INVITED PAPER

# Photonic Lanterns, 3-D Waveguides, Multiplane Light Conversion, and Other Components That Enable Space-Division Multiplexing

*This article reviews the mode-shaping technologies needed to perform mode-division multiplexing on few-mode fibers and discusses implementations and tradeoffs of four different techniques: multiplane light conversion, fused fiber devices, 3-D waveguides in glass, and free-space imaging systems.*

By Nicolas K. Fontaine, *Senior Member IEEE*, Joel Carpenter, *Member IEEE*, Simon Gross, Sergio Leon-Saval, Yongmin Jung, David J. Richardson, *Fellow IEEE*, and Rodrigo Amezcua-Correa

**ABSTRACT** | Four-mode multiplexing and manipulation technologies are reviewed in the context of space-division multiplexing (SDM) optical communication systems. These are multiplane light conversion (MPLC), fused fiber devices, such as photonic lanterns and tapered fiber bundles, 3-D waveguides fabricated using ultrafast laser inscription, and free-space imaging systems. Each device has its unique strengths and use cases. MPLC can create very complex transformations between two arbitrary sets of spatial modes and leverages mature gray-scale lithographic techniques. Photonic lanterns and tapered fiber bundles are all-fiber devices that can convert beams on single-mode fiber inputs into spatial modes. The all-fiber construction leads to very low losses and high-power handling. 3-D waveguides are inscribed into a glass block using ultrashort lasers and can arbitrarily route light in 3-D. Finally, free-space systems using lenses can also relay many multicore and multimode beams through a single free-space device, such as a thin film filter or an optical isolator. These four technologies have enabled hero transmission experiments in the multimode, multicore, and multimode fiber.

**KEYWORDS** | Multiplane light conversion (MPLC); photonic lanterns; ultrafast laser inscription (ULI).

Manuscript received 19 December 2021; revised 10 August 2022 and 25 August 2022; accepted 26 August 2022. Date of publication 12 October 2022; date of current version 11 November 2022. *(Corresponding author: Nicolas K. Fontaine.)*

**Nicolas K. Fontaine** is with Nokia Bell Labs, Murray Hill, NJ 07974 USA (e-mail: nicolas.fontaine@nokia-bell-labs.com).

**Joel Carpenter** is with the School of IT and Electrical Engineering (ITEE), The University of Queensland, Brisbane, QLD 4072, Australia (e-mail: j.carpenter@uq.edu.au).

**Simon Gross** is with the School of Engineering, Macquarie University, Sydney, NSW 2109, Australia (e-mail: simon.gross@mq.edu.au).

**Sergio Leon-Saval** is with the Institute of Photonics and Optical Science (IPoS), The University of Sydney, Camperdown, NSW 2006, Australia (e-mail: sergio.leon-saval@sydney.edu.au).

**Yongmin Jung** and **David J. Richardson** are with the Optoelectronics Research Centre, University of Southampton, SO17 1BJ Southampton, U.K. (e-mail: ymj@orc.soton.ac.uk; djr@orc.soton.ac.uk).

**Rodrigo Amezcua-Correa** is with the College of Optics and Photonics, University of Central Florida, Orlando, FL 32816 USA (e-mail: r.amezcua@creol.ucf.edu).

Digital Object Identifier 10.1109/JPROC.2022.3207046

## I. INTRODUCTION

Until recently, coupling light into multiple spatial modes has been limited to low mode counts, high losses, and bulky setups. In the last decade, the demand to increase capacity in telecommunication systems spurred researchers to use spatial modes in multimode and multicore fibers (MCFs) in addition to multiplexing in wavelength, polarization, quadrature, and time [1], [2], [3], [4], [5], [6], [7]. Supporting transmission capacities well beyond a petabit will require new spatial multiplexing devices supporting hundreds to thousands of spatial modes

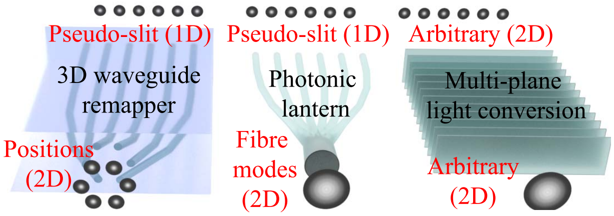


**Fig. 1.** ***Spatial multiplexing devices convert spatially separated beams (top) into spatial modes and/or a 2-D array of spots (bottom). Example devices from left to right are a 3-D waveguides remapper, a fiber-based photonic lantern, and an MPLC device. The color represents where the refractive index is modified. 3-D waveguides typically have one refractive index step, photonic lanterns have two or more, and MPLC mimics a 3-D refractive index profile through distributed phase planes.***

with low loss, broadband operation, and ease of fabrication. Following the historical scaling of space-division multiplexing (SDM) capacity, we could expect transmission demonstrations exceeding 1000 modes within the next decade. The applications for low-loss mode multiplexing and manipulating devices include free-space optical communications, SDM optical communications, imaging and endoscopy [8], [9], astronomy [10], quantum [11], [12], optical computation [13], [14], free-space optical communications [15], [16], [17], [18], [19], [20], and many other emerging fields.

Fig. 1 shows three examples of the main spatial multiplexing technologies: 3-D waveguides inscribed in a glass substrate [21], fused fiber photonic lanterns [22], and multiplane light conversion (MPLC) technology [23], [24]. They must take $N$ spatial separated Gaussian beams from multiple single-mode fibers and convert them into the $N$ spatially overlapping modes of multimode fiber (MMF). Quite often, these separated Gaussian beams are confined to a single spatial axis to ease interfacing with planar photonic integrated circuits and/or fiber arrays. In the figure, the color represents material where the refractive index can be modified.

Lossless spatial mode multiplexing is hard for two reasons: only lossless phase manipulations are allowed, and there is no extra orthogonal dimension available to demultiplex the modes. As an example, consider polarization and wavelength demultiplexing that both demultiplex into the unused spatial dimension. Lossless polarization demultiplexing uses birefringent materials to apply a different phase shift to the two polarizations, which causes the beams to separate in space. Demultiplexing wavelengths require gratings or dispersive materials that apply a unique phase shift to each wavelength, which causes them to separate in the spatial dimension. To demultiplex or multiplex spatial modes, phase shifts are applied to different spatial locations via refractive index modifications, and the modes are also separated in the spatial dimension. The spatial dimension is used for phase shifts, separation, and spatial modes. There is no orthogonal dimension to separate the spatial modes, and for this reason, devices that separate or combine spatial modes are more difficult to fabricate than a polarization or wavelength multiplexer. Other practical considerations to build useful multiplexers for communications are that they should be wavelength-independent and easily interface to single-mode fibers, that is, the outputs are Gaussian beams.

The two tools to demultiplex spatial modes are spatially varying phase shifts and wave propagation. Waveguide devices combine wave propagation and phase shifts in the same spatial location, whereas MPLC separates wave propagation from phase shifts by interleaving free-space propagation sections and phase masks. Fig. 1 shows the comparison of the three dimension refractive index profiles of lanterns, 3-D waveguides, and MPLC. Devices such as photonic lanterns [22] and 3-D waveguides [21] consist of smooth continuous 3-D refractive index profiles, through which spatial transformations can be implemented. Photonic lanterns are made by fusing and tapering different fibers and glass capillaries together. The all-fiber construction can build devices with losses well below 1 dB. 3-D waveguide devices modify the refractive index of a host material through multiphoton absorption using an ultrafast laser. Programmatically moving the material in $x$, $y$, and $z$ through a laser focus enables the 3-D placement of waveguides. MPLC [23], [24] systems consist of a series of discrete phase masks interleaved with bulk material propagation through which arbitrary spatial transformations can be implemented. They can be lithographically fabricated and, thus, can leverage mature fabrication technologies. In this sense, MPLC can be thought of as discretized 3-D refractive index profiles, whereby light is shaped at discrete planes separated by free-space propagation, rather than being shaped continuously through waveguide structures. Indeed, an MPLC device composed of infinitely many planes, infinitely closely spaced, yields an adiabatic refractive index transformation akin to a tapered fiber device, such as a photonic lantern.

This article is structured as follows. Section II defines the common terminology, SDM jargon, and performance metrics used throughout this article. Section III will describe the state of the art in fused fiber mode multiplexers that are typically called photonic lanterns or tapered fiber bundles. Section IV overviews the state of the art in inscribing waveguides into glass substrates using femtosecond lasers. Section V describes how to build multimode components, such as isolators and filters using lenses, free-space, and bulk optics. This section is important because free-space and bulk optic devices are used to build and interface with the rest. Section VI will describe MPLC devices and how they are similar to the other mode multiplexing technologies. Section VII summarizes the technologies.

## II. SPACE-DIVISON MULTIPLEXING TERMINOLOGY AND PERFORMANCE METRICS

Prior to delving into each type of device, here are the authors' thoughts on terminology common to all devices.

1) *MMF:* Any fiber supporting more than one spatial mode.
2) *Few-mode fiber (FMF):* When SDM systems employing MIMO first started, they could only support three spatial modes. To differentiate between regular 50-$\mu$m MMF that supports over 45 spatial modes, the authors called almost any fiber with fewer modes FMF. With recent demonstrations of 15 [25] and 45 spatial modes [26], it is unclear where to draw the line between FMF and MMF. In this article, less than ten is a few- or low-mode count.
3) *Space-division multiplexing (SDM):* Using spatial modes in fiber or free space for communications.
4) *Multiple-input–multiple-output (MIMO) processing:* Unscrambling mode mixing typically through linear combinations of all outputs. It could be done optically using interferometer networks but is typically accomplished in the digital domain. It can be thought of as backpropagating all outputs through the system back to individual inputs to figure out what the input signal was. Since MIMO can undo mode mixing, it is important to decide if your mode multiplexer needs to be mode-selective.
5) *Low loss:* Around 1 dB or below.
6) *Mode selectivity:* It refers to how well each single-mode input excites the intended mode. It can be expressed as a ratio between the power in the desired modes and the power in the unintended modes.
7) *Broadband:* A broadband mode multiplexer will have the same properties across the entire communication bands (e.g., typically 1500–1650 nm).
8) *Theoretical loss versus fabrication loss:* Some devices are theoretically lossless, but difficulties in the fabrication or assembly procedure create nontrivial losses. An example is an MPLC device that has theoretical losses well below 1 dB, but losses are typically 3–5 dB due to scattering of the phase planes.
9) *Mode-dependent loss (MDL):* Typically defined as the ratio of the transmission of the strongest mode and the weakest mode. In systems with strong mode mixing or devices with poor mode selectivity, MDL is the ratio of orthogonal combinations of modes that have maximal transmission and minimum loss. This requires a sophisticated measurement of the entire transfer matrix between each input and all output modes rather than just the measurement of the matrix diagonal. A device with poor mode selectivity and low MDL is better for SDM employing MIMO than a device with great mode selectivity and low MDL.

