SRC TR 87-148

Coded FH/SS Communications in the Presence of Combined Partial-Band Noise Jamming, Rician Nonselective Fading and Multi-User Interference

by

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CODED FH/SS COMMUNICATIONS IN THE PRESENCE OF COMBINED PARTIAL-BAND NOISE JAMMING, RICIAN NONSELECTIVE FADING AND MULTI-USER INTERFERENCE

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ABSTRACT

In this paper we address the problem of combatting combined interference in spread-spectrum communication links. We consider frequency-hopped spread-spectrum systems with M-ary FSK modulation and noncoherent demodulation which employ forward-error-control coding. The interference consists of partial-band noise jamming, nonselective Rician fading, other-user interference and thermal noise. The coding schemes which we analyze include: Reed-Solomon codes (with or without diversity and error-only, erasure-only or parallel erasure/error decoding), binary, nonbinary, and dual-k convolutional codes with and without side information (information about the state of the channel), and concatenated schemes (Reed-Solomon outer codes with either inner detection-only block codes or inner convolutional codes). In all cases we derive (i) the minimum signal-to-jammer energy ratio required to guarantee a desirable bit error rate as a function of ρ, the fraction of the band which is jammed, when the number of interfering users is fixed, and (ii) the maximum number of users that can be supported by the system as a function of ρ, when the signal-to-jammer energy ratio is fixed.

E. Geraniotis is also with the Systems Research Center at the University of Maryland, College Park. His research was supported by the Naval Research Laboratory.
J. W. Gluck is an NRL fellow under ONR Grant N00014-85-G-0207.

I. INTRODUCTION

Recently there has been an increasing interest in the design and performance analysis of frequency-hopped spread-spectrum (FH/SS) systems for, among several other qualities, they can combat hostile interference and provide multi-access capability. References [1] - [7] (we have not attempted to compile a complete list of references, but, rather to refer to the work most closely related to this paper) are concerned with FH/SS systems which operate in environments characterized by several distinct types of interference including partial-band noise jamming, nonselective fading, other-user interference and background (thermal) noise.

A common characteristic of the work described in [1] - [7] is that, although the effects of combined jamming and fading or combined other-user interference and fading on FH/SS systems have been studied, the effects of combined hostile and other-user interference have not been investigated.

This paper is concerned with the performance of FH/SS systems with binary or M-ary FSK modulation and noncoherent demodulation which employ forward-error-control (FEC) coding and operate in a combined partial-band noise jamming, other-user interference, Rician nonselective fading, and additive white Gaussian noise (AWGN) environment. Such a situation may arise in packet radio networks which employ FH/SS signaling. We will attempt to address problems related to the performance of a link in such a network. In particular, if the maximum number of transmitters operating in the vicinity (hearing range) of a receiver is a parameter of the network (which has been determined by some other network specifications) it is important to know the signal-to-jammer energy ratio which is required to guarantee a desirable bit error rate at the receiver in question. Similarly, when the signal-to-jammer energy ratio is a system parameter, we could be interested in knowing what is the maximum number of transmitters in the hearing range of a receiver that yields a bit error rate below a prespecified tolerable level.

The paper is organized as follows. The system and channel model are described in detail in Section II. Then, in Section III, the performance of several FEC coding schemes is analyzed. In particular, Reed-Solomon (RS) codes with error-only, erasure/error and parallel decoding are studied in Section III.1. Binary and M-ary repetition codes with and without side information (i.e., information about the state of the channel: the presence or abasence of hostile or other-user interference) are considered in Section III.2. Binary as well as nonbinary convolutional codes and dual-k convolutional codes with and wihout side information are examined in Sections III.3 and III.4. Concatenated codes with inner detection-only block codes or binary or dual-k convolutional inner codes and RS outer codes with error-only erasure/error and parallel decoding are analyzed in Section III.5. Finally, RS codes with time diversity and errors-only, erasures/errors and parallel decoding are investigated in Section III.6. In all cases only hard decisions on the channel output are implemented. In Section IV numerical results are presented on the minimum required signal-to-jammer energy ratio and the maximum number of transmitting users for all the FEC coding schemes enumerated above and comparisons are made. Finally, in Section V the key results of the report are summarized and conclusions are drawn.

II. SYSTEM AND CHANNEL MODEL

The FH/SSMA system model considered is that of [1] in an environment characterized by partial-band jamming and Rician nonselective fading. M-ary FSK data modulation with noncoherent demodulation is employed. The hopping rate is no larger than the data rate (slow-hopping). N_s M-ary symbols (and thus $N_b = N_s \log_2 M$ bits) are transmitted during each hop (dwell time), where $N_s \geq 1$.

It is assumed that in the vicinity of a particular receiver there are K asynchronous transmitted signals, all of which share the same channel, and the receiver can acquire synchronization with the frequency-hopping pattern and time of one of the K signals; then the other K-1 signals interfere with the reception of the signal that was singled out. Our model of other-user interference is that of [1]. There is no restriction on the power levels (or the communication range) of the transmitted signals, since we use bounds on the conditional probability of a receiver error when other-user interference is present which are independent of the power levels, phase angles, or time delays of the interfering signals.

The model for the partial-band noise interference is the one commonly used in the literature (e.g., [2]-[6]), except that thermal noise [modeled as additive white Gaussian noise (AWGN)] is also assumed to be present at the receiver. Therefore, if N_J denotes the effective one-sided spectral density of the partial-band Gaussian noise (i.e., $N_J = P_J / W$ where P_J, the power available to the jammer, is assumed to be fixed and W is the total bandwidth of the FH/SS system), N_0 denotes that of the AWGN, and ρ $(0 \leq \rho \leq 1)$ is the probability that a particular dwell time (frequency slot) is jammed, then the one-sided spectral density of the Gaussian noise is $N_0 + \dfrac{N_J}{\rho}$ with probability ρ, and it is N_0 with probability $1-\rho$. The density of the noise remains constant over the duration of the frequency slot (dwell time). Different dwell times are jammed independently.

The nonselective Rician fading channel model is that of [3] and [12]. The received signal consists of a nonfaded component and an attenuated, phase-shifted faded component (termed scatter component) whose delay with respect to the nonfaded component is negligible. The amplitude of the received signal has a Rician distribution and the probability of error of an M-ary FSK system with noncoherent demodulation is given by [3]

$$P_{e,M}(\eta) = \sum_{m=1}^{M-1} \binom{M-1}{m} \frac{(-1)^{m+1}}{m+1+m\,\beta(\eta)} \exp\left(-\frac{m\,\delta(\eta)}{m+1+m\,\beta(\eta)}\right) \tag{1}$$

where $\beta(\eta) = \Lambda(\eta)/(1+\gamma^{-2})$, $\delta(\eta) = \Lambda(\eta)/(1+\gamma^{2})$, $\Lambda(\eta) = \bar{E}_b \log_2 M/\eta$ is the received signal-to-noise ratio, i.e., $\bar{E}_b$ is the received energy per information bit and η is the spectral density of the AWGN. (For coded systems $\Lambda(\eta) = r\ \bar{E}_b \log_2 M/\eta$ where r is the code rate). Finally γ^2 is the ratio of the expected relative strength of the scatter component to the expected relative strength of the nonfaded component. Notice that $\gamma^2=0$ implies that $\beta(\eta)=0$ and $\delta(\eta)=\Lambda(\eta)$, so that the Rician fading channel reduces to an AWGN channel. Similarly, $\gamma^2=\infty$ implies that $\beta(\eta)=\Lambda(\eta)$ and $\delta(\eta)=0$, so that the channel becomes a Rayleigh fading channel.

III. PERFORMANCE EVALUATION OF CODED SYSTEMS

In this paper we consider several forward error-control coding schemes, in particular Reed-Solomon (RS) codes, binary convolutional codes (CC), dual-k convolutional codes, several concatenated coding schemes (e.g., block inner code/RS outer code, binary CC inner code/RS outer code, dual-k CC inner code/RS outer code), and Reed-Solomon coding schemes with time diversity are analyzed.

III.1 Reed-Solomon Codes

We consider three distinct cases: *(i) error correction, (ii) erasure/error correction,* and *(iii) parallel erasure/error correction* [5]. In *case (i)* there is no information about the state of the channel, and thus, the RS decoder only attempts to correct the errors. In *case (ii),* we assume that channel monitoring provides information about the state of the channel (presence or absence of jamming or other-user interference), which can be used by the RS decoder to erase the symbols which are subject to heavy interference. In this case the decoder attempts to correct the erasures and the few errors due to thermal noise. In *case (iii)* the decoder corrects erasures and errors due to thermal noise when the number of observed erasures is less than or equal to $e = n-k$; otherwise, it only attempts to correct errors. See [5] for a detailed description of this algorithm when partial-band jamming is the only form of interference in the channel

Next, we proceed with the evaluation of the average error probability for the cases described above. We assume that RS codes over GF(M^m) are employed, thus there are m M-ary symbols in each RS symbol. For *case (i),* we upper-bound the probability of a symbol error for the uncoded system by

$$p_s \leq 1 - (1-P_h)^{K-1}[(1-\rho)(1-P_0)^m + \rho(1-P_{J,0})^m]. \tag{2}$$

In (2) P_h denotes the probability of a *hit* from another user (i.e., both users use the same frequency for part of their dwell times). The probability of a hit for any RS symbol and M-ary FH/SS asynchronous systems has been shown [1] to be upper-bounded by

$$P_h = \left(1+\frac{m}{N_s}\right)\frac{1}{q} \tag{3}$$

(the m in the numerator accounts for the fact that each RS symbol contains m M-ary FSK symbols and thus it is more likely to be hit than a single M-ary FSK symbol); q is the number of available frequencies. The frequency-hopping patterns are assumed to be memoryless random sequences [1] and independent for distinct users. The probabilities P_0 and $P_{J,0}$ in (2) denote the error probabilities of an M-ary FSK system with non-coherent demodulation disturbed by AWGN of one-sided spectral densities N_0 and $N_0 + \frac{N_J}{\rho}$, respectively. Thus $P_0 = P_{e,M}(N_0)$ and $P_{J,0} = P_{e,M}(N_0 + \frac{N_J}{\rho})$, where $P_{e,M}(\cdot)$ is defined in (1). In (2) $(1-\rho)(1-P_0)^m + \rho(1-P_{J,0})^m$ is the probability of no error due to Gaussian noise in m M-ary symbols; $(1-P_h)^{K-1}$ is a lower bound on the probability of no error due to any of the other $K-1$ users. Notice that the conditional probability of error given that a hit from another user occurred has been upper-bounded by 1; in this way p_s does not depend on the power levels, time delays, or phase angles of the different users. For $M=2$ (binary FSK modulation) it has been shown [8] that this conditional error probability is upper-bounded by $\frac{1}{2}$; however, no such result has, so far, been established for $M>2$.

When bounded distance decoding of RS codes with hard decisions is employed, the symbol error probability for the coded system is given by [9]

$$P_{e,s} = \sum_{j=t+1}^{n} \frac{j}{n}\binom{n}{j} p_s^j (1-p_s)^{n-j}, \tag{4}$$

where $t = \lfloor (n-k)/2 \rfloor$ is the error-correction capability of the RS(n,k) code (k information symbols in a codeword of length n). Eq. (4) is valid when all of the RS symbols in the same hop (dwell time) are subject to independent errors. This can be achieved by interleaving to a depth of N_s/m. Eq. (4) can serve as an upper bound for the M-ary symbol error probability and the bit error probability of the coded system.

For *case (ii)* the probability of an erasure is

$$\epsilon_s = \rho + [1-(1-P_h)^{K-1}] - \rho[1-(1-P_h)^{K-1}], \tag{5}$$

since we assumed that the decoder erases a symbol if the jammer is present and/or if interference from other users is present. Then the probability of a symbol error is

$$p_s = [1-(1-P_0)^m](1-\epsilon_s), \tag{6}$$

since $1-\epsilon_s$ is the probability of no interference from the jammer or from the other users, and $1-(1-P_0)^m$ is the probability of error due to the thermal noise alone. Notice that neither ϵ_s nor p_s depend on the signal-to-jammer ratio E_b/N_J. In this case, the probability of a RS symbol error at the decoder is [9]

$$P_{e,s} = \sum_{\substack{j+l \leq n \\ e+1 \leq 2l+j}} \frac{j+l}{n} \binom{n}{j} \binom{n-j}{l} p_s^l \epsilon_s^j (1-p_s-\epsilon_s)^{n-l-j}. \tag{7}$$

where $e = n-k$ is the erasure-correction capability of the RS code.

For *case (iii)*, we can apply the parallel erasures/errors decoding algorithm of [5] discussed above, where partial-band noise jamming is the only source of interference, and that of [7], where partial-time jamming and thermal noise are the sources of interference, to the case in which partial-band noise jamming, multiple-access interference, Rician nonselective fading and thermal noise are present.

When the number of erasures is less than or equal to $e = n-k$, the erasures correction capability of the code, the contribution to the decoder's error probability is

$$P_{e,s;1} = \sum_{j=0}^{e} \binom{n}{j} \epsilon_s^j (1-\epsilon_s)^{n-j} \sum_{\substack{e+1 \le 2l+j \\ l+j \le n}} \frac{j+l}{n} \binom{n-j}{l} P_0^l (1-P_0)^{n-j-l}, \tag{8}$$

where the probability of an erasure ϵ_s was defined in (5) and P_0 is the error probability of an M-ary symbol due to thermal noise, discussed above. In (8), j is the number of erased symbols, $n-j$ is the number of symbols which are not erased, and l is the number of symbols out of those $n-j$ symbols that resulted in a receiver error due to the thermal noise alone.

When the number of erasures is larger than $e=n-k$, the contribution to the decoder's error probability becomes

$$P_{e,s;2} = \sum_{j=e+1}^{n} \binom{n}{j} \epsilon_s^j (1-\epsilon_s)^{n-j}$$

$$\cdot \sum_{\substack{t+1 \le l_1+l_2 \\ l_1 \le j \\ l_2 \le n-j}} \frac{l_1+l_2}{n} \binom{j}{l_1} \bar{p}^{\,l_1} (1-\bar{p})^{j-l_1} \binom{n-j}{l_2} P_0^{l_2} (1-P_0)^{n-j-l_2}. \tag{9}$$

In (9), $\bar{p}$ denotes the probability of error given that there is partial-band or multiple-access interference and is given by

$$\bar{p} = \frac{\epsilon_1}{\epsilon_s}[1 - (1 - P_{J,0})^m] + \frac{\epsilon_2}{\epsilon_s}(1 - \frac{1}{M^m}), \tag{10}$$

where ϵ_1 denotes the probability of being jammed but not hit by other users and ϵ_2 denotes the probability of being hit by other users. These quantities are given by

$$\epsilon_1 = \rho(1-P_h)^{K-1} \tag{11a}$$

and

$$\epsilon_2 = 1-(1-P_h)^{K-1} \tag{11b}$$

Furthermore, in (9) j is the number of symbols in an RS codeword which are subject to either partial-band interference or multiple-access interference, whereas $n-j$ is the number of symbols subject only to AWGN. Then l_1 out of the j symbols which are subject to interference are received in error, whereas $j-l_1$ are not; and l_2 out of the $n-j$ symbols subject to only AWGN are received in error, whereas $n-j-l_2$ are received correctly. Therefore, the decoder commits an error when the total number of errors l_1+l_2 exceeds t, the error-correction capability of the RS code. Finally, the total error probability at the output of the RS decoder is given by $P_{e,s} = P_{e,s;1} + P_{e,s;2}$.

III.2 Repetition Codes

We will consider two cases: (i) when channel monitoring reveals the presence or absence of interference (other than AWGN) in the channel, and (ii) when no information about the state of the channel is available. Both binary and M-ary FSK modulation with noncoherent demodulation are examined. In the case of binary FSK, the N_b bits of a dwell time are interleaved. In the second case, the $N_s = N_b / log_2 M$ M-ary symbols are interleaved.

When information about the state of the channel is *not available,* majority vote decoding (where the decoder decides in favor of the symbol which was received most times) with hard decisions is the maximum likelihood decoding algorithm. For *binary* repetition codes of block length n, the bit error rate (BER) is

$$P_{e,b} = P_2(n;p) = \begin{cases} \sum_{l=(n+1)/2}^{n} \binom{n}{l} p^l (1-p)^{n-l} & ; n \text{ odd} \\ \sum_{l=n/2+1}^{n} \binom{n}{l} p^l (1-p)^{n-l} + \frac{1}{2}\binom{n}{n/2}[p(1-p)]^{n/2} & ; n \text{ even} \end{cases} \quad (12)$$

where p denotes the error probability for a binary channel with combined multiple-access interference, partial-band jamming, Rician nonselective fading and thermal noise, which can be obtained from (2) by using $m=1$ and $M=2$. For *M-ary* repetition codes ($M>2$) the symbol error probability can be obtained from the formula

$$P_{e,s} = P_M(n;p) = 1 - \sum_{i=0}^{n-1} a_i p^i (1-p)^{n-i} \quad (13)$$

where for n smaller than 10 the coefficients a_i are provided in the Appendix A of [6] and p is as above.

When information about the state of the channel *is available* the requirement for maximum likelihood decoding results in a complicated rule which for $M=2$ is described in [6]; for $M>2$ the decoding rule becomes too complicated to be useful for implementation.

Therefore, we will consider the following suboptimal rule, which for moderately large values of E_b/N_0 (the signal-to-AWGN ratio) gives a performance very close to that of the optimal (maximum-likelihood) decoding rule (see Appendix B of [6]). Assuming availability of information about the state of the channel (presence or absence of interference other than AWGN) for each code symbol, the decoder counts the number of symbols hit by interference (multiple-access or jamming). If this number is equal to n, then it executes majority vote decoding of all n symbols; if the number is smaller than n, then it executes majority vote decoding of the symbols which were not hit by interference. For the channel model considered in this paper the bit error rate for this decoding scheme can be expressed as

$$P_{e,s} = \overline{P}_M(n) = \epsilon_s^n P_M(n;\overline{p}) + \sum_{l=0}^{n-1} \binom{n}{l} \epsilon_s^l (1-\epsilon_s)^{n-l} P_M(n-l;P_0) \tag{14}$$

In (14), ϵ_s, the probability that a symbol is hit by multiple-access or partial-band interference, is given by (5), and $\overline{p}$, the symbol error probability given that interference is present, is given by (10) and (11a)-(11b) for P_h defined by (3) for $m=1$ (N_b should replace N_s for $M=2$). The quantity $P_M(\cdot;\cdot)$ can be obtained from (12) and (13) for $M=2$ and $M>2$, respectively. For $M=2$, (14) provides the bit error probability; for $M>2$ it provides the symbol error probability and can serve as in upper bound for the bit error probability.

