Correction to "An Efficient Game Form for Multi-Rate Multicast Service Provisioning"

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THIS note corrects an error in our paper, "An Efficient Game Form for Multi-Rate Multicast Service Provisioning" (reference **[1]**), which presents a rate allocation mechanism for multi-rate multicast service provisioning in networks with arbitrary topology and strategic users. The mechanism presented in **[1]** includes a tax function which is not differentiable with respect to the rate allocations. To obtain a Nash implementation, however, we need a tax function that is differentiable with respect to these allocations. We correct this error as follows.

We consider the problem formulated in **[1]**. We use the same notation as in **[1]**. To help the reader, we repeat here the notation used in this note.

$$\begin{aligned}
G_i &:= \text{Multicast group } i.\\
\mathcal{N} &:= \{G_1, G_2, \cdots, G_N\}.\\
(j, G_i) &:= \text{User } j \text{ in multicast group } i.\\
\mathcal{R}_{(j,G_i)} &:= \text{Route of user } (j, G_i).\\
\Xi^l_{(j,G_i)} &:= \text{The term that appears in the tax function } t^l_{(j,G_i)}\\
&\qquad \text{and is expressed in terms of indicator functions.}\\
G_i^{\max}(l) &:= \text{Set of users in } G_i \text{ that request the maximum}\\
&\qquad \text{amount of bandwidth at link } l \text{ among all}\\
&\qquad \text{users in } G_i.\\
Q_l &:= \text{Set of multicast groups that use link } l.\\
x_{G_i}(l) &:= \text{Bandwidth requested by } G_i \text{ in link } l.
\end{aligned}$$

Specification of the game form/mechanism:

Message space: The message space is the same as that of the mechanism presented in **[1]**. A message of user $(j, G_i), j \in G_i, G_i \in \mathcal{N}$ is of the form

$$m_{(j,G_i)} = (x_{(j,G_i)}, \pi^{l_{j_1}}_{(j,G_i)}, \pi^{l_{j_2}}_{(j,G_i)}, \cdots, \pi^{l_{i_{|\mathcal{R}_{(j,G_i)}|}}}_{(j,G_i)}),$$

where $x_{(j,G_i)}$ denotes the (non-negative) bandwidth user (j, G_i) requests at all the links of his route $\mathcal{R}_{(j,G_i)}$, and $\pi^{l_{i_k}}_{(j,G_i)} \geq 0$ denotes the price user (j, G_i) is willing to pay at link l_{j_k} of his route $\mathcal{R}_{(j,G_i)}$.

Outcome function: For any $m \in \mathcal{M}$, the outcome function is defined as follows:

$$\begin{aligned}
f(m) &= ((x_{(i,G_1)}, t_{(i,G_1)}), \cdots, (x_{(k,G_N)}, t_{(k,G_N)}))\\
t_{(j,G_i)} &= \sum_{l \in \mathcal{R}_{(j,G_i)}} t^l_{(j,G_i)},
\end{aligned}$$

where $t^l_{(j,G_i)}$ is tax paid by user (j, G_i) for using link l. The form of $t^l_{(j,G_i)}$ is the same as the tax function defined in **[1]** excluding the terms denoted by $\Xi^l_{(j,G_i)}$. For example, in **Part DI** where $|G_i{}^{\max}(l)| \geq 2$, the tax function in **Eq. (14) of [1]** is modified as follows:
Let the label of (j, G_i) in $G_i^{\max}(l)$ be $(k, G_i^{\max}(l))$. Then:

(i). If $(j, G_i) \in G_i^{\max}(l)$,

$$\begin{aligned}
t^l_{(j,G_i)} &= \pi_{(k+1,G_i{}^{\max}(l))} x_{(j,G_i)}\\
&+ \frac{(P_{G_i{}^{\max}(l)} - P_{-G_i{}^{\max}(l)} - \eta^l_+)^2}{|G_i{}^{\max}(l)|}\\
&- 2\frac{P_{-G_i{}^{\max}(l)}}{|G_i{}^{\max}(l)|}(P_{G_i{}^{\max}(l)}\\
&- P_{-G_i{}^{\max}(l)})\left[\frac{\mathcal{E}_{-G_i{}^{\max}(l)} + x_{(j,G_i)}}{\gamma}\right]\\
&+ \frac{\Gamma^l_{G_i}}{|G_i^{\max}(l)|},
\end{aligned} \tag{1}$$

where

$$\eta^l_+ = \max\{0, \frac{\sum_{G_i \in Q_l} x_{G_i}(l) - c_l}{\hat{\gamma}}\},$$

c_l is the capacity of link l, $\mathcal{E}_{-G_i{}^{\max}(l)}, P_{G_i{}^{\max}(l)}, P_{-G_i{}^{\max}(l)}$ and $\Gamma^l_{G_i}$ are defined by equations **(18)-(21) in [1]**, and $\gamma, \hat{\gamma}$ are positive constants.

(ii). If $(j, G_i) \notin G_i^{\max}(l)$ then

$$t^l_{(j,G_i)} = \pi_{(k+1,G_i{}^{\max}(l))} \left(\mathcal{E}_{-G_i{}^{\max}(l)} + x_{G_i}(l)\right). \tag{2}$$

This completes the specification of the mechanism.

Based on the above specification, the proof of Lemma **3 in [1]** is updated as follows.