## III. OPTICAL FIBER-BASED PHOTONIC LANTERN MULTIPLEXERS AND DEVICES

Photonic lanterns allow for a low-loss transformation of a few-mode system into a discrete number of single-mode ones and vice versa, thus enabling their use as mode multiplexers/demultiplexers in SDM communications. They can be built in many waveguide technologies; this section will focus on lanterns fabricated using optical fibers, and Section IV will also discuss photonic lanterns built from waveguides inscribed into the glass. Furthermore, they provide seamless integration with the rest of single-mode photonic technologies through splices in telecommunication systems.

In this section, we will discuss the technology of the photonic lantern, along with several variants with a primary focus on the optical fiber-based photonic lantern type. Current fabrication technology can reliably produce fiber-based mode-selective lanterns with approximately ten spatial modes with sub-dB losses that are directly applicable to optical fibers, which is ideal for SDM systems with low mode counts. With fabrication improvements, mainly automation of the time-consuming process of pushing fibers into a capillary, we can expect future lanterns for SDM to scale similar to devices used in astronomy that have hundreds of modes.

### A. Photonic Lantern Technology and Fabrication

Most generally, photonic lanterns consist of a collection of single-mode waveguides (the single-mode end) that are interfaced to a multimode waveguide (multimode end) through a physical waveguide transition. The single-mode end of a photonic lantern comprises an array of isolated uncoupled single-mode waveguides. The array of single-mode waveguides is effectively a multimode system in which the spatial modes are the supermodes of the array. Hence, the number of degenerate or nondegenerate supermodes is equal to the number of single-mode waveguides [22], [27]. By matching the number of modes, i.e., degrees of freedom, the lossless coupling becomes possible by conserving the entropy of the system; light can couple between this array of single-mode waveguides and a multimode waveguide via a gradual transition. This is a necessary but not sufficient condition; other factors such as single-mode array geometry, transition length, operation wavelengths, and single-mode waveguide used do also govern the overall transition efficiency [28], [29]. Photonic lanterns can be realized by a physical transition in which the single- or few-mode waveguides cannot reasonably guide a nice mode and form a composite waveguide, i.e., ceasing to behave as independent uncoupled waveguides.

The aim of a photonic lantern is to adiabatically form a multimode waveguide in which the single-mode waveguides either vanish or form a composite waveguide formed by strong coupling between them. Photonic lanterns can be manufactured by many techniques where three are shown in Fig. 2: the first two using tapering techniques with individual optical fibers [30] or multicore optical fibers [31] and the third by using ultrafast laser inscription (ULI) techniques [32], [33]. Air-clad lanterns can be made without using a capillary [34], [35], which can simplify the fabrication process but add other challenges, such as dealing with a tip with a diameter of a few

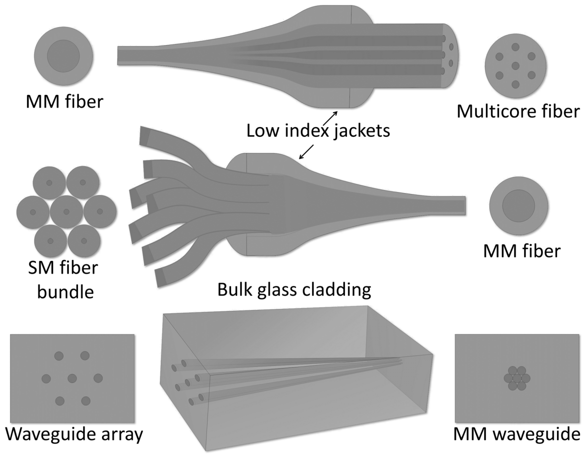


**Fig. 2.** ***Schematics of the three different photonic lantern fabrication approaches (top) MCF, (middle) fiber bundle, and (bottom) ULI (integrated photonic lantern).***

tens of microns. As will be discussed in Section VI, MPLC technology can mimic the adiabatic transition of lanterns through many closely spaced phase planes.

## B. Mode-Selective Devices

Individual modes in SDM systems represent an individual channel; hence, the loss of a mode is the loss of a data channel. Thus, it is important to have mode multiplexers/demultiplexers with negligible MDLs and coupling losses [36], [37]. Furthermore, since modes act as independent channels, mode control and selectivity on the mode multiplexers/demultiplexers are highly desirable to equalize MDL or differential group delays (DGDs). SDM systems can use MIMO processing to undo mode mixing occurring in the transmission MMF [37], [38]. Some fibers, such as the coupled-core fiber [39], have so much coupling that all launched signals spend equal amounts of time on all modes such that no signal sees an extreme loss or extreme gain. This averaging way of any mode dependence improves transmission performance due to reductions in the MDL and DGD. Without MIMO processing, any amount of mode mixing degrades transmission performance.

Due to mode mixing, in principle, a photonic lantern multiplexer/demultiplexer does not need to excite each independent mode but can excite orthogonal combinations of modes. However, MIMO processing has limitations in correcting for MDL or large delays caused by the DGD in the MMF or indeed differential mode gain in multimode amplifiers. In short-reach communications with very limited mode mixing, mode-selective transmission is possible without MIMO processing [34], [40], [41]. Therefore, mode input and output control in SDM systems are important for equalization and short-reach applications.

Mode selectivity can be added and tailored in photonic lanterns by using dissimilar single-mode waveguides at the single mode end rather than identical ones as in standard photonic lanterns. Along the adiabatic transition, the single-mode waveguides' dissimilarity will govern the coupling between the single-mode waveguides forcing light initially launched into certain single-mode waveguides to evolve into specific mode groups and not others [42], [43], [44]. Basically, an adiabatic transition from the single-mode end to the multimode end will ensure that light in the input waveguide of the $n$th greatest propagation constant will most likely excite only the output mode of the $n$th greatest propagation constant at the MMF end and vice versa. In most fiber photonic lanterns, the mode-selective bandwidth is several hundreds of nanometers, limited on the long wavelengths where the output multimode core no longer supports the correct number of modes and on the short wavelengths when the light remains well guided by the individual cores.

Fig. 3 shows a typical example of three- and six-mode-selective photonic lantern multiplexers/demultiplexers with their near-field LP mode profiles. Mode profiles are clearly influenced by the noncircular core symmetries of the fabricated photonic lanterns obtained by fusing fibers together during the tapering procedure. However, those asymmetries do not increase insertion losses significantly while splicing to the few-mode transmission fibers [45], [46]. The shown three-mode-selective photonic lantern, reported in [45], was fabricated by inserting three input fibers into a fluorine-doped capillary with an index difference of $4 \times 10^{-3}$ (compared to the silica forming the cladding of the single-mode fibers). Of the three input fibers, two of them are single-mode fibers with 8.2 $\mu$m core diameters, while the other has a slightly larger core of 15 $\mu$m and a similar core refractive index to an SMF-28 fiber. The cross section of the multimode end of the photonic lantern has a near-triangular 27-$\mu$m diameter core, as shown in Fig. 3(a). For the six-mode-selective photonic lantern, as shown in Fig. 3(b) and reported in [47], six step-index fibers with the following core diameters were used: one 15 $\mu$m for the $LP_{01}$, two 10 $\mu$m for the $LP_{11}$a/b, two 8 $\mu$m for the $LP_{21}$a/b, and one 5 $\mu$m for the $LP_{02}$. The

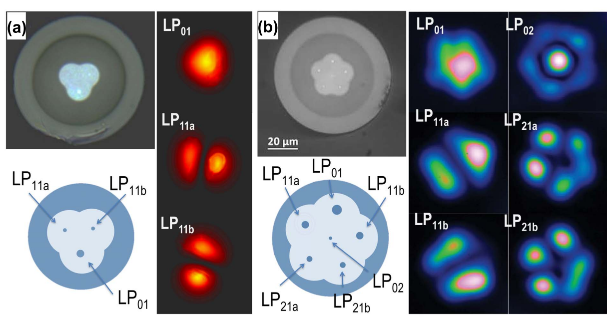


**Fig. 3.** ***Mode-selective photonic lantern devices. Photonic lantern few-mode end microscope image, single-mode waveguide geometry schematics, and near-field images at the few-mode end at 1550 nm for (a) three-mode-selective photonic lantern and (b) six-mode-selective photonic lantern.***

core refractive index difference with respect to undoped silica is $5 \times 10^{-3}$ for all step-index fibers. The multimode end core diameter of the six-mode photonic lantern is 29 $\mu$m.

## C. Scalability and Multicore Fan-Out Devices

Fiber-bundle photonic lanterns are made by placing multiple single-mode fibers inside a glass capillary with a lower refractive index than the core-cladding material of the fibers. The assemblage is then tapered down and fused together at one end of the structure, such as the low-index jacket and fused single-mode fiber cladding material form the multimode waveguide [see Fig. 2 (middle)]. Since the desired single-mode waveguide array geometries for efficient mode conversion described by Fontaine *et al.* [36] are not supported by simple fiber capillary bundling and/or stacking (i.e., hexagonal structures or rounded assemblies), scaling the fabrication process to more than six input fibers or modes (e.g., ten, 12, and 15) is challenging.

A new photonic lantern mode multiplexer/demultiplexer fabrication approach has overcome the geometry scaling issues using microstructured preforms [48]. This fabrication method is based on the stack-and-draw technique for photonic crystal fiber manufacturing. A microstructured preform with the right size, geometry, and refractive index layers is used as a template to insert the single-mode fibers instead of a plain lower index capillary, simplifying the fabrication process and improving the repeatability. Mode-selective photonic lanterns multiplexers/demultiplexers for the first ten and 15 spatial modes (six and nine LP modes, respectively) have been demonstrated by Velázquez-Benítez *et al.* [48]. Fig. 4 shows how the two different ten- and 15-mode-selective photonic lanterns were fabricated from the templates made by the stack-and-draw technique. The photonic lantern templates were fabricated inside fluorine-doped tubes using thin silica glass capillaries with a high air filling fraction, limiting the extra silica material around the fibers, which could affect the waveguide properties of the transition. The templates fix the positioning of the single-mode fibers (dissimilar or not) enabling geometric scaling as it can be appreciated in Fig. 4(c).

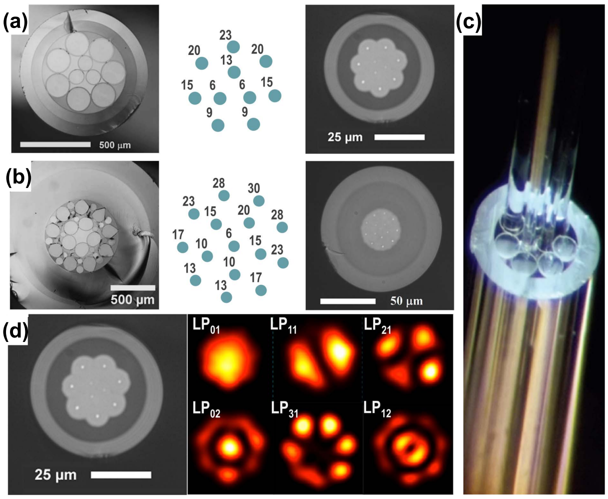


**Fig. 4.** *Geometry and core size arrangement, microstructured template, and photonic lantern multimode end face microscope image for (a) ten-mode-selective photonic lantern and (b) 15-mode-selective photonic lantern (numbers indicate core size diameters of the single-mode fibers). (c) Photonic lantern glass template filled with fibers accurately positioned by the microstructure. (d) Photonic lantern multimode end microscope image and near-field images of the six-LP modes (only one LP a/b type mode is shown for illustration purposes) at the few-mode end for the typical ten-mode-selective photonic lantern.*

To further help scalability of photonic lanterns, Huang *et al.* [46] recently demonstrated higher mode selectivity and relaxed adiabaticity requirements using few-mode graded index fibers as the single-mode input waveguides. The lantern typically brings fibers with claddings of 125 $\mu$m diameters to a final core size of 10–20 $\mu$m requiring a taper ratio typically between 20 and 30. Most of the taper, typically a reduction of the pitch by 3–5, brings the cores closer together without light escaping the cores and the quicker and more adiabatically this can be done will drastically reduce the lantern length. The local modes' "weak power transfer" criterion for adiabaticity in a taper transition dictates that: 1) the tapering rate (i.e., length of the transition) is proportional to the differences in propagation constants and 2) mode profiles that change slowly (i.e., more gradually) along the transition will lead to less crosstalk between the modes.