III.3 Binary and Nonbinary Convolutional Codes

For binary convolutional codes (CC's) binary FSK is employed ($M=2$) and all binary FSK symbols within the hop (dwell time) are assumed to be interleaved (the interleaving depth is N_b). The input to the Viterbi hard decisions decoder consists of bits with error probabililty p given by (2) for $m=1$, $M=2$, and $P_h = \left(1+\frac{1}{N_b}\right)\frac{1}{q}$.

The bit error probability of the coded system is [10] (for CC's with rates $\frac{1}{b}$)

$$P_{e,b} \leq \frac{1}{b} \sum_{j=d_{free}}^{\infty} w_j P_j, \tag{15}$$

where d_{free} is the free distance of the code, P_j is the probability of the error event that the decoder chooses a path at distance j from the correct path, and w_j is the total information weight of all sequences which produce paths of weight j. The weights for binary CC' s of various rates can be found in [10] and [11], and for nonbinary CC' s in [12]. For binary (or nonbinary) codes, P_j defined above coincides with the error probability of a binary (or nonbinary) repetition code of length j. Therefore, if there is *no* *side information* (i.e., channel monitoring to reveal the presence or absence of interference), P_j is given by [11]-[12]

$$P_j = P_M(j;p) \leq (M-1)\, Q_M(j;p)\,, \tag{16}$$

where $P_M(\cdot;\cdot)$ is defined in (12) for $M = 2$ (binary codes) and in (13) for $M > 2$ (nonbinary codes) and $Q_M(j;p)$ denotes the probability of error between two codewords of an M-ary repetition code of rate $1/j$ when side information is absent. The inequality in (16) follows from an application of the union bound and it has the advantage that it involves $Q_M(\cdot;\cdot)$ which for large j (and since $j \geq d_{free}$ it can be much larger than 10) is easier to compute than $P_M(j;p)$ (which is provided in [6] only for $j \leq 10$).

In the absence of side information, the probability of error between two codewords of an M-ary repetition code of rate $1/n$ is given by [6]

$$q_n = Q_M(n;p) = \sum_{\substack{j<k \\ j+k\leq n}} \binom{n}{j}\binom{n-j}{k}(1-p)^j \left(\frac{p}{M-1}\right)^k \left(\frac{M-2}{M-1}p\right)^{n-j-k}$$

$$+\frac{1}{2}\sum_{j=0}^{[n/2]} \binom{n}{j}\binom{n-j}{j}(1-p)^j \left(\frac{p}{M-1}\right)^j \left(\frac{M-2}{M-1}p\right)^{n-2j}, \tag{17}$$

If *side information* is available (i.e., channel monitoring provides information about the presence or absence of interference), then, instead of (16), we should use

$$P_j = \overline{P}_M(j) \leq (M-1)\, \overline{Q}_M(j), \tag{18}$$

where $\overline{P}_M(\cdot)$ is defined in (14) for $\overline{p}$ obtained from (10), (11a)-(11b) with $m=1$, $M=2$, and $N_s = N_b$ for binary codes and from (10), (11a)-(11b) with $m=1$ and $M > 2$ for the nonbinary codes, and $\overline{Q}_M(j)$ is the probability of error between two codewords of an M-ary repetition code of rare $1/j$. When side information is available this is given by

$$\overline{q}_n = \overline{Q}_M(n) = \epsilon_s^n Q_M(n;\overline{p}) + \sum_{l=0}^{n-1} \binom{n}{l} \epsilon_s^l (1-\epsilon_s)^{n-l} Q_M(n-l;P_0), \qquad (19)$$

where $Q_M(\cdot;\cdot)$ is defined in (17) and $\overline{p}$ can be obtained from (10) by using $m=1$ and $M>2$.

III.4 Dual-k Convolutional Codes

For dual-k convolutional codes of constraint length k [13]-[14] the alphabet size is 2^k and codes are used with M-ary FSK modulation ($M=2^k$). All M-ary FSK symbols within the frequency slot are assumed to be interleaved (the interleaving depth is $N_b/\log_2 M$). The performance of a Viterbi decoder for these codes can be evaluated using the equation [13]

$$P_{e,b} \leq 2^{k-1} \sum_{j=0}^{\infty} (j+1) \sum_{l=0}^{j} \binom{j}{l} a^l b^{j-l} q_{2v+vj-l}, \qquad (20)$$

where $1/v$ (v is a positive integer) is the rate of the code, $a = v$, $b = 2^k-1-v$, and q_n is the error probability between two codewords of a repetition code of length n on an M-ary symmetric channel.

In [13] and, more recently, in [6] upper bounds on (17) were obtained and they are cited here for reference. The bound in (19a) (taken from [6]) is tighter than the bound in (19b) (taken from [13]):

$$P_{e,b} \leq 2^{k-1} \left\{ \sum_{j=0}^{J} (j+1) \sum_{l=0}^{j} \binom{j}{l} a^l b^{j-l} q_{2v+vj-l} + D^{2v} \frac{c^{J+1}[1+(J+1)(1-c)]}{(1-c)^2} \right\} \qquad (21a)$$

$$\leq \frac{2^{k-1}D^{2v}}{[1-vD^{v-1}-(2^k-1-v)D^v]^2}, \tag{21b}$$

where $c = aD^{v-1}+bD^v < 1$. The quantity D, which is used in (19a) and (19b), is the Bhattacharyya distance and it was used in upperbounding q_j in the following way: $q_j \leq D^j$.

When *side information is not available* q_n, the probability of error between two codewords of an M-ary repetition code of rate $1/n$, is given by (17), and the Bhattacharyya distance is given by [15]

$$D = \frac{M-2}{M-1} p_s + 2\sqrt{(1-p_s)p_s/(M-1)}. \tag{22}$$

For our channel model p in (20) and (21) can be obtained from (2) by using $m=1$ and $M>2$.

When *side information is available*, the majority vote decision rule described in the corresponding section of III.3 is employed in deciding to which path of weight n in the trellis the received word is closer. Now, $\overline{q}_n$ is given by (19) and the Bhattacharyya distance becomes

$$\overline{D} = (1-\epsilon_s)\left[\frac{M-2}{M-1}P_0+2\sqrt{(1-P_0)P_0/(M-1)}\right] + \epsilon_s\left[\frac{M-2}{M-1}\overline{p}+2\sqrt{(1-\overline{p})\overline{p}/(M-1)}\right] \tag{23}$$

where $\overline{p}$ is given by (10) by using $m=1$ and $M>2$.

III.5 Concatenated Codes

In this section we consider two basic concatenated coding schemes: (i) inner detection-only block codes/outer RS codes and (ii) inner convolutional codes (binary or dual-k)/outer RS codes. In all cases we will assume that there is no channel monitoring, so that no side information is available. Bounded distance decoding is used for the outer RS codes.

(i) Inner Detection-Only Block Codes / Outer RS Codes

This concatenation scheme uses the inner code to *detect* errors within a hop (see [16] for the case when partial-band jamming is the only form of interference). One codeword of the inner code is used in one RS symbol. When an error is detected, every symbol of the codeword of the inner code (and thus, the corresponding symbol of the RS outer code) is erased. There are, however, errors that are not detected, and which result in errors at the output of the inner decoder. The outer code then attempts to correct the errors and erasures of the inner code. We consider two decoding strategies for the outer RS decoder: erasure/error decoding (see Section III.1 *case (ii)*) and parallel erasure/error decoding (see section III.1 *case (iii)*).

Let n be the block length of the inner code and k the number of information symbols in a codeword. The probability of an *undetected error* P_{ud} (i.e., when a nonzero error pattern satisfies all of the parity-check equations) is given by [10]

$$P_{ud} = \sum_{i=1}^{n} A_i \; \alpha_{n,i} , \tag{24}$$

where A_i is the number of codewords with Hamming weight i and $\alpha_{n,i}$ is the probability that in a sequence of n received symbols a particular pattern of i symbol errors occurred. Furthermore, the probability of a *detected error* P_d is given by [10]

$$P_d = 1 - \alpha_{n,0} - P_{ud} , \tag{25}$$

since $1-\alpha_{n,0}$ is the probability of at least one symbol error. For our system and channel model, $\alpha_{n,i}$ is upper-bounded by

$$\alpha_{n,i} < \begin{cases} (1-P_h)^{K-1}\left[(1-\rho)\left(\dfrac{P_0}{M-1}\right)^i (1-P_0)^{n-i} + \rho\left(\dfrac{P_{J,0}}{M-1}\right)^i (1-P_{J,0})^{n-i}\right]; 0 \le i < n \\ (1-P_h)^{K-1}\left[(1-\rho)\left(\dfrac{P_0}{M-1}\right)^n + \rho\left(\dfrac{P_{J,0}}{M-1}\right)^n\right] + \dfrac{1-(1-P_h)^{K-1}}{(M-1)^n} \quad ; i=n \end{cases}, \quad (26)$$

where P_0 and $P_{J,0}$ are the conditional error probabilities of an M-ary FSK system given that thermal noise or jammer plus thermal noise is present (P_0, $P_{J,0}$ were defined in Section III.1).

For high rate codes ($r=k/n$ close to 1), the A_i, $0 \le i \le n$ are easily calculated using the McWilliams identities (since a high rate code has more codewords than its dual). If $A(z)$ is the weight enumerator of the (n,k) code and $B(z)$ (B_i, $0 \le i \le n$) is that of its dual, then [17]

$$A(z) = M^{-(n-k)}[1+(M-1)z]^n B\left(\frac{1-z}{1+(M-1)z}\right). \quad (27)$$

For codes with a single parity-check symbol, $B_0 = 1$, $B_n = M-1$, $k = n-1$ so that $B(z) = 1+(M-1)z^n$, and we obtain

$$A_n = [(M-1)^n + (-1)^n(M-1)]/M \quad (28a)$$

and

$$P_{ud} = (1-P_h)^{K-1}\left[\rho\left\{\frac{1}{M}\left[1+(M-1)\left(1-\frac{MP_{J,0}}{M-1}\right)^n\right] - (1-P_{J,0})^n\right\} + (1-\rho)\left\{\frac{1}{M}\left[1+(M-1)\left(1-\frac{MP_0}{M-1}\right)^n\right] - (1-P_0)^n\right\}\right] + \frac{1-(1-P_h)^{K-1}}{(M-1)^n} A_n . \quad (28b)$$

To find the error probability at the output of the outer (RS) decoder in the case of *erasure/error decoding,* we need only substitute P_{ud} from (28b)for p_s and P_d from (25) for ϵ_s in (7).

In the case of *parallel erasure/error decoding*, the outer decoder attempts to correct both erasures (the detected errors of the inner decoder) and errors (the undetected errors of the inner decoder) when the number of the erasures is less than $\tilde{e} = \tilde{n} - \tilde{k}$ (the erasure correction capability of the outer RS $(\tilde{n}, \tilde{k})$ code); if the number of erasures is larger than $\tilde{e}$, the RS decoder switches to the error-correction mode.

To evaluate the probability of a symbol error at the output of the outer decoder for the erasure/error-correction mode (denoted by $P_{e,s;1}$), we need only substitute P_d from (25) and P_{ud} from (28b) for e_s and p_s of (7), respectively, and replace n and e of (7) with $\tilde{n}$ and $\tilde{e}$.

To evaluate the probability of a decoder symbol error for the error-correction mode (denoted by $P_{e,s;2}$), we observe that, given that more than $\tilde{e}$ errors were detected by the inner decoder, a codeword error at the outer decoder occurs when the number of detected and undetected errors is greater than $\tilde{t} = \left\lfloor (\tilde{n} - \tilde{k})/2 \right\rfloor$ (the error-correction capability of the outer RS code). Since the detected and undetected error events are mutually exclusive, we obtain

$$P_{e,s;2} = \sum_{j=\tilde{e}+1}^{\tilde{n}} \binom{\tilde{n}}{j} P_d^j (1-P_d)^{\tilde{n}-j} \sum_{0 \le l \le \tilde{n}-j} \frac{j+l}{\tilde{n}} \binom{\tilde{n}-j}{l} P_{ud}^l (1-P_{ud})^{\tilde{n}-j-l}, \tag{29}$$

and, finally, the overall symbol error probability is $P_{e,s} = P_{e,s;1} + P_{e,s;2}$.

(ii) Inner Convolutional Code / Outer RS Code

In this case, the binary or dual-k convolutional inner codes employ binary or M-ary $(M = 2^k)$ FSK modulation, respectively, with noncoherent demodulation, hard decisions, and Viterbi decoding. To evaluate the performance of these two coding schemes when they are concatenated with RS outer codes, we proceed in the following way. Let P_e be the prespecified value of the tolerable error probability. Let $(\tilde{n}, \tilde{k})$ be the parameters of the outer RS code. Using the error-correction capability $\tilde{t} = \left\lfloor (\tilde{n} - \tilde{k})/2 \right\rfloor$ of the RS code and P_e, one can determine [either from existing tables or from the expressions (4) and (2)] the maximum outer symbol error probability p_s (for error-only decoding) that yields a BER smaller than or equal to P_e. Once p_s has been obtained, the expressions for binary or dual-k CC (found in Sections III.3 and III.4, respectively) can be used to find $(E_b/N_J)_{\min}$ for given K or $K_{\max}$ for given E_b/N_J.

III.6 Reed-Solomon Codes with Diversity

In this section we consider the coupling of diversity of order L with Reed-Solomon (n,k) codes. Each M-ary symbol is transmitted L times; each RS symbol contains m M-ary symbols. All three cases examined in Section III.1 are also examined here. In all cases it is assumed that with the use of interleaving the L diversity transmissions take place through independent channels (e.g., during different dwell times). The FH/SS system with diversity is thus equivalent to a fast-frequency-hopping system. Bounded distance decoding of the RS code is employed.

For *case (i), error-only decoding* without information about the state of the channel available, majority vote combining of the L diversity transmissions is employed. The bit error probability can be upper-bounded by (4) where p_s should be replaced by

$$p_s(L) = 1 - [1 - P_M(L;p)]^m \tag{30}$$

where $P_M(\cdot;\cdot)$ is defined in (13) and p can be obtained from p_s of (2) for $m=1$.

For *case (ii), erasure/error decoding* with perfect information about the state of the channel available, the decoder erases a symbol of the RS code if and only if all L diversity transmissions of at least one out of the m M-ary symbols within the RS symbol are hit by interference; otherwise, the decoder attempts to correct errors and uses majority vote decoding on the diversity tranmissions which were not hit by interference for each M-ary symbol within the RS symbol. The bit error probability is now upper-bounded by (7), where ϵ_s should be replaced by

$$\epsilon_s(L) = 1-(1-\epsilon_s^L)^m, \tag{31}$$

in which ϵ_s can be obtained from (5) by setting $m=1$ and $M>2$, and p_s should be replaced by

$$\overline{p}_{s,0}(L) = 1 - \left[1 - \sum_{l=0}^{L-1} \binom{L}{l} \epsilon_s^l (1-\epsilon_s)^{L-l} P_M(L-l\,;\,P_0)\right]^m, \tag{32}$$

where $P_M(\cdot;\cdot)$ can be obtained from (12) for $M=2$ and (13) for $M>2$.

Finally, for *case (iii): parallel erasure/error* decoding with channel information available, the decoder erases symbols according to the same rule as for *case (ii)* and if the number or erased symbols is smaller than $e=n-k$, it uses majority vote decoding in an identical way. However, if the number of erased symbols becomes larger than e, then the decoder switches to the error-correcting mode. It uses majority vote decoding based on all L diversity transmissions of a particular M-ary symbol when all L transmissions are hit by interference and majority vote decoding based on the diversity transmissions which are not hit by interference when fewer than L transmissions are hit. For this scheme which is an extension of the scheme of [5] to our channel model (and is optimal in the absence of AWGN or multi-user interference as shown in [6]), the symbol, and thus, the bit error probability is upper-bounded by $P_{e,s} = P_{e,s;1} + P_{e,s;2}$ where

$$P_{e,s;1} = \sum_{j=0}^{e} \binom{n}{j} (\epsilon_s^L)^j (1-\epsilon_s^L)^{n-j} \sum_{\substack{e+1 \le 2l+j \\ l+j \le n}} \frac{l}{n} \binom{n-j}{l} \overline{p}_{s,0}(L)^l \, [1-\overline{p}_{s,0}(L)]^{n-l}, \tag{33}$$

in which $\overline{p}_{s,0}$ is given by (32) and

$$P_{e,s;2} = \sum_{j=e+1}^{n} \binom{n}{j} (\epsilon_s^L)^j (1-\epsilon_s^L)^{n-j} \sum_{\substack{l+1 \le l_1+l_2 \\ l_1 \le j \\ l_2 \le n-j}} \frac{l_1+l_2}{n} \binom{j}{l_1} \overline{p}_s(L)^{l_1} [1-\overline{p}_s(L)]^{j-l_1}$$

$$\binom{n-j}{l_2} \overline{p}_{s,0}(L)^{l_2} [1-\overline{p}_{s,0}(L)]^{n-j-l_2}, \tag{34}$$

in which $\overline{p}_s(L)$ is given by

$$\overline{p}_s(L) = 1-[1-P_M(L;\overline{p})]^m, \tag{35}$$

where $P_M(\cdot;\cdot)$ is defined by (12) and (13) for $M=2$ and $M>2$, respectively, and $\overline{p}$ can be obtained from (10) by setting $m=1$ and $M>2$.

IV. NUMERICAL RESULTS

The numerical results presented in this section are separated into two groups. The first group provides the minimum signal (actually information bit)-to-jammer energy ratio (E_b/N_J) required to achieve a desirable bit error probability of P_e (typically 10^{-3}) for a variety of error-control coding schemes. E_b/N_J is given as a function of the percentage of the band jammed (ρ), when K--the total number of users transmitting in the vicinity of a particular receiver--and the other channel parameters are fixed. The second group provides the maximum number of transmitting users ($K_{\max}$) that can be tolerated in the vicinity of a receiver, so that the bit error probability is below a desirable level P_e for a variety of error-control coding schemes. $K_{\max}$ is given as a function of ρ, when the signal-to-jammer power ratio (E_b/N_J) and the other system and channel parameters are held fixed.

