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Coordinated Active Power Dispatch for a Microgrid via Distributed Lambda Iteration

Jianqiang Hu, Michael Z. Q. Chen, *Senior Member, IEEE*, Jinde Cao, *Fellow, IEEE*, Josep M. Guerrero, *Fellow, IEEE*

***Abstract*—A novel distributed optimal dispatch algorithm is proposed for coordinating the operation of multiple micro units in a microgrid, which has incorporated the distributed consensus algorithm in multi-agent systems and the λ-iteration optimization algorithm in economic dispatch of power systems. Specifically, the proposed algorithm considers the global active power constraint by adding a virtual pinner and it can deal with the optimization problem with any initial states. That is, it can realize the global optimization and avoid the defect of the initial conditions' sensitivity in the optimization problem. On the other hand, the proposed optimization algorithm can either be used for off-line calculation or be utilized for on-line operation and has the ability to survive single-point failures and shows good robustness in the iteration process. Numerical studies in a seven bus microgrid demonstrate the effectiveness of the proposed algorithm.**

***Index Terms*—Microgrid, Distributed λ-iteration, Pinning consensus, Optimal dispatch, Markets**

Nomenclature

N_1 Number of DGs in the microgrid
N_2 Number of ESUs in the microgrid
N Number of micro units in the microgrid (equals to $N_1 + N_2$)
P_D Power demand of all bus load in the mircogrid
P_E Exchanged power (KW) of the tie line between the microgrid and the main grid
ρ_d Retail price (\$/KWh) for electricity in the microgrid
ρ_e Wholesale market price (\$/KWh) for electricity in the market
$P_{gi}(0)$ Initial active power output of ith DG
P_{gi} Active power output of ith DG
$P_{gi}^{\max}$ Maximum power output of ith DG
$P_{gi}^{\min}$ Minimum power output of ith DG
P_{gi}^* Optimal active power output of ith DG
$P_{bj}(0)$ Initial active power output of jth ESU
P_{bj} Active power output of jth ESU
$P_{bj}^{dch,\max}$ Maximum discharging power of jth ESU
$P_{bj}^{ch,\max}$ Maximum charging power of jth ESU
P_{bj}^* Optimal active power output of jth ESU
λ^* Optimal incremental cost of all micro units
λ_0 Estimated optimal incremental cost of the MGCC
$\hat{\lambda}_i$ Estimated optimal incremental cost of ith micro unit
$\bar{\lambda}_i$ Upper bound of the incremental cost of ith micro unit
$\underline{\lambda}_i$ Lower bound of the incremental cost of ith micro unit
P_i Active power output of ith micro unit
P_i^m Maximal or minimal power output of ith micro unit
κ Regulation gain for the pinning signal system.
μ Coupled strength of the distributed communication topology.
$\alpha_i(\beta_i, \gamma_i)$ Cost coefficients of micro units.

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I. Introduction

MICROGRID is a controllable microsystem formed by distributed generators (DGs), energy storage units (E-SUs) and local loads with the ability to operate connected to the main grid or as an island mode [1], [2], [3], [4]. When connecting to the main grid, microgrid serves as a prosumer and can sell/buy electric power to/from the main grid by participating in the electricity market. In practice, microgrid can be an alternating current (AC) microgrid, direct current (DC) microgrid, or hybrid AC/DC mircogrid according to different transmission modes [5].

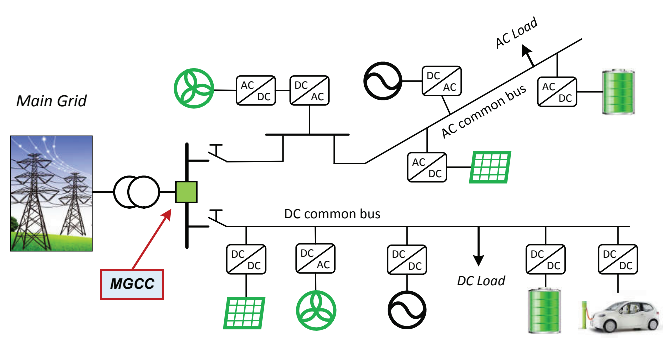

Fig. 1. A typical hybrid AC/DC microgrid structure.

As can be seen from Fig. 1 (a typical structure for a microgrid), there is a microgrid central controller (MGCC) in the system, which is responsible for the maximization of the profit by optimizing power outputs of local DGs/ESUs and the power exchange with the main distribution grid on different time scales. The optimized operating scenario is achieved by sending control signals to the micro source controllers and load controllers in a centralized manner [6], [7], [8].

One significant feature of microgrids is that the high penetration of flexible distributed micro units to the microsystem, and the traditional centralized dispatch control system becomes too complicated and low efficiency when processing a diversity of optimization and control problems. Meanwhile, a bidirectional communication structure is needed between the MGCC and the terminal units in order to connect sampled data from micro units and the MGCC is responsible for calculating and issuing control signals to all terminal units. Distributed technique have emerged with many merits, such as robustness, reliability and lower cost to be implemented, which has the good scalability and can survive single-point failures [9], [10]. Recently, distributed strategies have been utilized to solve different problems in microgrid.

Complex operational tasks managed at a centralized level can be decomposed into multiple undemanding operations implemented at a component level by distributed strategies. For example, distributed economic dispatch was considered in [11], [12], [13] on the basis of the distributed optimization theory. The authors in [14] investigated distributed active and reactive power dispatch and demand response control problems by the population game theory. Secondary voltage control of microgrids was studied in [15], [16] based on the distributed consensus algorithm of multi-agent systems. Distributed cooperative control was utilized in [17], [18] to coordinate multiple distributed generators for secondary frequency and voltage control in microgrids. The authors in [19], [20] proposed a distributed cooperative control strategy for coordinating multiple energy storage units to support the frequency control in microgrid systems.

In this manuscript, we consider the operation optimization problem of a microgrid to maximize its profit via optimizing the outputs of DGs and ESUs by proposing a distributed λ-iteration algorithm. The basic idea of λ-iteration is that there is an independent system operator (MGCC) who is responsible for estimating the optimal incremental cost and broadcasting the estimated value to all units, and then collects the power outputs of all units to calculate next estimation value, which has been utilized to solve economic dispatch problems in power systems [21]. If the total power output is too low (high), λ value will be increased (decreased) until finding the desired operating point [22]. In the microgrid, MGCC is the central decision-maker and perform all calculations in a central level by the λ-iteration algorithm. Here, we introduce a distributed λ-iteration algorithm to reduce the communication and computation burden of MGCC and mathematically prove the stability of the iteration algorithm.

Since micro units in the microgrid have the flexility to be involved in or out the active power dispatch control process, we illustrate the operation optimization problem in a distributed way. The reasons why we introduce a distributed algorithm can be stated in three aspects. Firstly, the centralized algorithm is time-consuming when dealing with the optimization problem with a large number of micro units for the information processing, analyzing, calculating and issuing in every iteration process. Secondly, the robustness of the centralized algorithm is poor compared with the distributed one under the scenario of single-point failure. Thirdly, the distributed algorithm has good scalability for easy implementation of plugging-in and plugging-out of the micro unit clusters [23], which is also easy to be maintained under a lower operation cost.

The principal contribution of this paper lie in that: (1) We generalize the traditional centralized λ-iteration algorithm to a distributed one and illustrate the stability and the convergence of the proposed distributed algorithm; (2) The distributed λ-iteration algorithm is applied for the active power coordination allocation process of multiple micro units in a microgrid, which can realize the maximal profit of the microgrid in the electricity market environment. The algorithm can guarantee the convergence of economic operations with any initial states of micro units; meanwhile the distributed calculation can be implemented using a simple communication network, such as local WiFi connections.

The rest of the paper is organized as follows. In Section II, some preliminaries and problem formulation of microgrids are provided. Section III presents the detailed distributed algorithm and parameters designing steps. Section IV illustrates the simulation results on a seven-bus microgrid system. Discussions on the proposed distributed λ-iteration algorithms are provided in Section V. Finally, conclusion and future work are presented in Section VI.

