Over-the-air Computation for IoT Networks: Computing Multiple Functions with Antenna Arrays

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***Abstract*—Over-the-air computation combines communication and computation efficiently by utilizing the superposition property of wireless channels, when Internet of Things (IoT) networks focus more on the computed functions than the individual messages. In this work, we study the computation of multiple linear functions of Gaussian sources over-the-air using antenna arrays at both the IoT devices and the IoT access point (AP). The key challenges in this study are the intra-node interference of multiple functions, the non-uniform fading between different IoT devices and the massive channel state information (CSI) required at the IoT AP. We propose a novel transmitter design at the IoT devices with zero-forcing beamforming to cancel the intra-node interference and uniform-forcing power control to compensate the non-uniform fading. In order to avoid massive CSI requirement, receive antenna selection is adopted at the IoT AP and a corresponding signaling procedure is proposed utilizing the "OR" property of the wireless channel. The performance of the proposed transceiver design is analyzed. The closed-form expression for the mean squared function error (MSFE) outage is derived. Due to the complexity of the expression, an asymptotic analysis of the MSFE outage is further provided to demonstrate the diversity order in terms of the transmit power constraint and the number of IoT devices. Simulation results are presented to show the performance of the proposed design.**

***Index Terms*—antenna array, distributed data aggregation, function computing, Internet of Things, multiple access scheme, wireless sensor network**

I. Introduction

The 5th generation cellular system is predicted to provide an Internet of Things (IoT) that interconnects up to 1 trillion devices with a million connections per square kilometer [1]. This raises new challenges to the distributed data aggregation for IoT networks [2].

Unlike conventional wireless networks whose main objective is to provide end-to-end information transmission, IoT networks are more interested in the functions of the observations rather than the individual observations. For example, an IoT-based monitoring system does not care about the abundant individual observations but the computed functions thereof, such as the sum and the mean [3]. The big data computing was made to extract meaningful data from a large dataset for large-scale smart grid in [4], which deleted a large amount meaningless data before communication. In [5], the traffic volume was predicated before making service degradability to alleviate the communication pressure if the network has a heavy load. A statistical machine learning approach was employed to identify the anomalies within the incoming dataset collected via various probes in the network [6]. Many event-driven IoT applications define the triggering event based on the functions of observations, such as the weighted linear combination [7]. This makes the traditional "communication-and-computation separation" method inefficient. Reconstructing a function over wireless multiple-access channel (MAC), referred to as "over-the-air computation", provides great potential for IoT network to compute the target function using a summation structure in an efficient way. It utilizes the superposition property of wireless channel instead of making the interference between different IoT devices orthogonal.

The study in over-the-air computation first started from the information-theoretic point of view. In the seminal work [8], B. Nazer and M. Gastpar pointed out that it is beneficial to compute the sum of Gaussian sources over a Gaussian MAC, which combines communication and computation efficiently and harnesses the interference between different nodes. The achievable aggregation rate of type-sensitive functions (e.g. mean, mode, median, etc.) and that of type-threshold functions (e.g. max, min, range, etc.) was defined and derived in [9]. These works lay formulation in over-the-air computation for IoT network. When the target functions match the algebraic structure of channel, there is significant performance gain can be obtained by jointly designing communication and computation [10]. When there is mismatch between the target functions and the channel structure, the impact on the achievable performance gains with joint communication and computation designs over separation-based designs has been analyzed in [11]. Considering the correlation of sources, the information-theoretic performance has been studied for linear functions

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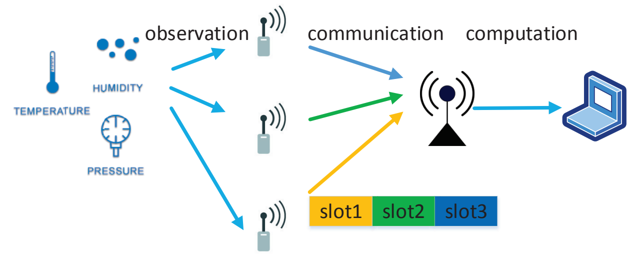
analog computation of two correlated Gaussian sources in [12].

In order to achieve reliable function, the use of channel coding in over-the-air computation has also been widely studied. Using nested lattice coding to compute the noisy modulo sum was investigated in [13] based on the linear property of nested lattice coding. Nested lattice coding has also been applied to compute-and-forward network to recover the combination of transmitted messages [14]. M. Goldenbaum *et al.* proposed a unified digital scheme to compute structured functions over-the-air in [15], where each node in the network first quantizes its real-valued pre-processed readings and then employs a nested lattice code to protect the sum of messages against Gaussian channel noise. Using randomized network coding through appropriate choice of the subspace codebooks at the source nodes was proposed for function computing in [16], where a lower bound on the number of transmissions required to ensure successful computation was provided. The over-the-air computation for a generalized IoT model consisting of multiple clusters was studied in [17], where the network was divided into several clusters with independent target functions computed. The risky virtual machines were captured in prior based on real data trace, thus guaranteeing high reliable virtual machines transferring among data centers in [18].

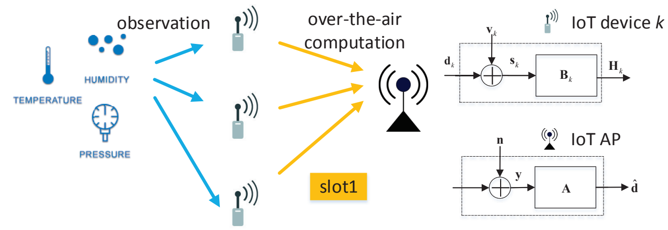
Since the nodes in IoT networks are generally low-power and low-cost, a practical way to realize over-the-air computation is through analog scheme. Uncoded transmission where the channel input of IoT devices is merely a scaled version of its noisy observation has been proved to be optimal for a standard Gaussian multiple-access channel in [19]. A robust analog function computation scheme was proposed in [20]. By employing random sequences, the proposed scheme is robust against synchronization errors. Utilizing retransmission to increase reliability, the achievable rate for analog computation was defined and analyzed in [21]. Considering the difficulty of gathering the channel state information (CSI) of all nodes, the effect of channel estimation error was studied in [22], [23]. The work in [24] selected a subset of sensors in an opportunistic way to improve the performance of function computation, which achieves a nonvanishing computation rate even when the number of sensors approaches infinity. Various experimental platforms have been built to verify the idea of analog over-the-air computation in [25]–[27].

To the best of our knowledge, the use of multiple functions in over-the-air computation has never been studied before. Although multi-antenna has been applied to compute-and-forward network [28], [29], it was intended to improve the communication rate using the multiplexity gain of multi-antenna and its key challenge is integer coefficient selection for adapting to the fading MAC [30], [31]. In the case of over-the-air computation, the coefficient of multiple functions becomes arbitrary and the key challenge becomes the transceiver design to create an equivalent MAC with the target coefficient.

Motivated by this observation, we study how to compute multiple functions over-the-air with antennas arrays at the IoT devices and the IoT access point (AP), where multiple linear combinations with arbitrary coefficient of Gaussian sources are computed over the Gaussian MAC. The main contributions of the work can be summarized as follows.



(a) Computation after data aggregation.



(b) Computation over-the-air.

Figure 1. The communication and computation model of the IoT network

- *A novel transceiver design*: The transmitter is designed to cancel the intra-node interference between multiple functions and compensate the non-uniform fading between different IoT devices. Also, receive antenna selection and its corresponding signaling procedure is proposed to avoid massive CSI requirement at the IoT AP.
- *The computation performance*: We define the mean squared function error (MSFE) to analyze the performance of the computation. The closed-form expression of MSFE outage is derived for Gaussian sources over Gaussian MAC. Due to its complexity, the asymptotic analysis with large transmit power constraint and large number of IoT devices is provided.
- *The diversity order*: The diversity order of MSFE outage is further derived in terms of the transmit power constraint and the number of IoT devices, which depends on the number of antennas, the number of functions and the correlation coefficient between different sources.

The remainder of this paper is organized as follows. Section II presents the system model. The transceiver and signaling procedure are designed in Section III. The performance is analyzed in Section IV. Simulation results and discussions are present in Section V, and conclusion is given in VI.