In both cases q denotes the number of frequences used for frequency-hopping, N_b the number of bits per dwell time, E_b/N_0 the signal-to-AWGN ratio, M the number of orthogonal signals (or the frequency tones) employed for the M-ary FSK modulation (with noncoherent demodulation), and m the number of M-ary symbols in each symbol of the Reed-Solomon code. All the results which follow are concerned with asynchronous FH/SSMA sytems.

The presentation of results from the first group starts with Figure 1, where M E_b/N_J versus ρ is plotted for a Reed-Solomon RS(64,32) code (rate 1/2) over the Galois field $GF(8^2)$ (i.e., 8-ary FSK signaling is employed and each RS 64-ary symbol consists of two 8-ary symbols) with either error-only or parallel erasure/errors decoding. The values of the system and channel parameters are $P_e = 10^{-3}$, $K = 5$, $E_b/N_0 = 12\ dB$ and the relative power of the faded component of the Rician channel γ^2 is varying

[$\gamma^2 = 0$ (AWGN), .01, .1, 1, and 10]. For the case of error-only decoding and $\gamma^2 = 1$ or 10, it is impossible to achieve a bit error probability smaller than or equal to 10^{-3} with this code rate (1/2) and that is why these two curves of E_b/N_J versus ρ are missing. As we will see later, a lower code rate can overcome this problem.

In the same figure notice the improvement that the parallel erasure/error decoding scheme offers over the error-only decoding scheme for both $(E_b/N_J)_{\max}$ (the maximum E_b/N_J required to achieve a bit error probability of $P_e = 10^{-3}$ (or 10^{-5}) as ρ varies between 0 and 1) and ρ^*. The quantity ρ^* is defined as the maximum value of ρ for which the specified performance is achieved independently of the power of the jammer. These two parameters are important in characterizing the performance of the FH/SS system against partial-band noise jamming (see [5]-[6]). Obviously, it is desirable to decrease the value of $(E_b/N_J)_{\max}$ and to increase the value of ρ^*. The improvement of $(E_b/N_J)_{\max}$ is more substantial for larger values of γ^2 (e.g., $\gamma^2 = 1.$ and 10). The improvement of ρ^* is about .2 (from .08 to .28). Parallel decoding of RS codes is superior to error-only decoding and should be preferred for combatting severe interference. Regarding the deterioration of the receiver performance as γ^2 increases from 0 (AWGN) to .01 and .1, notice that the increase in E_b/N_J is modest (it is .3dB and 1.9dB, respectively, at $\rho = .4$) but as γ^2 becomes 1. and 10. the increase in the required E_b/N_J becomes substantial.

In Figure 2 E_b/N_J is plotted versus ρ for a RS(32,8) code (rate 1/4) parallel error/erasures decoding and the cases ($M = 2$, $m = 5$), ($M = 32$, $m = 1$) for $K = 5$ and an Rician fading channel with varying γ^2. A comparison of the two cases indicates that ρ^* does not change considerably but $(E_b/N_J)_{\max}$ does: from 8.8 dB for $M=32$ to 15.6 dB for $M=2$ for the AWGN channel ($\gamma^2 = 0.$). As the relative power in the fading component increases from $\gamma^2 = 0.$ to .1, and then to 1., the difference in the required $(E_b/N_J)_{\max}$ increases substantially between the two schemes; the scheme with $M = 32$ requires substantially less signal-to-jammer power ratio to achieve the same bit error probability. Therefore it is preferable to employ M-ary instead of binary FSK modulation in this case.

In Figure 3 similar results are presented for an RS(64,16) code (rate 1/4), the same systems parameters, and for the cases ($M = 2$, $m = 6$), ($M = 4$, $m = 3$), and ($M = 8$, $m = 2$). Similar observations as for Figure 2 can be made.

In Figure 4 E_b/N_J is plotted versus ρ for repetition codes of rates $1/L = 1/4$, 1/5, 1/6, 1/7, 1/8, and 1/9 without side information and 32-ary FSK modulation. For the parameters given in the Figure caption it appears that the rate 1/8 repetition code is the optimal such code. For rates smaller than 1/8, the principle of diminishing returns manifests itself, due to the noncoherent combining loss.

In Figure 5 we illustrate the performance of the same repetition codes as in Figure 4, but with side information, for the same system parameters. Notice that now the rate 1/8 code is optimal for $(E_b/N_J)_{\max}$ but not for ρ^* (ρ^* increases as the rate decreases). A comparison of $(E_b/N_J)_{\max}$ and ρ^* for Figures 4 and 5 indicates that the former decreases slightly while the latter increases substantially, resulting in improvement in both directions, when side information is available.

In Figure 6 the performance of binary convolutional codes with and without side information is illustrated. Codes of constraint length 9 and rates 1/2 and 1/3 are being considered. The figure reveals the substantial improvement on ρ^* and E_b/N_J for fixed ρ provided by the reduction of the rate of the code and the utilization of side information. In the presence of fading ($\gamma^2 = .5$) the performance deteriorates considerably with respect to the E_b/N_J. The missing curves for the cases CC(9,1/2) without side information and $\gamma^2 = 0.$ or .5 and CC(9,1/3) without side information and $\gamma^2 = .5$ indicate that a bit error probability of 10^{-3} can not be achieved by these codes under the specified channel conditions.

In Figure 7 we illustrate the performance of nonbinary convolutional codes of constraint length 7 and code rates of 1/2 (information symbols per code symbol) with 4-ary FSK modulation and 1/3 (information symbols per code symbol) with $M = 8$-ary FSK modulation (i.e., the effective code rate for both codes is 1 bit per code symbol). For these results we used the union bound cited in formulas (17) and (19) for the cases of no side information and side information, respectively, to compute P_j in (15). The code which employs 8-ary FSK modulation performs better than the one which employs 4-ary FSK modulation for all the cases considered. The use of side information improves the performance of the codes considerably. The CC(7,1/2) $M = 4$ code cannot achieve a bit error probability of 10^{-3} for a fading channel with $\gamma^2 = .5$.

In Figure 8 the performance of dual-5 convolutional codes with or without side information is shown; 32-ary FSK modulation is employed and the upper bound of equation (19a) with $J = 5$ points is used. Notice the increase in ρ^* for the case without side information. Also notice that, in the case of side information, E_b/N_J becomes unbounded for ρ larger than .55. This is due to the expressions (19a)-(19b) of Section III.4, which are used to upperbound the performance of the coded FH/SS system: as ρ increases, c in eq. (19a) approaches 1, and the bound becomes arbitrarily large; this is a defficiency of the available bounds but does not imply that the actual perfromance follows this pattern of behavior.

In Figure 9 the performance of Reed-Solomon codes with varying diversity is illustrated; 32-ary FSK modulation and parallel erasure/error decoding is employed. Increasing the diversity causes a substantial improvement to the value ρ^* and a slight deterioration to the value of $(E_b/N_J)_{\max}$; this is of course provided at the expense of a lower overall code rate and subsequently of a larger bandwidth expansion.

In Figure 10 the performance of repetition codes and Reed-Solomon codes of rate 1/4 which employ 32-ary FSK modulation is compared. Repetition codes with or without side information and Reed-Solomon codes with error-only or parallel error/erasure decoding are considered. The superiority of Reed-Solomon codes over repetition codes and of parallel decoding over error-only decoding is clear for both the performance measures $(E_b/N_J)_{\max}$ and ρ^*. Notice that the repetition code with $n = 4$ and without side information cannot achieve a bit error probability of 10^{-3} for fading with $\gamma^2 = .5$.

In Figure 11 the performance of several error-control coding schemes of rate 1/8 is shown. Repetition codes (with and without side information), Reed-Solomon codes (with error-only or parallel decoding) and Reed-Solomon codes with diversity are considered. For this coding rate (1/8) and in the absence of side information the RS(32,4) code performs worse than the repetition code with $L = 8$ (this is in contrast to the comparison of the rate 1/4 codes presented in Figure 10) and the latter performs worse than the RS(32,16) code with diversity 4 (error-only decoding). However, when side information is available, the RS(32,16) code with diversity 4 and parallel decoding outperforms the RS(32,4) code with parallel decoding and the repetition code with $L = 8$ in both performance measures [ρ^* and $(E_b/N_J)_{\max}$] and between these two the former outperforms the latter in ρ^* but not in $(E_b/N_J)_{\max}$.

In Figure 12 we illustrate the performance of asynchronous FH/SSMA systems with 32-ary FSK, noncoherent demodulation and RS(32,8) coding with error-only or parallel decoding for various combinations of q (number of frequencies) and N_b (number of bits per dwell time), and an AWGN channel. As q increases from 100 to 1000, the jammer is assumed to maintain the same N_J (i.e., its total power P_J increases by a factor of 10 and 100, respectively), and all other system and channel parameters are held fixed, then there is a slight increase in ρ^* (about .08) and a more considerable decrease in $(E_b/N_J)_{\max}$ (about 2.8dB) for the case of error-only decoding. For the case of parallel decoding there is improvement in both performance measures as q increases but this is more modest. Furthermore, if q is held fixed and N_b increases from 5 to 12, then there is again some improvement in both performance measures. The improvement is more considerable for small q than for larger q and for the case of error-only decoding than the case of parallel decoding. Usually the values of q, N_b are determined by considerations other than the combat against the channel interference; in any case it is desirable to employ a large q and an N_b larger that 1, at least when the interference is expected to be of the form considered in this paper.

In Figure 13 the performance of a concatenated RS(64,38) outer code with a parity-check (PC) (6,5) inner block code (overall rate approximately 1/2) is shown. Both erasure/error decoding and parallel decoding (see Section III.4) are considered. For the system and channel parameters shown in the figure caption there is very little difference in the performance of the two decoding schemes.

In Figure 14 the performance of RS codes and concatenated codes using RS outer codes and inner PC codes is compared in the absence of side information. The RS(64,16) code uses error-only decoding; the RS(64,19) + PC(6,5) concatenated code with overall code rate approximately 1/4 uses either erasure/error decoding or parallel decoding. The comparison shows that the concatenation of RS outer codes with inner detection-only (here parity-check) codes improves the system performance with respect to $(E_b/N_J)_{\max}$ (the maximum required signal-to-jammer power ratio to achieve a bit error probability of 10^{-5} when the jammer's ρ varies between 0 and 1) substantially (up to 2.5 dB for $\gamma^2 = 0$), and with respect to ρ^* modestly. The improvement is more considerable when the parallel decoding scheme is employed.

In Figures 15 and 16 the performance of concatenated RS outer codes with inner binary convolutional codes and dual-k convolutional codes, respectively, is illustrated. In particular, in Figure 15 the performance of the RS(32,8) code is compared with that of the concatenated RS(32,16) + CC(9,1/2) code with overall code rate 1/4, when error-only decoding and binary FSK modulation is employed. The concatenated code provides a considerable improvement on the $(E_b/N_J)_{\max}$ and negligible improvement on the ρ^*. Similarly, in Figure 16 the performance of the RS(32,4) code is compared with that of the concatenated RS(32,16) + (dual-5,rate 1/4) code with overall rate 1/8 when error-only decoding and 32-ary FSK modulation are employed. The figure shows that the concatenated scheme offers a substantial improvement in both the aforementioned performance measures; the improvement is actually more considerable than that of the concatenated scheme of Figure 15.

The first group of results ends with Table 1. In this table we cite the aforementioned system performance indicators $(E_b/N_J)_{\max}$ and ρ^* for a FH/SS system with 32-ary FSK modulation (unless indicated otherwise), with noncoherent demodulation and other system parameters as described in the table caption. The desirable values of the bit error probability are 10^{-3} and 10^{-5}. From the coding schemes presented in this table the RS codes with diversity show the best performance, in particular when we seek to maximize ρ^*. Recall that as figures 15 and 16 show, concatenated codes provide very good performance for $(E_b/N_J)_{\max}$ but not a very good one for ρ^*.

The second group of results starts with Tables 2 and 3. In these tables we illustrate the multiple-access capability ($K_{\max}$) of FH/SS systems employing 32-FSK modulation [except for the binary and nonbinary convolutional codes (CC) which employ binary, 4-ary, and 8-ary FSK modulation] with noncoherent demodulation and a variety of error-control coding schemes. For the results of Table 2 it has been assumed that $\rho = 0$ and $E_b/N_0 = \infty$ so that the multiple-access interference is the only source of interference. Therefore, $K_{\max}$ is the absolute maximum multiple-access capability of the FH/SS system. For RS codes and RS codes with diversity we cite two numbers in each enrty correponding to erasure/error and parallel decoding, respectively. We see from the results of this table that the RS codes are superior in supporting a large number of users to most of the other codes of the same rate. In Table 3 we present similar results to those of Table 2 for the case in which the fraction of the band jammed by the jammer is $\rho = 50\%$ and $E_b/N_J = 10\ dB$. Similar observations as those for Table 2 are in order here.

In the remaining five figures we cite some values of $K_{\max}$ as a function of ρ for different error-control coding schemes and an AWGN channel. In Figure 17 the increase in the multiple-access capability ($K_{\max}$) is shown as the available signal-to-jammer power ratio (E_b/N_J) increases by a step of 5dB at a time. The error-control code employed is a binary convolutional code of constaint length 9 and code rate 1/3; side information is available.

Then in Figure 18 the performance of binary convolutional codes with different constraint lengths and code rates is shown. Codes with larger constaint lengths and lower code rates present uniformly (over all values of ρ in [0,1]) better performance than codes of smaller constraint lengths or higher code rates.

In Figure 19 the performance of dual-5 convolutional codes of different code rates ($= 1/v$) employing 32-ary FSK modulation is illustrated. Both situations are considered when side information is available and when it is not available. The multiple-access capability increases as the code rate decreases from 1/2 to 1/3 and then to 1/4; it also increases when side information is available.

In Figure 20 results similar to those of Figure 17 are presented for an RS(32,16) code employing 32-ary FSK modulation and error-only decoding for varying E_b/N_J. As E_b/N_J increases the multiple-access capability becomes insensitive to its actual value since the other-user interference dominates the jamming interference which becomes negligible.

Finally, in Figure 21 the multiple-access capability versus ρ is presented for several RS coded FH/SS systems. For small values of ρ, RS(32,16) with diversity 4 outperforms all the other systems; in this range of ρ the other-user interference is dominant. For large values of ρ, the RS(32,8) code outperforms the other schemes; in this range of ρ the jamming interference is dominant.

V. CONCLUSIONS

This paper provides an analytical framework for evaluating the performance of coded FH/SS systems operating in the presence of combined partial-band noise jamming, Rician nonselective fading, other-user interference, and AWGN. Several forward error-control coding schemes with or without side information at the decoder, with binary or nonbinary data modulation schemes, as well as combinations of coding schemes, have been analyzed. Numerical results have been presented for several cases and can be easily obtained from the available formulas for all the the remaining cases for which they were not presented.

Our conclusions are the following. For the combined interference considered in this paper, Reed-Solomon codes perform better than convolutional codes of the same rate when hard decisions are employed; no soft decisions decoding was considered in this paper. Similarly, RS codes outperform the repetition codes for code rates not lower than a critical rate (usually 1/4); a situation which is reversed for code rates lower than the critical rate. Employing RS codes with diversity and parallel decoding (when side information is available) provides excellent performance in terms of both performance measures $(E_b/N_J)_{\max}$ and ρ^*.

The availability and use of side information improves the system performance in all cases; in particular it increases the value of ρ^* more drastically than it decreases the value of $(E_b/N_J)_{\max}$. By contrast, increasing the value of M (i.e., employing codes with nonbinary alphabets) decreases the value of $(E_b/N_J)_{\max}$ more drastically than it increases the value of ρ^*. Similarly, lowering the code rate improves ρ^* more drastically than it improves $(E_b/N_J)_{\max}$.

In the absence of side information the use of concatenated codes, especially of RS outer codes (employing parallel decoding) with inner parity-check codes (to detect the presence of interference), retrieves most of the advantage that perfect knowledge of side information offers with respect to $(E_b/N_J)_{\max}$; to increase ρ^* we also need to lower the rate of the outer code. When RS outer codes (employing error-only decoding) with non-binary (preferably dual-k) convolutional inner codes are used, lowering the rate of the inner code retrieves most of the advantage that perfect side information offers with respect to both ρ^* and $(E_b/N_J)_{\max}$.

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Figure Captions

Figure 1. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing $RS(64,32)$ codes with error-only and parallel erasure/error decoding ($q = 100$, $N_b = 12$, $M = 8$, $m = 2$, $E_b/N_0 = 12\ dB$, $K = 5$); Rician fading channel with varying γ^2.

Figure 2. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing $RS(32,8)$ codes with parallel erasure/error decoding over various $GF(M^m)$, with corresponding M-ary FSK modulation ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $K = 5$); Rician fading channel with varying γ^2.

Figure 3. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing $RS(64,16)$ codes with parallel erasure/error decoding over various $GF(M^m)$, with corresponding M-ary FSK modulation ($q = 100$, $N_b = 12$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel.

Figure 4. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK and varying diversity without side information ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel; for $L = 8$, AWGN channel and Rician fading channel with $\gamma^2 = 1$.

Figure 5. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK and varying diversity with side information ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel; for $L = 8$, AWGN channel and Rician fading channel with $\gamma^2 = 1$.

Figure 6. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing binary convolutional codes of constraint length 9 and code rates 1/2 and 1/3 with and without side information; ($q = 100$, $N_b = 12$, $E_b/N_0 = 12\ dB$, $K = 5$); AWGN channel and Rician fading channel with $\gamma^2 = .5$.

Figure 7. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing nonbinary convolutional codes of constraint length 7 and code rates 1/2 ($M = 4$) and 1/3 ($M = 8$) with or without side information ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel and Rician fading channel with $\gamma^2 = 5$.

Figure 8. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing a dual convolutional code of rate 1/4 with or without side information ($q = 100$, $N_b = 15$, $M = 32$, $E_b/N_0 = 12\ dB$, $K = 5$); AWGN channel and Rician fading channel with $\gamma^2 = .5$.

Figure 9. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK with $RS(32,16)$ codes and parallel erasure/error decoding plus varying diversity ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel.

Figure 10. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK with various rate 1/4 coding schemes ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel and Rician fading channel for $\gamma^2 = .5$.

Figure 11. Minimum E_b/N_J required for $P_e = 10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK with various rate 1/8 coding schemes ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel.