Photonic lanterns made with graded-index few-mode fibers can satisfy both requirements by: 1) providing a larger difference (and variety) in propagation constants easing adiabaticity requirement and 2) providing a more gradual fundamental mode field change during tapering compared to their few-mode step-index counterparts [46]. Using few-mode/MMFs as the single-mode waveguides in the fabrication of photonic lanterns may seem counterintuitive. However, their use is realistic for two reasons. First, when the length of the multimode graded-index fiber is relatively short, mode coupling from the fundamental mode to higher order modes is negligible. Second, coupling and matching single-mode fibers to the fundamental mode of graded-index MMFs with minimal loss even with dissimilar diameters are relatively straightforward. Furthermore, the effective area of the fundamental mode of the graded-index MMF can be designed to be as approximately as that of the single mode, as shown in Ref [46]. Note that there are other graded index fiber profiles, such as the logarithmic profile, where the fundamental mode does not change during the taper [49], [50], potentially enabling even shorter photonic lanterns.

In SDM communications, the scaling in the number of modes of photonic lantern mode multiplexers/demultiplexers is indeed driven by the desired increase

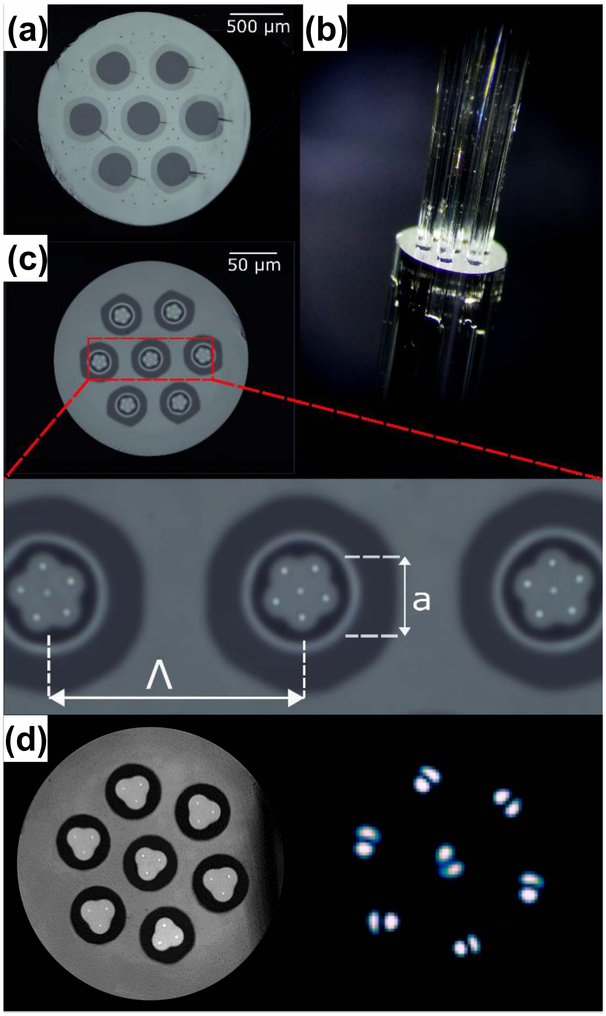


**Fig. 5.** *(a) Cross-sectional image of the microstructured glass template. (b) Image of the glass template after inserting seven initially tapered bundles. (c) Cross-sectional image of the fabricated 7 × 6 mode device after the microstructured template is adiabatically tapered and zoomed-in portion of the PL cores where ∧ represents the core-to-core distance and the parameter a represents the core diameter of the PL. (d) Cross-sectional image of the fabricated 7 × 3 mode device and near-field image of all excited $LP_{11}$ modes.*

in data bandwidth via new channels in SDM systems. Another, very active area in SDM is focused on using multicore few-mode fibers [51], [52] to further increasing spatial multiplexing possibilities. Additional mode scaling in few-mode MCFs is possible using photonic lanterns as mode multiplexers/demultiplexers. Few-mode multicore photonic lanterns can be fabricated using the microstructured template technique, where seven (or more) low refractive index fluorine-doped capillaries are used to insert the dissimilar single-mode fibers, thus forming the seven few-mode photonic lantern multimode ends, as shown in Fig. 5. Two seven-core, three-mode-selective, and six-mode-selective photonic lantern multiplexers/demultiplexers capable of selectively generating 21 and 42 spatial modes, respectively, on a few-mode MCF have been recently demonstrated [53], [54]. This new approach further extends the scalability of spatial channels in SDM systems using photonic lanterns multiplexers/demultiplexers. In the case shown here, two seven three- and six-mode cores in a hexagonal arrangement were demonstrated; however, the number of cores, the number of modes per core, and geometry arrangements are relatively straightforward to customize with the microstructured template method. To better match the mode structure of graded index fibers, photonic lanterns with a graded index core profile have been demonstrated [55].

The photonic lantern is a versatile and powerful technology, allowing transformation between optical multimode systems and single-mode ones. Photonic lantern multiplexers supporting three, six, ten, and 15 spatial modes have successfully been used in high-capacity transmission experiments with 100-TB/s data rates in single-core few-mode fibers [25], [56], [57]. Nowadays, photonic lanterns have shown their potential to perhaps be the best mode multiplexing technology for SDM up to ten spatial modes (limited by fabrication challenges) due to their sub-dB insertion losses, versatile broadband mode-selectivity well beyond 100 nm, and ease of integration due to direct splicing to fibers.

## IV. ULTRAFAST LASER INSCRIBED 3-D WAVEGUIDE DEVICES

3-D waveguide circuits are fabricated using a technique widely known as ULI [58], [59], [60]. A short-pulsed laser is tightly focused into a glass substrate to induce a localized and permanent refractive index change. Waveguides are created by translating the substrate in 3-D with respect to the stationary laser focus. The change in refractive index is typically caused by laser-induced defects that increase the polarizability of the substrate material and, thereby, its refractive index, or the laser causes a localized redistribution of the glass composition, which then changes the polarizability or density of the material [61], [62]. The index change is stable and can easily withstand temperatures in hundreds of °C [63]. The most common substrate materials for 3-D waveguide devices in optical communication are fused silica, borosilicate, or boro-aluminosilicate glasses. However, ULI can be used to fabricate waveguides in almost any transparent dielectric, ranging from polymers up to crystalline materials. The laser-induced refractive index change can range from $1 \times 10^{-3}$ to a few times $10^{-2}$, which compares well to the core-cladding index contrast of optical fibers. This ensures excellent mode-matching and, hence, low-loss coupling between 3-D waveguide devices and optical fibers. The index contrast and waveguide dimensions can be tuned through the inscription parameters, such as laser repetition rate, feedrate at which the sample is translated, the laser focusing conditions, the laser pulse duration, and the laser wavelength. The parameters have to be rigorously optimized for each material in order to achieve the desired index contrast and low optical losses. Laser-written waveguides feature propagation losses of less than $<$0.5 dB/cm [64] at telecommunication wavelengths, and losses similar or below 0.1 dB/cm have been reported [65], [66]. Moreover, ULI waveguides exhibit tunable birefringence with values as high as several times $10^{-4}$ [67] and down to $10^{-6}$ similar to optical fibers [68].

The attractiveness of ULI for SDM stems from its capability to create arbitrary 3-D waveguide patterns, ideally suited for interfacing with the 2-D core geometries of MCFs or multiplexing the LP modes of few-mode fibers. 3-D waveguide-based fan-outs for single-mode MCFs have been demonstrated for core counts ranging from four [69] up to 121 cores [70]. The 3-D waveguide chip acts as the intermediate between a standard fiber array with a linear arrangement of fibers and the 2-D core geometry of the MCF. Insertion losses close to 1 dB or even less are possible by ensuring good mode-matching to the fiber array and the MCF through tapering of the waveguides, and making the chip as short as possible while maintaining a waveguide bend radius above a certain value to limit bend losses [71]. Further integration is possible by for instance incorporating passive fiber alignment structures to create a single monolithic 84-channel interposer between silicon photonics and an array of 12 seven-core MCFs [72].

3-D waveguide technology has also produced mode multiplexers with up to six spatial modes. Multimode optical fibers have modes in both transverse directions, necessitating a 3-D waveguide geometry [73]. ULI photonic lanterns or integrated photonic lanterns were initially demonstrated for astronomical applications [32]. ULI's flexibility enables straightforward scaling to large port counts [74], [75]. To achieve mode-group-selective multiplexing, asymmetry in the waveguide propagation constants must be introduced by varying the inscription parameters. This has led to the demonstration of laser-written mode-group-selective three-mode [76] and six-mode photonic lanterns [77]. The challenge of ULI photonic lanterns is efficient coupling with low-MDLs to graded-index few-mode fibers due to the mismatch between the fiber and 3-D waveguide chip in terms of refractive index profile and index contrast [78]. Alternatively, tapered couplers where the multimode waveguide cores increases in size and the single-mode input cores decrease in size can be used for mode-multiplexing. These couplers are based on an adiabatic mode-evolution similar to the classic photonic lanterns but feature a single few-mode graded-index waveguide at the output [21]. This enables low coupling losses, broad operational bandwidth ($>$100 nm), as well as high mode-extinction ratios ($>$20 dB) [79]. Interference-based mode multiplexers are similar to directional couplers. However, the fundamental mode in one arm is phase-matched to the desired higher order mode in the waveguide of the coupler's second arm. Benefitting from the ability of ULI to tailor the refractive index contrast and waveguide dimensions, efficient couplers for the $C$-band have been realized [80]. However, maintaining phase-matching across a broad bandwidth is challenging. Thus, these interference couplers suffer from a narrower bandwidth compared to multiplexers based on an adiabatic modal evolution, such as tapered couplers or photonic lanterns.