II. Preliminaries and Problem Formulation

A. Network Theory for Distributed Computation

Since distributed computation is based on distributed communication, suppose the communication network among multiple interactive units can be modeled by a digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where the nodes' set $\mathcal{V} = \{1, 2, \dots, N\}$ in the network denotes the set of individual units and the links' set $\mathcal{E}$ denotes the set of communication lines. The edge $e_{ij} = (i, j) \in \mathcal{E}$ indicates that jth unit can receive the information from ith unit. A graph is said to be undirected if $e_{ij} \in \mathcal{E}$ implies $e_{ji} \in \mathcal{E}$. A directed graph is said to be strongly connected if there exists a path between any pair of two nodes with respect to the orientation of edges. A directed tree is a digraph, where every node, except the root, has exactly one parent node. A directed spanning tree of $\mathcal{G}$ is a directed tree whose node set is $\mathcal{V}$ and whose edge set is a subset of $\mathcal{E}$. For a digraph $\mathcal{G}$, the adjacency matrix $A \in \mathbb{R}^{N \times N}$ is defined as $a_{ij} \geq 0$, in which $a_{ij} = 1 \Leftrightarrow e_{ji}(j \hookrightarrow i) \in \mathcal{E}$ while $a_{ij} = 0$ if $e_{ji} \notin \mathcal{E}$, and it is further required that self links are not allowed, *i.e.*, $a_{ii} = 0$.

The *Laplacian matrix* L is defined as $L = D - A$, where D is a diagonal matrix with $d_{ii} = \sum_{j \neq i} a_{ij}$, thus L has nonnegative diagonal entries and zero row sums. Let $d_{\max} = \max\{d_{ii}\}$ denote the maximal node in-degree of digraph $\mathcal{G}$. Then, the matrix $P_\epsilon = I - \epsilon \mathcal{L}$ is a nonnegative and row stochastic matrix for all $\epsilon \in (0, 1/d_{\max})$, in which P_ϵ is called as the *Perron matrix* induced by digraph $\mathcal{G}$.

B. λ-iteration Solving Economic Dispatch

The conventional economic dispatch problem aims at minimizing the total generation cost of generating units and determining the power output levels of online generators, which can be formulated by the following optimization problem [24]:

$$\min F_{cost}(P) = \sum_{i=1}^{N} C_i(P_i) \tag{1}$$

$$\text{s.t.} \begin{cases} \sum\limits_{i=1}^{N} P_i = P_D \\ P_i^{\min} \le P_i \le P_i^{\max} \end{cases} \tag{2}$$

for $i = 1, \dots, N$, where $C_i(P_i)$ is the generation cost for ith generating unit; $\sum_{i=1}^{N} P_i$ is the total generated power which is consumed by the active load demand P_D; $P_i^{\min}$ and $P_i^{\max}$ are the lower power bound and upper power bound for each unit i, respectively.

The following provides the λ-iteration algorithm for the economic dispatch problem (1) and (2), see Fig. 2. In each step, the dispatch center estimates an incremental cost rate λ and sends it to all units, and then collect all the power outputs of units based on the issued λ. If the sum of all outputs of units is far low, then the dispatch center will increase the λ value and try another solution until finding the desired operating point λ^*, i.e., $|\varepsilon| \le$ TOLERANCE. Furthermore, the optimal output of each unit can be calculated.

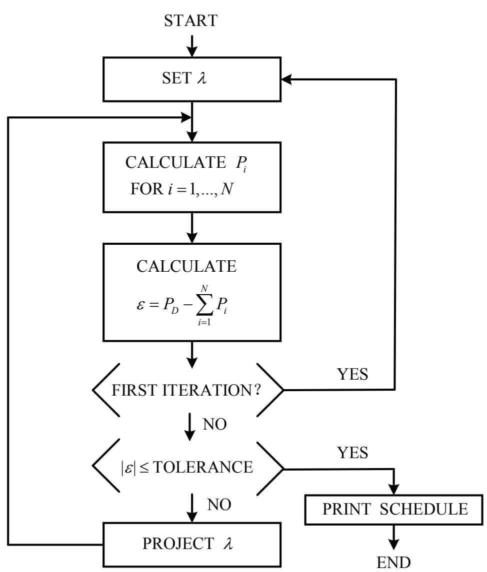

Fig. 2. Solving economic dispatch problems (1) and (2) by a centralized λ-iteration algorithm [22].

It has been shown that the λ-iteration procedure converges very rapidly for this particular type of cooperative optimization economic dispatch problem. The actual computational procedure is slightly more complex than the steps in Fig. 2, since it is necessary to observe the operating limits and the prohibited operating zones of each unit during the process of the computation [22].

C. Distributed (Pinning) Consensus Algorithm

For a large-scale multi-unit interactive system, there are always three kinds of control strategies: centralized control, decentralized control, and distributed control [25], [26]. While, distributed control can realize a cooperative group objective as centralized control by spare communication links among neighbors and consensus is the basic coordination problem in the distributed interactive system.

The coupled system for a one-dimensional continuous-time integrator multi-unit system is provided as

$$\dot{x}_i(t) = \sum_{j=1}^{N} a_{ij}\big(x_j(t) - x_i(t)\big), \tag{3}$$

and its discrete-time counterpart is

$$x_i(k+1) = x_i(k) + \epsilon \sum_{j=1}^{N} a_{ij}\big(x_j(k) - x_i(k)\big), \tag{4}$$

for $i = 1, 2, \dots, N$, where ϵ is the discrete-time step satisfying $\epsilon \in (0, 1/d_{\max})$.

In the vector notation, the discrete-time multi-unit system (4) takes the form $x(k+1) = P_\epsilon x(k)$, where P_ϵ is the *Perron matrix* of the communication topology $\mathcal{G}$. Is has been shown that [27] $x_i(k+1)$ converges to $\sum_{i=1}^{N} \omega_i x_i(0)$ under the assumption that the communication topology is strongly connected, where $\boldsymbol{\omega}^T = [\omega_1, \dots, \omega_N]$ is the left eigenvalue vector of matrix P_ϵ with the eigenvalue 1, i.e., $\boldsymbol{\omega}^T P_\epsilon = \boldsymbol{\omega}^T$ and $\mathbf{1}_N^T \boldsymbol{\omega} = 1$, here $\mathbf{1}_N = [1, \dots, 1]_N$.

The converged consensus value depends on the communication topology and the initial state values of each unit, i.e., the well-known weighted average consensus for the first-order discrete-time system. However, the average consensus value in the above formula is not always the desired final state in practice. In order to drive the multi-unit system to converge to a given objective value (Leader), the distributed pinning consensus protocol is introduced. The so-called "distributed pinning control" means only a small fraction of the nodes in the network are pinned by the control center to the objective trajectory and the rest of the nodes communicate with each other to reach the expected networked tracking.

The following distributed pinning protocol is a special case for the continuous-time distributed system in [28]:

$$\dot{x}_i(t) = \sum_{j=1}^{N} a_{ij}\big(x_j(t) - x_i(t)\big) - d_i\big(x_i(t) - \theta(t)\big), \tag{5}$$

where $i = 1, 2, \dots, N$; the pinning control gain $d_i \ge 0$, in which $d_i = 0$ indicates that ith unit is free of control; and θ is an expected consensus state which can be a static or dynamic trajectory.

If a node is pinned, i.e., $d_i > 0$, then it can access the global objective $\theta(t)$. That is, an aditional communication link is built between the pinner and the pinned nodes. By denoting the objective trajectory $\theta(t)$ as the dynamics of an islote node 0 and we use the union of the digraph $\mathcal{G}$ and the node $\{0\}$ ($\tilde{\mathcal{G}} \triangleq \mathcal{G} \cup \{0\}$) to denote the pinning joint communication topology. The Laplacian matrix of $\tilde{\mathcal{G}}$ is

$$\tilde{\mathcal{L}} = \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ -\tilde{d} & L + D \end{bmatrix},$$

in which $\tilde{d} = [d_1, d_2, \dots, d_N]^T$ and $D = \mathrm{diag}\{\tilde{d}\}$ is the pinning matrix. Before proposing the main results, we need the following lemma.