Figure 12. Minimum E_b/N_J required for $P_e = 10^{-5}$ versus ρ for asynchronous **FH/SSMA communications employing 32-ary FSK with RS(32,8) codes and various combinations of q and N_b ($E_b/N_0 = 15\ dB$, $K = 5$); AWGN channel.**

Figure 13. Minimum E_b/N_J required for $P_e = 10^{-3}$ and $P_e = 10^{-5}$ versus ρ for asynchronous FH/SSMA communications employing binary FSK with concatenated coding schemes (RS(64,38) error-only/parallel decoding + PC(6,5)) ($q = 100$, $N_b = 12$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel and Rician fading channel with $\gamma^2 = .5$.

Figure 14. Minimum E_b/N_J required for $P_e = 10^{-5}$ versus ρ for asynchronous FH/SSMA communications employing binary FSK with various rate 1/4 coding schemes ($q = 100$, $N_b = 12$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel and Rician fading channel with $\gamma^2 = .5$.

Figure 15. Minimum E_b/N_J required for $P_e = 10^{-5}$ versus ρ for asynchronous FH/SSMA communications employing binary FSK and various rate 1/4 coding schemes ($q = 100$, $N_b = 12$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel.

Figure 16. Minimum E_b/N_J required for $P_e = 10^{-5}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK and various rate 1/8 coding schemes ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $K = 5$); AWGN channel and Rician fading channel with $\gamma^2 = .5$.

Figure 17. $K_{\max}$ versus ρ for asynchronous FH/SSMA communications employing binary FSK and CC(9, 1/3) coding with side information for varying E_b/N_J ($q = 100$, $N_b = 12$, $E_b/N_0 = 20\ dB$, AWGN, $P_e = 10^{-5}$)

Figure 18. $K_{\max}$ versus ρ for asynchronous FH/SSMA communications employing binary FSK with various convolutional coding schemes with side information ($q = 100$, $N_b = 12$, $E_b/N_0 = 20\ dB$, $E_b/N_J = 20\ dB$, $P_e = 10^{-5}$); AWGN channel.

Figure 19. $K_{\max}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK with various dual-5 coding schemes ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, $P_e = 10^{-3}$); AWGN channel.

Figure 20. $K_{\max}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK and RS (32,16) coding with error-only decoding and varying E_b/N_J ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, AWGN, $P_e = 10^{-5}$).

Figure 21. $K_{\max}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK and Reed-Solomon coding with various rates and decoding methods with and without diversity ($q = 100$, $N_b = 10$, $E_b/N_0 = 20\ dB$, AWGN, $E_b/N_J = 10dB$, $P_e = 10^{-5}$).

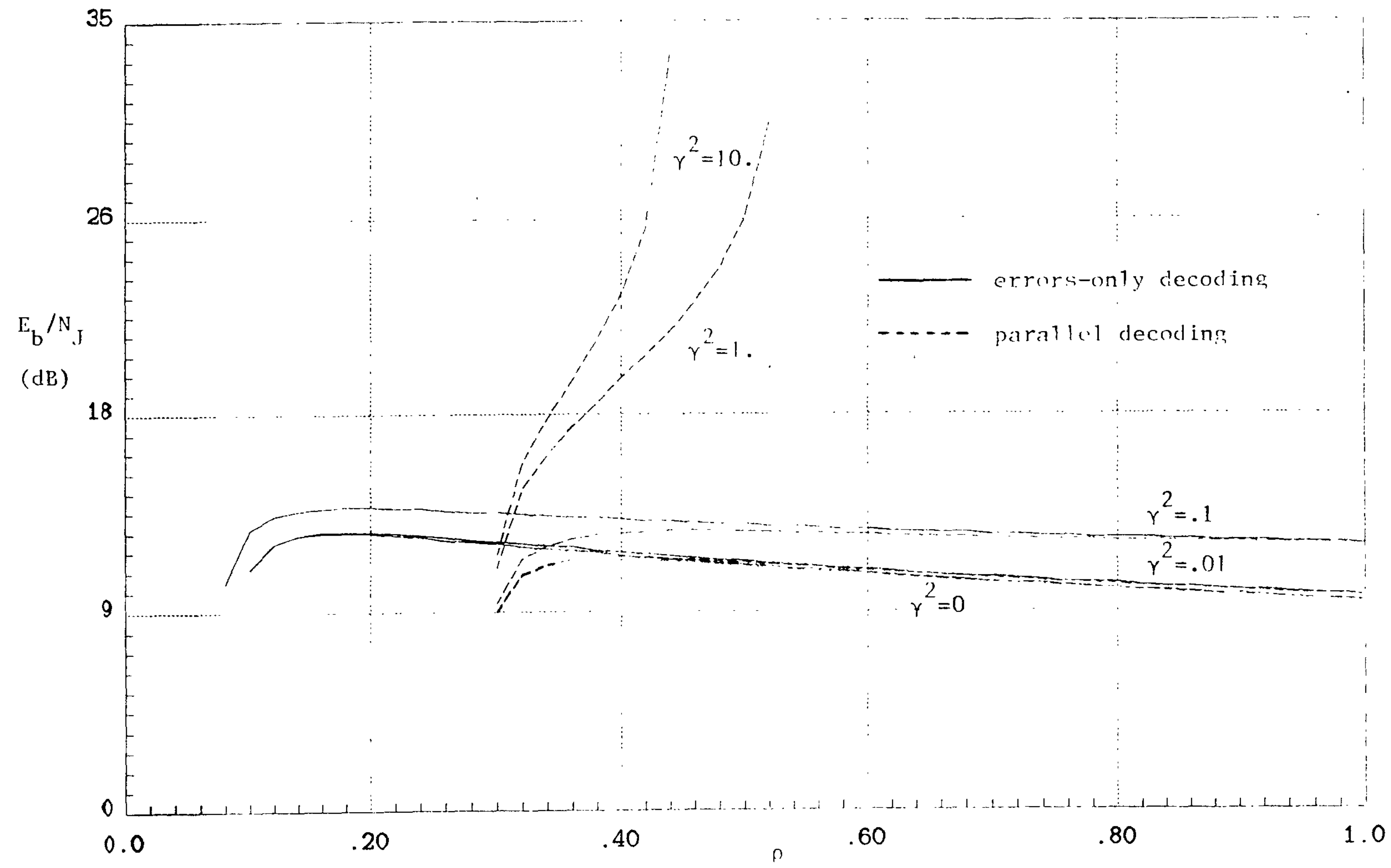


Figure 1. Minimum E_b/N_J required for $P_e=10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing RS(64,32) codes with **error-only** and parallel **erasure/error** decoding ($q=100$, $N_b=12$, $M=8$, $m=2$, $E_b/N_0=12$ dB, $K=5$); Rician fading channel with varying γ^2.

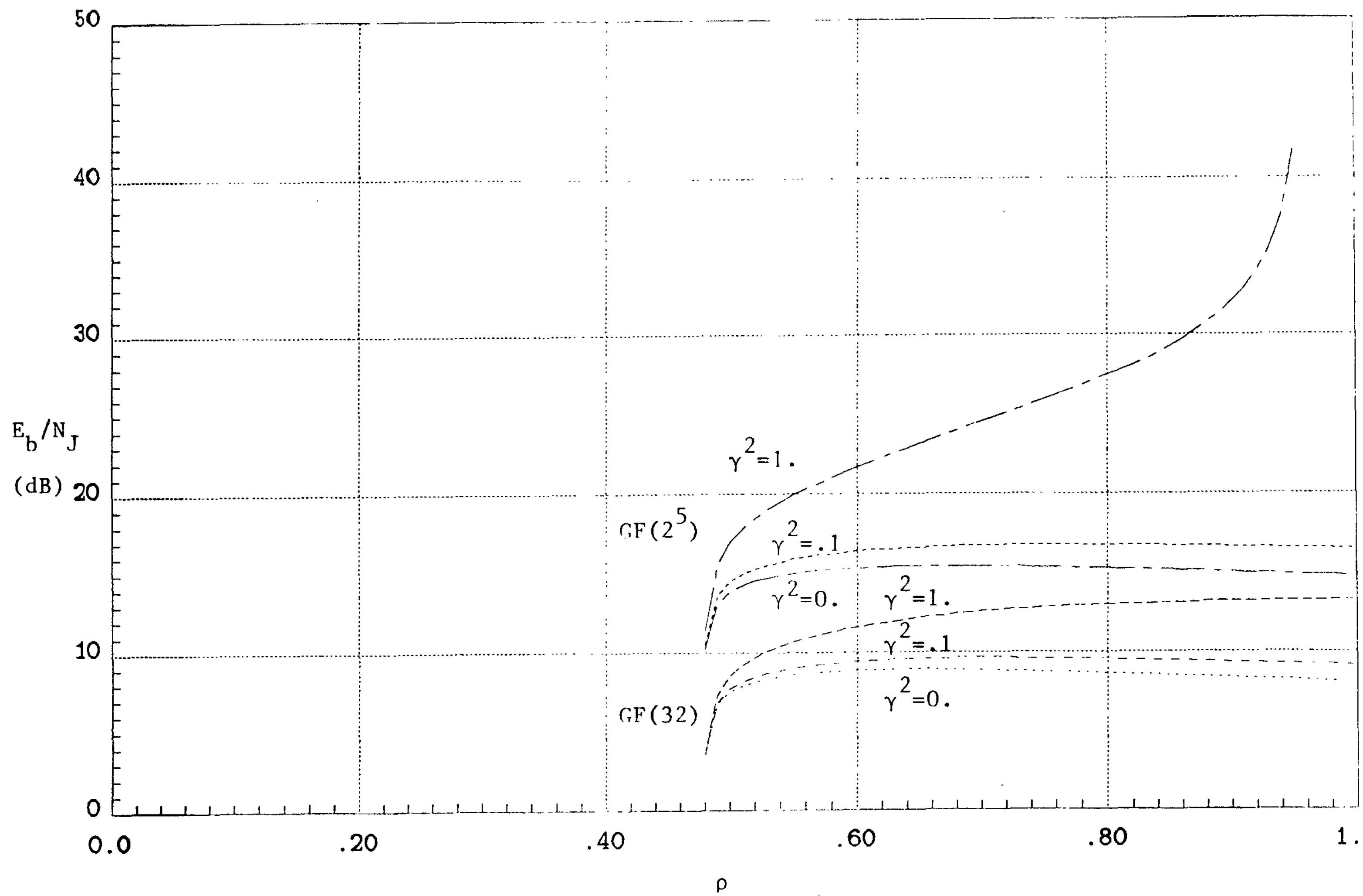
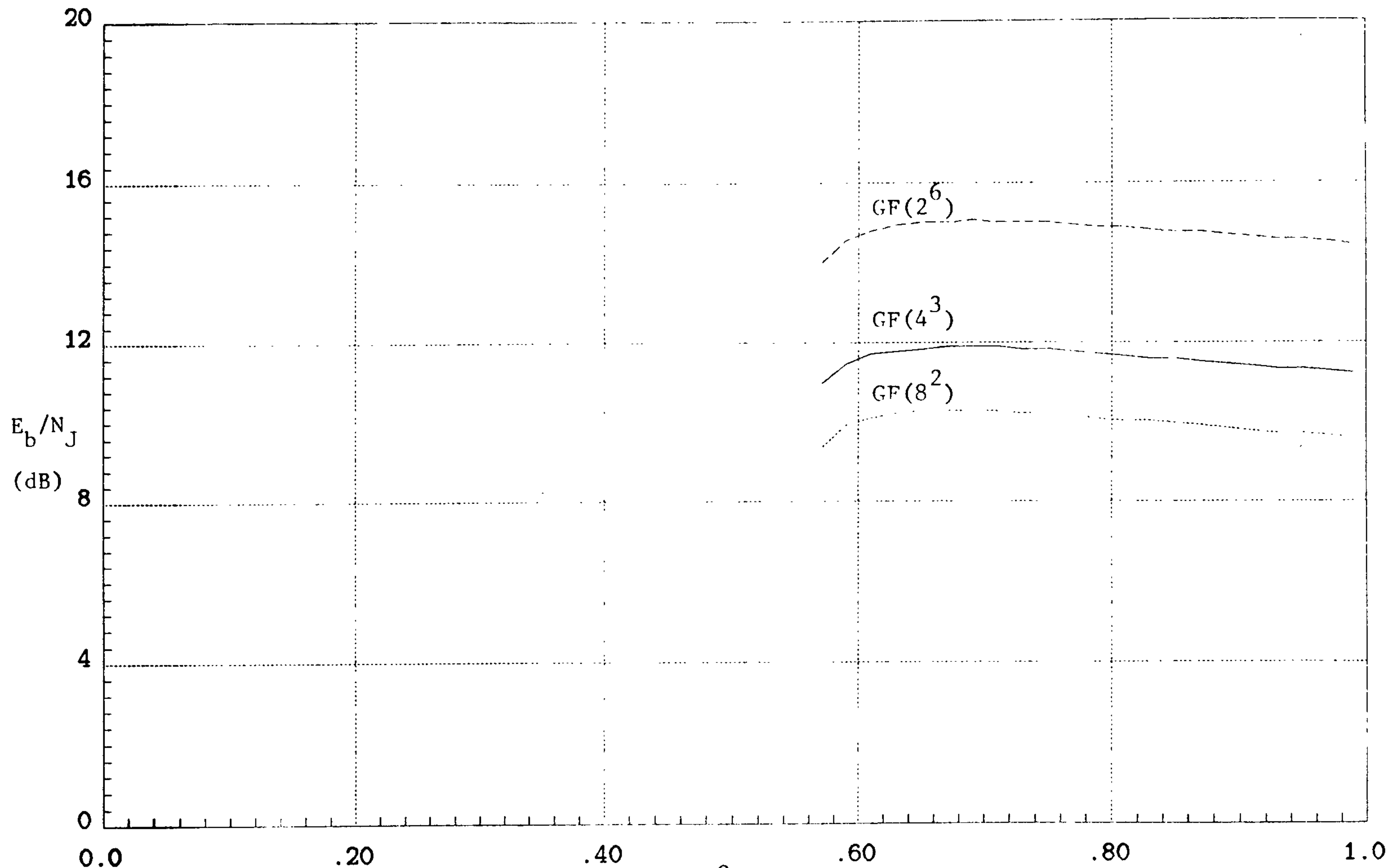


Figure 2. Minimum E_b/N_J required for $P_e=10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing RS(32,8) codes with parallel erasure/error decoding over various $GF(M^m)$, with corresponding M-ary FSK modulation (q=100, $N_b=10$, $E_b/N_0=20$ dB, K=5); Rician fading channel with varying γ^2.



