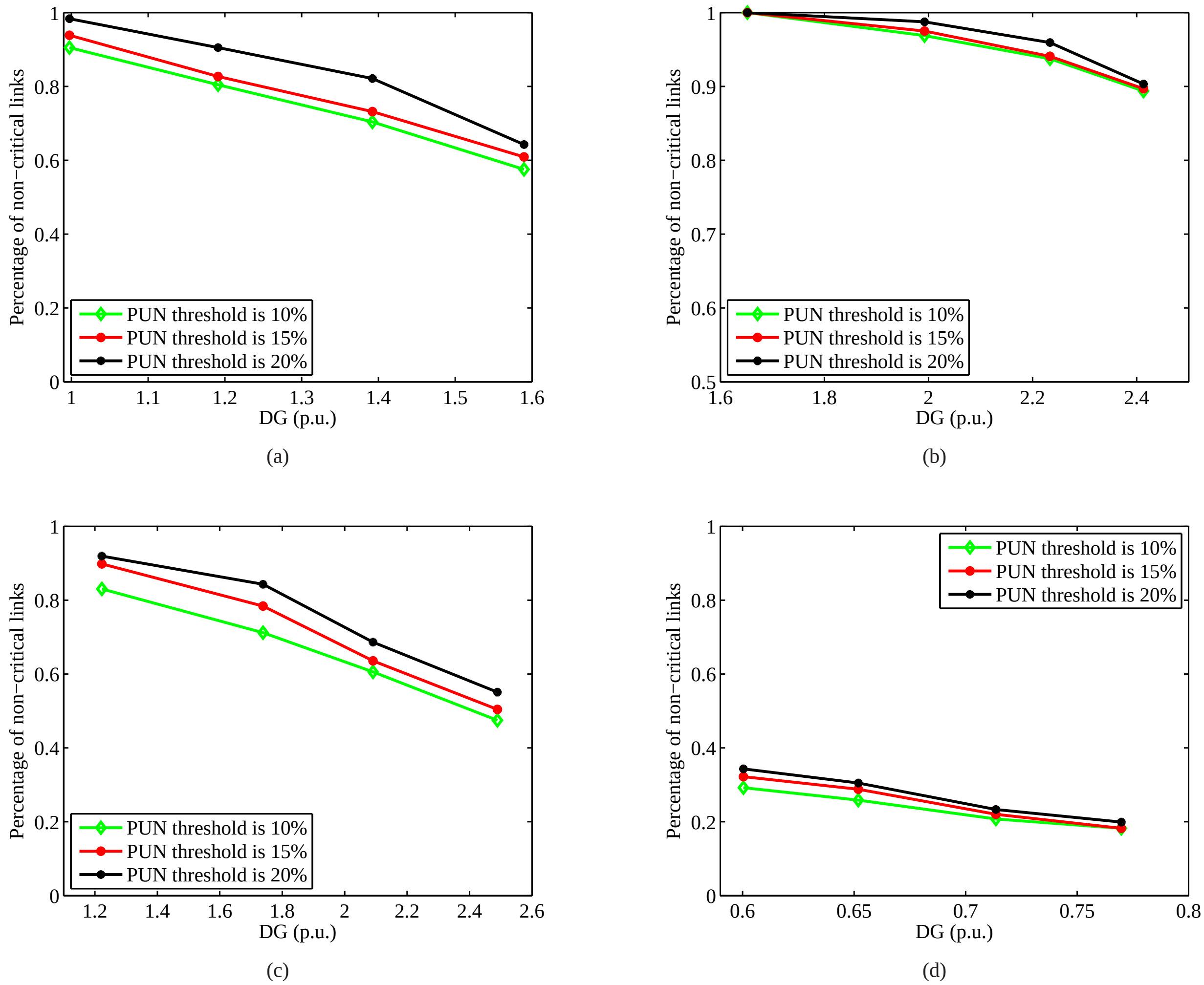


Fig. 11. Effect of locations of generators. (a) IEEE 118 Bus; (b) Northern European Grid; (c) regular network; (d) small-world network. DG measures nodes' distance to generators. Higher DG means less decentralised distribution of generators.