The largest number of parallel spatial channels in SDM can be achieved by combining the multicore and few-mode fiber approach through few-mode MCFs. These fibers feature multiple cores, where each core supports multiple modes. Nonmode-selective 3-D waveguide fan-outs have been demonstrated for fibers with seven cores/three modes ($LP_{01}$, $LP_{11a}$, and $LP_{11b}$) each (21 spatial channels) [52], 19 cores/three modes (57 spatial channels) [81], and even 36 cores/three modes (108 spatial channels) by using integrated photonic lanterns [82]. Mode-selective multiplexing has the benefit of enabling the compensation of differential mode-delay and MDLs. Using tapered couplers operating across the $S$-, $C$-, and $L$-bands, ULI has enabled a monolithic mode multiplexer for a four-core, three-mode fiber [83].

The strong potential of ULI 3-D waveguide devices for SDM-based optical communication has been validated by many long distance, high data-rate experiments, such as 159 Tbit/s across $>$1000 km of the three-mode graded-index few-mode fiber [84], 255 Tbit/s across a seven-core/three-mode few-mode MCF [52], 1.2 Pbit/s across a four-core/three-mode few-mode MCF [85], and recently 10.66 Pbit/s across a 36-core/three-mode MCF [86].

## V. MICRO-OPTIC DEVICES

Compact micro-optic devices based on free-space optics can, in principle, accommodate hundreds of spatial modes if they can be guided through the device without introducing MDL. Such integration can provide huge cost savings compared to individually packaged multiple single-mode fiber components. In addition, devices such as MMF collimators are essential to build MPLC devices described in Section VI. Numerous commercially available fiber optic devices already contain micro-optic lenses to guide optical beams between multiple fibers through dielectric thin film filters, isolators, or polarization elements. In a SDM system with multiple spatial modes (or cores), the spatial mode overlap should be precisely matched for all modes (or cores), which puts more constraints on the optical design. The easiest way to satisfy these design constraints is to use optics in a $4f$ lens configuration, which moves/relays both the amplitude and phase exactly from one plane to another plane. As shown in the subfigure of Fig. 6, a $4f$ system can be implemented with the same lens type [87], but it can also be implemented using the lens with different focal lengths ($f1$ and $f2$) if a scaling factor is needed. In this section, the basic building block of SDM fiber components based on micro-optic collimators is detailed.

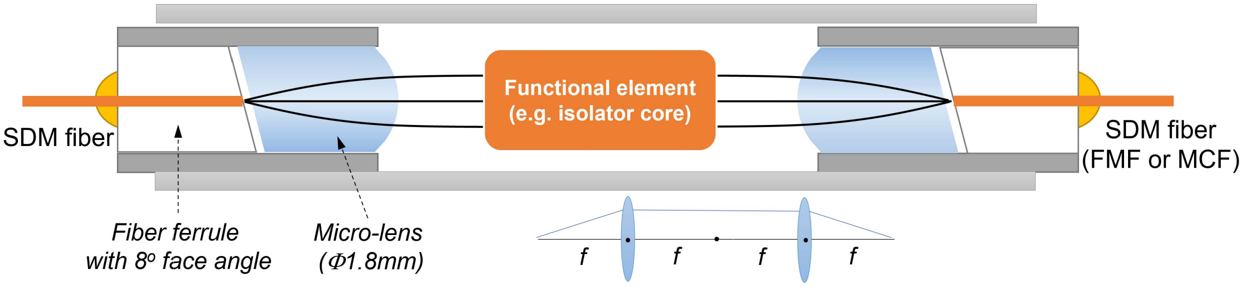


**Fig. 6.** ***Schematic of micro-optic collimator-based SDM fiber components.***

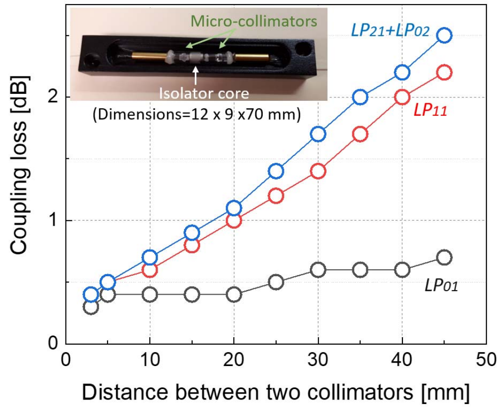


**Fig. 7.** ***Optical performance of the FMF isolator using two GRIN lenses and one free-space isolator. The lowest MDL occurs when the collimators are placed in a 4f configuration that leaves two focal lengths between the collimators (3.9 mm).***

## A. Multicore and Multimode Collimators

Micro-optic collimators are widely used to collimate the light from an optical fiber (or to launch the collimated light into a fiber) and are the basic elements for many active and passive single-mode fiber components, such as optical isolators, circulators, wavelength-division multiplexed (WDM) couplers, and optical switches. In order to provide a scalable and reliable platform to fabricate SDM fiber components, the micro-optic collimator technology has been extended to various SDM fibers (e.g., FMFs and MCFs) [88], [89], [90], [91]. It provides for an array of new and practical packaged components with good optical performance in terms of functionality and an insertion loss broadly comparable with equivalent existing single-mode fiber devices while, at the same time, ensuring low levels of mode (or core) dependent loss or intermodal (or intercore) crosstalk.

Fig. 6 shows a schematic of a representative SDM fiber component consisting of a pair of SDM fiber collimators and with a functional optical element in the middle. As compact micro-optic lens elements, C-lenses or gradient-index (GRIN) lenses having a diameter of 1.8 mm are typically used to transform the emergent light from the input SDM fiber into a collimated free-space beam that can then be refocused into another SDM fiber using a second identical assembly in reverse. Optical elements can then be inserted into the free-space region to provide the required in-line functionality. For example, one may insert an optical isolator core in order to obtain a fiberized optical isolator or a dielectric multilayer thin-film device as a gain flattening filter.

Initially, FMFs were incorporated into this micro-optic platform, and mode-dependent coupling loss, which is one of the most important optical specifications of FMF components, was examined as a function of the distance between the two collimators [88]. As shown in Fig. 7, higher order modes are found to experience slightly increased coupling losses relative to lower order modes due to their larger mode field diameters, and the MDL gradually increased as the gap distance was extended. For most fiber components, a gap distance of 5–10 mm, which is practically achievable in a $4f$ configuration, is sufficient to accommodate most functional elements of interest, and an MDL of 0.2–0.4 dB can readily be achieved with an average insertion loss of $<1$ dB. This platform can be readily extended to multiport FMF components [89], and various FMF devices have been developed, including optical isolators, gain flattening filters, optical circulators, beam splitters, and WDM filters.

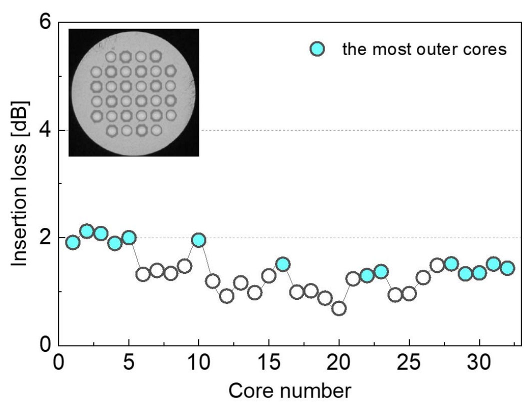


**Fig. 8.** ***Optical performance of the MCF isolator.***

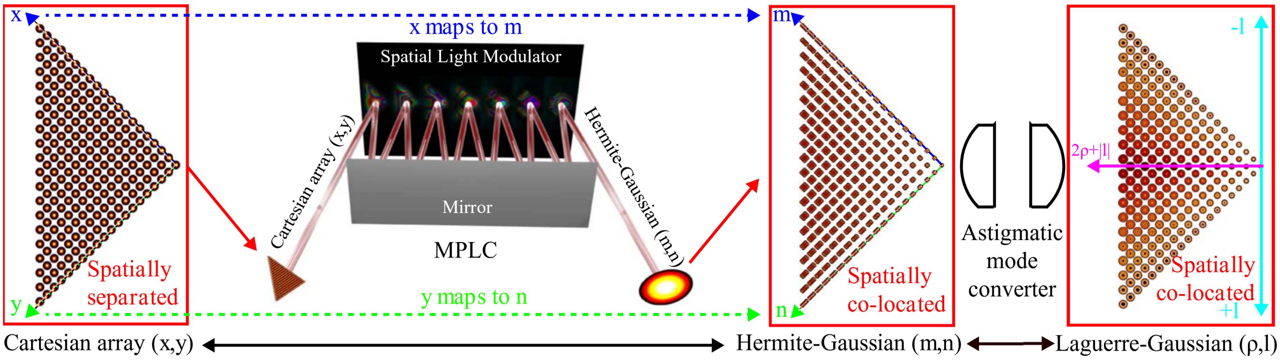


**Fig. 9.** *MPLC device to rearrange a triangle array of Gaussian beams to spatially overlapping HG modes. An astigmatic mode converter can convert between HG and LG modes.*

In the case of MCFs, the device fabrication process is similar to that of FMF components, but a precise rotational alignment of the MCF collimators is additionally required to fabricate low-loss devices. Rotational alignment accuracy of less than 1° is typically required to achieve less than 0.2-dB core-dependent loss [92]. As shown in Fig. 8, for the case of a 32-core fiber, the outer cores are highlighted as filled circles, and it is apparent that the outer cores experience a slightly larger insertion loss relative to the inner cores due to the imperfections in the angular orientation of the MCF collimator assembly. This is simply due to the larger transverse offset for the outer cores for a particular angular displacement. The initial measured device performance for a 32-core MCF isolator [88] looks most encouraging, yielding an average insertion loss of 1.5 dB, core-to-core loss variation of 1.5 dB, and an intercore crosstalk of less than −40 dB.

The results discussed above indicate that conventional micro-optic collimator technology can be successfully applied to both FMFs and MCFs such that the same optical function can be achieved on all spatial modes (or cores) simultaneously with nearly identical performance in a single device. The approach clearly demonstrates the potential benefits of SDM fiber components in terms of materials, space, and cost savings relative to the use of an equivalent number of conventional single-mode fiber components.

## VI. MULTIPLANE LIGHT CONVERSION

MPLC is built from multiple phase planes separated by free space, as shown in Figs. 9 and 10. Using multiple phase planes rather than a single plane can convert an orthogonal set of input modes into a completely different orthogonal set of output modes [23]. Most often, the input modes are spatially separated array of spots, and the output modes are the spatially overlapping modes of an optical fiber. Conceptually, MPLC approximates a 3-D refractive index device through a series a phase plates that provide position-dependent phase shifts. The phase profiles of the best devices will be smooth and appear to be made up of many arbitrary shaped and overlapping lens, such as structures resembling smooth rolling hills rather than having sharp phase jumps commonly seen in diffractive phase plates. MPLC can handle high powers [93], and since MPLC only needs a patterned mirror as a phase plate, they can potentially be built at challenging wavelengths, such as the UV or the far infrared, where integrated technologies may be too absorptive.

One of the attractive features of MPLC over waveguide-based devices is its ability to implement completely arbitrary spatial transformations, which would be impractical using a planar waveguide device. With MPLC, there is effectively complete freedom to implement any phase mask in any plane, whereas a waveguide-based platform has much more limited fabrication constraints. Theoretically, each phase plane of an MPLC device incurs zero loss, and in that sense, any spatial transformation could be implemented for an arbitrary optical bandwidth using a sufficiently large number of planes. Yet, the number of planes required for a given transformation in practice is an important consideration. Not only will the losses in the practice of a device be proportional to the number of optical interfaces but a large number of planes can also become impractical to build. A large number of planes place high demands on parameters such as the precision of the phase masks (as errors will cascade), as well as flatness and parallelism of optical components.