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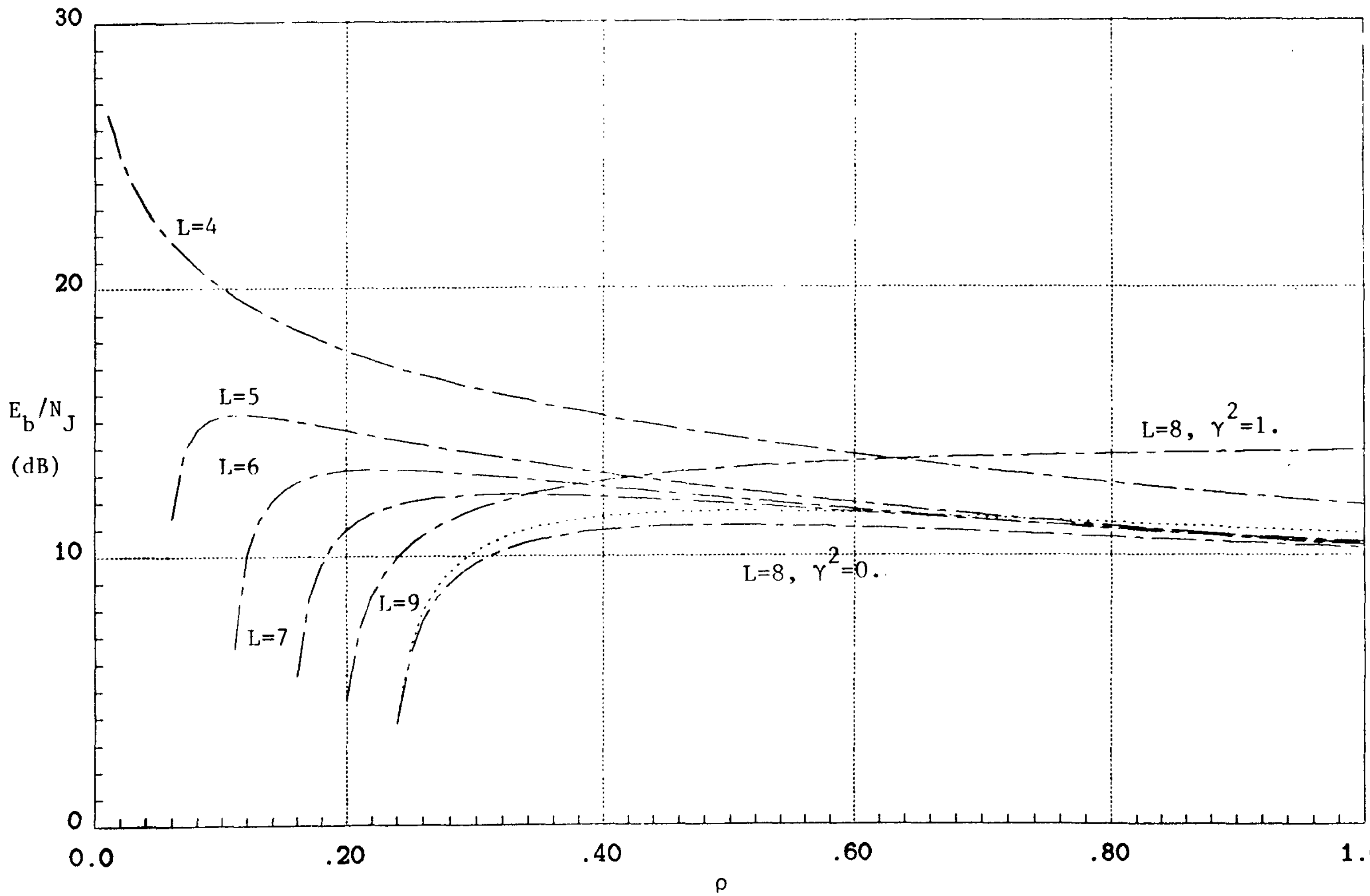
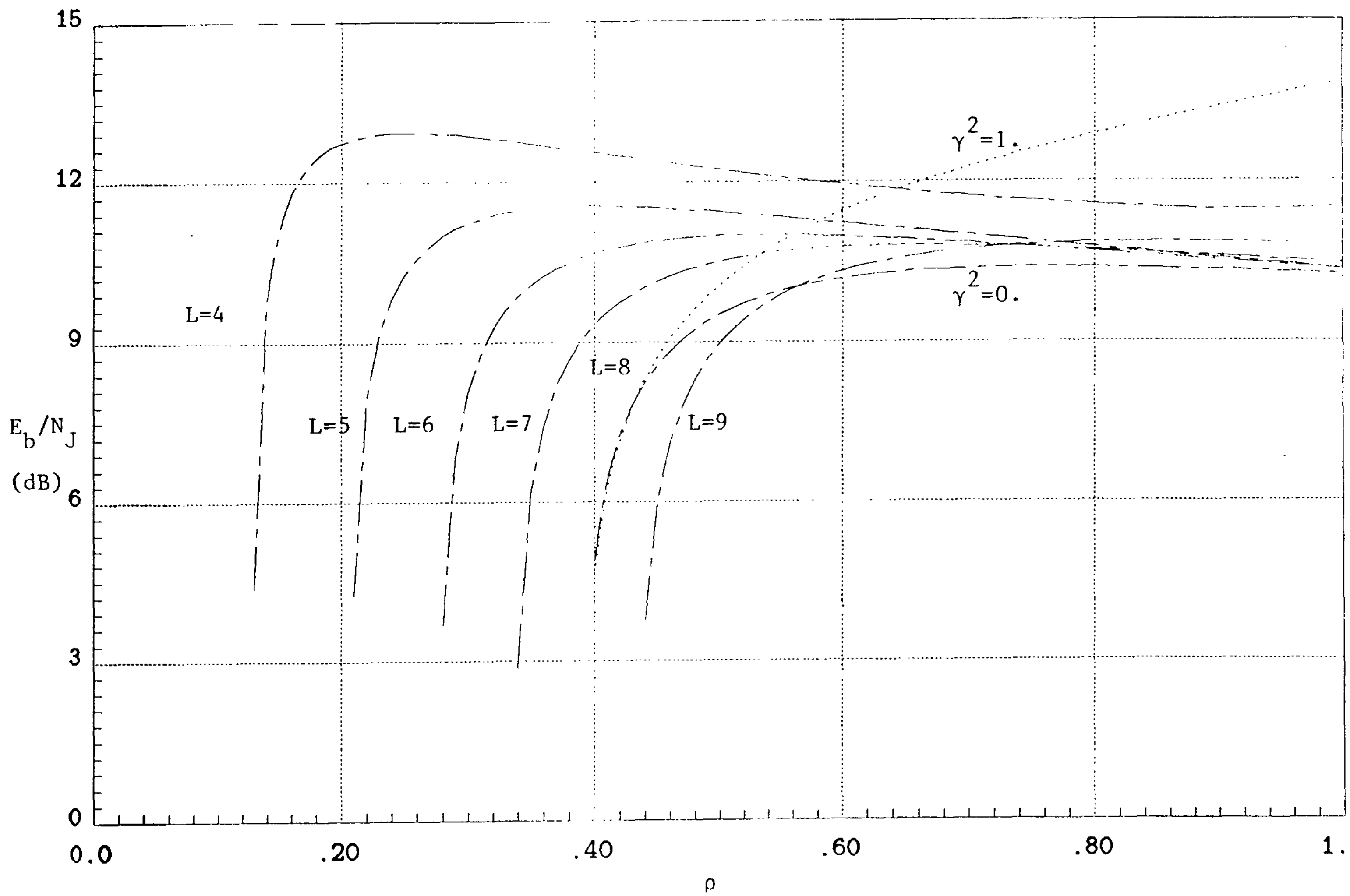


Figure 4. Minimum E_b/N_J required for $P_e=10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK and varying diversity without side information ($q=100$, $N_b=10$, $E_b/N_0=20$ dB, $K=5$); AWGN channel; for L=8, AWGN channel and Rician fading channel with $\gamma^2=1$.



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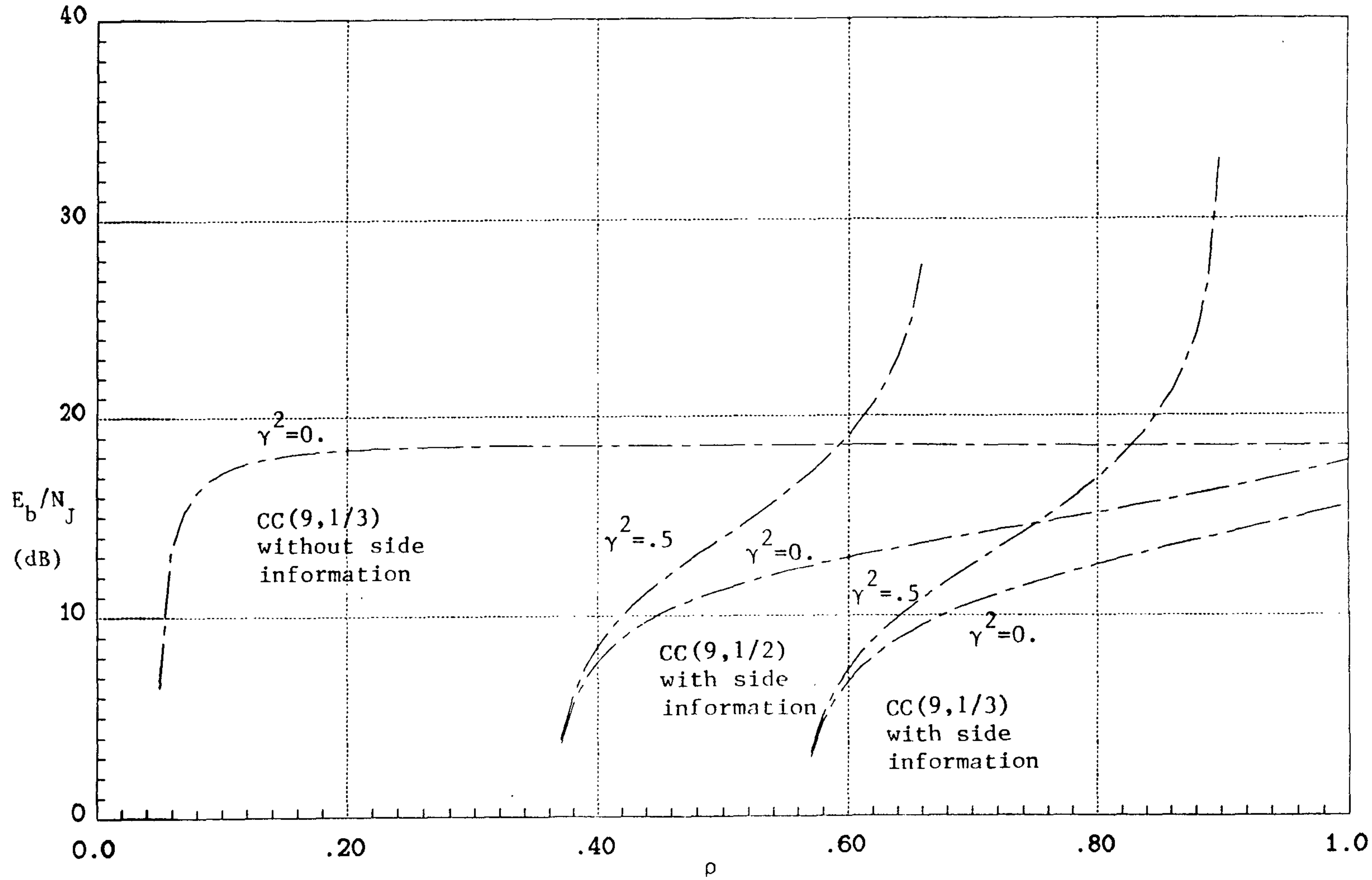

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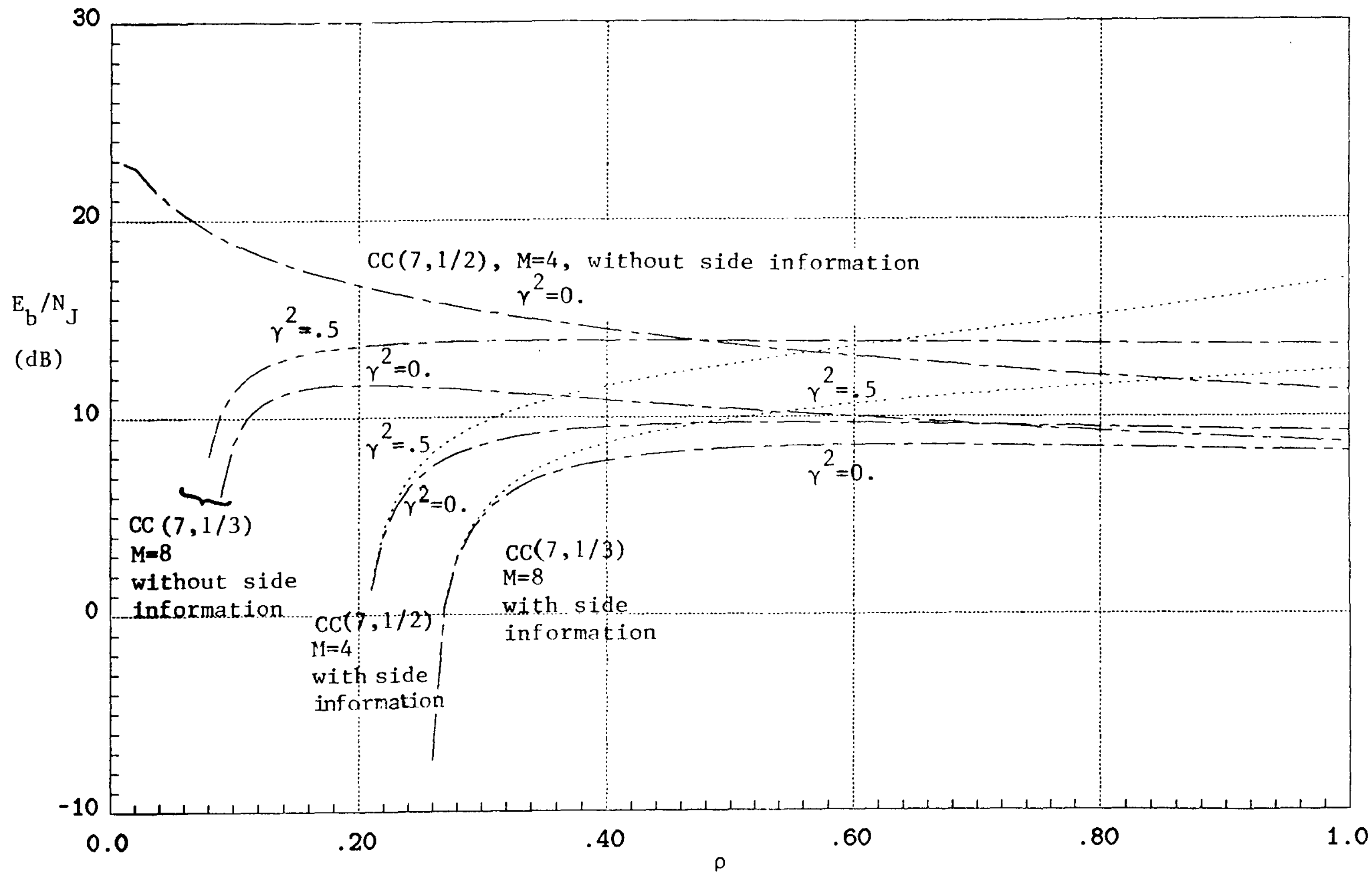

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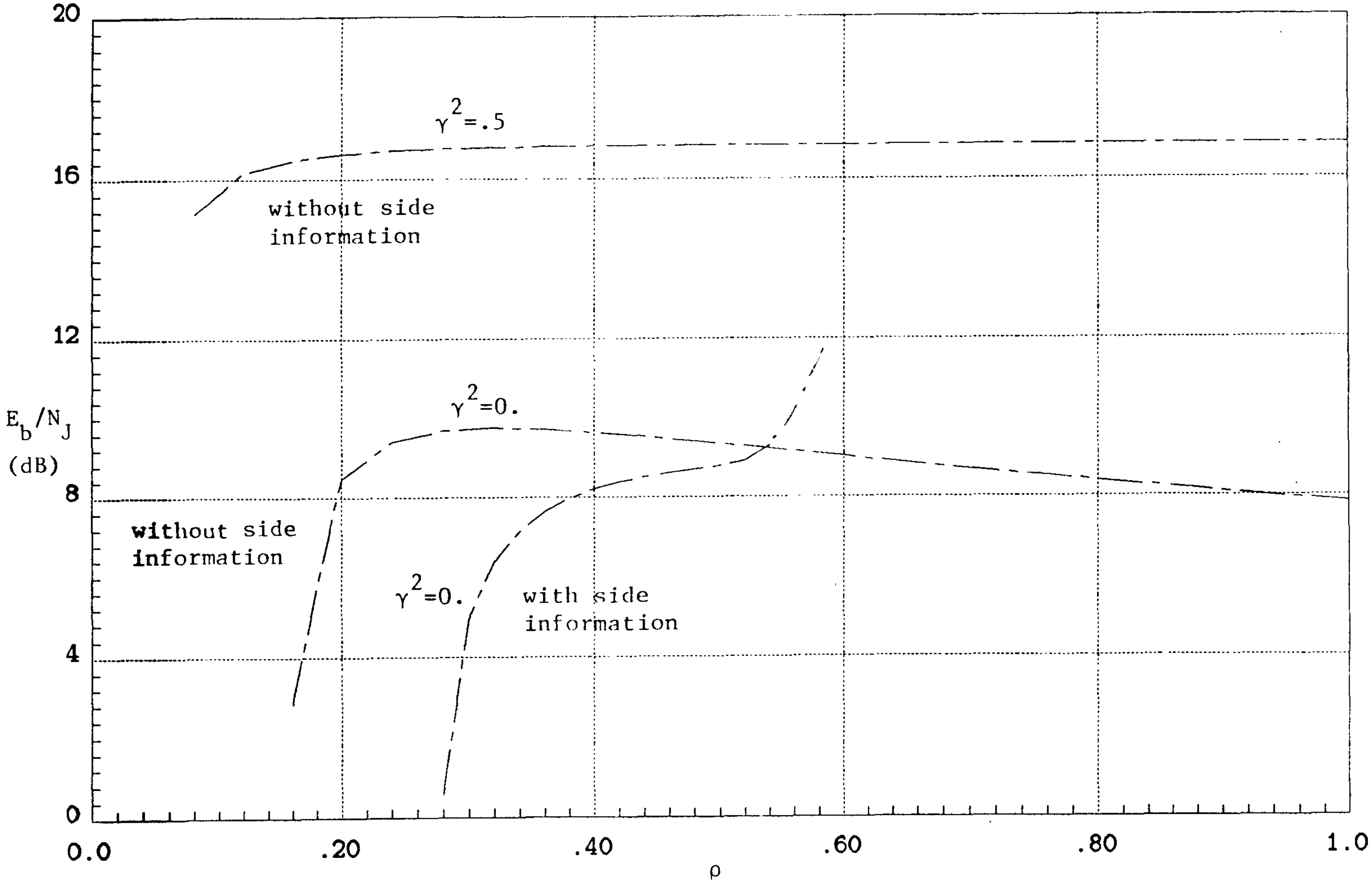
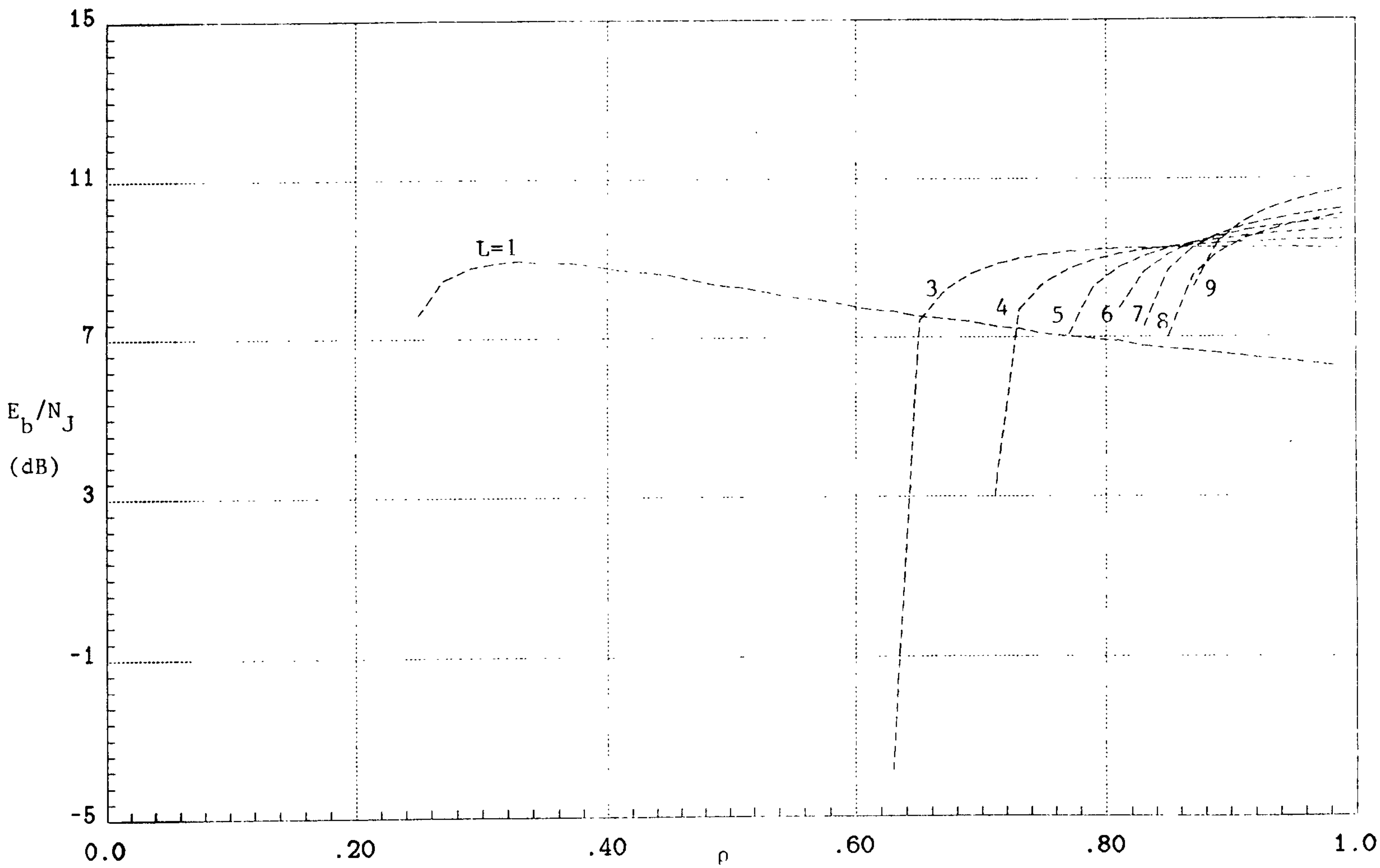


Figure 8. Minimum E_b/N_J required for $P_e=10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing a dual-5 convolutional code of rate 1/4 with or without side information ($q=100$, $N_b=15$, $M=32$, $E_b/N_0=12$ dB, $K=5$); AWGN channel and Rician fading channel with $\gamma^2=.5$.