Lemma 1. *[29] For the multi-unit system (5), if the pinning joint communication topology has a directed spanning tree, then $L+D$ is a nonsingular M-matrix and the group value will be synchronized to the leader's equilibrium θ_0 ($\lim_{t\to\infty}\theta(t) = \theta_0$), i.e., $\lim_{t\to\infty}(x_i(t)-\theta_0) = 0$.*

D. Problem Formulation

Microgrid is a mini autonomous source-grid-load system, which can operate in either grid-connected mode or islanded mode through a static transfer switch, such as the structure in Fig. 1. In the microgrid, MGCC serves as an independent system operator who is responsible for the maximization of total profit during interconnected operation by optimizing the active power outputs of local DGs/ESUs and the power exchange with the main grid.

The fluctuation of the tie line power between the main grid and the microgrid reflects the dynamic power injection from the distribution system to mircogrid or the injection from mircogrid to the main grid. Suppose the positive direction of the exchanged power of the tie line is the power injection from the main grid to the mircogrid. Therefore, each grid-connected mircogrid needs to purchase (or sell) electric power from (or to) the distributed system and sell the electric power to customers in the mircogrid. The tertiary frequency control of microgrid will maximze its profit by determining the power outputs of all distributed energy resources and energy storage units such that the supply and demand balance is maintained.

Suppose there are N_1 DGs and N_2 ESUs in the microgrid and the microgrid is operated in a grid-connected mode, then the optimization model of the profit maximization can be expressed as

$$\begin{aligned}&\max F_1(P_{gi}, P_{bj})\\ &= -\rho_e P_E + \rho_d P_D - \sum_{i=1}^{N_1} C_{gi}(P_{gi}) - \sum_{j=1}^{N_2} C_{bj}(P_{bj})\end{aligned} \quad (6)$$

subject to the active power balance constraint

$$\sum_{i=1}^{N_1} P_{gi} + \sum_{j=1}^{N_2} P_{bj} + P_E = P_D \quad (7)$$

where $C_{gi}(\cdot)$ and $C_{bj}(\cdot)$ are the cost functions of the ith DG and the jth ESU, which are always approximated by quadratic functions, provided as

$$\begin{aligned}C_{gi}(P_{gi}) &= \frac{(P_{gi}-\alpha_{1,i})^2}{2\beta_{1,i}} + \gamma_{1,i}, \qquad \forall 1 \le i \le N_1,\\ C_{bj}(P_{bj}) &= \frac{(|P_{bj}|-\alpha_{2,j})^2}{2\beta_{2,j}} + \gamma_{2,j}, \qquad \forall 1 \le j \le N_2,\end{aligned}$$

where $\alpha_{1,i}$, $\beta_{1,i}$, $\gamma_{1,i}$ are the cost coefficients of the ith DG and $\alpha_{2,j}$, $\beta_{2,j}$, $\gamma_{2,j}$ are the cost coefficients of the jth ESU.

The capacity constraint of ith DG is given as

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max} \quad (8)$$

with $P_{gi}^{\min}$ and $P_{gi}^{\max}$(KW) being the minimum and maximum regulation capacities.

Furthermore, each ESU has two operational states, i.e., the discharging state as a generating unit and the charging state as a controllable load. Thus, the constraints are divided into two categories with the discharging power constraint being

$$0 \le P_{bj} \le P_{bj}^{dch,max}, \quad (9)$$

and the charging power constraint being

$$-P_{bj}^{ch,max} \le P_{bj} \le 0. \quad (10)$$

Based on the established profit maximization optimization problem, MGCC in micrograms is responsible for optimizing the active power output of all micro units so as to acquire the maximal profit via distributing the power demand among micro units or tie line between the microgrid and main grid.

Remark 1. *The generation cost function of distributed generators is always expressed by a general quadratic form [30], [22], or a piecewise quadratic cost function [31], or a nonconvex cost function [32], and so on. While, the quadratic cost function is commonly utilized by power engineers or theoretical research. As for distributed energy storage, there are various types of electricity storage methods, such as pumped hydro storage, thermal energy storage, chemical storage, and flywheel energy storage [33]. The function of electricity storage is similar to the charging and discharging process of a battery. Initially, the power energy must be transformed into another form of storable energy and to be transformed back when needed. Taking the pumped hydro storage [34] as an example, the principle is generally well known: during periods when power demand is low, water stations use electricity to pump the water from the lower reservoir to the upper reservoir. When power demand is very high, the water flows out of the upper reservoir and activates the turbines to generate high-value electricity for peak hours. There are many possible techniques for energy storage, found in practically all forms of energy: mechanical, chemical, and thermal. Therefore, the operation cost of buffer storage should be considered.*

The operation cost of the ESU is just like a power plant and the difference lies in the fact that there are two stages, i.e., charging and discharging process. For a ESU plant with a specified energy capacity, the economic dispatch of it can be achieved by maximizing the fuel-cost savings of thermal units. Additional fuel cost is needed to supply the equivalent power without ESU. Therefore, we still assume the cost function of operating energy storage units follows a quadratic function, provided in the problem formulation subsection.

III. Distributed λ-Iteration Optimization

A. Solution without Power Constraints

If there are no capacity constraints for the DGs and ESUs, then the optimization model reduces to be the objective function (6) with the equality constraint (7). By eliminating the variable P_E in the objective function through the equality $P_E = P_D - \sum_{i=1}^{N_1} P_{gi} - \sum_{j=1}^{N_2} P_{bj}$, one has the following minimum optimization function

$$\begin{aligned}&\min F_2(P_{gi}, P_{bj})\\&=\sum_{i=1}^{N_1}[C_{gi}(P_{gi})-\rho_e P_{gi}]+\sum_{j=1}^{N_2}[C_{bj}(P_{bj})-\rho_e P_{bj}]\end{aligned} \quad (11)$$

subject to constraint (7). We furthermore simplify the optimization model by augmenting the optimization variable $P = [P_{g1}, \ldots, P_{gN_1}, P_{b1}, \ldots, P_{bN_2}]$ and denote $N = N_1+N_2$, then such an optimization problem (11) and (7) is equivalent to

$$\begin{cases}\min F_3(P)=\sum_{i=1}^{N}\tilde{C}_i(P_i)=\sum_{i=1}^{N}[C_i(P_i)-\rho_e P_i]\\ \text{s.t.}\quad \sum_{i=1}^{N}P_i+P_E=P_D\end{cases} \quad (12)$$

The well-known solution to such an optimization problem (12) is the equal incremental cost criterion, i.e., $\frac{\partial \tilde{C}_i(P_i)}{\partial P_i} = \frac{\partial \tilde{C}_j(P_j)}{\partial P_j} = \lambda^*$, $\forall 1 \le i,j \le N$ and $\sum_{i=1}^{N} P_i + P_E = P_D$, where λ^* is called the optimal incremental cost, which can be calculated by

$$\lambda^* = \frac{P_D - P_E - \sum_{i=1}^{N}\alpha_i}{\sum_{i=1}^{N}\beta_i} - \rho_e.$$

If the optimal value λ^* is shared with each unit, then the optimal power output for ith unit can be calculated as $P_i^* = \beta_i(\lambda^* + \rho_e) + \alpha_i$, $\forall 1 \le i \le N$, where $\alpha = [\alpha_1, \ldots, \alpha_N] = [\alpha_{1,1}, \ldots, \alpha_{1,N_1}, \alpha_{2,1}, \ldots, \alpha_{2,N_2}]$, similarly for parameters β_i, γ_i.