### A. How Many Phase Planes Are Required?

In the general case, in order to spatially transform between two arbitrary spatial bases each composed of $N$ spatial modes, an order of $N$ phase planes is required. This has similarities to implementing an arbitrary 1-D spatial transformation using $N$ layers of Mach–Zehnder interferometer meshes [95]. The first MPLC-based devices [96], [97] followed this trend. However, some spatial transformations are easier than others and require substantially fewer planes of phase manipulation to accomplish. For example, a simple lens (parabolic phase profile) is able to implement a spatial Fourier transform consisting of a

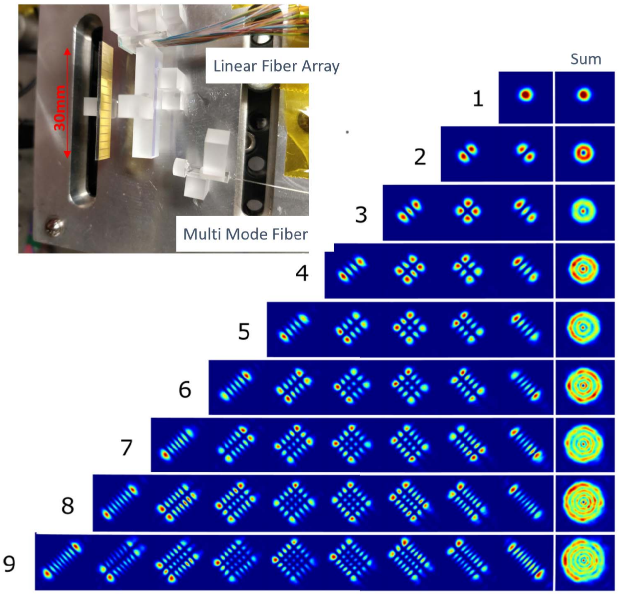


**Fig. 10.** ***Assembled MPLC device on an aluminum base with fiber array, phase mask (gold), dielectric mirror, and output MMF. Plots show the output HG modes before launching into an MMF arranged by mode group (rows). The sum column is the intensity summation of all modes in the group.***

very large number of modes using just a single plane of phase manipulation. As it turns out, Hermite–Gaussian (HG) and Laguerre–Gaussian (LG) spatial transformations are another special case, in which a relatively large number of modes can be transformed using a small number of planes [24], [98], [99]. A device arranging a linear spot array to the first 45 HG modes is shown in Fig. 10. 210 and 325 modes have been demonstrated using just seven phase planes implemented on a single spatial light modulator. The LG mode sorter is illustrated in Fig. 9. The transformation that it performs maps Gaussian spots on a Cartesian grid $(x, y)$ to HG modes of the corresponding Cartesian indices $(m, n)$. These HG modes can then be converted to LG modes using two cylindrical lenses. It is important to note that the drastic reduction in the required number of phase planes is only possible for this specific configuration, whereby the Cartesian spot position maps to the Cartesian index of the HG modes. Some different mappings between the triangular spot array and the HG mode basis would, in general, require a number of planes that scale with the number of modes, rather than in this case, where the number of planes scales like the square root of $N$. There have been some speculations that the HG transform is efficient due to its separability [98], but it also strongly depends on arranging the inputs in increasing mode order to form a triangle.

## B. Applications

Within the context of optical communications, these HG and LG mode sorters can serve as spatial mode multiplexers [26], [100] with the potential to scale to very large mode counts. Designs up to 1035 modes have been demonstrated [101]. The devices themselves can be prototyped using spatial light modulators [24]; however, they are best implemented by lithographically etching fixed phase masks that are then glued and assembled in place [100]. As mode multiplexers/demultiplexers, these devices could also be used as part of SDM wavelength-selective switches [102] and amplifiers. Recently, an HG mode sorter was used in conjunction with a modified wavelength-selective switch to create an arbitrary spatiotemporal beam shaper [103]. A type of spectral pulse shaper is able to independently control the

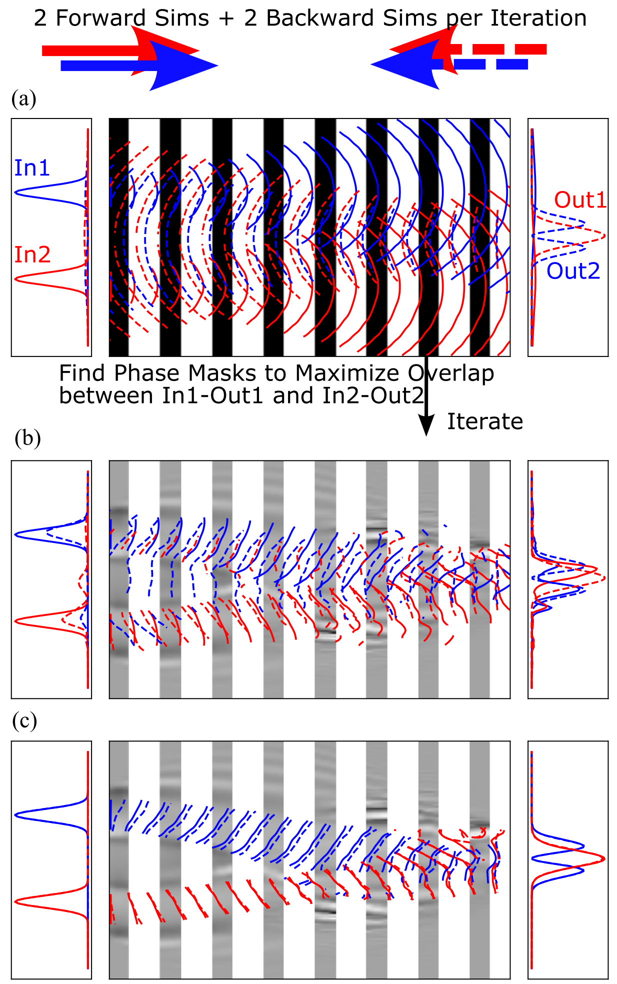


**Fig. 11.** ***Illustration of wavefront matching algorithm between forward modes (solid) and backward/output modes (dashed) at (a) initialization, (b) couple iterations, and (c) fully converged. Launched modes are propagated from the input to a phase plane. Target output modes are propagated backward in time from the output to the input. The overlaps between the pairs of input and output modes are compared to find a phase correction that minimizes the error. A detailed example of walking through the algorithm can be found from the timestamp of 3 min and 30 s in [94]. (a) Zeroed phase masks. (b) Partially matched wavefronts. (c) Matched wavefronts.***

**Table 1** High-Level Comparison of Different SDM Multiplexing Technologies

| Device | Best Application | Scalability | Fabrication Maturity | Loss | Bandwidth | Size |
|---|---|---|---|---|---|---|
| Photonic Lantern | low loss and broadband | 100s of fibers | labor intensive | $< 1$ dB | 100-500nm | a few cm long |
| MPLC | complex modes | 1035 HG modes | waferscale | $> 3$ dB | 100 nm | 100 $mm^3$ to 10 $cm^3$ |
| 3D Waveguide | spot remapper | 100s of cores | mature | 1 dB | >100 nm | 10 mm$\times$10 mm$\times$1 mm |

temporal/spectral features of up to 90 spatial/polarization modes.

### C. Wavefront Matching Algorithm

Ultimately, any lossless optical system is constructed using a series of phase manipulations. Be it a lens relay, a waveguide, or dispersion by a prism. In that sense, MPLC can implement arbitrary optical systems by approximating the desired phase profiles using discrete planes. Designing an MPLC device is done through the use of an inverse design algorithm process [104] based on a wavefront matching approach [99], [105]. Fig. 11 describes the wavefront matching procedure, and [94] is an excellent illustrative video available on YouTube. The final phase masks produced by the design process are often quite nonintuitive, yet the underlying principles of the design process are straightforward. At its core, the approach of wavefront matching simply acknowledges that, in order for a lossless transformation to occur between a set of $N$ input (forward) spatial modes and a corresponding set of $N$ output (backward) spatial modes, there must be complete overlap between each pair of forward and backward modes everywhere in space. That is, the spatial amplitude and phase of each pair of forward and backward modes must be the same throughout the entire transition. Hence, the goal of the MPLC device is to spatially match the phases between the forward and backward modes until they become identical, and hence, a lossless transformation has occurred. Fig. 11 shows the wavefront matching algorithm for two spatial modes. The algorithm itself starts by propagating all $N$ spatial modes in both directions [see Fig. 11(a)]. The wavefronts do not match anywhere, and changing the phase at each phase will improve the matching. That phase planes are then updated to become the average phase difference between all pairs of forward and backward modes. This maximizes the coupling between the forward and backward modes, and moves the solution toward a lower loss result. The forward/backward modes are propagated through the system again to provide a better optimized set of phase masks [see Fig. 11(b)]. The forward and backward modes match better at the inputs and outputs, and throughout the device. More iterations are continued until the process converges to the lowest loss solution that it can find [see Fig. 11(c)]. Here, notice that the wavefronts between forward and backward (in/out modes) are the same with respect to a phase shift. The process could also be done sequentially by moving modes forward and backward sequentially through phase planes and updating one phase plane at a time.

The design process shares some similarities with the approach of the Gerchberg–Saxton algorithm for designing single-phase masks' systems. In that, like the Gerchberg–Saxton algorithm, there is no error function or gradients that are directly calculated. Rather, a solution is found by continual feedback and reinforcement of solutions that align with the goal. The basic wavefront matching algorithm simply optimizes the coupling between pairs of input/output modes and cannot directly optimize MDL, mask smoothness, or the number of phase planes. For this, more advanced optimization techniques from the machine learning field can be applied to wavefront matching to optimize additional cost functions [106].

MPLC is an exciting area of research with potential use cases not only in optical telecommunications but also areas such as quantum optics [107], [108], spatiotemporal beam shaping [103], and astrophotonics [109]. All of which could benefit from MPLCs' often surprising ability to implement optical systems with functionalities that have not previously been available.

## VII. CONCLUSION AND DISCUSSION

This article reviewed four multiplexing technologies that can convert spatially separated beams into the modes of optical fibers. These are photonic lanterns, MPLC, 3-D waveguides, and free-space devices. There are many other types of less used mode multiplexing devices, such as cascaded directional couplers [110], which were not discussed. Which technology to deploy in your application will depend on numerous factors, such as the number of modes, the type of modes, the loss requirements, and the quantity required. A comparison is tabulated in Table 1.

Each device has a key strength and challenge. Photonic lanterns are applicable to MMF and the SMF inputs producing devices with losses well below 1 dB and bandwidths well over 100 nm. Unfortunately, their fabrication is labor intensive requiring special doped optical glass preforms, capillaries, and fibers followed by stacking and numerous tapering steps. There are also few companies that can make these devices. Theoretically, they could scale to hundreds of modes, but the fabrication challenges practically limit lanterns to smallish mode counts for SDM. Lanterns for astronomy support hundreds of modes, but they often can have high MDL even if the insertion losses are sub-dB (i.e., one mode is missing), which is an issue for optical communications.