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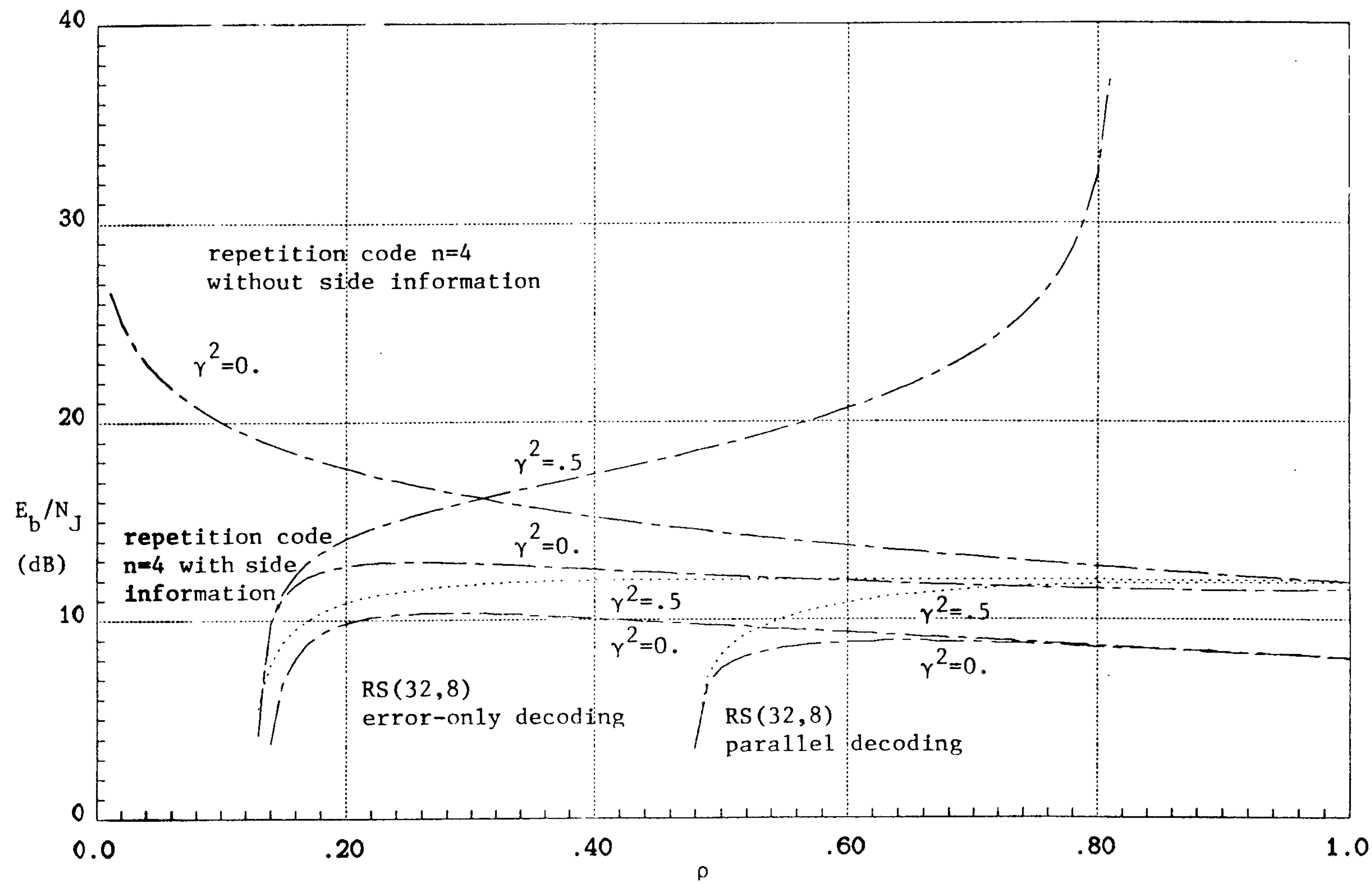
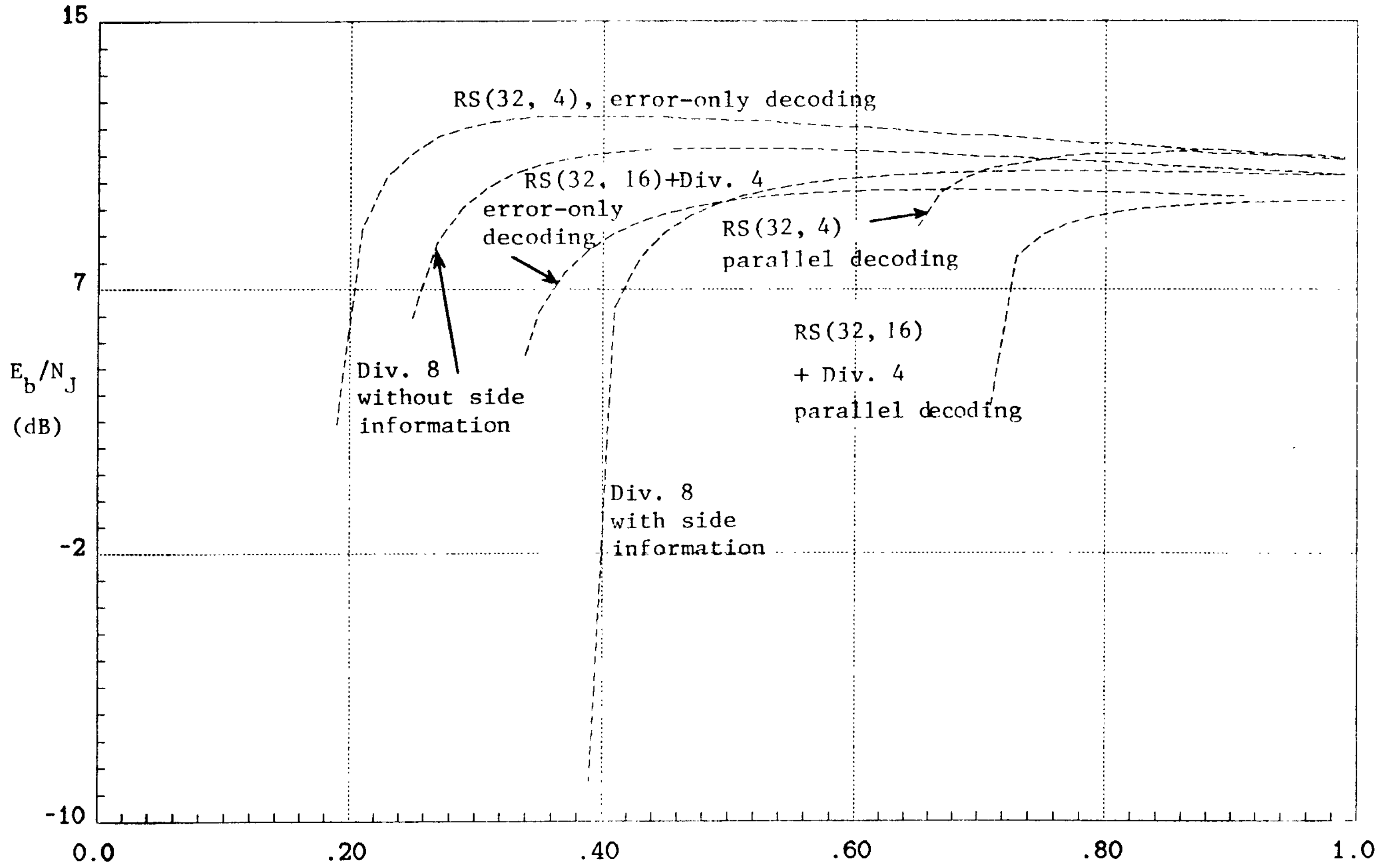


Figure 10. Minimum E_b/N_J required for $P_e=10^{-3}$ versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK with various rate 1/4 coding schemes (q=100, N_b=10, E_b/N_0=20 dB, K=5); AWGN channel and Rician fading channel with γ^2=.5.



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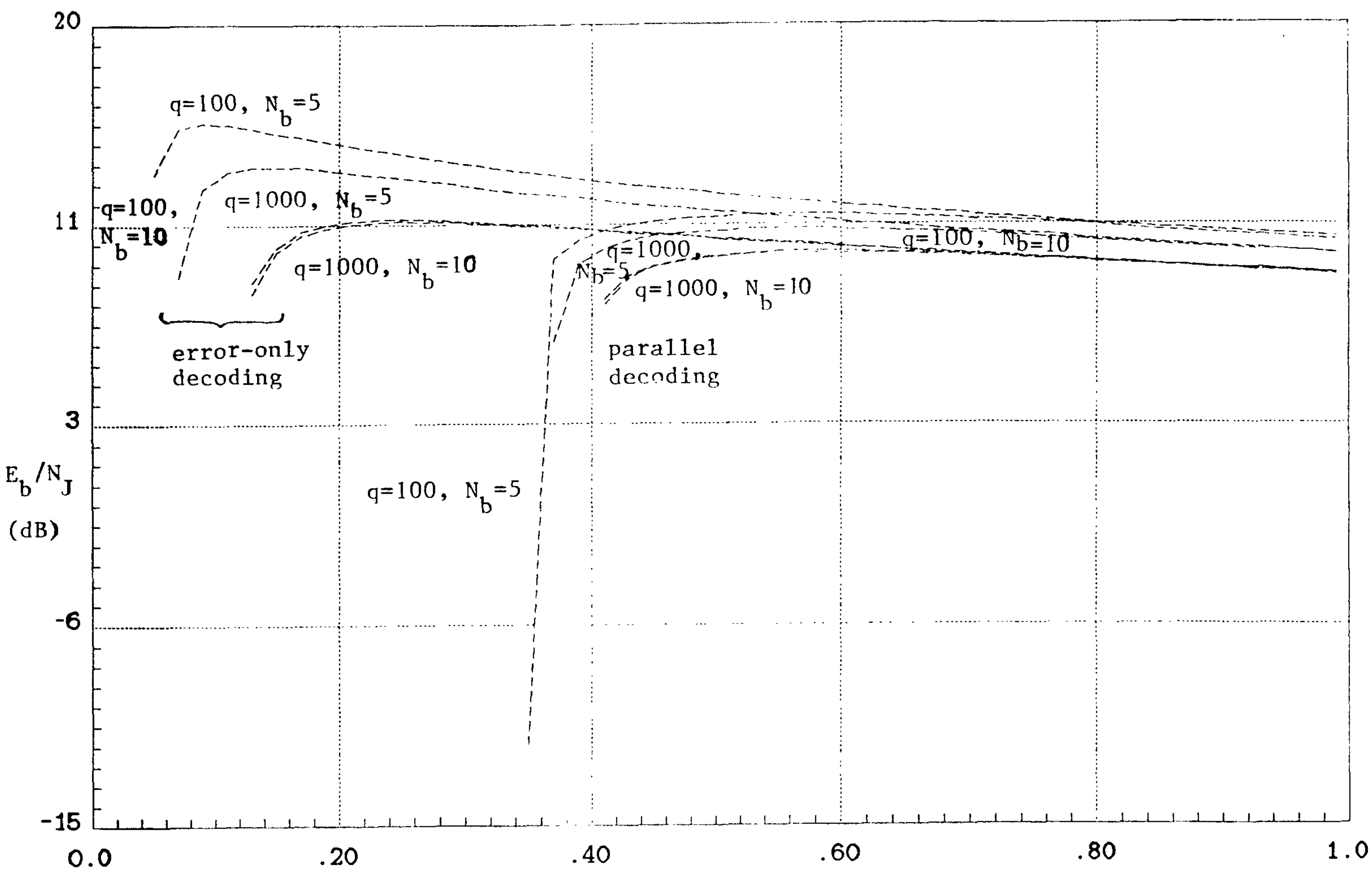

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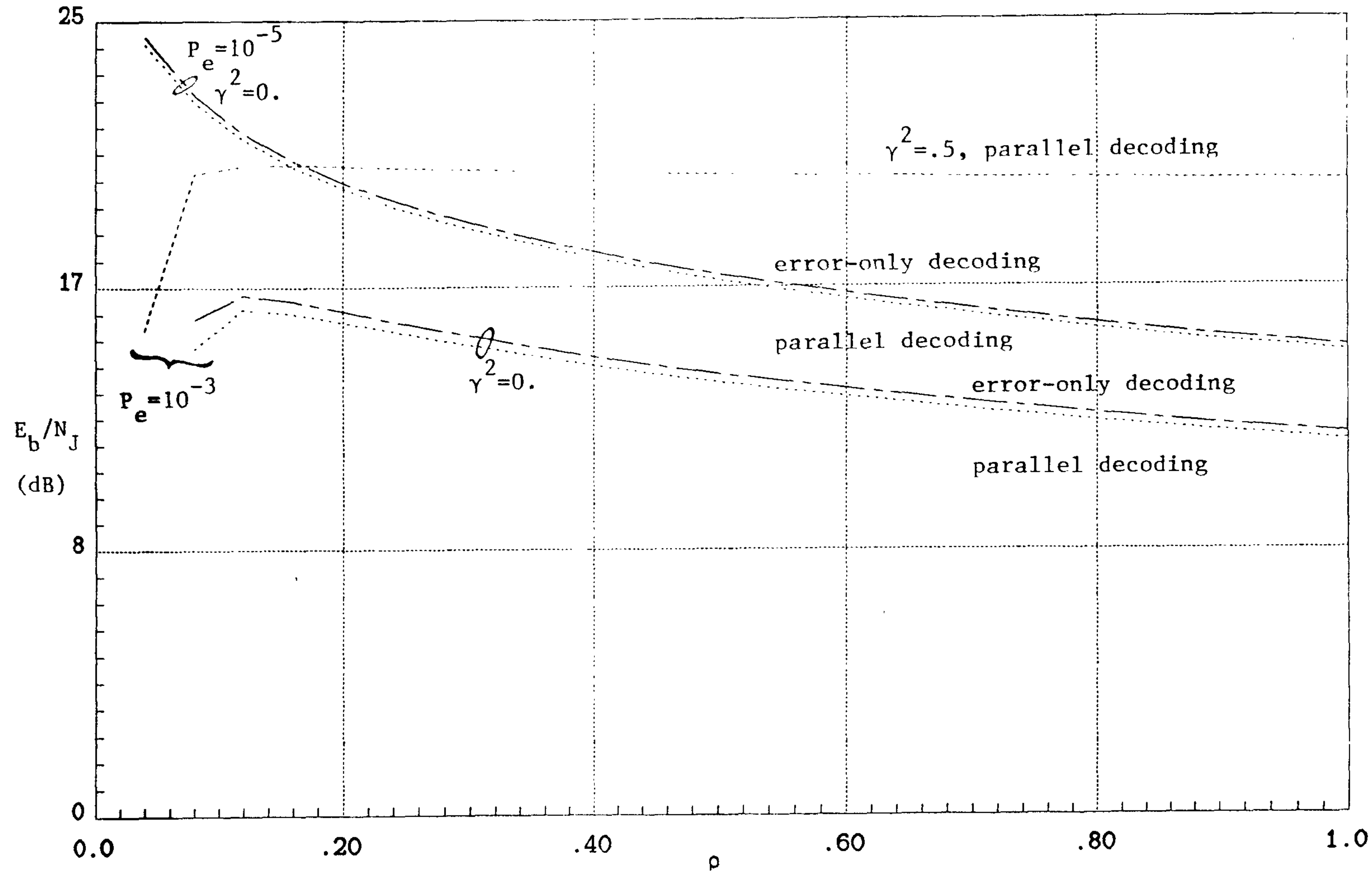

