# Secret Key Generation from Sparse Wireless Channels: Ergodic Capacity and Secrecy Outage

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#### Abstract

This paper investigates generation of a secret key from a reciprocal wireless channel. In particular we consider wireless channels that exhibit sparse structure in the wideband regime and the impact of the sparsity on the secret key capacity. We explore this problem in two steps. First, we study key generation from a *state-dependent discrete memoryless multiple source*. The state of source captures the effect of channel sparsity. Secondly, we consider a wireless channel model that captures channel sparsity and correlation between the legitimate users' channel and the eavesdropper's channel. Such dependency can significantly reduce the secret key capacity.

According to system delay requirements, two performance measures are considered: (i) ergodic secret key capacity and (ii) outage probability. We show that in the wideband regime when a white sounding sequence is adopted, a sparser channel can achieve a higher ergodic secret key rate than a richer channel can. For outage performance, we show that if the users generate secret keys at a fraction of the ergodic capacity, the outage probability will decay exponentially in signal bandwidth. Moreover, a larger exponent is achieved by a richer channel.

#### **Index Terms**

Secret key generation, public discussion, reciprocal wireless channel, channel sounding, ergodic capacity, secrecy outage.

## I. INTRODUCTION

The fundamental limit of secret key generation from discrete memoryless multiple source (DMMS) is developed by Ahlswede, Csiszár [1] and Maurer [2]. Their results show that if X, Y, Z (respectively observed by Alice, Bob and Eve) are correlated with a known distribution, it is possible to generate a secret key between Alice and Bob at a positive rate through use of a public discussion. The resulting information rate leaked to Eve can be made arbitrarily small. The supremum of achievable secret key rates is called the *secret key capacity*.

Since their work, there have been many extensions to explore the secret key capacity of more complicated models. In [3, 4], users observe DMMS and also transmit information via wiretap channel [5], but there is no access to public channel for discussion. The authors in [6–9] consider a wiretap channel influenced by a random channel state, known by one (or both) of the legitimate users. In such models, the random channel state can be viewed as a kind of correlated source shared by transmitter/receiver which also influences the transmission.

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In [10, 11], key generation from DMMS is considered where the DMMS is excited by a deterministic source [10] or by a random source [11]. This sender-excited model is motivated by an application in which key generation is based on the inherent randomness of reciprocal wireless channel. Consider a situation where Alice and Bob transmit a sounding signal to each other over a reciprocal wireless channel. Due to the channel reciprocity, Alice and Bob observe a pair of correlated sources. The source turns out to be a good source for secret key generation because it can be the case that (i) the source is correlated, (ii) the source is ubiquitous since it is from wireless channel, and (iii) it is hard to eavesdrop because the wireless channel varies quickly in the spatial and temporal domains. This issue has received much attention in terms of theoretical and practical research [12–19]. However, most of this work is subject to the assumption that the eavesdropper channel is statistically independent of the main channel (the channel between Alice and Bob). This is true when the environment has rich scattering such that the correlation between channel coefficients decreases rapidly in the spatial domain.

However, there is growing experimental evidence (e.g., [20–23]) and physical arguments (e.g., [24–26]) which show that realistic wireless channels are sparse at large bandwidths. The effect of channel sparsity on secret key capacity is twofold: (i) it reduces the degrees-of-freedom (DoF) of the main (Alice-Bob) channel and (ii) it induces spatial correlation [27], thereby increasing Eve's ability to observe the main channel.

We revisit the key generation problem when the channel exhibits sparsity in the wideband regime. This channel characteristic can be captured by a *sparsity pattern* that defines the non-zero support of the channel coefficients. Depending on the environment, the sparsity pattern could experience fast or slow time variations. The channel model also captures the correlation between the main channel and Eve's observations. To study secret key generation in this context we capture these characteristics by defining a *state-dependent* discrete multiple memoryless source (SD-DMMS). We specialize this model to the statistical characterization of sparse wireless channels were the sparsity pattern plays a role of the channel state and, as we discuss next, develop ergodic capacity and secrecy outage results.

In analogy to communication over a fading channel, two regimes are studied according to the system delay constraint:

- *Ergodic regime (the delay tolerant regime)*: If the key is generated based on a large number of observations across multiple states, the secret key capacity is well-defined in the Shannon sense. We call the capacity in this case the *ergodic* secret key capacity. The main problem is that the system suffers from an excessive delay.
- *Non-ergodic regime (the delay stringent regime)*: If the observed source sequence is not long enough or the state changes slowly so that the key generation is forced to occur within a period of constant state, the capacity is not defined in general. In this case, we consider the *secrecy outage probability* which measures the probability that the instantaneous state condition cannot support the key rate to fulfill the secrecy condition (this will be defined later).

Secrecy outage is also considered in other research regarding state-dependent (fading) wiretap channel (e.g., [28, 29]). We show that when a white sounding sequence is adopted in the wideband (low power) regime, a sparser channel can achieve a higher secret key rate than a richer channel can. This is analogous to capacity behavior in sparse multi-antenna channels in [30]. Furthermore, at each signal-to-noise ratio (SNR), there is an adequate bandwidth that maximizes the secret key rate. For the outage performance, we show that the system can achieve an exponential decaying outage probability by using an  $\alpha$ -backoff scheme ( $0 < \alpha \leq 1$ ) in which secret key rate is a fraction  $\alpha$  of the ergodic capacity. Unlike the ergodic case, now a richer channel always has a larger exponent characterizing the decay of the outage probability. In a similar vein as communication over a fading channel, this demonstrates that a large number of DoF helps to smooth out the effect of the unknown state.