Theoretically, MPLC can build any unitary transformation between two sets of spatial modes. In practice, this is true for infinite planes, and only certain transforms, such as the HG mode multiplexer, work extremely well. Practically, for fiber-to-fiber losses around 3 dB, the plane count should be less than 10 as losses increase and packaging becomes more difficult. MPLC leverages wafer scale gray-scale lithography and, therefore, can be relatively inexpensive and mass-produced. MPLC should be chosen when a very specific transform is required, and slightly elevated losses are allowed. It is clear that MPLC produces the best HG mode multiplexers and is ideal to couple to graded index fibers. For losses similar to photonic lanterns, the phase mask fabrication requires extremely smooth phase masks that do not scatter light.

3-D waveguides excel at making fan-ins or remappers. Many devices can be fabricated on a wafer programmatically and, therefore, are mass-producible. Multiple companies, such as Optoscribe and Modular Photonics, can produce excellent 3-D-waveguide devices. Due to the limitations in refractive index modification, it is difficult to produce mode multiplexers.

This tradeoff discussion has indicated that there is still much development and research needed to improve the performance of all these devices. In the future, we believe that all these devices will have losses below 1-dB loss and can be mass-produced, which will enable numerous exciting new applications. ■

## REFERENCES


[1] D. Richardson, J. Fini, and L. Nelson, "Space-division multiplexing in optical fibres," *Nature Photon.*, vol. 7, no. 5, pp. 354–362, 2013.

[2] G. Li, N. Bai, N. Zhao, and C. Xia, "Space-division multiplexing: The next frontier in optical communication," *Adv. Opt. Photon.*, vol. 6, no. 4, pp. 413–487, Oct. 2014.

[3] R. Ryf *et al.*, "Space-division multiplexing over 10 km of three-mode fiber using coherent 6 × 6 MIMO processing," in *Proc. Nat. Fiber Opt. Eng. Conf.* Washington, DC, USA: Optical Society of America, 2011, pp. 1–3, Paper PDPB10.

[4] P. J. Winzer, "Optical networking beyond WDM," *IEEE Photon. J.*, vol. 4, no. 2, pp. 647–651, Apr. 2012.

[5] B. J. Puttnam, G. Rademacher, and R. S. Luís, "Space-division multiplexing for optical fiber communications," *Optica*, vol. 8, no. 9, pp. 1186–1203, 2021.

[6] D. Soma *et al.*, "2.05 peta-bit/s super-Nyquist-WDM SDM transmission using 9.8-km 6-mode 19-core fiber in full c band," in *Proc. Eur. Conf. Opt. Commun. (ECOC)*, Sep. 2015, pp. 1–3.

[7] M. Suzuki, D. Soma, T. Tsuritani, and I. Morita, "Ultra-high capacity optical transmission over 10-Pbit/s based on optical space division multiplexing technologies," in *Proc. 22nd Int. Conf. Transparent Opt. Netw. (ICTON)*, Jul. 2020, pp. 1–4.

[8] Y. Choi *et al.*, "Scanner-free and wide-field endoscopic imaging by using a single multimode optical fiber," *Phys. Rev. Lett.*, vol. 109, no. 20, Nov. 2012, Art. no. 203901.

[9] T. Čižmár and K. Dholakia, "Shaping the light transmission through a multimode optical fibre: Complex transformation analysis and applications in biophotonics," *Opt Exp.*, vol. 19, no. 20, pp. 18871–18884, 2011.

[10] N. Jovanovic *et al.*, "Integrated photonic building blocks for next-generation astronomical instrumentation I: The multimode waveguide," *Opt. Exp.*, vol. 20, no. 15, pp. 17029–17043, 2012.

[11] G. B. Xavier and G. Lima, "Quantum information processing with space-division multiplexing optical fibres," *Commun. Phys.*, vol. 3, no. 1, pp. 1–11, Dec. 2020.

[12] M. Ureña, I. Gasulla, F. J. Fraile, and J. Capmany, "Modeling optical fiber space division multiplexed quantum key distribution systems," *Opt. Exp.*, vol. 27, no. 5, pp. 7047–7063, 2019.

[13] U. Teğin, M. Yıldırım, İ. Oğuz, C. Moser, and D. Psaltis, "Machine learning with multimode fibers," in *Proc. Conf. Lasers Electro-Opt.*, 2021, pp. 1–2.

[14] U. Teğin, M. Yıldırım, İ. Oğuz, C. Moser, and D. Psaltis, "Scalable optical learning operator," *Nature Comput. Sci.*, vol. 1, no. 8, pp. 542–549, Aug. 2021.

[15] H. Song *et al.*, "Simultaneous turbulence mitigation and channel demultiplexing using a single multi-plane light convertor for a free-space optical link with two 100-Gbit/s OAM channels," *Opt. Commun.*, vol. 501, Dec. 2021, Art. no. 127359. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0030401821006088

[16] A. E. Willner, K. Pang, H. Song, K. Zou, and H. Zhou, "Orbital angular momentum of light for communications," *Appl. Phys. Rev.*, vol. 8, no. 4, 2021, Art. no. 041312.

[17] A. Billaud *et al.*, "Optimal coherent beam combining based on multi-plane light conversion for high throughput optical feeder links," in *Proc. IEEE Int. Conf. Space Opt. Syst. Appl. (ICSOS)*, Oct. 2019, pp. 1–5.

[18] N. K. Fontaine *et al.*, "Digital turbulence compensation of free space optical link with multimode optical amplifier," Tech. Rep., 2019. [Online]. Available: https://ieeexplore.ieee.org/document/9125743

[19] N. K. Fontaine, C. R. Doerr, and L. L. Buhl, "Efficient multiplexing and demultiplexing of free-space orbital angular momentum using photonic integrated circuits," in *Proc. Opt. Fiber Commun. Conf.* Washington, DC, USA: Optica Publishing Group, 2012, pp. 1–3, Paper OTu1I-2.

[20] D. J. Geisler, T. M. Yarnall, M. L. Stevens, C. M. Schieler, B. S. Robinson, and S. A. Hamilton, "Multi-aperture digital coherent combining for free-space optical communication receivers," *Opt. Exp.*, vol. 24, no. 12, pp. 12661–12671, Jun. 2016.

[21] S. Gross, N. Riesen, J. D. Love, and M. J. Withford, "Three-dimensional ultra-broadband integrated tapered mode multiplexers," *Laser Photon. Rev.*, vol. 8, no. 5, pp. L81–L85, Sep. 2014, doi: 10.1002/lpor.201400078.

[22] S. G. Leon-Saval, A. Argyros, and J. Bland-Hawthorn, "Photonic lanterns: A study of light propagation in multimode to single-mode converters," *Opt. Exp.*, vol. 18, no. 8, pp. 8430–8439, Apr. 2010. [Online]. Available: http://www.opticsexpress.org/abstract.cfm?URI=oe-18-8-8430

[23] J.-F. Morizur *et al.*, "Programmable unitary spatial mode manipulation," *J. Opt. Soc. Amer. A, Opt. Image Sci.*, vol. 27, no. 11, pp. 2524–2531, Nov. 2010. [Online]. Available: http://josaa.osa.org/abstract.cfm?URI=josaa-27-11-2524

[24] N. K. Fontaine, R. Ryf, H. Chen, D. T. Neilson, K. Kim, and J. Carpenter, "Laguerre–Gaussian mode sorter," *Nature Commun.*, vol. 10, no. 1, pp. 1–7, Dec. 2019.

[25] N. K. Fontaine *et al.*, "30 × 30 MIMO transmission over 15 spatial modes," in *Proc. Opt. Fiber Commun. Conf.* Washington, DC, USA: Optical Society of America, 2015, pp. 1–3, Paper Th5C-1.

[26] R. Ryf *et al.*, "High-spectral-efficiency mode-multiplexed transmission over graded-index multimode fiber," in *Proc. Eur. Conf. Opt. Commun. (ECOC)*, Sep. 2018, pp. 1–3.

[27] F. Ladouceur and J. D. Love, "Multiport single-mode fibre splitters," *Opt. Quantum Electron.*, vol. 22, no. 5, pp. 453–465, Sep. 1990.

[28] T. A. Birks, I. Gris-Sánchez, S. Yerolatsitis, S. G. Leon-Saval, and R. R. Thomson, "The photonic lantern," *Adv. Opt. Photon.*, vol. 7, no. 2, pp. 107–167, Jun. 2015. [Online]. Available: http://www.osapublishing.org/aop/abstract.cfm?URI=aop-7-2-107

[29] S. G. Leon-Saval, N. K. Fontaine, and R. Amezcua-Correa, "Photonic lantern as mode multiplexer for multimode optical communications," *Opt. Fiber Technol.*, vol. 35, pp. 46–55, Feb. 2017. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1068520016300797

[30] D. Noordegraaf, P. M. W. Skovgaard, M. D. Nielsen, and J. Bland-Hawthorn, "Efficient multi-mode to single-mode coupling in a photonic lantern," *Opt. Exp.*, vol. 17, no. 3, pp. 1988–1994, Feb. 2009. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-17-3-1988

[31] T. A. Birks, B. J. Mangan, A. Díez, J. L. Cruz, and D. F. Murphy, "'Photonic lantern' spectral filters in multi-core fibre," *Opt. Exp.*, vol. 20, no. 13, pp. 13996–14008, Jun. 2012. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-20-13-13996

[32] R. R. Thomson, T. A. Birks, S. G. Leon-Saval, A. K. Kar, and J. Bland-Hawthorn, "Ultrafast laser inscription of an integrated photonic lantern," *Opt. Exp.*, vol. 19, no. 6, pp. 5698–5705, Mar. 2011. [Online]. Available: http://www.ncbi.nlm.nih.gov/pubmed/21445210

[33] I. Spaleniak, N. Jovanovic, S. Gross, M. J. Ireland, J. S. Lawrence, and M. J. Withford, "Integrated photonic building blocks for next-generation astronomical instrumentation II: The multimode to single mode transition," *Opt. Exp.*, vol. 21, no. 22, pp. 27197–27208, Nov. 2013. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-21-22-27197

[34] L. Gruner-Nielsen, N. M. Mathew, M. Lillieholm, M. Galili, and K. Rottwitt, "Modeling of MIMO less mode division multiplexed systems," *IEEE Photon. Technol. Lett.*, vol. 32, no. 18, pp. 1191–1194, Sep. 15, 2020.

[35] N. M. Mathew, L. Grüner-Nielsen, M. Galili, M. Lillieholm, and K. Rottwitt, "MDM transmission using air-clad photonic lanterns," *IEEE Photon. Technol. Lett.*, vol. 32, no. 17, pp. 1049–1052, Sep. 1, 2020.