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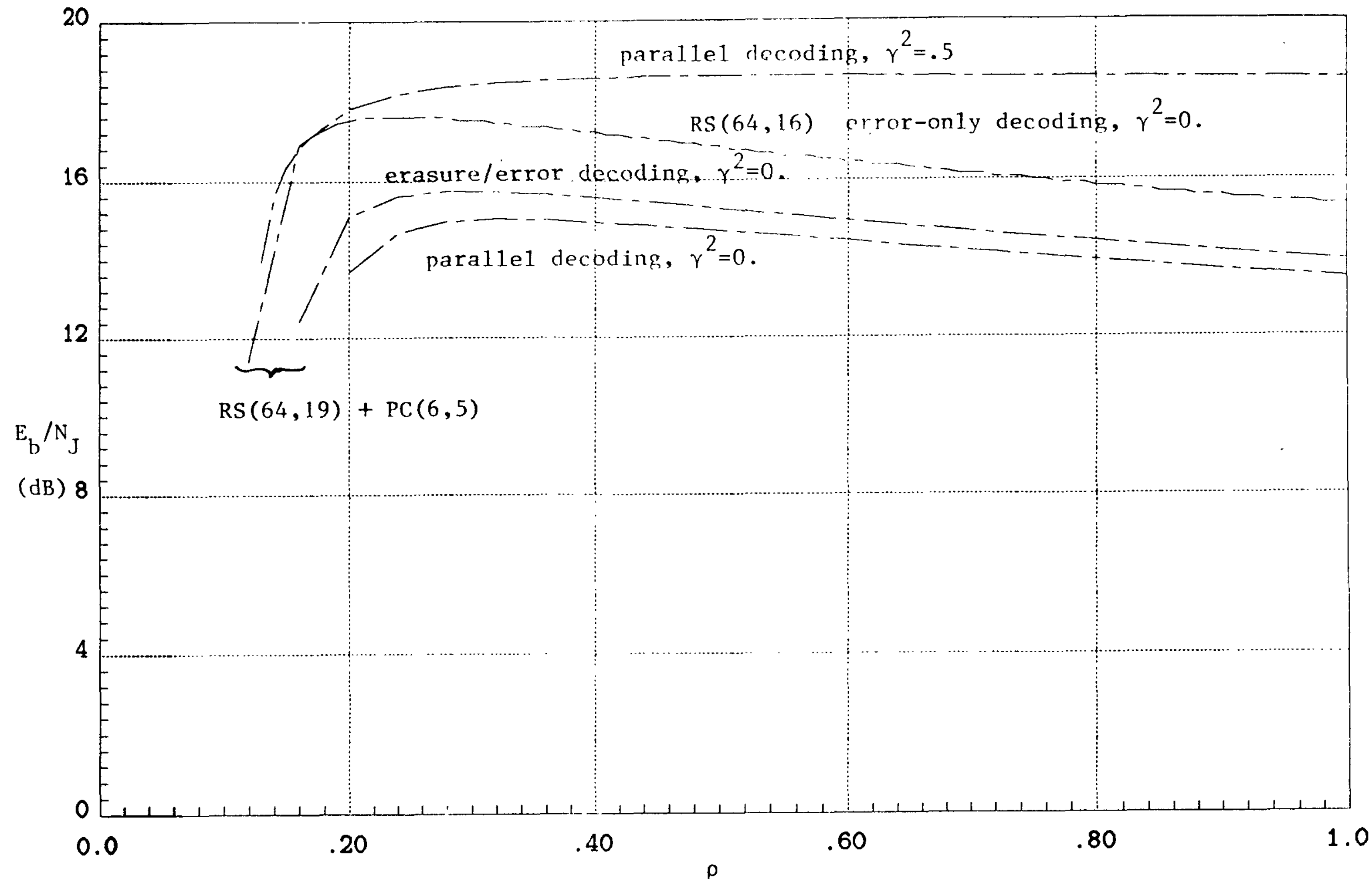

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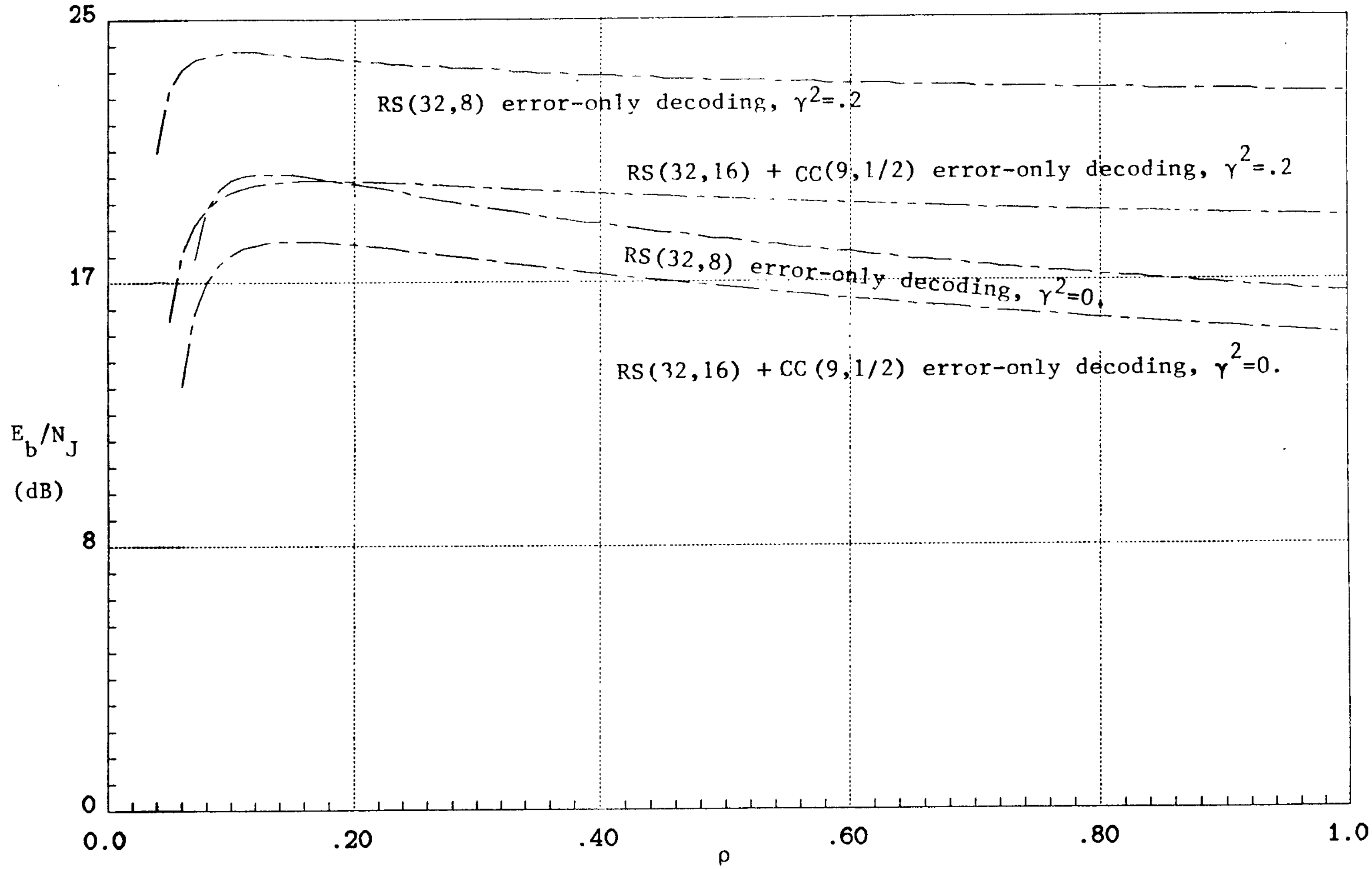

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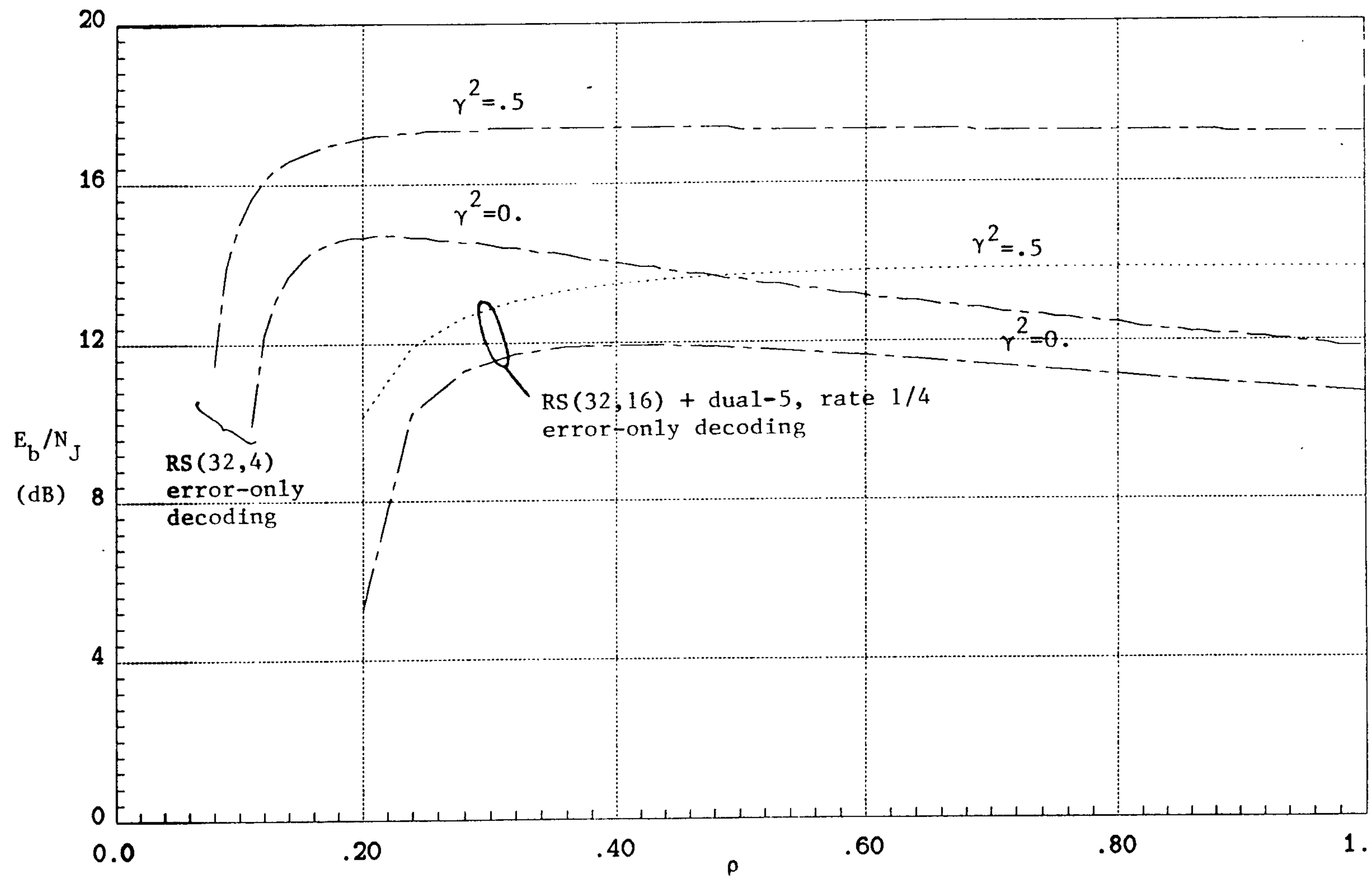

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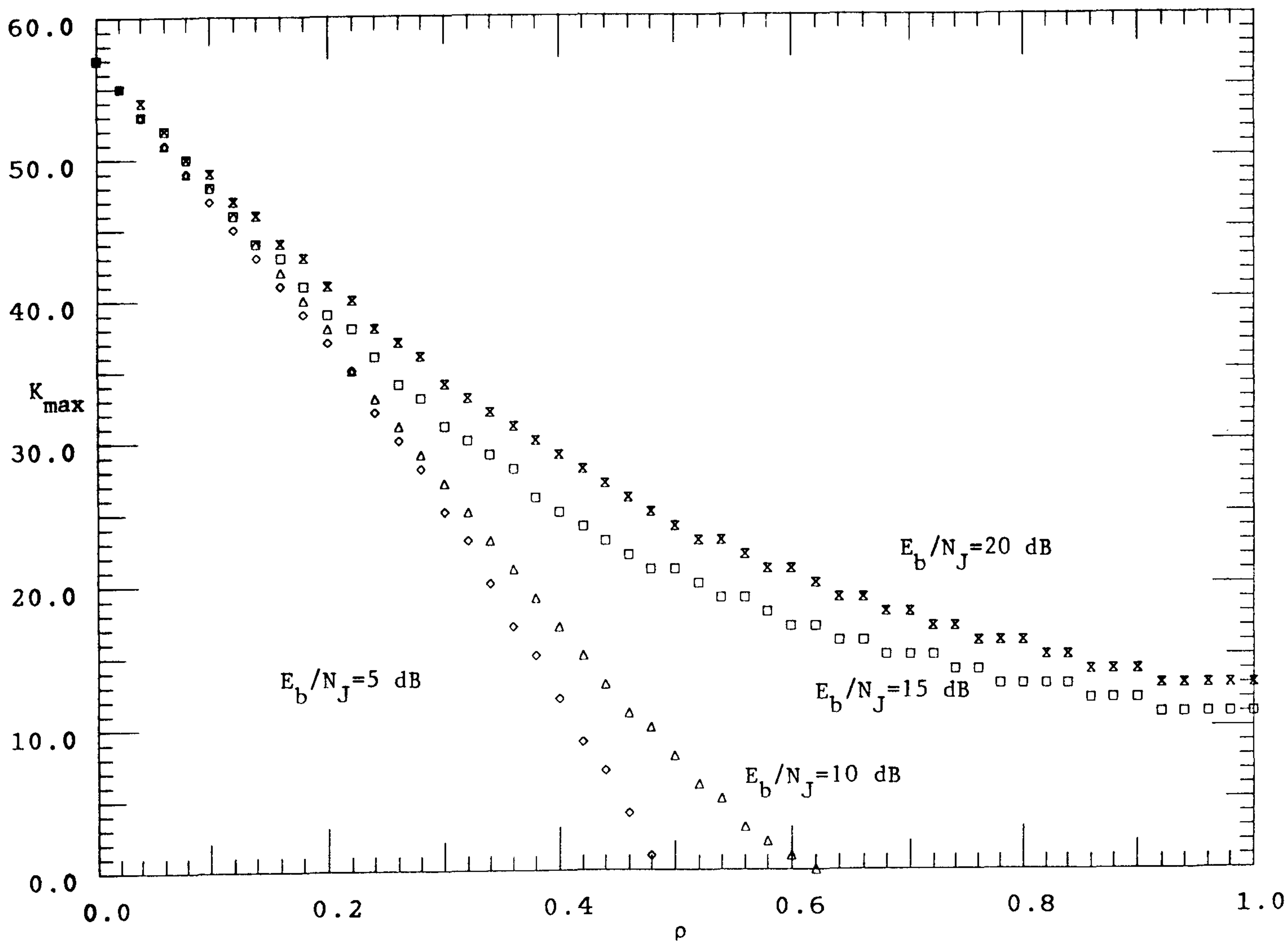


Figure 17. K_{max} versus ρ for asynchronous FH/SSMA communications employing binary FSK and CC(9, 1/3) coding with side information for varying E_b/N_J (q=100, N_b=12, E_b/N_0=20 dB, AWGN, $P_e=10^{-5}$).

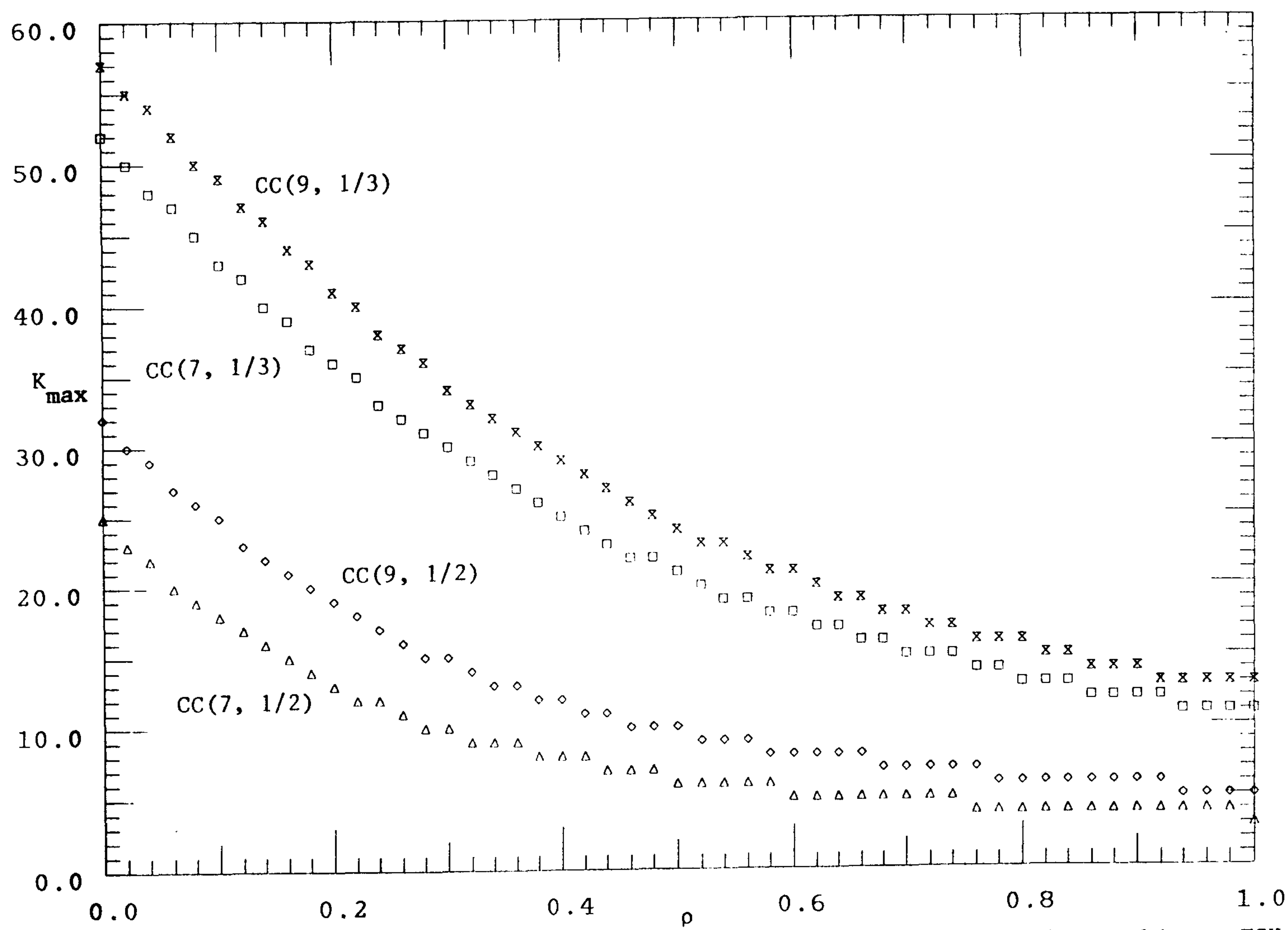


Figure 18. K_{max} versus ρ for asynchronous FH/SSMA communications employing binary FSK with various convolutional coding schemes with side information (q=100, N_b=12, E_b/N_0=20 dB, E_b/N_J=20 dB, $P_e=10^{-5}$); AWGN channel.