Throughout the paper, we will use boldface lowercase to refer to vectors and boldface uppercase to refer to matrices. The real and complex numbers are denoted as $\mathcal{R}$ and $\mathcal{C}$ respectively. Let $\mathbf{A}^H$ denote the conjugate transpose of a matrix $\mathbf{A}$ and let $\mathbf{A}^{-1}$ denote inverse of a matrix $\mathbf{A}$. Let $\|\mathbf{a}\|$ denote the norm of a vector $\mathbf{a}$, and let $\mathbf{a}^T$ denote the transpose of a vector $\mathbf{a}$.

II. System Model

We consider a IoT network composed of K IoT devices indexed by $k \in \{1, \cdots, K\}$ as illustrated in Fig. 1. Each IoT device observes L sources (e.g. temperature, humidity, pressure, etc.) indexed by $l \in \{1, \cdots, L\}$. The observations of IoT device k are expressed as an L-dimensions vector $\mathbf{d}_k \in \mathcal{R}^L$. And the target functions at the IoT AP are linear combinations of the observations which is expressed as

$$\mathbf{d} = \sum_{k=1}^{K} \mathbf{w}_k \mathbf{d}_k, \tag{1}$$

where $\mathbf{w}_k = \operatorname{diag}\{w_{k1} \cdots w_{kL}\}$ is the function coefficient matrix.

As illustrated in Fig. 1(a), the AP can aggregate the observations first and then computes the target function. The distributed data aggregation requires multiple access schemes (e.g. time division multiple access (TDMA), carrier sense multiple access (CSMA)), which requires multiple time slots and incurs a high latency.

If we utilize the summation property of wireless MAC to reconstruct the target function, we can avoid the multiple access scheme for data aggregation and compute the target function in one time slot as illustrated in Fig. 1(b).

The transmit vector $\mathbf{s}_k \in \mathcal{R}^L$ of IoT device k is

$$\mathbf{s}_k = \mathbf{d}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{v}_k \in \mathcal{R}^L$ is the observe noise vector. The elements of $\mathbf{d}_k$ and $\mathbf{v}_k$ are assumed to be Gaussian distributed, i.e., $d_{kl} \sim \mathcal{N}\left(0, \sigma_{dl}^2\right)$ and $v_{kl} \sim \mathcal{N}\left(0, \sigma_{vl}^2\right)$. Thus, the elements of $\mathbf{s}_k$ are also Gaussian distributed, i.e., $s_{kl} \sim \mathcal{N}\left(0, \sigma_{dl}^2 + \sigma_{vl}^2\right)$.

Each IoT device is equipped with N_t antennas and the AP is equipped with N_r antennas. We assume that $N_t \geq L$ and $N_r \geq L$. After coherent MAC, the estimated functions at the AP can be written as

$$\hat{\mathbf{d}} = \mathbf{A} \sum_{k=1}^{K} \mathbf{H}_k \mathbf{B}_k \mathbf{s}_k + \mathbf{A}\mathbf{n}, \tag{3}$$

where $\mathbf{B}_k \in \mathcal{C}^{N_t \times L}$ is the transmitter matrix of IoT device k, $\mathbf{H}_k \in \mathcal{C}^{N_r \times N_t}$ is the channel matrix between IoT device k and the AP with each element distributed as $\mathcal{CN}(0, 1)$, $\mathbf{A} \in \mathcal{C}^{L \times N_r}$ is the receiver matrix of the AP, and $\mathbf{n} \in \mathcal{C}^{N_r}$ is the receive noise vector with each element distributed as $\mathcal{CN}\left(0, \sigma_n^2\right)$.

Comparing the target functions in (1) with the estimated ones in (3), the corresponding error vector is $\mathbf{e} = \hat{\mathbf{d}} - \mathbf{d}$. We define the metrics of MSFE and MSFE outage to evaluate the performance of over-the-air computation.

Definition 1. (MSFE and MSFE outage) Given the target functions $\mathbf{d}$ and the estimated ones $\hat{\mathbf{d}}$, the estimation error vector can be calculated as $\mathbf{e} = \hat{\mathbf{d}} - \mathbf{d}$. Then, the MSFE is defined as

$$\text{MSFE} = \frac{E\left(\|\mathbf{e}\|^2\right)}{E\left(\|\mathbf{d}\|^2\right)}. \tag{4}$$

Given a MSFE threshold $\xi \in [0, 1]$, the corresponding MSFE outage is defined as

$$\mathrm{P}_{out} = \Pr\left(\text{MSFE} > \xi\right). \tag{5}$$

III. TRANSCEIVER DESIGN FOR MULTIPLE FUNCTIONS

According to the system model, the key challenges to compute multiple functions over-the-air are the intra-node interference of multiple functions, the non-uniform fading between different IoT devices, and the massive CSI gathering at the AP. In this section, we design the transceiver to combat these challenges. The signaling procedure is proposed to avoid massive CSI gathering at the AP.

A. The case that $N_r = L$

We first consider the case that $N_t \geq L$ and $N_r = L$. The case that $N_r > L$ will be discussed in the subsection III-B. In this case, the pseudo-inverse matrix of channel matrix exists. The transmitter matrix of IoT device k is designed as

$$\mathbf{B}_k = \sqrt{\eta} {\mathbf{H}_k}^H \left(\mathbf{H}_k {\mathbf{H}_k}^H\right)^{-1} \mathbf{w}_k, \tag{6}$$

where $\mathbf{w}_k$ is the function coefficient matrix in (1), and η is the power control factor considering transmit power constraint of the IoT device. Then the estimated functions in (3) can be rewritten as

$$\begin{aligned} \hat{\mathbf{d}} &= \mathbf{A} \sum_{k=1}^{K} \sqrt{\eta} \mathbf{H}_k {\mathbf{H}_k}^H \left(\mathbf{H}_k {\mathbf{H}_k}^H\right)^{-1} \mathbf{w}_k \mathbf{s}_k + \mathbf{A}\mathbf{n} \\ &= \sqrt{\eta} \mathbf{A} \sum_{k=1}^{K} \mathbf{w}_k \mathbf{s}_k + \mathbf{A}\mathbf{n} \end{aligned}, \tag{7}$$

where the non-uniform fading is compensated to the uniform level $\sqrt{\eta}$. If

$$\mathbf{A} = \frac{\mathbf{I}_L}{\sqrt{\eta}}, \tag{8}$$

we have that

$$\hat{\mathbf{d}} = \sum_{k=1}^{K} \mathbf{w}_k \mathbf{d}_k + \sum_{k=1}^{K} \mathbf{w}_k \mathbf{v}_k + \frac{\mathbf{n}}{\sqrt{\eta}}. \tag{9}$$

The transmit power of IoT device k is

$$\begin{aligned} P_k &= \|\mathbf{B}_k \mathbf{s}_k\|^2 \\ &= \eta \left(\mathbf{w}_k \mathbf{s}_k\right)^H \left(\mathbf{H}_k {\mathbf{H}_k}^H\right)^{-1} \left(\mathbf{w}_k \mathbf{s}_k\right) \\ &= \frac{\eta}{\tau_k} \end{aligned} \tag{10}$$

where

$$\tau_k = \left[\left(\mathbf{w}_k \mathbf{s}_k\right)^H \left(\mathbf{H}_k {\mathbf{H}_k}^H\right)^{-1} \left(\mathbf{w}_k \mathbf{s}_k\right)\right]^{-1}, \tag{11}$$

and τ_k is "the ratio between the channel power gain to the effective signal power". Considering a special case that $N_t = N_r = 1$, τ_k can be rewritten as $\tau_k = |h_k|^2 / |w_k s_k|^2$, where $|h_k|^2$ and $|w_k s_k|^2$ are the channel power gain and the effective signal power of IoT device k respectively.

With a instantaneous power constraint considered, i.e., $P_k \leq P_0$, the power control factor η can be calculated as

$$\eta = P_0 \min_k (\tau_k), \tag{12}$$

which depends on the IoT device with the minimum ratio between the channel power gain to the effective signal power.

Remark 1. (To avoid massive CSI gathering) The transceiver design avoids massive CSI gathering at the AP. According to transmitter matrix in (6), each IoT device needs its own CSI $\mathbf{H}_k$ and the power control level η to determine $\mathbf{B}_k$. $\mathbf{H}_k$ can be estimated based on the broadcasting pilots. And η is determined at the AP and broadcated to all IoT devices subsequently. According to receiver matrix in (8), the AP only needs η to determine $\mathbf{A}$. It seems that the AP requires all IoT devices' CSI to determine η in (12), which may incur massive CSI gathering at the AP. In the subsection III-C, we will propose a novel signaling procedure to determine η without gathering all IoT devices' CSI.