[36] N. K. Fontaine, R. Ryf, J. Bland-Hawthorn, and S. G. Leon-Saval, "Geometric requirements for photonic lanterns in space division multiplexing," *Opt. Exp.*, vol. 20, no. 24, pp. 27123–27132, Nov. 2012. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-20-24-27123

[37] P. J. Winzer and G. J. Foschini, "MIMO capacities and outage probabilities in spatially multiplexed optical transport systems," *Opt. Exp.*, vol. 19, no. 17, pp. 16680–16696, Aug. 2011. [Online].

Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-19-17-16680
[38] R. Ryf *et al.*, "Mode-division multiplexing over 96 km of few-mode fiber using coherent $6 \times 6$ MIMO processing," *J. Lightw. Technol.*, vol. 30, no. 4, pp. 521–531, Feb. 2012. [Online]. Available: http://www.osapublishing.org/jlt/abstract.cfm?URI=jlt-30-4-521
[39] R. Ryf *et al.*, "Space-division multiplexed transmission over 4200-km 3-core microstructured fiber," in *Proc. Nat. Fiber Opt. Eng. Conf.* Washington, DC, USA: Optica Publishing Group, 2012, pp. 1–3, Paper PDP5C.2. [Online]. Available: http://opg.optica.org/abstract.cfm?URI=NFOEC-2012-PDP5C.2
[40] S. Jiang, C. Liang, L. Ma, J. Xiong, W. Zhang, and Z. He, "Ultra-low-loss broadband all-fiber mode selective couplers for MIMO-less MDM transmission," *J. Lightw. Technol.*, vol. 38, no. 8, pp. 2376–2382, Apr. 15, 2020.
[41] H. Liu *et al.*, "Demonstration of stable $3 \times 10$ Gb/s mode group-multiplexed transmission over a 20 km few-mode fiber," in *Proc. Opt. Fiber Commun. Conf. (OFC)*, 2018, pp. 1–3.
[42] K. Lai, S. G. Leon-Saval, A. Witkowska, W. J. Wadsworth, and T. A. Birks, "Wavelength-independent all-fiber mode converters," *Opt. Lett.*, vol. 32, no. 4, pp. 328–330, 2007. [Online]. Available: http://www.osapublishing.org/ol/abstract.cfm?URI=ol-32-4-328
[43] S. G. Leon-Saval *et al.*, "Mode-selective photonic lanterns for space-division multiplexing," *Opt. Exp.*, vol. 22, no. 1, pp. 1036–1044, Jan. 2014. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-22-1-1036
[44] S. Yerolatsitis, I. Gris-Sánchez, and T. A. Birks, "Adiabatically-tapered fiber mode multiplexers," *Opt. Exp.*, vol. 22, no. 1, pp. 608–617, Jan. 2014. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-22-1-608
[45] C. Xia *et al.*, "Time-division-multiplexed few-mode passive optical network," *Opt. Exp.*, vol. 23, no. 2, pp. 1151–1158, Jan. 2015. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-23-2-1151
[46] B. Huang *et al.*, "All-fiber mode-group-selective photonic lantern using graded-index multimode fibers," *Opt. Exp.*, vol. 23, no. 1, pp. 224–234, Jan. 2015. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-23-1-224
[47] A. M. Velazquez-Benitez *et al.*, "Six mode selective fiber optic spatial multiplexer," *Opt. Lett.*, vol. 40, no. 8, pp. 1663–1666, Apr. 2015. [Online]. Available: http://www.osapublishing.org/ol/abstract.cfm?URI=ol-40-8-1663
[48] A. M. Velázquez-Benítez *et al.*, "Scaling photonic lanterns for space-division multiplexing," *Sci. Rep.*, vol. 8, no. 1, p. 8897, Jun. 2018, doi: 10.1038/s41598-018-27072-2.
[49] K. Harrington, S. Yerolatsitis, D. Van Ras, D. M. Haynes, and T. Birks, "Endlessly adiabatic fiber with a logarithmic refractive index distribution," *Optica*, vol. 4, no. 12, pp. 1526–1533, 2017.
[50] Y. Jung, K. Harrington, S. Yerolatsitis, D. J. Richardson, and T. A. Birks, "Adiabatic higher-order mode microfibers based on a logarithmic index profile," *Opt. Exp.*, vol. 28, no. 13, pp. 19126–19132, 2020.
[51] Y. Sasaki, K. Takenaga, S. Matsuo, K. Aikawa, and K. Saitoh, "Few-mode multicore fibers for long-haul transmission line," *Opt. Fiber Technol.*, vol. 35, pp. 19–27, Feb. 2017. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S106852001630116X
[52] R. G. H. van Uden *et al.*, "Ultra-high-density spatial division multiplexing with a few-mode multicore fibre," *Nature Photon.*, vol. 8, no. 11, pp. 865–870, Nov. 2014. [Online]. Available: http://www.nature.com/doifinder/10.1038/nphoton.2014.243
[53] Z. S. Eznaveh, J. E. Antonio-Lopez, J. C. A. Zacarias, A. Schülzgen, C. M. Okonkwo, and R. A. Correa, "All-fiber few-mode multicore photonic lantern mode multiplexer," *Opt. Exp.*, vol. 25, no. 14, pp. 16701–16707, Jul. 2017. [Online]. Available: http://www.osapublishing.org/oe/abstract.cfm?URI=oe-25-14-16701
[54] J. C. Alvarado-Zacarias *et al.*, "7-core$\times$6-mode photonic lantern mode multiplexer," in *Proc. 45th Eur. Conf. Opt. Commun.*, 2019, pp. 1–4.
[55] J. C. Alvarado-Zacarias *et al.*, "Mode selective photonic lantern with graded index core," in *Proc. Opt. Fiber Commun. Conf.*, 2018, pp. 1–3.
[56] R. Ryf *et al.*, "Photonic-lantern-based mode multiplexers for few-mode-fiber transmission," in *Proc. Opt. Fiber Commun. Conf.* Washington, DC, USA: Optical Society of America, 2014, pp. 1–3, Paper W4J-2.
[57] R. Ryf *et al.*, "10-mode mode-multiplexed transmission over 125-km single-span multimode fiber," in *Proc. Eur. Conf. Opt. Commun. (ECOC)*, Sep. 2015, pp. 1–3.
[58] K. M. Davis, K. Miura, N. Sugimoto, and K. Hirao, "Writing waveguides in glass with a femtosecond laser," *Opt. Lett.*, vol. 21, no. 21, pp. 1729–1731, Nov. 1996. [Online]. Available: http://www.ncbi.nlm.nih.gov/pubmed/19881782
[59] R. Osellame, G. Cerullo, and R. Ramponi, Eds., *Femtosecond Laser Micromachining* (Topics in Applied Physics), vol. 123. Berlin, Germany: Springer, 2012. [Online]. Available: http://link.springer.com/10.1007/978-3-642-23366-1
[60] S. Gross and M. J. Withford, "Ultrafast-laser-inscribed 3D integrated photonics: Challenges and emerging applications," *Nanophotonics*, vol. 4, no. 3, pp. 332–352, 2015. [Online]. Available: http://www.degruyter.com/view/j/nanoph.2015.4.issue-1/nanoph-2015-0020/nanoph-2015-0020.xml
[61] T. T. Fernandez *et al.*, "Bespoke photonic devices using ultrafast laser driven ion migration in glasses," *Prog. Mater. Sci.*, vol. 94, pp. 68–113, May 2018. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0079642517301433, doi: 10.1016/j.pmatsci.2017.12.002.
[62] D. Tan, K. N. Sharafudeen, Y. Yue, and J. Qiu, "Femtosecond laser induced phenomena in transparent solid materials: Fundamentals and applications," *Prog. Mater. Sci.*, vol. 76, pp. 154–228, Mar. 2016, doi: 10.1016/j.pmatsci.2015.09.002.
[63] A. Arriola *et al.*, "Low bend loss waveguides enable compact, efficient 3D photonic chips," *Opt. Exp.*, vol. 21, no. 3, pp. 2978–2986, Feb. 2013. [Online]. Available: http://www.opticsinfobase.org/abstract.cfm?URI=oe-21-3-2978
[64] T. Meany, S. Gross, N. Jovanovic, A. Arriola, M. Steel, and M. Withford, "Towards low-loss lightwave circuits for non-classical optics at 800 and 1,550 nm," *Appl. Phys. A, Mater. Sci. Process.*, vol. 114, no. 1, pp. 113–118, 2014.
[65] S. Huang, M. Li, S. M. Garner, M.-J. Li, and K. P. Chen, "Flexible photonic components in glass substrates," *Opt. Exp.*, vol. 23, no. 17, pp. 22532–22543, 2015. [Online]. Available: https://www.osapublishing.org/abstract.cfm?URI=oe-23-17-22532
[66] J. Lapointe, M. Gagné, M.-J. Li, and R. Kashyap, "Making smart phones smarter with photonics," *Opt. Exp.*, vol. 22, no. 13, Jun. 2014, Art. no. 15473. [Online]. Available: http://www.opticsinfobase.org/abstract.cfm?URI=oe-22-13-15473
[67] L. A. Fernandes, J. R. Grenier, P. R. Herman, J. S. Aitchison, and P. V. S. Marques, "Stress induced birefringence tuning in femtosecond laser fabricated waveguides in fused silica," *Opt. Exp.*, vol. 20, no. 22, p. 24103, Oct. 2012. [Online]. Available: http://www.opticsinfobase.org/abstract.cfm?URI=oe-20-22-24103
[68] G. Corrielli, S. Atzeni, S. Piacentini, I. Pitsios, A. Crespi, and R. Osellame, "Symmetric polarization-insensitive directional couplers fabricated by femtosecond laser writing," *Opt. Exp.*, vol. 26, no. 12, p. 15101, Jun. 2018. [Online]. Available: https://www.osapublishing.org/abstract.cfm?URI=oe-26-12-15101
[69] R. R. Thomson *et al.*, "Ultrafast-laser inscription of a three dimensional fan-out device for multicore fiber coupling applications," *Opt. Exp.*, vol. 15, no. 18, pp. 11691–11697, Sep. 2007. [Online]. Available: http://www.ncbi.nlm.nih.gov/pubmed/19547529
[70] R. R. Thomson, R. J. Harris, T. A. Birks, G. Brown, J. Allington-Smith, and J. Bland-Hawthorn, "Ultrafast laser inscription of a 121-waveguide fan-out for astrophotonics," *Opt. Lett.*, vol. 37, no. 12, p. 2331, Jun. 2012. [Online]. Available: http://www.opticsinfobase.org/abstract.cfm?URI=ol-37-12-2331
[71] A. Ross-Adams, S. Gross, B. J. Puttnam, R. S. Luís, G. Rademacher, and M. J. Withford, "Enabling future fiber networks using integrated ultrafast laser-written multicore fiber fan-outs," in *Proc. 14th Pacific Rim Conf. Lasers Electro-Opt. (CLEO PR)*, 2020, pp. 22–23.
[72] G. Djogo *et al.*, "Femtosecond laser additive and subtractive micro-processing: Enabling a high-channel-density silica interposer for multicore fibre to silicon-photonic packaging," *Int. J. Extreme Manuf.*, vol. 1, no. 4, Nov. 2019, Art. no. 045002. [Online]. Available: https://iopscience.iop.org/article/10.1088/2631-7990/ab4d51
[73] J. D. Love and N. Riesen, "Mode-selective couplers for few-mode optical fiber networks," *Opt. Lett.*, vol. 37, no. 19, p. 3990, Sep. 2012. [Online]. Available: http://www.opticsinfobase.org/abstract.cfm?URI=ol-37-19-3990
[74] D. G. MacLachlan *et al.*, "Development of integrated mode reformatting components for diffraction-limited spectroscopy," *Opt. Lett.*, vol. 41, no. 1, p. 76, Jan. 2016. [Online]. Available: https://www.osapublishing.org/abstract.cfm?URI=ol-41-1-76, doi: 10.1364/OL.41.000076.
[75] B. Guan *et al.*, "15-spatial-mode photonic lanterns based on ultrafast laser inscription," in *Proc. ECOC*, Sep. 2015, pp. 1–3.
[76] B. Guan, B. Ercan, N. K. Fontaine, R. P. Scott, and S. J. B. Yoo, "Mode-group-selective photonic lantern based on integrated 3D devices fabricated by ultrafast laser inscription," in *Proc. Opt. Fiber Commun. Conf.*, vol. 1, 2015, pp. 1–3, Paper W2A.16. [Online]. Available: http://www.osapublishing.org.acces.bibl.ulaval.ca/abstract.cfm?uri=OFC-2015-W2A.16
[77] S. Gross, M. Ams, S. G. Leon-Saval, and M. J. Withford, "Ultrafast laser inscribed mode-group-selective 6-mode photonic lanterns for mode-division multiplexing," in *Proc. Eur. Conf. Lasers Electro-Opt. Eur. Quantum Electron. Conf.* Optica Publishing Group, 2017, Paper CM_4_1.
[78] H. Chen, N. K. Fontaine, R. Ryf, B. Guan, S. J. B. Yoo, and T. Koonen, "Design constraints of photonic-lantern spatial multiplexer based on laser-inscribed 3-D waveguide technology," *J. Lightw. Technol.*, vol. 33, no. 6, pp. 1147–1154, Mar. 15, 2015. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=6955710
[79] N. Riesen, S. Gross, J. D. Love, and M. J. Withford, "Three-dimensional C + L band and EDFA pump band integrated tapered mode couplers," in *Proc. Austral. New Zealand Conf. Opt. Photon. (ANZCOP)*, 2015, pp. 48–50. [Online]. Available: https://search.informit.org/doi/10.3316/informit.950844653256016
[80] N. Riesen, S. Gross, J. D. Love, and M. J. Withford, "Femtosecond direct-written integrated mode couplers," *Opt. Exp.*, vol. 22, no. 24, pp. 29855–29861, 2014. [Online]. Available: http://www.opticsinfobase.org/oe/abstract.cfm?