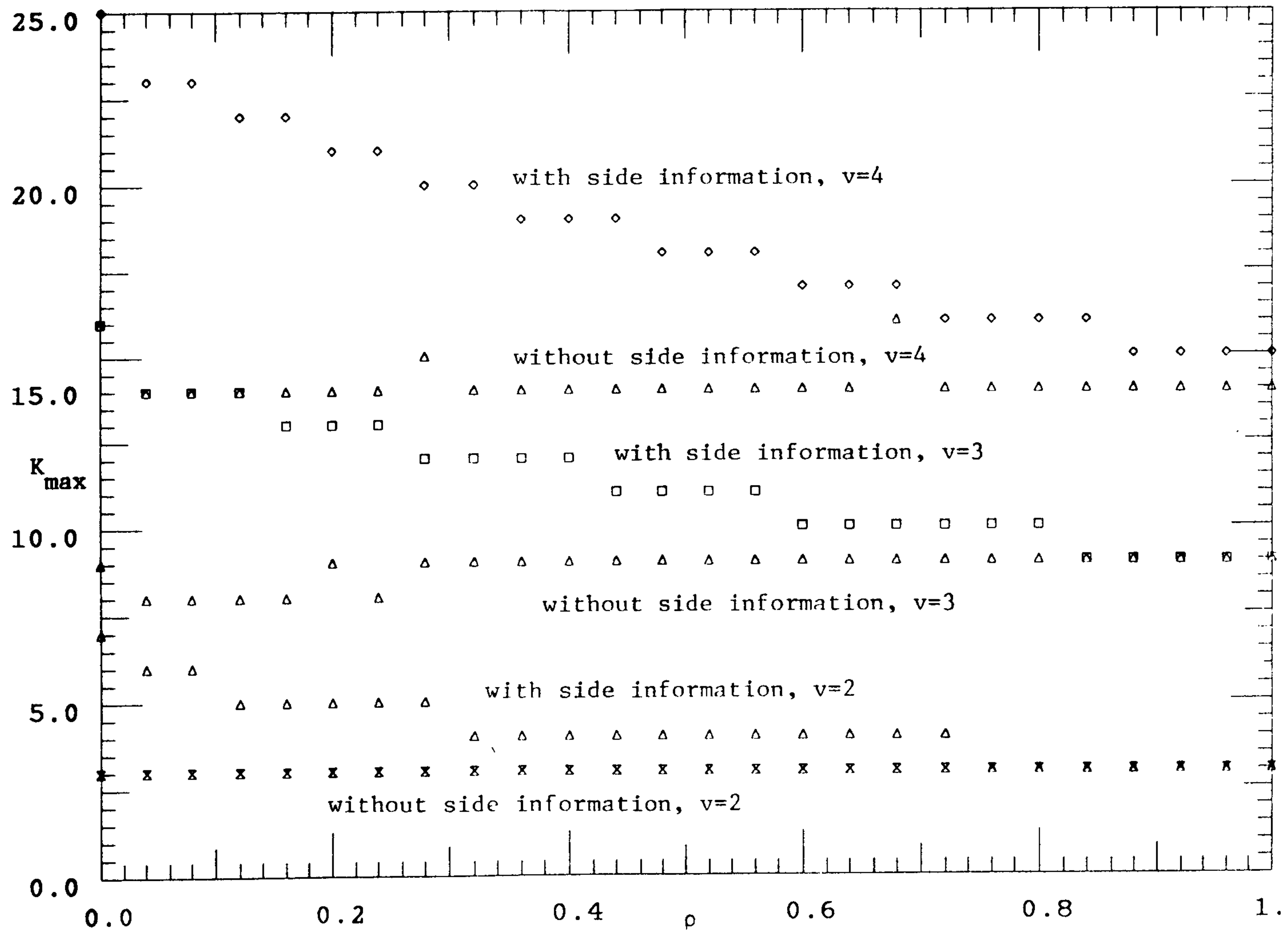
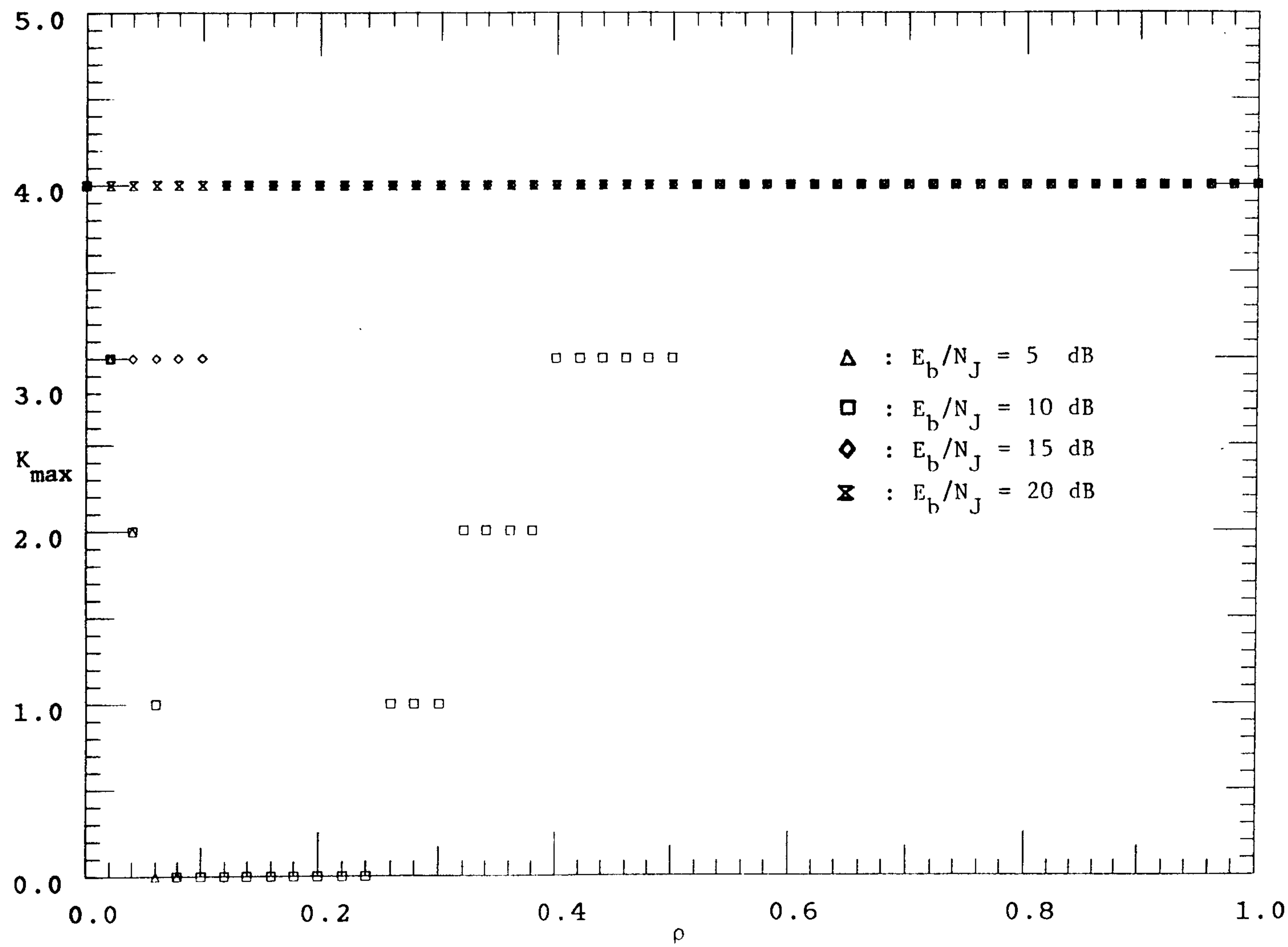


Figure 19. K_{max} versus ρ for asynchronous FH/SSMA communications employing 32-ary FSK with various dual-5 coding schemes ($q=100$, $N_b=10$, $E_b/N_0=20$, $E_b/N_J=20$ dB, $P_e=10^{-3}$); AWGN channel.



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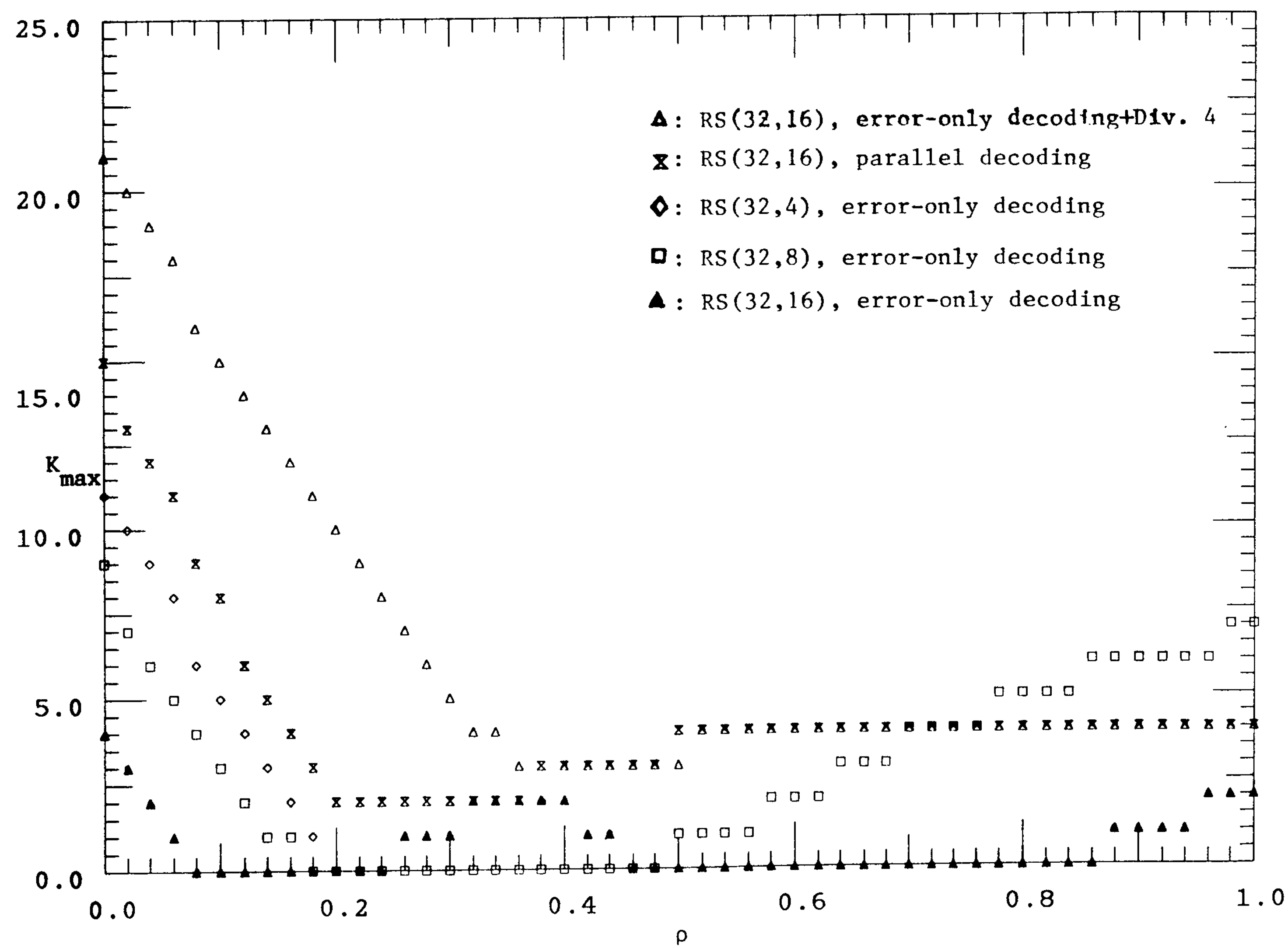

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Table Captions

Table 1. Maximum value of signal-to-jammer power ratio $E_b/N_J)_{\max}$ (dB) required and ρ^* for an FH/SS system employing 32-ary FSK with noncoherent demodulation ($K = 5$ asynchronous users, $q = 100$, $N_b = 10$, and AWGN with $E_b/N_0 = 20\ dB$).

Table 2. Maximum number of asynchronous users that can be supported by an FH/SS system employing 32-ary FSK with noncoherent demodulation ($q = 100$, $N_b = 10$, $\rho = 0$, and AWGN with $E_b/N_0 = \infty$).

Table 3. Maximum number of asynchronous users that can be supported by an FH/SS system employing 32-ary FSK with noncoherent demodulation ($q = 100$, $N_b = 10$, $\rho = .5$, $E_b/N_J = 10\ dB$, and AWGN with $E_b/N_0 = \infty$).

Table 1

Maximum value of signal-to-jammer power ratio $(E_b/N_J)_{\max}$ (dB) required and ρ^* for an FH/SS system employing 32-ary FSK with noncoherent demodulation ($K = 5$ asynchronous users, $q = 100$, $N_b = 10$, and AWGN with $E_b/N_0 = 20\ dB$)

Code	No Side Information		With Side Information	
	$P_e = 10^{-5}$	$P_e = 10^{-3}$	$P_e = 10^{-5}$	$P_e = 10^{-3}$
Repetition code L=3	--- (0.00)	--- (0.00)	--- (0.00)	--- (0.05)
Repetition code L=4	--- (0.00)	26.58 (0.01)	--- (0.00)	12.93 (0.05)
Repetition code L=5	--- (0.00)	15.28 (0.06)	--- (0.05)	11.56 (0.21)
Repetition code L=7	21.32 (0.02)	12.30 (0.16)	14.72 (0.15)	10.80 (0.34)
Repetition code L=9	16.44 (0.08)	11.65 (0.24)	13.65 (0.24)	10.86 (0.44)
RS(32,8)	13.71 (0.07)	10.34 (0.14)	10.47 (0.37)	8.91 (0.48)
RS(32,24)+diversity 3	17.36 (0.03)	12.09 (0.09)	10.87 (0.34)	9.17 (0.44)
RS(32,16)+diversity 3	12.55 (0.13)	10.52 (0.20)	10.33 (0.56)	9.30 (0.63)
RS(32,4)	14.72 (0.11)	12.15 (0.19)	12.22 (0.52)	10.97 (0.64)
RS(32,16)+diversity 4	11.09 (0.24)	9.82 (0.32)	10.27 (0.65)	9.47 (0.71)
CC(7,1/2) M=2	--- (0.00)	--- (0.00)	--- (0.20)	12.33 (0.32)
CC(7,1/3) M=2	20.51 (0.03)	13.80 (0.10)	12.89 (0.39)	11.20 (0.52)
CC(9,1/2) M=2	--- (0.00)	--- (0.00)	14.45 (0.26)	11.55 (0.36)
CC(9,1/3) M=2	17.44 (0.05)	13.07 (0.12)	12.33 (0.45)	10.90 (0.56)
CC(7,1/2) M=4	--- (0.00)	22.88 (0.01)	12.64 (0.11)	9.71 (0.21)
CC(7,1/3) M=8	21.49 (0.01)	11.67 (0.09)	11.15 (0.14)	8.57 (0.26)

Table 2

Maximum number of asynchronous users that can be supported by an FH/SS system employing 32-ary FSK with noncoherent demodulation ($q = 100$, $N_b = 10$, $\rho = 0$, and AWGN with $E_b/N_0 = \infty$)

Code	No Side Information		With Side Information	
	$P_e = 10^{-5}$	$P_e = 10^{-3}$	$P_e = 10^{-5}$	$P_e = 10^{-3}$
Repetition code L=3	0	2	2	8
Repetition code L=4	0	5	4	14
Repetition code L=5	2	8	8	20
Repetition code L=7	6	15	15	32
Repetition code L=9	10	21	22	42
RS(32,8)	9	14	34 34	47 47
RS(32,24)+diversity 3	6	11	32 32	42 42
RS(32,16)+diversity 3	13	18	57 57	70 70
RS(32,4)	11	17	52 52	71 71
RS(32,16)+diversity 4	21	28	72 72	86 86
CC(7,1/2) M=2	0	0	25	38
CC(7,1/3) M=2	5	9	49	70
CC(9,1/2) M=2	0	0	31	44
CC(9,1/3) M=2	6	10	57	78
CC(7,1/2) M=4	2	5	17	29
CC(7,1/3) M=8	5	11	25	45
Dual-5 Rate=1/2	0	3	3	6
Dual-5 Rate=1/3	4	8	8	15
Dual-5 Rate=1/4	8	15	14	24
Dual-5 Rate=1/5	13	21	20	32
Dual-5 Rate=1/6	17	26	26	40

Dual-5 Rate=1/7	21	31	32	46
Dual-5 Rate=1/8	25	36	37	53
Dual-5 Rate=1/9	29	40	42	59
Dual-5 Rate=1/10	**32**	**44**	**47**	**64**

Table 3

Maximum number of asynchronous users that can be supported by an FH/SS system employing 32-ary FSK with noncoherent demodulation ($q = 100$, $N_b = 10$, $\rho = .5$, $E_b/N_J = 10\ dB$, and AWGN with $E_b/N_0 = \infty$)

Code	No Side Information		With Side Information	
	$P_e = 10^{-5}$	$P_e = 10^{-3}$	$P_e = 10^{-5}$	$P_e = 10^{-3}$
Repetition code L=3	0	0	0	0
Repetition code L=4	0	0	0	0
Repetition code L=5	0	0	0	0
Repetition code L=7	0	0	0	3
Repetition code L=9	0	0	0	8
RS(32,8)	1	6	0 4	1 11
RS(32,24)+diversity 3	0	3	0 3	0 9
RS(32,16)+diversity 3	0	4	12 14	24 26
RS(32,4)	0	0	6 8	25 26
RS(32,16)+diversity 4	1	8	26 28	40 41
CC(7,1/2) M=2	0	0	0	1
CC(7,1/3) M=2	0	0	4	21
CC(9,1/2) M=2	0	0	0	5
CC(9,1/3) M=2	0	0	9	27
CC(7,1/2) M=4	0	1	0	7
CC(7,1/3) M=8	0	4	0	18
Dual-5 Rate=1/2	0	2	0	3
Dual-5 Rate=1/3	1	5	2	7
Dual-5 Rate=1/4	1	7	3	11
Dual-5 Rate=1/5	1	9	4	14
Dual-5 Rate=1/6	2	11	6	18

Dual-5 Rate=1/7	3	13	8	21
Dual-5 Rate=1/8	4	15	11	25
Dual-5 Rate=1/9	6	17	13	28
Dual-5 Rate=1/10	**7**	**19**	**16**	**32**