The maximal profit $F_1^{\max}$ can be calculated by $F_1^{\max} = (\rho_d - \rho_e)P_D - F_3^{\min}$ through the optimal solution of the optimization problem (12).

We introduce a distributed λ-iteration algorithm to solve the centralized optimization problem. Each unit cannot acquire the optimal incremental cost in the distributed scenario. In order to update its power output, units try to estimate the optimal value of λ^*. Suppose the real-time estimation of ith unit is $\hat{\lambda}_i(t)$, which is characterized by the following distributed differential equation

$$\dot{\hat{\lambda}}_i(t)=\mu\sum_{j=1}^{N}a_{ij}\big(\hat{\lambda}_j(t)-\hat{\lambda}_i(t)\big)-d_i\mu\big(\hat{\lambda}_i(t)-\lambda_0(t)\big), \quad (13)$$

where μ is the coupling strength of the distributed protocol, a_{ij} is the element of the adjacent matrix A of the communication topology among the participating units; and $d_i = 1$ if the ith unit is pinned by the MGCC, otherwise $d_i = 0$. The initial estimation value $\hat{\lambda}_i(0) = \frac{P_i(0)-\alpha_i}{\beta_i} - \rho_e$, and $P_i(0)$ is the initial state of ith unit. And $\lambda_0(t)$ is the pinning signal generated from MGCC, which is updated by

$$\begin{cases}\dot{\lambda}_0(t)=\kappa\big(P_D-P_E-\sum_{i=1}^{N}P_i(t)\big),\\ P_i(t)=\beta_i(\hat{\lambda}_i(t)+\rho_e)+\alpha_i,\end{cases} \quad (14)$$

and the initial value $\lambda_0(0)$ is set to be the average value of all the pinned units, i.e., $\lambda_0(0) = \mathrm{ave}\big(\hat{\lambda}_i^{pin}(0)\big)$.

Remark 2. *If all units are pinned by the MGCC, then $d_i = 1, \forall 1 \le i \le N$, which reduces to be the centralized optimization algorithm (traditional λ-iteration algorithm). On the other hand, the optimal incremental cost λ^* can be calculated by MGCC under a cooperative scenario where the private cost information of all units are reported to MGCC, that is, $\lambda_0(t) \equiv \lambda^*$. Then, the estimation value of each unit will converge to the optimal value even with the distributed algorithm (13) under the assumption that the joint communication topology has a directed spanning tree.*

In the implementation of the distributed iteration algorithm, continuous-time differential equations (13) and (14) need to be transformed to discrete-time difference equations. Here, we utilize the Euler method to derive the discrete-time system:

$$\begin{cases}\hat{\lambda}_i(k+1)=\hat{\lambda}_i(k)+\mu h\sum_{j=1}^{N}a_{ij}\big(\hat{\lambda}_j(k)-\hat{\lambda}_i(k)\big)\\ \qquad\qquad -d_i\mu h\big(\hat{\lambda}_i(k)-\lambda_0(k)\big)\\ \lambda_0(k+1)=\lambda_0(k)+\kappa h\big(P_D-P_E-\sum_{i=1}^{N}P_i(k)\big)\end{cases} \quad (15)$$

and the output power of ith unit is calculated by

$$P_i(k)=\beta_i(\hat{\lambda}_i(k)+\rho_e)+\alpha_i, \quad (16)$$

where h is the discretization step, i.e., the sampling period for the practical operation system.

Theorem 1. *Suppose the pinning joint communication topology between MGCC and participate units is connected for an undirected topology or has a directed spanning tree for a directed topology, the proposed distributed λ-iteration algorithm 1 solves the optimization problem (12), i.e., the estimated incremental cost $\hat{\lambda}_i$ and output power P_i asymptotically converge to the optimal values λ^* and P_i^* globally, respectively.*

Proof: To begin with, we define two error variables, the estimation error variable $e_i(t) = \hat{\lambda}_i(t) - \lambda_0(t)$ and the translation variable $r(t) = \lambda_0(t) - \lambda^*$. According to the equilibrium equation of (14), one has

$$P_D - P_E - \sum_{i=1}^{N}[\beta_i(\lambda^*+\rho_e)+\alpha_i] = 0. \quad (17)$$

Furthermore, it is easy to derive the error system

$$\begin{cases}\dot{e}_i(t)=\mu\sum_{j=1}^{N}a_{ij}\big(e_j(t)-e_i(t)\big)-d_i\mu e_i(t)-\dot{r}(t),\\ \dot{r}(t)=-\kappa\sum_{i=1}^{N}\beta_i\big(e_i(t)+r(t)\big).\end{cases} \quad (18)$$

Algorithm 1 Distributed λ-iteration Optimization

Require: Load demand P_D and exchange power of the tie-line P_E;
Laplacian matrix L and Pinning matrix D;
Initial states of units $P_i(0)$;
Value for stopping rule TOLERANCE;
Pinning joint communication topology has a directed spanning tree.

Ensure: Optimal incremental cost λ^* and output power P_i^*.

1: Solve the linear matrix equation $(L+D)\xi = \mathbf{1}_N$ to get a positive column vector $\xi = [\xi_1, \ldots, \xi_N]$ and set $\Theta = \mathrm{diag}\{\xi\}$ and calculate $\lambda_{\min} = \lambda_{\min}\{\Theta(L+D) + (L+D)^T\Theta\}$.

2: Set a positive regulation gain κ and then choose the gain μ by

$$\mu > \frac{2\kappa}{\lambda_{\min}}\Theta\mathbf{1}_N B + \frac{\kappa}{2\lambda_{\min}B\mathbf{1}_N}M,$$

where $M = [\Theta\mathbf{1}_N B\mathbf{1}_N - B^T][(\Theta\mathbf{1}_N B\mathbf{1}_N)^T - B]$ and $B = [\beta_1, \ldots, \beta_N]$.

3: Run the distributed iterations (15) and (16) until

$$|P_D - P_E - \sum_{i=1}^{N} P_i(k)| < \text{TOLERANCE}.$$

4: Acquire the steady-state values $\lambda^* \approx \lambda_0(k)$;

5: **return** Optimal solutions $P_i^* \approx P_i(k)$.

By denoting $e(t) = [e_1(t), \ldots, e_N(t)]^T$, the error system (18) can be transformed into the following augmented one

$$\begin{cases} \dot{e}(t) = -\mu(L+D)e(t) + \kappa\mathbf{1}_N\big(Be(t) + B\mathbf{1}_N r(t)\big), \\ \dot{r}(t) = -\kappa B\big(\mathbf{1}_N r(t) + e(t)\big). \end{cases} \tag{19}$$

Then, consider the following Lyapunov candidate:

$$V(t) = \frac{1}{2}r^2(t) + \frac{1}{2}e^T(t)\Theta e(t), \tag{20}$$

where Θ is a positive definite matrix given in Algorithm 1. Then the time derivative of $V(t)$ along the solution of error system (19) is

$$\begin{aligned} \dot{V}(t) =& -\kappa r(t)(B\mathbf{1}_N)r(t) - \kappa r(t)Be(t) \\ &+ \kappa e^T(t)(\Theta\mathbf{1}_N B\mathbf{1}_N)r(t) + \kappa e^T(t)(\Theta\mathbf{1}_N B)e(t) \\ &- \frac{\mu}{2}e^T(t)\big(\Theta(L+D) + (L+D)^T\Theta\big)e(t) \\ \triangleq& z^T(t)\Omega z(t), \end{aligned}$$

where $z(t) = [r^T(t), e^T(t)]^T$ and Ω is given as follows:

$$\Omega = \begin{bmatrix} -\kappa B\mathbf{1}_N & \frac{\kappa}{2}[(\Theta\mathbf{1}_N B\mathbf{1}_N)^T - B] \\ * & \kappa\Theta\mathbf{1}_N B - \frac{\mu}{2}[\Theta(L+D) + (L+D)^T\Theta] \end{bmatrix}. \tag{21}$$