B. The case that $N_r > L$

When $N_r > L$, we adopt receive antenna selection to select L receive antennas from N_r. Although it is not optimal, it also avoids massive CSI gathering at the AP and only needs the power control level η. The selected subset from N_r receive antennas is Φ_i, where $i \in \left\{1, \cdots, C_{N_r}^L\right\}$. The corresponding receiver matrix $\mathbf{A}_{\Phi_i}$ is composed of L rows of $\mathbf{I}_{N_r}/\sqrt{\eta}$, where the index set of selected rows is Φ_i. The equivalent channel matrix after receiver is

$$\mathbf{H}_{k\Phi_i} = \mathbf{A}_{\Phi_i} \mathbf{H}_k. \tag{13}$$

Then the transmitter matrix for IoT device k can be rewritten as

$$\mathbf{B}_k = \sqrt{\eta} \mathbf{H}_{k\Phi_i}{}^H \left(\mathbf{H}_{k\Phi_i} \mathbf{H}_{k\Phi_i}{}^H \right)^{-1} \mathbf{w}_k, \tag{14}$$

The optimal selection criterion for subset Φ_i is

$$\Phi^{opt} = \arg \max_{\Phi_i} \min_k \tau_{k\Phi_i}, \tag{15}$$

where

$$\tau_{k\Phi_i} = \left[(\mathbf{w}_k \mathbf{s}_k)^H \left(\mathbf{H}_{k\Phi_i} \mathbf{H}_{k\Phi_i}{}^H \right)^{-1} (\mathbf{w}_k \mathbf{s}_k) \right]^{-1}. \tag{16}$$

The optimal antenna selection algorithm is based on exhaustive search and sort, whose complexity is related to the search space. Thus, the complexity of the optimal antenna selection algorithm in terms of the search space will be $\mathcal{O}\left(C_{N_r}^L\right)$, where $C_{N_r}^L$ is the size of the search space. In order to avoid the prohibitive complexity, we adopt a sub-optimal algorithm as illustrated in Algorithm 1. Instead of comparison and selection over $C_{N_r}^L$ possible subsets, we simplify the search and sort into $M = \lfloor N_r/L \rfloor$ disjoint subsets. The corresponding receive antenna subset Φ_m consists of antennas from $(m-1)L+1$ to mL, where $m \in \{1, 2, \cdots, M\}$. Thus, the complexity of the Algorithm 1 in terms of the search space will be $\mathcal{O}(\lfloor N_r/L \rfloor)$. Also, due to the independence between antennas subsets, the analytical evaluation of Algorithm 1 becomes tractable, which will be provided in Proposition 6.

Algorithm 1 Antenna selection with disjoint subsets

- **Step 1** IoT device k estimates its own CSI $\mathbf{H}_k$ based on the broadcasting pilots of AP. It further calculates $\tau_{k\Phi_m}$ for receive antennas subset Φ_m according to (16).
- **Step 2** The AP determines the $\tau_{\Phi_m} = min_k \, \tau_{k\Phi_m}$ for each selected antennas subsets m respectively. Then it sorts τ_{Φ_m}, and selects the Φ_m with the largest τ_{Φ_m}.

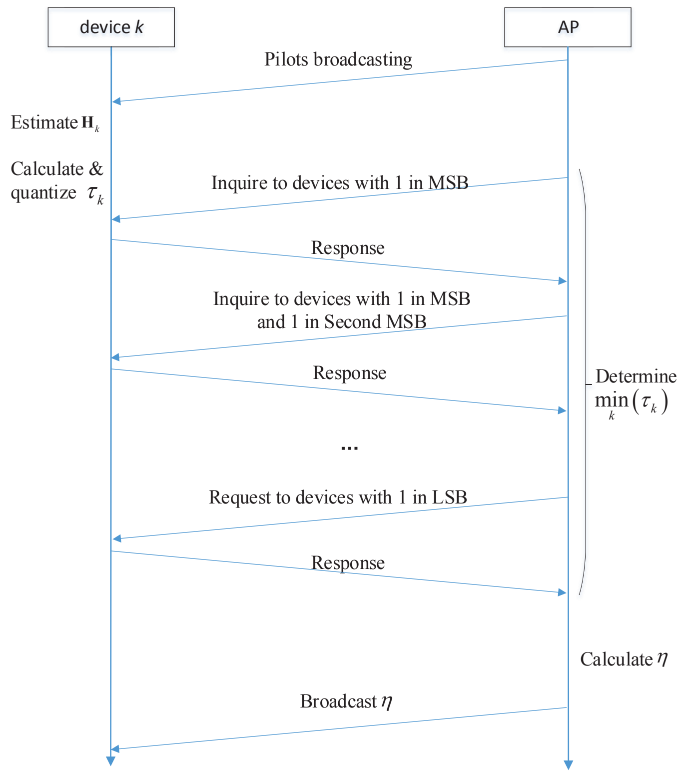


Figure 2. The signaling procedure to avoid massive CSI gathering

C. Signaling Procedure

According to the transceiver design above, both the IoT devices and the AP requires the power control level η in (12). In order to avoid gathering all IoT devices' CSI to calculate η, we utilize the "OR" property of the wireless channel [32] to determine the $\max(1/\tau_k)$, i.e., $\min(\tau_k)$.

Firstly, each IoT device k locally calculates its own τ_k based on its own CSI according to (11) and quantizes the $(1/\tau_k)$ into a binary representation as

$$\frac{1}{\tau_k} = \sum_{b=-b_L}^{b_M} \nu_b 2^b, \tag{17}$$

where $\nu_b \in \{0, 1\}$, b_M is the most significant bit (MSB) and b_L is the least significant bit (LSB). Then the AP uses several rounds of inquiry from MSB b_M to LSB b_L in order to determine the $\max(1/\tau_k)$, i.e., $\min(\tau_k)$. It can be described in Algorithm 2.

The whole signaling procedure is illustrated in Fig. 2. The time complexity of Algorithm 2 is related to the length of the quantized $1/\tau_k$. According to the definition of τ_k in (11), the value range of τ_k can be defined as $\tau_k \in [\tau_{\min}, \tau_{\max}]$,

Algorithm 2 Utilizing "OR" property of the wireless channel to determine $\min(\tau_k)$

- **Step 1** In the first inquiring round, IoT devices with 1 in the MSB respond, while IoT devices with 0 in the MSB keep silent. The AP detects the signal to determine whether the MSB of $\max(1/\tau_k)$ is 1. If so, the MSB of $\max(1/\tau_k)$ is set as 1. Otherwise, it is set as 0.
- **Step 2** In the second inquiring round, if MSB is set as 1, the AP inquires the IoT devices with MSB as 1 whether they have 1 in the second MSB. Otherwise, the AP inquires all IoT devices whether they have 1 in the second MSB. Then the second MSB is determined.
- **Step 3** The AP inquires in this way until the LSB is determined. Then $\max(1/\tau_k)$, i.e., $\min(\tau_k)$, can be determined according to (17).

where $\tau_{\min}$ and $\tau_{\max}$ are the minimum value and maximum value of τ_k . The MSB b_M is determined by the $1/\tau_{\min}$, i.e., $2^{b_M} \geqslant 1/\tau_{\min}$. And the LSB b_L is determined by the maximum tolerable quantization error Δ, i.e., $2^{-b_L} \leqslant \Delta$. Thus, the length of the quantized $1/\tau_k$ satisfies

$$b = b_M + b_L \geqslant \log_2 \frac{1}{\tau_{\min}\Delta}. \tag{18}$$

The pilot signal for each IoT device should be an $N_t \times N_t$ matrix for estimating an $N_r \times N_t$ channel matrix. It takes at least N_t symbol slots to complete the channel training process in Algorithm 2. According to the length of the quantized $1/\tau_k$, it takes b symbol slots to determine the $\min(\tau_k)$. Thus, it takes $N_t + b$ symbol slots for Algorithm 2.

In contrast, the conventional channel training process for each IoT device takes at least N_t symbol slots for estimating an $N_r \times N_t$ channel matrix. Thus, it takes KN_t symbol slots to obtain all IoT devices CSI at the AP. Consider a typical dense sensor network with $K = 100$ and $N_t = 2$, it takes 200 time slots for conventional channel training process. Assuming $b = 18$, it only takes 20 time slots for the proposed channel training process in Algorithm 2, which achieves 10-time of time complexity reduction in this example.