For a given network structure, if the generators are evenly distributed and the percentage of generator nodes is adequately high, all consumers can reach a power source within a short resistance distance, i.e., all nodes have ready access to a power source. This will reduce the total power load imposed on the transmission lines, making the system more robust. Here, we define *average effective resistance (distance) to a nearest generator of all consumer nodes* (DG) as a measure of the accessibility to generators of all consumers, i.e.,

$$\mathrm{DG} = \frac{1}{(n-g)} \sum_{i \in N \backslash G} d(i) \tag{13}$$

where $N \backslash G$ is the set of nodes excluding the generator nodes, n is the total number of nodes, and g is the number of generators in the network. Small DG indicates better accessibility to power sources to generators. A network has a smaller DG if its generators are more decentralized or has a sufficiently large number (percentage) of generators. Thus, in terms of basic network design, DG offers an effective measure of accessibility to power, which is the combined effect of the distribution of generators and the percentage of generators in a network.

TABLE III
DG AND PG OF IEEE 118 BUS A AND B. PERCENTAGE OF GENERATORS IS FIXED AT 8% FOR COMPARISON.

	IEEE 118 Bus A	IEEE 118 Bus B
DG (p.u.)	0.9977	1.5334

A large percentage of generator nodes with decentralized locations will make DG small. It is obvious that a power system could be very robust if there exist a large percentage of generator nodes. We therefore focus on the influence of the locations of generators on a system's robustness. Table III gives the DG values of IEEE 118 Buses A and B, with the percentage of generators fixed at 8%. We see that the DG of IEEE 118 Bus A is smaller than that of Bus B, which indicates that the generators in IEEE 118 Bus A are more decentralized than in IEEE 118 Bus B.

We now study the effect of varying DG in the IEEE 118 Bus, Northern European Grid, regular and small-world networks. For the IEEE 118 Bus system, a series of tests are performed, with generators' locations randomly chosen while keeping the same structure and fixing the percentage of generators at 8%. Then, we sort the results into five groups according to the values of DG. Ten test results are chosen in each group, and we average their PNLs and DGs. Fig. 11 (a) shows the PNLs with different PUN thresholds for the IEEE 118 Bus. It is obvious that the value of PNL drops significantly as DG increases. We then apply the same test procedure to assess the Northern European Grid, regular network and small-world network. The regular network is the same network generated in Section VI-A, and the small-world network is generated by rewiring the links of the regular network with a probability of 0.3. Figs. 11(b), (c) and (d) show consistent results. Thus, the metric DG proposed here is an effective design parameter for guiding the power engineers to choose appropriate locations for generators in a given network structure to achieve a more robust power system.

It should be emphasized that our conclusion here has been drawn on the condition that the network structure is fixed. If the network structure is varied, small-world connectivity may also make DG very small. In that case, a small DG does not necessarily describe a decentralized distribution of the generators. In Section VI-A we has observed that small-world connectivity can degrade a power system's robustness even though the DG value is small. The reason for this is that small resistance distances among nodes make the overall sensitivity of all components to a failure relatively high.

VII. Conclusion

We assess the robustness of power systems using a model that is derived from consideration of electrical laws and network connectivity. Taking into consideration the properties of the components and their mutual effects, this model offers realistic assessment of the power grid compared to other previously proposed complex-network based models. We define effective robustness metrics to quantitatively describe a system's robustness. Our key conclusion is that the robustness of a power system can be significantly affected by (i) the average shortest path length; and (ii) the consumers' accessibility to generators.

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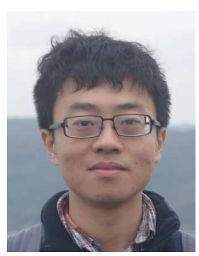

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