By the Schur complement Lemma, one knows that $\Omega < 0$ is equivalent to $\kappa > 0$ and

$$\kappa\Theta\mathbf{1}_N B - \frac{\mu}{2}[\Theta(L+D) + (L+D)^T\Theta] + \frac{\kappa M}{4B\mathbf{1}_N} < 0,$$

from which one can derive the scope of the parameter μ. This completes the proof, i.e., the estimated incremental cost $\hat{\lambda}_i$ asymptotically converge to the optimal values λ^* globally. ■

B. Solution with Power Constraints

When considering the regulation capacities of DGs and ESUs, expressed as

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \tag{22}$$

the optimal incremental cost λ^* satisfied the following optimum condition,

$$\begin{cases} \dfrac{P_i - \alpha_i}{\beta_i} = \lambda^*, & \text{for } P_i^{\min} \leq P_i \leq P_i^{\max}, \\ \dfrac{P_i - \alpha_i}{\beta_i} < \lambda^*, & \text{for } P_i = P_i^{\max}, \\ \dfrac{P_i - \alpha_i}{\beta_i} > \lambda^*, & \text{for } P_i = P_i^{\min}. \end{cases} \tag{23}$$

By denoting G_p as the set of units whose optimal outputs are their maximal or minimal capacities. Then the optimal incremental cost λ^* can be calculated by

$$\lambda^* = \frac{P_D - P_E - \sum_{i\in G_P} P_i^m - \sum_{i\notin G_p}\alpha_i}{\sum_{i\notin G_p}\beta_i} - \rho_e.$$

The distributed algorithm (15) considered the output constraint of $P_i(k)$ by the following updating equation:

$$\begin{aligned} P_i(k) &= g_i\big(\hat{\lambda}_i(k)\big), \\ &= \begin{cases} P_i^{\max}, & \text{if } \hat{\lambda}_i(k) > \overline{\lambda}_i \\ \beta_i(\hat{\lambda}_i(k) + \rho_e) + \alpha_i, & \text{if } \underline{\lambda}_i \leq \hat{\lambda}_i(k) \leq \overline{\lambda}_i \\ P_i^{\min}, & \text{if } \hat{\lambda}_i(k) < \underline{\lambda}_i \end{cases} \end{aligned} \tag{24}$$

where $\underline{\lambda}_i = (P_i^{\min} - \alpha_i)/\beta_i$ and $\overline{\lambda}_i = (P_i^{\max} - \alpha_i)/\beta_i$.

Algorithm 2 Distributed λ-iteration Constrainted Optimization

Require: Load demand P_D and exchange power of the tie-line P_E;
Laplacian matrix L and Pinning matrix D;
Initial states of units $P_i(0)$;
Value for stopping rule TOLERANCE;
Pinning joint communication topology has a directed spanning tree.

Ensure: Optimal incremental cost λ^* and output power P_i^*.

1: Set the gains μ and κ in the distributed iteration (15) by Algorithm 1 and update the output $P_i(k)$ by (24) until

$$|P_D - P_E - \sum_{i=1}^{N} P_i(k)| < \text{TOLERANCE}.$$

2: Acquire the steady-state values $\lambda^* \approx \lambda_0(k)$;

3: **return** Optimal solutions $P_i^* \approx P_i(k)$.

Proposition 1. *Suppose the joint communication topology between MGCC and participate units is connected for an undirected topology or has a directed spanning tree for a directed topology, the proposed distributed λ-iteration algorithm (15) based on (24) solves the optimization problem (12) with the additional constraint (22), i.e., the output power P_i converge to the optimal value P_i^* with finite steps.*

This Proposition is a generalization of Theorem 1, where the capacity constraints have been taken into account in the optimization iteration. When the microgird is operated in a grid-connected mode, the coordinated active power dispatch problem of the microgid is always solvable due to the existence of the exchanged power with the main grid. When the microgird is operated in an isolated mode, the coordinated active power dispatch problem of the microgid is solvable under the assumption that all the power supply from the sources is able to satisfy the demand of the load. Therefore, the optimization problem is always solvable.

Proposition 1 under these two operating scenarios can be classified as the following two cases: (1) If the intersection of the incremental cost intervals $\bigcap_{1\leq i\leq N}[\underline{\lambda}_i, \bar{\lambda}_i] \neq \emptyset$, then there exists a common optimal incremental cost λ^*. And all the incremental costs $\lambda_i (1 \leq i \leq N)$ will converge to the optimal value within finite steps; (2) If the intersection of the incremental cost intervals $\bigcap_{1\leq i\leq N}[\underline{\lambda}_i, \bar{\lambda}_i] = \emptyset$, then partial incremental costs $\lambda_i, i \in [1, N]$ converge to the boundary of the incremental cost interval, i.e., some units with lower operating costs operate in their maximum output powers' states. Thus, according to Theorem 1, the iteration updating in Proposition 1 is valid in the practical operation.

Remark 3. *Distributed consensus algorithms of multi-agent systems have been adopted extensively to solve the economic dispatch problems in power systems, such as [35], [36], [37], [38]. Compared with these published works, the main difference of this paper lies in that we introduce a pinning term in the distributed iteration such that the distributed algorithm turns out to be a leader-following consensus algorithm [39], [40], [41] and coincides with the distributed λ-iteration algorithm. This algorithm has no restrictions on the initial states of the participating units and can be applicable to any initial states of micro units, which can either be utilized for off-line calculation or be used for on-line operation.*

Remark 4. *The distributed algorithm (13) can be replaced with a fixed-time or a finite-time one, which can speed up the convergence speed of the algorithm, given as*

$$\begin{aligned}\dot{\hat{\lambda}}_i =& \mu_1 \sum_{j=1}^{N} a_{ij} \mathrm{sig}^p\big(\hat{\lambda}_j - \hat{\lambda}_i\big) - d_i \mu_1 \mathrm{sig}^p\big(\hat{\lambda}_i - \lambda_0\big) \\ &+ \mu_2 \sum_{j=1}^{N} a_{ij} \mathrm{sig}^q\big(\hat{\lambda}_j - \hat{\lambda}_i\big) - d_i \mu_2 \mathrm{sig}^q\big(\hat{\lambda}_i - \lambda_0\big),\end{aligned}$$

where μ_1, μ_2 are the coupling strengths of the distributed protocol, a_{ij} is the element of the adjacent matrix A of the communication topology among the participating units; and $d_i = 1$ if the ith unit is pinned by the MGCC, otherwise $d_i = 0$. The function $\mathrm{sig}^p(\cdot) = \mathrm{sign}(\cdot)|\cdot|^p$. Especially, when $p = 1$, $q = 0$ and $\mathrm{sig}(x) = x$, the above algorithm reduces to be the previous one (13).

IV. Case Studies

In this section, the proposed distributed λ-iteration algorithms are tested on a seven-bus microgrid system and the physical topology structure is given in Fig. 3, where there are seven DGs and three ESUs and seven-bus loads. The red dashed lines represent the communication links among MGCC and units. The cost coefficients and capacity constraints for these units are provided in Tab. I.

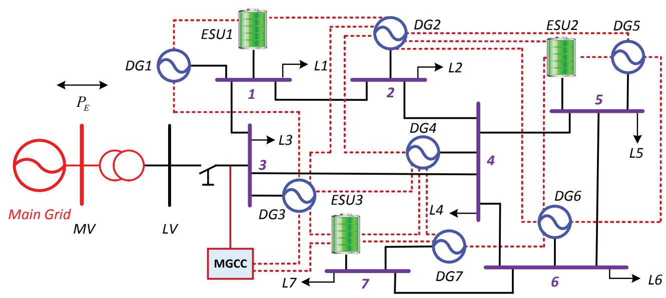

Fig. 3. Seven-bus microgrid with seven DGs and three ESUs.

TABLE I
DG and ESU's private parameters.