IV. Performance of Multiple Functions Computation

In this section, we further provide the performance of multiple functions computed over the air based on the defined MSFE in Definition 1. Both the exact and asymptotic analysis is given, and the diversity order in term of the transmit power and the number of devices is also derived.

A. Exact analysis of MSFE

Proposition 1. (The expression of MSFE) Assuming that different observation sources are i.i.d. and the observation sources of different IoT devices are correlated, the MSFE of multiple functions computed over-the-air with the transceiver designed above can be calculated as

$$\text{MSFE} = \frac{\sum\limits_{l=1}^{L}\sum\limits_{k=1}^{K} {w_{kl}}^2 \sigma_{vl}^2 + L\sigma_n^2/\eta}{\sum\limits_{l=1}^{L}\sum\limits_{k_1=1}^{K}\sum\limits_{k_2=1}^{K} w_{k_1 l} w_{k_2 l} \rho_{k_1 k_2}^{(l)} \sigma_{dl}^2}, \tag{19}$$

where $\rho_{k_1 k_2}^{(l)}$ is the correlation coefficient between IoT device k_1 and IoT device k_2 for the observation source l.

Proof. According to the target functions in (1) and the estimated ones in (9), the error vector is

$$\mathbf{e} = \sum_{k=1}^{K} \mathbf{w}_k \mathbf{v}_k + \frac{\mathbf{n}}{\sqrt{\eta}}. \tag{20}$$

Due to the distribution of $\mathbf{v}_k$ and $\mathbf{n}$, the lth element of $\sum_{k=1}^{K} \mathbf{w}_k \mathbf{v}_k$ satisfies $\mathcal{N}\left(0, \sum_{k=1}^{K} {w_{kl}}^2 \sigma_{vl}^2\right)$ and the lth element of $\mathbf{n}/\sqrt{\eta}$ satisfies $\mathcal{CN}\left(0, \sigma_n^2/\eta\right)$. For $\mathbf{d}_k$, the target function d_l satisfies $\mathcal{N}\left(0, \sum_{k_1=1}^{K}\sum_{k_2=1}^{K} w_{k_1 l} w_{k_2 l} \rho_{k_1 k_2}^{(l)} \sigma_{dl}^2\right)$. Then according to Definition 1, the expression of MSFE in (19) can be given. □

Remark 2. (A case that MSFE is a function of K) We consider a special case that the target function has the same coefficient, and the data sources have the same spacial correlation coefficient, i.e., $w_{kl} = 1$ and $\rho_{k_1 k_2}^{(l)} = \rho_c$ for $k_1 \neq k_2$. Also, the observation data and noise for different sources obey the same distribution, i.e., $\sigma_{dl}^2 = \sigma_d^2$ and $\sigma_{vl}^2 = \sigma_v^2$. Then the corresponding MSFE in (19) is a function of the number of IoT devices K and can be simplified as

$$\text{MSFE} = \frac{\sigma_v^2 + \sigma_n^2/K\eta}{\sigma_d^2 + (K-1)\rho_c \sigma_d^2}. \tag{21}$$

The MSFE is a function of power control factor η, and η in (12) is a function of τ_k. We first provide the following lemma about the distribution of τ_k.

Lemma 1. (The distribution of τ_k) τ_k in (11) is a transformation of Wishart distributed matrix $\mathbf{H}_k {\mathbf{H}_k}^H$, which obeys a chi-square distribution with $2(N_t - L + 1)$ degrees of freedom. Specifically,

$$2\mu_k \tau_k \sim \chi^2\left(2\left(N_t - L + 1\right)\right), \tag{22}$$

where μ_k is defined as

$$\mu_k = \left\|\mathbf{w}_k \mathbf{s}_k\right\|^2 = \sum_{l=1}^{L} (w_{kl} s_{kl})^2. \tag{23}$$

Proof. The proof of Lemma 1 is provided in Appendix A. □

Then with fixed μ_k, the cumulative distribution function (CDF) of τ_k can be given as

$$F_{\tau_k}(\tau_k, \mu_k) = \frac{\gamma\left(N_t - L + 1, \mu_k \tau_k\right)}{\Gamma\left(N_t - L + 1\right)}, \tag{24}$$

where $\gamma(\cdot)$ is the lower incomplete gamma function, $\Gamma(\cdot)$ is the gamma function, and μ_k in (23) can be regard as "the effective signal power gain" with the distribution approximated in the following lemma.

Lemma 2. (The distribution of μ_k) μ_k in (23) is the linear combinations of independent chi-square random variables, which can be approximated as a chi-square distribution. Specifically,

$$\frac{b_k \mu_k}{a_k} \sim \chi^2 \left(b_k\right) \tag{25}$$

where the constant a_k and b_k are given as

$$a_k = \sum_{l=1}^{L} {w_{kl}}^2 \left(\sigma_{dl}^2 + \sigma_{vl}^2\right) \tag{26}$$

and

$$b_k = \frac{\left(\sum\limits_{l=1}^{L} {w_{kl}}^2\right)^2}{\sum\limits_{l=1}^{L} {w_{kl}}^4}. \tag{27}$$

Proof. The proof of Lemma 2 is provided in Appendix B. □

Then the probability distribution function (PDF) of μ_k is

$$f_{\mu_k}\left(\mu_k\right) = \frac{1}{2^{b_k/2}\Gamma\left(b_k/2\right)} \left(\frac{b_k}{a_k}\right) \left(\frac{b_k \mu_k}{a_k}\right)^{b_k/2-1} e^{-\frac{b_k \mu_k}{2a_k}}, \tag{28}$$

where a_k and b_k are given in (26) and (27) respectively.

According to Lemma 1 and Lemma 2, we can further derive the closed-form expression of MSFE outage.

Proposition 2. (The expression of MSFE outage) The MSFE outage can be calculated as

$$\begin{aligned} \mathrm{P}_{out} &\overset{(a)}{=} \Pr\left[P_0 \min_k \left(\tau_k\right) < \psi\right] \\ &\overset{(b)}{=} 1 - \left[1 - F_{\tau_k}\left(\frac{\psi}{P_0}\right)\right]^K, \end{aligned} \tag{29}$$

where

$$\psi = \frac{L\sigma_n^2}{\xi \sum\limits_{l=1}^{L} \sum\limits_{k_1=1}^{K} \sum\limits_{k_2=1}^{K} w_{k_1 l} w_{k_2 l} \rho_{k_1 k_2}^{(l)} \sigma_{dl}^2 - \sum\limits_{l=1}^{L} \sum\limits_{k=1}^{K} {w_{kl}}^2 \sigma_{vl}^2}, \tag{30}$$

$F_{\tau_k}(\cdot)$ is the CDF of τ_k given as (31) at the bottom and ${}_2F_1(\cdot)$ is the Gauss hypergeometric function.

Proof. The procedure (a) is calculated according to (19). The procedure (b) is because the ordered distribution of the minimum one of K i.i.d. variables. The CDF of τ_k can be derived by

$$\begin{aligned} F_{\tau_k}\left(\tau_k\right) &= \int_0^\infty F_{\tau_k}\left(\tau_k, \mu_k\right) f_{\mu_k}\left(\mu_k\right) d\mu_k \\ &\overset{(c)}{=} \frac{\left(\frac{b_k}{a_k}\right)^{\frac{b_k}{2}} \int_0^\infty \mu_k^{\frac{b_k}{2}-1} e^{-\frac{b_k \mu_k}{2a_k}} \gamma\left(N_t - L + 1, \mu_k \tau_k\right) d\mu_k}{2^{\frac{b_k}{2}} \Gamma\left(\frac{b_k}{2}\right) \Gamma\left(N_t - L + 1\right)}, \end{aligned} \tag{32}$$

where $F_{\tau_k}\left(\tau_k, \mu_k\right)$ and $f_{\mu_k}\left(\mu_k\right)$ are given in (24) and (28) respectively. The integral (c) can be calculated according to [33, 6.455.2]. That is

$$\begin{aligned} &\int_0^\infty x^{\mu-1} e^{-\beta x} \gamma\left(\nu, \alpha x\right) dx \\ &= \frac{\alpha^\nu \Gamma\left(\mu + \nu\right)}{\nu (\alpha + \beta)^{\mu+\nu}} {}_2F_1\left(1, \mu + \nu; \nu + 1; \frac{\alpha}{\alpha + \beta}\right) \end{aligned} \tag{33}$$

□

B. Asymptotic Analysis of MSFE

The exact closed-form expression of MSFE outage in Proposition 2 is too complex to give us any insights. Thus, we will provide some asymptotic analysis to illustrate the diversity order in terms of the transmit power constraint and the number of IoT devices.