Proof of Lemma 3 in [1]: Let m^* be a NE of the game induced by the mechanism. Consider $G_i \in Q_l$, and $(k, G_i^{\max}(l)) \in G_i^{\max}(l)$. Since user $(k, G_i^{\max}(l))$ does not control $\Gamma^l_{G_i}$ (as in **[1]**, page **2101**), $\frac{\partial \Gamma_{(k,G_i^{\max}(l))}}{\partial \pi_{(k,G_i^{\max}(l))}} = 0$. By following the same steps as in equations **(32-40) of [1]**, we

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obtain:

$$\left.\frac{\partial t^l_{(k,G_i^{\max}(l))}}{\partial \pi_{(k,G_i{}^{\max}(l))}}\right|_{\boldsymbol{m}=\boldsymbol{m}^*} = \frac{-2}{|G_i{}^{\max}(l)|} P^*_{-G_i{}^{\max}(l)} \left(\frac{\mathcal{E}^*_{-G_i{}^{\max}(l)} + x^*_{G_i}(l)}{\gamma}\right) + \frac{2}{|G_i{}^{\max}(l)|}\left(P^*_{G_i{}^{\max}(l)} - P^*_{-G_i{}^{\max}(l)} - \eta^{*l}_{+}\right) = 0. \tag{3}$$

Furthermore (as in Eq. **(36) in [1]**) we note that $\sum_{G_i \in Q_l} P_{G_i{}^{\max}(l)} = \sum_{G_i \in Q_l} P_{-G_i{}^{\max}(l)}$. Thus, summing (3) over all $G_i \in Q_l$, and $(k, G_i^{\max}(l)) \in G_i{}^{\max}(l)$, we get

$$\sum_{G_i \in Q_l} \sum_{(k,G_i^{\max}(l)) \in G_i{}^{\max}(l)} \frac{\partial t^l_{(k,G_i^{\max}(l))}}{\partial \pi_{(k,G_i{}^{\max}(l))}} = -|Q_l|\eta^{*l}_{+} - \sum_{G_i \in Q_l} P^*_{-G_i{}^{\max}(l)} \left(\frac{\mathcal{E}^*_{-G_i{}^{\max}(l)} + x^*_{G_i}(l)}{\gamma}\right) = 0. \tag{4}$$

Suppose $\sum_{G_i \in Q_l} x^*_{G_i}(l) - c_l > 0$. Then we must have $\eta^*_{+} > 0$ and $\sum_{G_i \in Q_l} P^*_{-G_i{}^{\max}(l)} \left(\frac{\mathcal{E}^*_{-G_i{}^{\max}(l)} + x^*_{G_i}(l)}{\gamma}\right) \geq 0$. But this contradicts (4). Therefore, we must have

$$\sum_{G_i \in Q_l} x^*_{G_i}(l) \leq c_l. \tag{5}$$

Inequality (5) implies,

$$\eta^{*l}_{+} = 0. \tag{6}$$

Combining (6) along with (4) we obtain

$$\sum_{G_i \in Q_l} P^*_{-G_i{}^{\max}(l)} \left(\frac{\mathcal{E}^*_{-G_i{}^{\max}(l)} + x^*_{G_i}(l)}{\gamma}\right) = 0. \tag{7}$$

Moreover, combining (6) and (7) we obtain

$$P^*_{-G_i{}^{\max}(l)} \left(\frac{\mathcal{E}^*_{-G_i{}^{\max}(l)} + x^*_{G_i}(l)}{\gamma}\right) = 0. \tag{8}$$

for any $G_i \in Q_l$. Using (8) and (6) in (3) we obtain

$$P^*_{-G_i{}^{\max}(l)} = P^*_{G_i{}^{\max}(l)}. \tag{9}$$

Since (9) is true for all $G_i \in Q_l$, it implies

$$P^*_{-G_i{}^{\max}(l)} = P^*_{G_i{}^{\max}(l)} = P^*_{G_j{}^{\max}(l)} =: P^*_{G^{\max}(l)}, \ \forall\, G_i \in Q_l \tag{10}$$

and along with (8) it implies

$$P^*_{G^{\max}(l)} \left(\mathcal{E}^*_{-G_i{}^{\max}(l)} + x^*_{G_i}(l)\right) = 0. \tag{11}$$

Furthermore, because of

$$\frac{\partial \Gamma_{(k,G_i^{\max}(l))}}{\partial x_{G_i}(l)} = 0 \tag{12}$$

(as in **[1]**, page **2101**), and (6), (8), (10), Eqs. (1) and (2) give

$$\left.\frac{\partial t^l_{(k,G_i^{\max}(l))}}{\partial x_{G_i}(l)}\right|_{\boldsymbol{m}=\boldsymbol{m}^*} = \pi^*_{(k+1,G_i{}^{\max}(l))}. \tag{13}$$

■

I. Properties of the mechanism

Existence of Nash equilibria (NE): The proof of existence of NE of the game induced by the mechanism is the same as in Theorem **1** of **[1]**.

Feasibility of allocations at NE: Because of the specification of the mechanism and Eq. (6), the allocations corresponding to all NE are in the feasible set.

Budget Balance at any NE: Budget balance at any NE follows by Lemma **4** of **[1]**.

Individual Rationality: Individual rationality follows by Theorem **5** of **[1]**.

Nash implementation: Nash implementation follows by Theorem **6** of **[1]**.

References

[1] A. Kakhbod and D. Teneketzis, "An efficent game form for multi-rate multicast service provisioning", *IEEE J. on Selected Areas in Communications: Special Issue on the Economics of Communication Networks and Systems*, vol. 30, no. 11, pp. 2093-2104, December 2012.