	DGs' parameters $(\alpha, \beta, \gamma(\times 10^3))$; (unit: KW)				
DG	$\alpha_{1,i}$	$\beta_{1,i}$	$\gamma_{1,i}$	$P_{g,i}^{\min}$	$P_{g,i}^{\max}$
DG1	-0.0593	0.0069	-0.2032	10	60
DG2	-0.0313	0.0050	-0.0502	10	60
DG3	-0.0219	0.0064	-0.0063	8	60
DG4	-0.0120	0.0048	0.0628	3.8	40
DG5	-0.0245	0.0061	-0.0073	5.4	45
DG6	-0.0345	0.0048	-0.0616	4.2	18
DG7	-0.0065	0.0053	0.0470	7.8	45
	ESUs' parameters $(\alpha, \beta, \gamma(\times 10^3))$; (unit: KW)				
ESU	$\alpha_{2,i}$	$\beta_{2,i}$	$\gamma_{2,i}$	$P_{b,i}^{dch,max}$	$P_{b,i}^{ch,max}$
ESU1	-0.3170	0.0333	-1.4153	25	20
ESU2	-0.1331	0.0156	-0.4891	30	25
ESU3	-0.1677	0.0192	-0.6931	45	40

A. Case 1: The full participation of DGs and ESUs

We consider the discharging mode of ESUs, i.e., the storage units serve as generating units to provide the active power sharing and the communication topology (red dashed lines) is provided in Fig. 3, where only the third DG and the third ESU are pinned by MGCC. Suppose the power demand in this MG is $P_D = 370$ kW and the injection power from the main grid is $P_E = 120$ kW; the wholesale market price $\rho_e = 1.2$ \$/kWh, and the retail price $\rho_d = 1.8$ \$/kWh. The initial states of DGs are given by $P_g(0) = [30, 20, 40, 15, 18, 6, 20]$ KW and the initial states of ESUs are given by $P_b(0) = [10, 12, 21]$ kW. By setting $\kappa = 0.01$, one can derive that $\mu > 1.7773$ and we set $\mu = 3.82$ in the simulation. The sampling period is set to be $h = 0.02$ and the stop rule TOLERANCE $= 0.4$, one can solve the profit maximal problem by the proposed distributed λ-iteration algorithms.

Then, it is easy to derive the optimal incremental cost $\lambda^* = 8.6431$ (see Fig. 4) by utilizing the distributed algorithm 1 without considering the capacities of each units and the corresponding active power outputs for DGs are $P_{g1}^* = 9.0418$ kW, $P_{g2}^* = 17.3866$ kW, $P_{g3}^* = 41.2305$ kW, $P_{g4}^* = 34.8194$ kW, $P_{g5}^* = 35.5010$ kW, $P_{g6}^* = 12.8470$ kW, $P_{g7}^* = 45.8619$ kW and the discharging power of ESUs are $P_{b1}^* = 11.0689$

kW, $P^*_{b2} = 20.6586$ kW, $P^*_{b3} = 21.5775$ kW, which are shown in Fig. 5.

Here, Figs. 4 and 5 illustrate the dynamic response curves for the distributed system (15) and (16), but the distributed λ-iteration algorithm breaks out the circulation by the stop rule TOLERANCE $= 0.4$ (see Fig. 6).

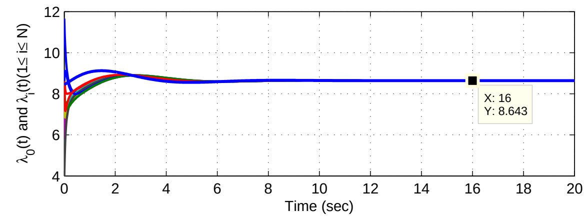

Fig. 4. Case 1: The estimated incremental costs by Algorithm 1.

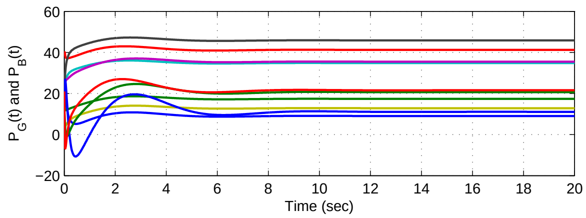

Fig. 5. Case 1: The active power outputs of DGs/ESUs by Algorithm 1.

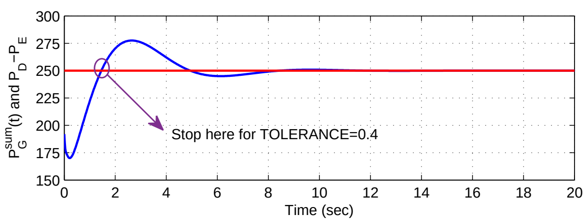

Fig. 6. Case 1: The supply and demand balance by Algorithm 1.

Furthermore, we utilize the distributed iterations (15) and (24) by considering the capacity constraints of the units, it can be derived that the corresponding active power outputs for DGs are $P^*_{g1} = 9.3565$ kW, $P^*_{g2} = 17.4053$ kW, $P^*_{g3} = 41.6542$ kW, $P^*_{g4} = 34.7387$ kW, $P^*_{g5} = 35.5236$ kW, $P^*_{g6} = 12.8647$ kW, $P^*_{g7} = 45$ kW and the discharging powers of ESUs are $P^*_{b1} = 11.8866$ kW, $P^*_{b2} = 20.7166$ kW, $P^*_{b3} = 20.8538$ kW, which are shown in Fig. 8 and the incremental cost for each unit is given in Fig. 7. They illustrate the dynamic response curves for the distributed system (15) and (24), but the distributed λ-iteration algorithm breaks out the circulation by the stop rule TOLERANCE $= 0.4$ (see Fig. 9).

Employing the proposed algorithms, all units can coordinate each other to minimize the total cost while converging to the optimal operation point subject to the power balance constraint. On the other hand, as can be seen form the simulation results, there is a common consensus state λ^* for all participating units in the scenario of without capacity constraints and all participating units do not have a common consensus state λ^* due to the fact that part of units operate at their boundaries.

B. Case 2: Exit of ESUs and Time-varying Demand

In this simulation scenario, the cost and capacity parameters of DGs and ESUs are the same as in Case 1. The main

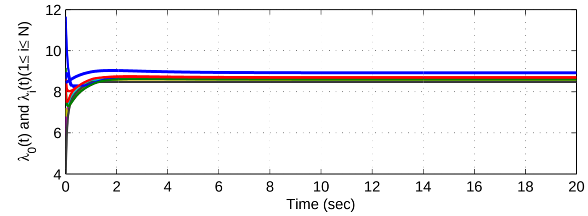

Fig. 7. Case 1: The estimated incremental costs by Algorithm 2.

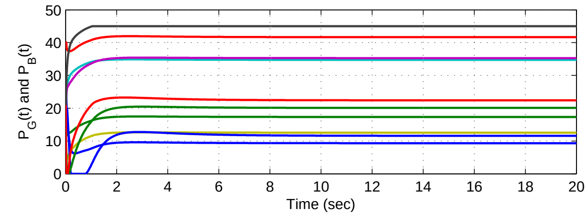

Fig. 8. Case 1: The active power outputs of DGs/ESUs by Algorithm 2.

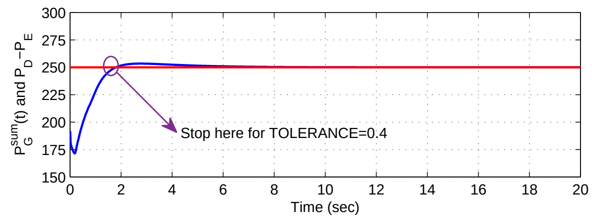

Fig. 9. Case 1: The supply and demand balance by Algorithm 2.

difference is that we consider the case that the load demand is a time-varying one in different dispatch periods and the participating unit may be plugged-in or plugged-out. Here, we only consider two dispatch periods in the simulation. Suppose the load demand is $P_D = 370$ kW and the injection power from the main grid is $P_E = 120$ kW; the wholesale market price $\rho_e = 1.2$ \$/kWh and the retail price $\rho_d = 1.8$ \$/kWh in the first dispatch period and in the second dispatch period (such as 15 min dispatch period), the load demand turns out to be $P_D = 400$ kW and the injection power from the main grid is $P_E = 170$ kW; the wholesale market price $\rho_e = 1.4$ \$/kWh and the retail price $\rho_d = 2.0$ \$/kWh. Meanwhile, suppose all DGs and ESUs participate the active power allocation in the first dispatch period and all ESUs exit in the second dispatch period and the load demand will be shared by all DGs. The communication topology of all DGs in this scenario is given in Fig. 10 and only the third DG is pinned by the MGCC.