We first give the definition about the diversity order of MSFE outage in terms of the transmit power constraint and the number of IoT devices.

Definition 2. (The diversity order) The MSFE outage P_{out} is a function of the transmit power constraint P_0 and the number of IoT devices K, the diversity order of MSFE outage in terms of the transmit power constraint is defined as

$$D_P = \lim_{P_0 \to \infty} \frac{\log \mathrm{P}_{out}\left(P_0\right)}{\log P_0}, \tag{34}$$

and the diversity order of MSFE outage in terms of the number of IoT devices is defined as

$$D_K = \lim_{K \to \infty} \frac{\log \mathrm{P}_{out}\left(K\right)}{\log K}. \tag{35}$$

Then we provide the asymptotic analysis with a large transmit power constraint P_0. Based on series expansion of the MSFE outage expression in Proposition 2, we have the following results.

Proposition 3. (Asymptotic MSFE outage with large P_0) As the transmit power constraint P_0 is sufficiently large, the MSFE outage can be approximated as

$$\log \mathrm{P}_{out} \overset{a}{\to} -\left(N_t - L + 1\right) \log P_0. \tag{36}$$

$$F_{\tau_k}\left(\tau_k\right) = \frac{\Gamma\left(N_t - L + 1 + \frac{b_k}{2}\right) {\tau_k}^{N_t - L + 1} \left(\frac{b_k}{2a_k}\right)^{\frac{b_k}{2}}}{\Gamma\left(N_t - L + 2\right) \Gamma\left(\frac{b_k}{2}\right) \left(\tau_k + \frac{b_k}{2a_k}\right)^{N_t - L + 1 + \frac{b_k}{2}}} {}_2F_1\left(1, N_t - L + 1 + \frac{b_k}{2}; N_t - L + 2; \frac{\tau_k}{\tau_k + \frac{b_k}{2a_k}}\right) \tag{31}$$

where $\xrightarrow{a}$ means asymptotically converging to (as P_0 becomes large).

Proof. The Proposition 3 is proved in Appendix C. □

Remark 3. (The diversity order D_P) According to Definition 2, the diversity order of MSFE outage in terms of transmit power constraint P_0 is $D_P = N_t - L + 1$, which depends on the number of the transmit antennas and the number of computed functions.

Then we provide the asymptotic analysis with a large number of IoT devices. According to the extreme value theory of ordered statistics, if some specific convergence conditions are satisfied, the distribution of $\min_k(\tau_k)$ with large K approaches to Weibull-α distribution with CDF given by

$$F_W(x) = 1 - \exp\left(-x^{\alpha}\right), x > 0, \tag{37}$$

where $\alpha > 0$ is the shape parameter.

The specific convergence conditions are provided by the following lemma, and the value of α is also determined accordingly.

Lemma 3. (The extreme value theory of ordered statistics) $F(\cdot)$ is the CDF of X. iff $F^{-1}(0)$ is finite and

$$\lim_{\varepsilon \to 0^+} \frac{F\left(F^{-1}(0) + \varepsilon x\right)}{F\left(F^{-1}(0) + \varepsilon\right)} = x^{\alpha} \tag{38}$$

for all $x > 0$. When n is sufficiently large, one can choose $a_n^* = F^{-1}(0)$ and $b_n^* = F^{-1}(1/n) - F^{-1}(0)$ such that

$$(X_{n:n} - a_n^*)/b_n^* \xrightarrow{d} W, \tag{39}$$

where W is a Weibull distribution variable with CDF given in (37), $X_{n:n}$ is the nth large (minimum) variable from n i.i.d. random x, and $\xrightarrow{d}$ means convergence in distribution.

Proof. [34, Theorem 8.3.5 & 8.3.6] □

Proposition 4. (Weibull approximation of $\min_k(\tau_k)$) The distribution of $\min_k(\tau_k)$ satisfies the convergence condition in Lemma 3. And we have

$$\frac{\min\limits_k(\tau_k)}{K^{-\frac{1}{N_t - L + 1}} C^{-1}} \xrightarrow{d} W(N_t - L + 1), \tag{40}$$

where

$$C = \left[\frac{\Gamma(N_t - L + 1 + b_k/2)}{\Gamma(N_t - L + 2)\Gamma(b_k/2)}\right]^{\frac{1}{N_t - L + 1}} \left(\frac{2a_k}{b_k}\right). \tag{41}$$

Proof. The Proposition 4 is proved in Appendix D. □

In order to make the analysis tractable, we consider the MSFE in Remark 2, which is an explicit expression of the number of IoT devices K. Based on the Weibull approximation, the asymptotic MSFE outage for large number of IoT devices can be given as follows.

Proposition 5. (Asymptotic MSFE outage with large K) As the number of IoT devices K is sufficiently large, the MSFE outage in (21) can be approximated as

$$\log \mathrm{P}_{out} \xrightarrow{a} \begin{cases} -(N_t - L)\log K, & \rho_c = 0 \\ -(2N_t - 2L + 1)\log K \\ -(N_t - L + 1)\log \rho_c, & \rho_c > 0 \end{cases}. \tag{42}$$

Proof. The Proposition 5 is proved in Appendix E. □

Remark 4. (The diversity order D_K) When $\rho_c > 0$, the diversity order of MSFE outage in terms of the number of IoT devices K is $D_K = 2N_t - 2L + 1$ according to Definition 2. The increasing of correlation coefficient ρ_c will achieve the diversity gain with the diversity order unchanged. When $\rho_c = 0$, the diversity order of MSFE outage in terms of the number of IoT devices K decreases to $D_K = N_t - L$.

Finally, we discuss the selection diversity for the sub-optimal selection algorithm proposed in Algorithm 1. Due to the independence between antenna subsets, the analytical evaluation of selection is tractable. The asymptotic MSFE outage of the proposed algorithm can be given as follows.

Proposition 6. (Asymptotic MSFE outage with antenna selection) For the proposed sub-optimal Algorithm 1, as the transmit power constraint P_0 is sufficiently large, we have the MSFE outage in Proposition 3 asymptotically converges as

$$\log \mathrm{P}_{out} \xrightarrow{a} -M(N_t - L + 1)\log P_0. \tag{43}$$

And as the number of IoT devices K is sufficiently large, we have the MSFE outage in Proposition 5 asymptotically converges as

$$\log \mathrm{P}_{out} \xrightarrow{a} \begin{cases} -M(N_t - L)\log K, & \rho_c = 0 \\ -M(2N_t - 2L + 1)\log K \\ -M(N_t - L + 1)\log \rho_c, & \rho_c > 0 \end{cases}, \tag{44}$$

where $M = \lfloor N_r/L \rfloor$ is the number of antenna subsets.

Proof. The Proposition 6 is proved in Appendix F. □

Remark 5. (The diversity order of antenna selection) The diversity order of MSFE outage in terms of transmit power constraint and that in terms of the number of IoT devices both increase $M = \lfloor N_r/L \rfloor$ times for the proposed Algorithm 1 due to antenna selection diversity.

V. Simulation Results and Discussions

In this section, we provide some simulation results to illustrate the performance of multiple functions computed over-the-air. The simulation parameters are set as follows unless specified otherwise. The number of transmit and receive antennas $N_t = N_r = 2$, the number of computed functions $L = 2$, the number of IoT devices $K = 10$, the observation data of source l for IoT device k $d_{kl} \sim \mathcal{N}(0, 1)$, the threshold of MSFE outage $\xi = 0.1$, the target function coefficient matrix for IoT device k $\mathbf{w}_k = \mathbf{I}_L$, the signal to observe noise ratio (SONR) $\sigma_d^2/\sigma_o^2 = 10$dB and the signal to receive noise ratio (SRNR) $\sigma_d^2/\sigma_n^2 = 10$dB.