uri=oe-22-24-29855

[81] P. Mitchell, G. Brown, R. R. Thomson, N. Psaila, and A. K. Kar, "57 channel ($19 \times 3$) spatial multiplexer fabricated using direct laser inscription," in *Proc. Opt. Fiber Commun. Conf.*, no. 1, 2014, pp. 1–3, Paper M3K.5. [Online]. Available: http://www.opticsinfobase.org/abstract.cfm?URI=OFC-2014-M3K.5

[82] S. Rommel, J. M. D. Mendinueta, W. Klaus, J. Sakaguchi, J. J. V. Olmos, Y. Awaji, I. T. Monroy, and N. Wada, "Characterization of a fiber-coupled 36-core 3-mode photonic lantern spatial multiplexer," in *Adv. Photon., IPR, NOMA, Sensors, Netw., SPPCom, PS, OSA Tech. Dig.* Optica Publishing Group, 2017, Paper NeW3B.2.

[83] N. Riesen, S. Gross, J. D. Love, Y. Sasaki, and M. J. Withford, "Monolithic mode-selective few-mode multicore fiber multiplexers," *Sci. Rep.*, vol. 7, no. 1, p. 6971, Aug. 2017. [Online]. Available: http://www.nature.com/articles/s41598-017-06561-w

[84] G. Rademacher *et al.*, "159 Tbit/s C+L band transmission over 1045 km 3-mode graded-index few-mode fiber," in *Proc. Opt. Fiber Commun. Conf. Postdeadline Papers*. Washington, DC, USA: OSA, 2018, pp. 1–3, Paper Th4C.4. [Online]. Available: https://www.osapublishing.org/abstract.cfm?URI=OFC-2018-Th4C.4

[85] R. S. Luis *et al.*, "1.2 Pb/s throughput transmission using a 160 $\mu$m cladding, 4-core, 3-mode fiber," *J. Lightw. Technol.*, vol. 37, no. 8, pp. 1798–1804, Apr. 2019.

[86] G. Rademacher *et al.*, "10.66 peta-bit/s transmission over a 38-core-three-mode fiber," in *Proc. Opt. Fiber Commun. Conf. (OFC)*. Washington, DC, USA: OSA, 2020, Paper Th3H.1, pp. 1–3. [Online]. Available: https://www.osapublishing.org/abstract.cfm?URI=OFC-2020-Th3H.1

[87] M. Kozhevnikov *et al.*, "Micromechanical optical crossconnect with 4-F relay imaging optics," *IEEE Photon. Technol. Lett.*, vol. 16, no. 1, pp. 275–277, Jan. 2004.

[88] Y. Jung, A. Wood, S. Jain, Y. Sasaki, S.-U. Alam, and D. J. Richardson, "Fully integrated optical isolators for space division multiplexed (SDM) transmission," *APL Photon.*, vol. 4, no. 2, Feb. 2019, Art. no. 022801.

[89] A. Wood, Y. Jung, and D. J. Richardson, "Multiport fiber optic beam splitters for space division multiplexed (SDM) systems," *IEEE Photon. Technol. Lett.*, vol. 32, no. 13, pp. 795–798, Jul. 1, 2020.

[90] Y. Jung *et al.*, "Compact micro-optic based components for hollow core fibers," *Opt. Exp.*, vol. 28, no. 2, pp. 1518–1525, 2020.

[91] Y. Jung, S.-U. Alam, D. J. Richardson, S. Ramachandran, and K. S. Abedin, "Multicore and multimode optical amplifiers for space division multiplexing," in *Optical Fiber Telecommunications VII*. Amsterdam, The Netherlands: Elsevier, 2020, pp. 301–333.

[92] Y. Amma, K. Takenaga, S. Matsuo, and K. Aikawa, "Fusion splice techniques for multicore fibers," *Opt. Fiber Technol.*, vol. 35, pp. 72–79, Feb. 2017.

[93] M. Meunier *et al.*, "Fully reflective annular laser beam shaping for 1.03 $\mu$m ultra-high throughput laser beam welding (conference presentation)," *Proc. SPIE*, vol. 11273, Mar. 2020, Art. no. 112730H.

[94] J. Carpenter. (2017). *Multi-Plane Light Conversion With Low Plane Count*. [Online]. Available: https://youtu.be/iXjFQj15Xzg

[95] W. Bogaerts *et al.*, "Programmable photonic circuits," *Nature*, vol. 586, no. 7828, pp. 207–216, 2020.

[96] G. Labroille, B. Denolle, P. Jian, P. Genevaux, N. Treps, and J.-F. Morizur, "Efficient and mode selective spatial mode multiplexer based on multi-plane light conversion," *Opt. Exp.*, vol. 22, no. 13, pp. 15599–15607, 2014.

[97] N. Barré, B. Denolle, P. Jian, J.-F. Morizur, and G. Labroille, "Broadband, mode-selective 15-mode multiplexer based on multi-plane light conversion," in *Proc. Opt. Fiber Commun. Conf.* Washington, DC, USA: Optical Society of America, 2017, pp. 1–3, Paper Th2A-7.

[98] S. Bade *et al.*, "Fabrication and characterization of a mode-selective 45-mode spatial multiplexer based on multi-plane light conversion," in *Proc. Opt. Fiber Commun. Conf.* Washington, DC, USA: Optica Publishing Group, 2018, pp. 1–3, Paper Th4B-3.

[99] N. K. Fontaine, R. Ryf, H. Chen, D. Neilson, and J. Carpenter, "Design of high order mode-multiplexers using multiplane light conversion," in *Proc. Eur. Conf. Opt. Commun. (ECOC)*, Sep. 2017, pp. 1–3.

[100] N. K. Fontaine *et al.*, "Packaged 45-mode multiplexers for a 50-$\mu$m graded index fiber," in *Proc. Eur. Conf. Opt. Commun. (ECOC)*, 2018, pp. 1–3.

[101] N. K. Fontaine *et al.*, "Hermite–Gaussian mode multiplexer supporting 1035 modes," in *Proc. Opt. Fiber Commun. Conf. (OFC)*. Washington, DC, USA: Optical Society of America, 2021, pp. 1–3, Paper M3D-4.

[102] N. K. Fontaine *et al.*, "Wavelength selective switch with optimal steering element utilization," in *Proc. Opt. Fiber Commun. Conf. Exhib. (OFC)*, 2016, pp. 1–3.

[103] M. Mounaix *et al.*, "Time reversed optical waves by arbitrary vector spatiotemporal field generation," *Nature Commun.*, vol. 11, no. 1, pp. 1–7, Dec. 2020.

[104] S. Molesky, Z. Lin, A. Y. Piggott, W. L. Jin, J. Vucković, and A. W. Rodriguez, "Inverse design in nanophotonics," *Nature Photon.*, vol. 12, no. 11, pp. 659–670, 2018.

[105] Y. Sakamaki, T. Saida, T. Hashimoto, and H. Takahashi, "New optical waveguide design based on wavefront matching method," *J. Lightw. Technol.*, vol. 25, no. 11, pp. 3511–3518, Nov. 1, 2007.

[106] N. Barré and A. Jesacher, "Inverse design of gradient-index volume multimode converters," 2021, *arXiv:2111.15461*.

[107] F. Brandt, M. Hiekkamäki, F. Bouchard, M. Huber, and R. Fickler, "High-dimensional quantum gates using full-field spatial modes of photons," *Optica*, vol. 7, no. 2, pp. 98–107, 2020.

[108] D. S. Dahl, J. Romero, M. Ploschner, N. K. Fontaine, and J. Carpenter, "High-dimensional quantum gates for azimuthal modes," in *Proc. 14th Pacific Rim Conf. Lasers Electro-Opt. (CLEO PR)*, 2020, pp. 1–2.

[109] J. Carpenter, N. K. Fontaine, B. R. M. Norris, and S. Leon-Saval, "Spatial mode sorter coronagraphs," in *Proc. 14th Pacific Rim Conf. Lasers Electro-Opt. (CLEO PR)*. Washington, DC, USA: Optical Society of America, 2020, pp. 1–2, Paper C6G_3.

[110] Z. Wu *et al.*, "$3\times4\times10$-Gb/s MDM-WDM transmission over 21-km OM3 MMF with OOK modulation and direct detection," in *Proc. Opt. Fiber Commun. Conf.* Washington, DC, USA: Optica Publishing Group, 2018, pp. 1–3, Paper W4J-3.