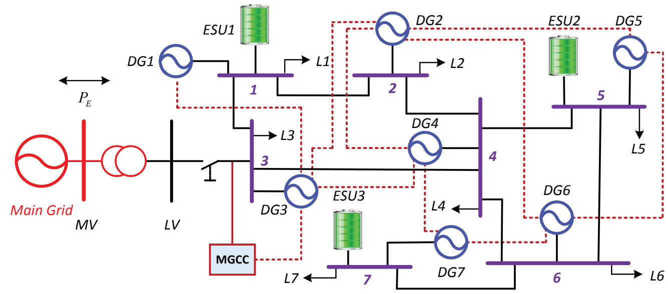

Fig. 10. The communication topology among seven DGs.

Next, the initial states of all DGs and ESUs in the first dispatch period are the same as in Case 1 and the initial states of all DGs in the second dispatch period are the steady-state of the result of first dispatch period. By setting $\kappa_1 = 0.01$ and $\mu_1 = 3.82$ in the first dispatch period; and $\kappa_2 = 0.01$ and $\mu_2 = 6.82$ in the next dispatch period; and the sampling period $h = 0.02$; the stop rule $\text{TOLERANCE}_1 = 0.4$ and $\text{TOLERANCE}_2 = 0.2$, one can solve the profit maximal problem by the proposed distributed λ-iteration algorithms.

The simulation results of all DGs without and with capacity constraints via the distributed λ-iteration algorithms 1 and 2 are provided in Figs. 11–16.

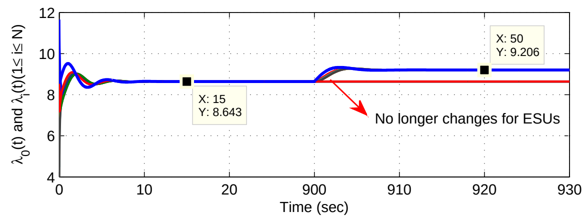

Fig. 11. Case 2: The estimated incremental costs by Algorithm 1.

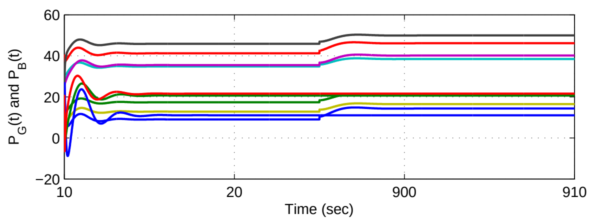

Fig. 12. Case 2: The active power outputs of DGs/ESUs by Algorithm 1.

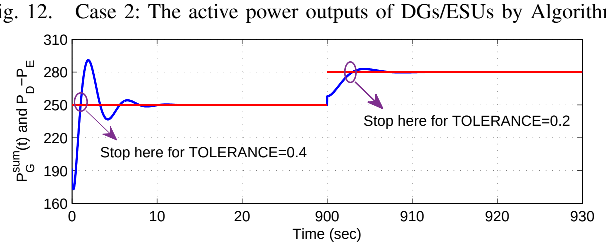

Fig. 13. Case 2: The supply and demand balance by Algorithm 1.

In the second dispatch period, all ESUs exit the dispatch system and therefore their output powers stay at the steady-states of the previous dispatch period. The DGs will accomplish the active power demand by updating their output according to the distributed λ-iteration algorithms. As can be seen from the simulation results, all DGs operate to the new equilibrium under the time-varying load demand.

C. Case 3: The Plugging-in DGs to Share the Active Power

In this mode, we consider the scenario that two additional DGs and bus loads are connected to the microgrid, labeled by $DG8, DG9$ and $L8, L9$. The new communication topology after the pulgging in the additional DGs is shown in Fig. 17, and the private coefficients for DGs are provided in Table II.

Suppose the load demand is $P_D = 410$ kW and the injection power from the main grid is $P_D = 120$ kW; the whole market price $\rho_e = 1.2$ \$/kWh and the retail price $\rho_d = 1.8$ \$/kWh are the same before and after the plugging-in of the additional DGs and loads. While, after the connecting of DGs, the new load

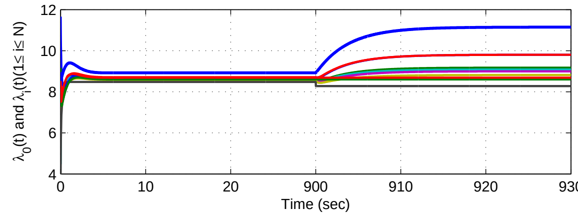

Fig. 14. Case 2: The estimated incremental costs by Algorithm 2.

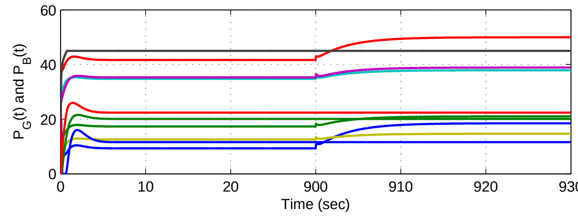

Fig. 15. Case 2: The active power outputs of DGs/ESUs by Algorithm 2.

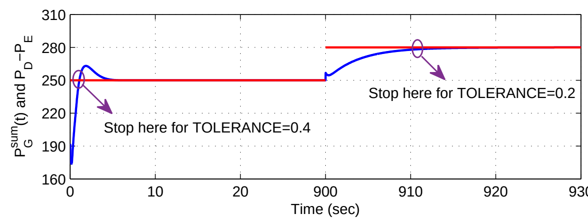

Fig. 16. Case 2: The supply and demand balance by Algorithm 2.

TABLE II
THE PRIVATE PARAMETERS FOR DG8 AND DG9.

	DGs' parameters ($\alpha, \beta, \gamma, \times 10^3$; *unit: KW*)				
DG	$\alpha_{1,i}$	$\beta_{1,i}$	$\gamma_{1,i}$	$P_{g,i}^{\min}$	$P_{g,i}^{\max}$
DG8	-0.0550	0.0596	-0.1861	8	42
DG9	-0.0281	0.0379	-0.0456	8	40

demand increases to $P_D = 445$ kW and the injection power from the main grid reduces to $P_D = 115$ kW. The initial states of the DGs and ESUs are the same with values in Case 1, and the initial states of $DG8$ and $DG9$ are $P_{g8}(0) = 10$ kW, $P_{g9}(0) = 12$ kW.

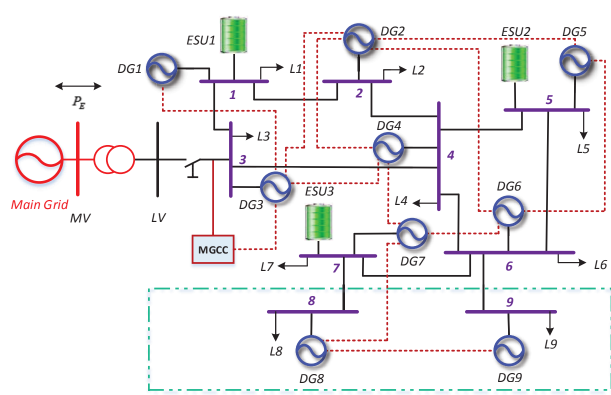

Fig. 17. The communication topology among nine DGs and three ESUs.