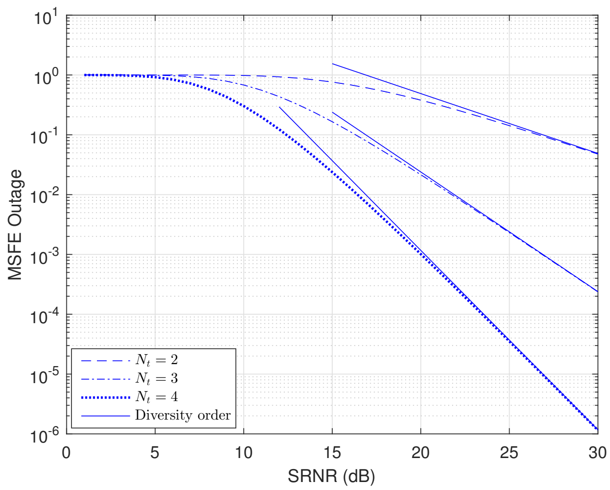


Figure 3. The MSFE outage versus different SRNR with different numbers of transmit antennas

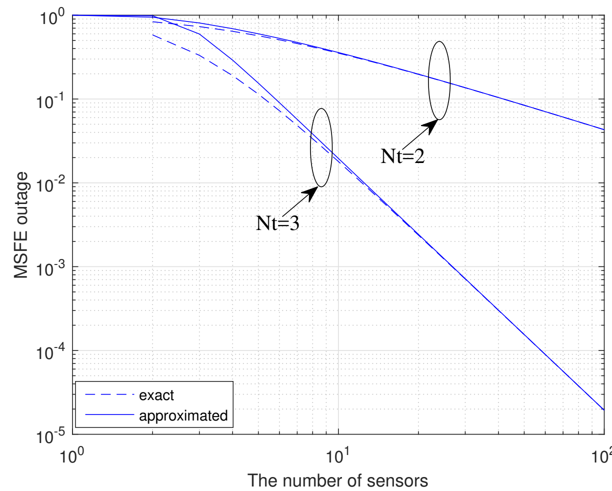


Figure 5. The MSFE outage versus different numbers of IoT devices with different numbers of transmit antennas

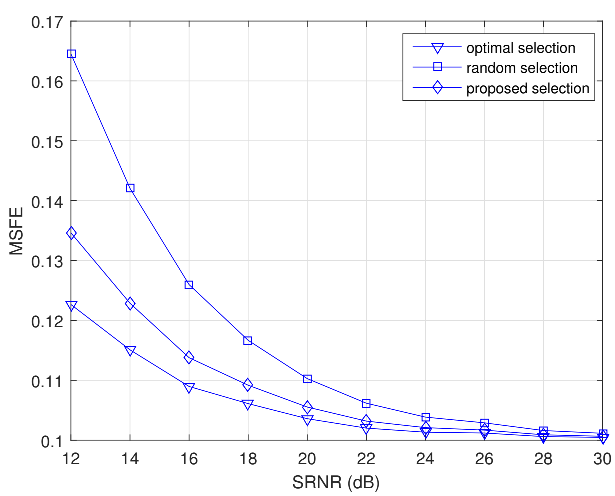


Figure 4. The mean MSFE of different receive antenna selection schemes

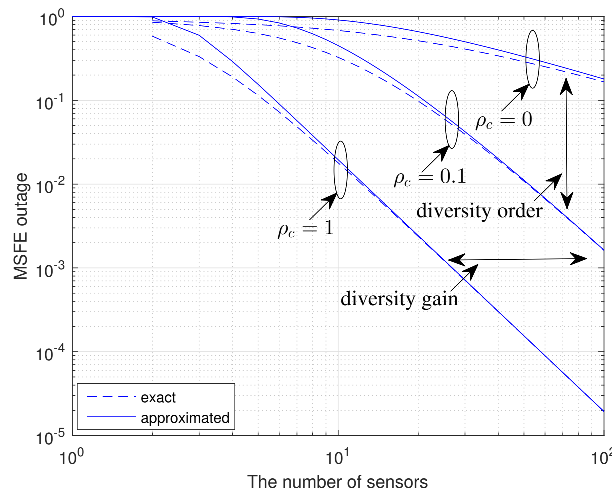


Figure 6. The MSFE outage versus different numbers of IoT devices with different correlation coefficient ρ_c

The MSFE outage versus different SRNR from 0dB to 30dB with different numbers of transmit antennas is illustrated in Fig. 3. Firstly, the MSFE outage is a monotone-decreasing function of SRNR. That is because the computed error caused by receive noise decreases. Then, the log function of MSFE outage almost linearly decreases with the increase of SRNR at the high SRNR regime. And the decreasing rates increases with the increase of the number of transmit antennas N_t. It verifies the Proposition 3 which reveals that the diversity order of MSFE outage in terms of transmit power constraint is $N_t - L + 1$.

The performance of different receive antenna selection schemes is present in Fig. 4, where the number of receive antennas N_r is 6. The mean MSFE of 10^6 Monte Carlo simulations is shown. The random selection scheme, the optimal selection scheme based on brute-force search, and the sub-optimal scheme proposed in Algorithm 1 are all illustrated with different SRNR from 10dB to 30dB. For random selection, we have no prior knowledge. Any selection chosen L elements from the set with N_r elements can be regarded as random selection. In our simulated results, we just chose the first L antennas from all N_r antennas. It can be seen that the performance of Algorithm 1 is between the performance of the optimal selection and the performance of the random selection. The reason can be explained from the search space point of view. There is no doubt that the optimal selection based on brute force search will go over the entire search space. It can obtain the optimal performance at the price of the high complexity. The motivation of the proposed algorithm is to make a tradeoff between the complexity and the performance, where the search space is limited to $M = \lfloor N_r/L \rfloor$ disjoint subsets. Thus, its performance is always inferior to the optimal one. We have provided the theoretical performance of the proposed algorithm in Proposition 6, which can obtain a selection diversity gain of $M = \lfloor N_r/L \rfloor$. Thus, its performance is superior to the random one. As the

SRNR becomes large, the performance gap between different schemes decreases. That is because receive antenna selection only affects the transmission error. As the SRNR increases, the MSFE caused by transmission error decreases and the observation error gradually dominates the whole errors.

The MSFE outage versus different numbers of IoT devices from 2 to 100 is present in Fig. 5 and Fig. 6 with different numbers of transmit antennas N_t and different correlation coefficient ρ_c. Both the exact expression and Weibull approximation are shown. According to these two figures, the MSFE outage is a monotone-decreasing function of the number of IoT devices. On one hand, it is due to the increase of the combined received signal power, which decreases the transmission error caused by the receive noise. On the other hand, it is due to the decrease of the observation error when the observe sources of different IoT devices are correlated with each other. Also, it can be observed that the Weibull approximation proposed in Proposition 4 is accurate, especially when the number of IoT device is in a large regime. In Fig.5, the MSFE outage almost linearly decreases when the number of IoT devices K exponential increases. And the decreasing rate increases with the increase of the number of transmit antennas N_t. It verifies the Proposition 5 that the diversity order in terms of the number of IoT devices increases with N_t. In Fig.6, when $\rho_c > 0$, the decreasing rate with the increase of the number of IoT devices is the same for different correlation coefficient ρ_c. And the increase of ρ_c will bring diversity gain. When $\rho_c = 0$, the diversity order decreases. That verifies the Proposition 5.

VI. Conclusion and future work

In this paper, we propose a novel transceiver design for multiple functions computed over-the-air. The transmitter matrix is designed to cancel the intra-node interference of multiple functions and compensated the non-uniform fading between different IoT devices. The receive antenna selection is adopted and the corresponding signaling procedure is proposed to avoid massive CSI gathering problem by utilizing the "OR" property of the wireless channel. The performance of MSFE outage is derived based on the signal and channel distributions. Due to the complexity of the expression, asymptotic analysis is provided by series expansion and Weibull distribution approximation. The diversity order are defined and derived in terms of transmit power constraint and the number of IoT devices, which are determined by the number of transmit antennas, the number of functions, and the correlation coefficient between different sources.

In the future work, we will study the robust design for multiple functions computed over-the-air, where the CSI estimation error and the synchronization error will be considered. Also, the network model will be extended to the IoT network with multiple clusters and experimental platforms will be built.