Here, the dispatch period before the connecting of additional DGs and loads is labeled as the first dispatch period and the latter dispatch period is labeled as the second dispatch period. Along with the active power changes of load and units, the power output of each units need to adjust its outputs so as to achieve the new equilibrium. By setting $\kappa_1 = 0.01$, one can derive that $\mu_1 > 1.7773$; and $\kappa_2 = 0.01$, one can derive that $\mu_1 > 1.4732$ and we set $\mu_1 = 3.82$ in the first dispatch period and $\mu_2 = 6.82$ in the second dispatch period; and the sampling period $h = 0.02$; the stop rule $\text{TOLERANCE}_1 = 0.4$ and $\text{TOLERANCE}_2 = 0.2$, one can solve the profit maximal problem by the proposed distributed λ-iteration algorithms. The optimal output power can be derived by the proposed distributed algorithm through the optimal incremental cost.

The simulation results of all DGs without and with capacity constraints via the distributed λ-iteration algorithms 1 and 2 are provided in Figs. 18–23.

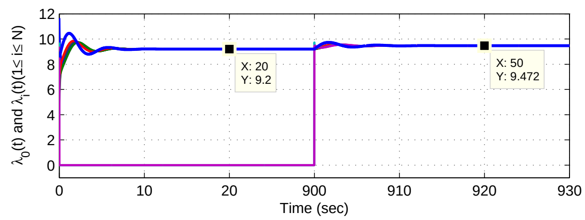

Fig. 18. Case 3: The estimated incremental costs by Algorithm 1.

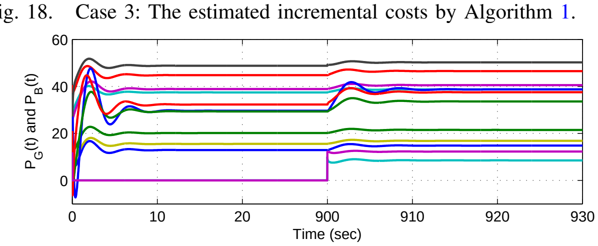

Fig. 19. Case 3: The active power outputs of DGs/ESUs by Algorithm 1.

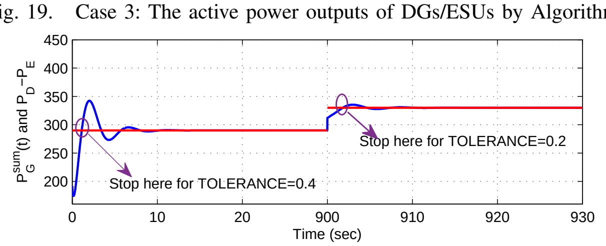

Fig. 20. Case 3: The supply and demand balance by Algorithm 1.

In the second dispatch period, additional loads and DGs are connected to the microgird suddenly, which has caused a shock to the microgrid and the steady operation state of the microgrid is broken at this time. As can be seen form the simulation results, the proposed distributed λ iteration algorithm has shown good scalability and robustness in the whole dispatch process. The proposed distributed algorithm can restore the steady operation of the microgrid and the new equilibrium is achieved after the disturbance.

V. Discussion

This paper generalizes the traditional centralized λ-iteration algorithm to a distributed one, which is coinsided with leader-following consensus control in multi-agent systems. In the centralized λ-iteration algorithm, the total power response and

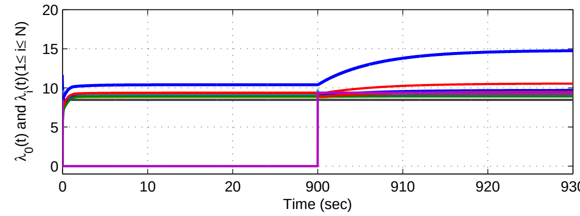

Fig. 21. Case 3: The estimated incremental costs by Algorithm 2.

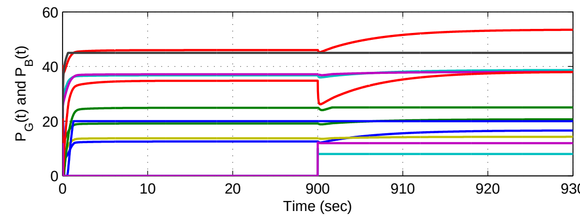

Fig. 22. Case 3: The active power outputs of DGs/ESUs by Algorithm 2.

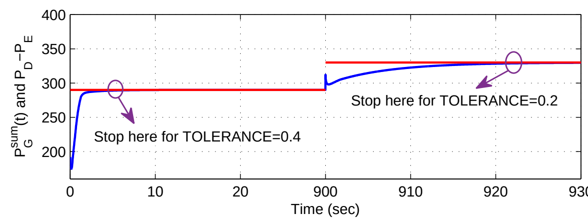

Fig. 23. Case 3: The supply and demand balance by Algorithm 2.

the estimation value of the optimal incremental cost are both global information. The MGCC is in charge of the data collection and information broadcasting. All the iteration calculation and information processing are performed by the MGCC. However, in the proposed distributed λ-iteration algorithm, the global information is only shared with pinned units instead of all micro units. On the other hand, the optimal incremental cost is estimated by each units through local interactions. Thus, local WiFi connections are sufficient for the communication requirement. Compared with the traditional centralized optimization algorithm for the main grid, distributed strategy is more suitable for microgrid since the number of DGs in microgrids is more flexible as a result of disconnecting or adding units.

1) Implementation: In the proposed distributed λ-iteration algorithm, a communication network is needed to realize the information sharing among multiple DGs and each DG can connect to its neighbors by WiFi connection or wired connection as long as the directed communication topology has a directed spanning tree rooted at the centralized pinner or the undirected communication topology is connected. On the other hand, the MGCC only needs to pin its neighboring DGs and the initial value for the centralized pinner is set to be the averaged initial states of the pinned DGs; the initial value for each estimator is initialized by the active power output of each DG.

2) Operation Cost: In the proposed distributed lambda iteration algorithm, the computation cost of the MGCC is greatly reduced compared with the traditional centralized one. The iteration computation in the distributed algorithm has

been decomposed to all micro units and each unit updates it estimation value according to the distributed protocol by communicating with its neighbours. In the centralized lambda iteration algorithm, MGCC is the central controller and all units are connected to it by setting bidirectional communication links with MGCC; while in the distributed lambda iteration algorithm, the communication structure has been changed to a more general one and not all units have to be connected to the MGCC and only the pinned units are connected to the MGCC.

3) Applications: The distributed λ-iteration algorithm can be used in different application scenarios, such as active power allocation among multiple DGs and ESUs in a microgrid in this paper; energy optimization among multiple microgrids to provide ancillary service; charging and discharging control for multiple ESUs to provide microgrid tie-line smoothing services; load shedding among multiple load aggregators to provide frequency regulation service and so on.

In summary, the proposed distributed λ-iteration algorithm based on the leader-following consensus strategy is an efficient active power allocation algorithm for multiple interactive units in a microgrid.

VI. Conclusion and Future Works

This paper proposed a distributed λ-iteration algorithm to solve economic operation problem of microgirds with/without unit constraints, which could be used to deal with economic dispatch problems in power systems as well. Compared with the existing distributed optimization methods, our proposed algorithm considers the global active power constraint by adding a virtual pinner and it can deal with optimization problem with any initial states. The optimization algorithm is partially distributed such that each unit in the mircrogrid only communicates with its neighbors and part of the units have direct communications with MGCC as long as the joint communication topology between MGCC and participate units is connected for an undirected topology or has a directed spanning tree for a directed topology. The distributed algorithm can also enable the plug-and-play of some extra units in microgrids. Lastly, simulation results validated the effectiveness of the proposed algorithm.

Future works will be focused on the reference tracking implementation problems of source controllers in the terminal DGs or ESUs.

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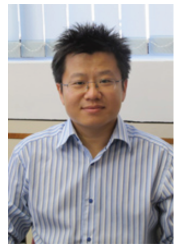

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