Appendix A
Proof of Lemma 1

Considering τ_k defined in (11), $\mathbf{H}_k{\mathbf{H}_k}^H$ obeys a complex Wishart distribution with N_r dimensions and $2N_t$ degrees of freedom with Rayleigh fading assumed, i.e.,

$$\mathbf{H}_k{\mathbf{H}_k}^H \sim \mathcal{CW}\left(\frac{1}{2}\mathbf{I}_{N_r}, L, 2N_t\right) \tag{45}$$

Then according to [35, Proposition 8.9] about the transformation of Wishart distributed matrix. That is suppose $\mathbf{S}_0$ has a nonsingular Wishart distribution, say $\mathcal{W}(\mathbf{\Sigma}, p, n)$, and let $\mathbf{A}$ be an $r \times p$ matrix of rank r. We have

$$\left(\mathbf{A}\mathbf{S}_0^{-1}\mathbf{A}^H\right)^{-1} \sim \mathcal{W}\left(\left(\mathbf{A}\mathbf{\Sigma}^{-1}\mathbf{A}^H\right)^{-1}, r, n-p+r\right). \tag{46}$$

Thus, $\tau_k = \left[(\mathbf{w}_k\mathbf{s}_k)^H\left(\mathbf{H}_k{\mathbf{H}_k}^H\right)^{-1}(\mathbf{w}_k\mathbf{s}_k)\right]^{-1}$ also obeys a complex Wishart distribution, i.e.,

$$\tau_k \sim \mathcal{CW}\left(\left((\mathbf{w}_k\mathbf{s}_k)^H 2\mathbf{I}_{N_r}(\mathbf{w}_k\mathbf{s}_k)\right)^{-1}, 1, 2(N_t - L + 1)\right). \tag{47}$$

The above one-dimensional complex-valued Wishart distribution is actually a chi-square distribution with $2(N_t - L + 1)$, i.e., $2\mu_k\tau_k \sim \chi^2(2(N_t - L + 1))$. □

Appendix B
Proof of Lemma 2

We adopt the Welch-Satterthwaite approximation to approximate the linear combinations of independent chi-square random variables [36].

That is let $M_1, \cdots, M_n$ be independent random variables, and let $a_1, \cdots, a_n$, $b_1, \cdots, b_n$ and $k_1, \cdots, k_n$ be positive numbers. If we have that

$$\frac{b_j M_j}{a_j} \sim \chi^2(b_j) \tag{48}$$

Then the distribution of $M = k_1 M_1 + \cdots + k_n M_n$ can be approximated as

$$\frac{bM}{a} \sim \chi^2(b) \tag{49}$$

with

$$a = k_1 a_1 + \cdots + k_n a_n \tag{50}$$

and

$$b = \frac{a^2}{\frac{(k_1 a_1)^2}{b_1} + \cdots + \frac{(k_n a_n)^2}{b_n}}. \tag{51}$$

Because $s_{kl} \sim \mathcal{N}\left(0, \left(\sigma_{dl}^2 + \sigma_{vl}^2\right)\right)$, we have

$$\frac{(s_{kl})^2}{\sigma_{dl}^2 + \sigma_{vl}^2} \sim \chi(1) \tag{52}$$

The distribution of $\mu_k = \sum_{l=1}^{L}(w_{kl})^2(s_{kl})^2$ is approximated as chi-square distribution according to Welch-Satterthwaite approximation, i.e.,

$$\frac{b_k\mu_k}{a_k} \sim \chi^2(b_k) \tag{53}$$

where

$$a_k = \sum_{l=1}^{L} {w_{kl}}^2 \left(\sigma_{dl}^2 + \sigma_{vl}^2\right) \tag{54}$$

and

$$\begin{aligned} b_k &= \frac{\left[\sum\limits_{l=1}^{L} {w_{kl}}^2 \left(\sigma_{dl}^2 + \sigma_{vl}^2\right)\right]^2}{\sum\limits_{l=1}^{L} \left[{w_{kl}}^2 \left(\sigma_{dl}^2 + \sigma_{vl}^2\right)\right]^2} \\ &= \frac{\left(\sum\limits_{l=1}^{L} {w_{kl}}^2\right)^2}{\sum\limits_{l=1}^{L} {w_{kl}}^4}. \end{aligned} \tag{55}$$

□

Appendix C
Proof of Proposition 3

The series expansion of the Gauss hypergeometric function ${}_2F_1(\cdot)$ in [33, 9.100] is

$${}_2F_1(\alpha, \beta; \gamma; z) = 1 + \frac{\alpha\beta}{\gamma} z + O\left(z^2\right), \tag{56}$$

As the transmit power constraint P_0 is large, $\psi/P_0 \to 0$. The closed-from expression of $F_{\tau_k}(\tau_k)$ in (38) is approximated as

$$F_{\tau_k}\left(\frac{\psi}{P_0}\right) \approx \frac{\Gamma\left(N_t - L + 1 + \frac{b_k}{2}\right)}{\Gamma\left(N_t - L + 2\right)\Gamma\left(\frac{b_k}{2}\right)} \left(\frac{\frac{2a_k\psi}{b_k}}{\frac{2a_k\psi}{b_k} + P_0}\right)^{N_t - L + 1} \tag{57}$$

by ignoring the higher order terms. And the MSFE outage in (29) is approximated as

$$\mathrm{P}_{out} \approx K F_{\tau_k}\left(\frac{\psi}{P_0}\right). \tag{58}$$

Combining (57) and (58), we have that

$$\begin{aligned} \log \mathrm{P}_{out} \approx &- (N_t{-}L{+}1) \log P_0 + \log \frac{K\Gamma\left(N_t{-}L{+}1 + \frac{b_k}{2}\right)}{\Gamma\left(N_t{-}L{+}2\right)\Gamma\left(\frac{b_k}{2}\right)} \\ &+ (N_t - L + 1) \log \frac{2a_k\psi}{b_k} \end{aligned} \tag{59}$$

□

Appendix D
Proof of Proposition 4

According to the series expansion of ${}_2F_1(\cdot)$ [33, 9.100] in (56) and $F_{\tau_k}(\tau_k)$ in (58), we have that

$$\lim_{\varepsilon \to 0^+} \frac{F_{\tau_k}(\varepsilon x)}{F_{\tau_k}(\varepsilon)} = \lim_{\varepsilon \to 0^+} \frac{(\varepsilon x)^{N_t - L + 1}}{(\varepsilon)^{N_t - L + 1}} = x^{N_t - L + 1} \tag{60}$$

for large number of IoT devices.

According to the extreme value theory of ordered statistics in Lemma 3, the distribution of $\min_k(\tau_k)$ converge to a Weibull distribution with $\alpha = N_t - L + 1$,

$$a_n^* = {F_{\tau_k}}^{-1}(0) = 0, \tag{61}$$

and

$$b_n^* = F_{\tau_k}^{-1}(1/K) \approx K^{-\frac{1}{N_t - L + 1}} C^{-1} \tag{62}$$

with C given in (41). □

Appendix E
Proof of Proposition 5

According to the Weibull approximation in Proposition 4, we get that

$$\Pr\left(\min(\tau_k) \le x\right) \approx 1 - \exp\left(-C^{N_t - L + 1} K x^{N_t - L + 1}\right) \tag{63}$$

Then the corresponding MSFE outage in (29) can be further approximated as

$$\begin{aligned} \mathrm{P}_{out} &= \Pr\left(\min(\tau_k) < \frac{\psi}{P_0}\right) \\ &\overset{(a)}{\approx} 1 - \exp\left(-C^{N_t - L + 1} K \frac{1}{P_0} \frac{L\sigma_n^2}{K\xi\left[1 + (K-1)\rho_c\right]\sigma_d^2}\right) \\ &\overset{(b)}{\approx} C^{N_t - L + 1} K \left(\frac{1}{P_0} \frac{L\sigma_n^2}{K\xi\left[1 + (K-1)\rho_c\right]\sigma_d^2}\right)^{N_t - L + 1}, \end{aligned} \tag{64}$$

where the procedure (a) is due to (12) and (21), the procedure (b) is due to the series expansion of exponential function for $\psi \to 0$. Then we have that

$$\log \mathrm{P}_{out} \overset{a}{\to} -(N_t{-}L) \log K - (N_t{-}L{+}1) \log\left[1 + (K{-}1)\rho_c\right]. \tag{65}$$

Then the proposition is proved. □

Appendix F
Proof of Proposition 6

Because of the independence between antenna subsets, the MSFE outage with the selected subset Φ_m can be calculated according to ordered distribution, i.e.,

$$\begin{aligned} \mathrm{P}_{out} &= \Pr\left(\max_{\Phi_m} \min_k \tau_{k\Phi_m} < \frac{\psi}{P_0}\right) \\ &= \left[1 - \left[1 - F_{\tau_k}\left(\frac{\psi}{P_0}\right)\right]^K\right]^M. \end{aligned} \tag{66}$$

Then the asymptotic analysis for large transmit power constraint in (58) can be rewritten as

$$\mathrm{P}_{out} \approx K^M \left[F_{\tau_k}\left(\frac{\psi}{P_0}\right)\right]^M. \tag{67}$$

And the asymptotic analysis for large number of IoT devices in (64) can be rewritten as

$$\mathrm{P}_{out} \approx C^{M(N_t-L+1)} K^M \left(\frac{1}{P_0} \frac{L\sigma_n^2}{K\xi \left[1+(K-1)\,\rho_c\right] \sigma_d^2} \right)^{M(N_t-L+1)} \tag{68}$$

Then the MSFE outage with large P_0 and large K can be given in (43) and (44), respectively. □

References

[1] M. Agiwal, A. Roy, and N. Saxena, "Next generation 5G wireless networks: A comprehensive survey," *IEEE Commun. Surv. Tutorials*, vol. 18, no. 3, pp. 1617–1655, thirdquarter 2016.

[2] K. Yang, "Interference management in lte wireless networks [industry perspectives]," *IEEE Wireless Commun.*, vol. 19, no. 3, pp. 8–9, June 2012.

[3] F. Zhang, M. Liu, Z. Zhou, and W. Shen, "An IoT-based online monitoring system for continuous steel casting," *IEEE Internet Things J.*, vol. 3, no. 6, pp. 1355–1363, Dec 2016.

[4] W. Hou, Z. Ning, L. Guo, and X. Zhang, "Temporal, functional and spatial big data computing framework for large-scale smart grid," *IEEE Trans. Emerging Top. Comput.*, 2017, to be published, doi: 10.1109/TETC.2017.2681113.

[5] W. Hou, Z. Ning, L. Guo, and M. S. Obaidat, "Service degradability supported by forecasting system in optical data center networks," *IEEE Syst. J.*, 2018, to be published, doi: 10.1109/JSYST.2018.2821714.

[6] K. Yang, R. Liu, Y. Sun, J. Yang, and X. Chen, "Deep network analyzer (DNA): A big data analytics platform for cellular networks," *IEEE Internet Things J.*, vol. 4, no. 6, pp. 2019–2027, Dec 2017.

[7] P. Kolios, C. Panayiotou, G. Ellinas, and M. Polycarpou, "Data-driven event triggering for IoT applications," *IEEE Internet Things J.*, vol. 3, no. 6, pp. 1146–1158, Dec 2016.

[8] B. Nazer and M. Gastpar, "Computation over multiple-access channels," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3498–3516, 2007.

[9] A. Giridhar and P. R. Kumar, "Computing and communicating functions over sensor networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 4, pp. 755–764, April 2005.

[10] N. Karamchandani, R. Appuswamy, and M. Franceschetti, "Time and energy complexity of function computation over networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 12, pp. 7671–7684, Dec 2011.

[11] N. Karamchandani, U. Niesen, and S. Diggavi, "Computation over mismatched channels," in *2012 50th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Oct 2012, pp. 1042–1048.

[12] R. Soundararajan and S. Vishwanath, "Communicating linear functions of correlated gaussian sources over a mac," *IEEE Trans. Inf. Theory*, vol. 58, no. 3, pp. 1853–1860, March 2012.

[13] B. Nazer and M. Gastpar, "Compute-and-forward: Harnessing interference through structured codes," *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6463–6486, Oct 2011.

[14] J. Zhan, B. Nazer, M. Gastpar, and U. Erez, "MIMO compute-and-forward," in *2009 IEEE International Symposium on Information Theory*, June 2009, pp. 2848–2852.

[15] M. Goldenbaum, H. Boche, and S. Staczak, "Nomographic functions: Efficient computation in clustered gaussian sensor networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2093–2105, April 2015.

[16] N. Karamchandani, L. Keller, C. Fragouli, and M. Franceschetti, "Function computation via subspace coding," *Phys. Commun.*, vol. 6, pp. 134 – 141, 2013.

[17] M. Goldenbaum, H. Boche, and S. Staczak, "Harnessing interference for analog function computation in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 61, no. 20, pp. 4893–4906, Oct 2013.

[18] W. Hou, Z. Ning, L. Guo, Z. Chen, and M. S. Obaidat, "Novel framework of risk-aware virtual network embedding in optical data center networks," *IEEE Syst. J.*, 2017, to be published, doi: 10.1109/JSYST.2017.2673828.

[19] M. Gastpar, "Uncoded transmission is exactly optimal for a simple gaussian sensor network," *IEEE Trans. Inf. Theory*, vol. 54, no. 11, pp. 5247–5251, Nov 2008.

[20] M. Goldenbaum and S. Stanczak, "Robust analog function computation via wireless multiple-access channels," *IEEE Trans. Commun.*, vol. 61, no. 9, pp. 3863–3877, September 2013.

[21] M. Goldenbaum, S. Staczak, and H. Boche, "On achievable rates for analog computing real-valued functions over the wireless channel," in *2015 IEEE International Conference on Communications (ICC)*, June 2015, pp. 4036–4041.

[22] J. Huang, Q. Zhang, Q. Li, and J. Qin, "Robust parallel analog function computation via wireless multiple-access mimo channels," *IEEE Signal Process Lett.*, vol. 22, no. 9, pp. 1297–1301, Sept 2015.

[23] M. Goldenbaum and S. Stanczak, "On the channel estimation effort for analog computation over wireless multiple-access channels," *IEEE Wireless Commun. Lett.*, vol. 3, no. 3, pp. 261–264, June 2014.

[24] S. W. Jeon and B. C. Jung, "Opportunistic function computation for wireless sensor networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 6, pp. 4045–4059, June 2016.

[25] S. Sigg, P. Jakimovski, and M. Beigl, "Calculation of functions on the RF-channel for IoT," in *2012 3rd IEEE International Conference on the Internet of Things*, Oct 2012, pp. 107–113.

[26] A. Kortke, M. Goldenbaum, and S. Staczak, "Analog computation over the wireless channel: A proof of concept," in *IEEE SENSORS 2014 Proceedings*, Nov 2014, pp. 1224–1227.

[27] O. Abari, H. Rahul, and D. Katabi, "Over-the-air function computation in sensor networks," *CoRR*, vol. abs/1612.02307, 2016. [Online]. Available: http://arxiv.org/abs/1612.02307

[28] J. Zhan, B. Nazer, U. Erez, and M. Gastpar, "Integer-forcing linear receivers," *IEEE Trans. Inf. Theory*, vol. 60, no. 12, pp. 7661–7685, Dec 2014.

[29] T. Yang, "Distributed mimo broadcasting: Reverse compute-and-forward and signal-space alignment," *IEEE Trans. Wireless Commun.*, vol. 16, no. 1, pp. 581–593, Jan 2017.

[30] W. Liu and C. Ling, "Efficient integer coefficient search for compute-and-forward," *IEEE Trans. Wireless Commun.*, vol. 15, no. 12, pp. 8039–8050, Dec 2016.

[31] Q. Huang and A. Burr, "Low complexity coefficient selection algorithms for compute-and-forward," *IEEE Access*, vol. 5, pp. 19 182–19 193, 2017.

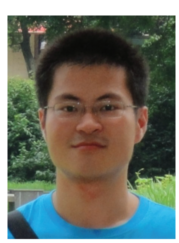
[32] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Médard, and J. Crowcroft, "XORs in the air: Practical wireless network coding," in *Proceedings of the 2006 Conference on Applications, Technologies, Architectures, and Protocols for Computer Communications(SIGCOMM)*, 2006, pp. 243–254.

[33] A. Jeffrey and D. Zwillinger, *Table of Integrals, Series, and Products, Fifth edition*. Elsevier Science, 2007.

[34] B. Arnold, N. Balakrishnan, and H. Nagaraja, *A First Course in Order Statistics*, ser. Classics in Applied Mathematics. Society for Industrial and Applied Mathematics, 2008.

[35] M. Eaton, *Multivariate statistics: a vector space approach*, ser. Lecture notes-monograph series. Institute of Mathematical Statistics, 1983.

[36] F. E. Satterthwaite, "An approximate distribution of estimates of variance components," *Biometrics Bulletin*, vol. 2, no. 6, pp. 110–114, 1946. [Online]. Available: http://www.jstor.org/stable/3002019



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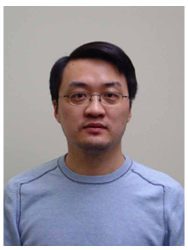

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