The paper is organized as following. In Section II we give some definitions and describe the system model. This includes the correlated sparse wireless channel model, the definition of the SD-DMMS, and the one-way discussion key generation protocol. In Section III, we investigate the ergodic secret key capacity of SD-DMMS and apply this to key generation from a sparse wireless channel. Outage is defined in Section IV. We give a necessary and sufficient condition for an outage event and explore the outage probability when an  $\alpha$ -backoff scheme is used. Detailed proofs are deferred for the Appendix.

#### **II.** DEFINITIONS AND SYSTEM MODEL

In this paper we are motivated by key generation based on wireless channel that exhibits sparsity in the delay domain. We first develop our model of a sparse wireless channel in Section II-A. While in earlier works on modeling sparse wireless channels, e.g., see [20, 31–35], there is only a single channel to model, in Section II-A we need to model the main (Alice-to-Bob) channel as well as Eve's correlated observations of that main channel. Following our wireless motivations, in Section II-B we develop an abstracted *state-dependent discrete multiple source* (SD-DMMS) model. In this model the "state" captures the effect of the slowly varying sparsity pattern while the key itself is extracted from the conditionally-generated (conditioned on the sparsity pattern) channel fades. Finally, in Section II-C the *one-way public discussion* key generation protocol is formally presented.

### A. Sparse reciprocal wireless channel

Consider a wireless communication system with bandwidth W. Say that the channel exhibits sparsity in the delay domain<sup>1</sup> where  $\tau_{\text{max}}$  is the maximum delay spread of the channel. Then  $L_{\text{max}} = \lceil \tau_{\text{max}} W \rceil$  is the maximum number of resolvable paths. A sounding sequence  $\mathbf{d} = \lfloor d_1, d_2, \cdots, d_{N_d} \rfloor^T$  is transmitted over time period T, where  $N_d = \lceil TW \rceil$ . The sounding sequence is a known sequence with power  $\mathbf{d}^H \mathbf{d} = P$ . We assume each two-way (Alice  $\leftrightarrows$  Bob) sounding is done within a channel coherence period (i.e.,  $T_{coh} \gg 2T$ ). Further multiple channel soundings (indexed by t) are performed within non-overlapping coherence periods meaning that each set of soundings are independent.

The channel outputs in sounding interval t are

$$\boldsymbol{X}[t] = \mathbf{D}\boldsymbol{H}_{ab}[t] + \boldsymbol{W}_{1}[t] \qquad \text{(Alice)} , \qquad (1a)$$

$$\boldsymbol{Y}[t] = \mathbf{D}\boldsymbol{H}_{ab}[t] + \boldsymbol{W}_{2}[t] \qquad (\text{Bob}) , \qquad (1b)$$

where  $H_{ab}[t] = (H_1[t], \dots, H_{L_{max}}[t])^T$  is the sampled (virtual) channel coefficient [24, 36]

<sup>&</sup>lt;sup>1</sup>In this paper, we consider channel sparsity in the delay domain. It is not difficult to extend the result to the sparsity in either the Doppler or spatial domains, e.g., [24–26, 35].

vector, and **D** is an N-by- $L_{\text{max}}$  Toeplitz matrix with  $N = N_d + L_{\text{max}} - 1$ :

$$\mathbf{D} = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ d_2 & d_1 & \cdots & 0 \\ \vdots & d_2 & \cdots & d_1 \\ d_{N_d} & \vdots & & d_2 \\ \vdots & d_{N_d} & \ddots & \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & d_{N_d} \end{bmatrix} = \begin{bmatrix} \mathbf{d}_1, \mathbf{d}_2, \cdots, \mathbf{d}_{N_d} \end{bmatrix}.$$

A widely used sounding signal is a sequence whose spectrum is asymptotically white in  $N_d$ . In this case **D** is a full column-rank matrix such that<sup>2</sup>

$$\mathbf{D}^{H}\mathbf{D} \doteq P\mathbf{I}_{L_{\max}} \tag{2}$$

when  $N_d$  is sufficiently large. One such example is  $\mathbf{d} = \sqrt{P}\mathbf{e} = \sqrt{P}(1, 0, \dots, 0)^T$ . Another such example is pseudo-random (PN) sequence in spread spectrum system [37]. The noise terms  $W_1[t]$  and  $W_2[t]$  in (1) are independent  $\mathcal{CN}(\mathbf{0}, \sigma_a^2 \mathbf{I}_N)$  and  $\mathcal{CN}(\mathbf{0}, \sigma_b^2 \mathbf{I}_N)$  vectors, respectively.

1) Sparse channel model: Most channels that have a small number of physical paths will exhibit sparsity in the delay domain as the signal bandwidth W increases. In particular, in some delay bin  $\ell$ , the corresponding channel coefficient  $H_{\ell}[t]$  will be zero. In this paper, we adopt the *sub-linear* law model considered in previous work [34, 35] to capture the sparse channel characteristic. In this model, the channel is called  $\delta$ -sparse if the average number of non-zero channel coefficients scales as

$$L = (\tau_{\max}W)^{\delta} = L^{\delta}_{\max}, \qquad \delta \in (0,1) .$$
(3)

The parameter L is also the mean number of channel DoF.

The *channel sparsity pattern* of the main channel in sounding interval t is

$$\boldsymbol{S}_{ab}[t] = \left(S_{ab,1}[t], \cdots, S_{ab,L_{\max}}[t]\right) \in \mathcal{S}^{L_{\max}},$$

where  $S = \{0, 1\}$  and  $E\left[\sum_{\ell=1}^{L_{\text{max}}} S_{ab,\ell}[t]\right] = L$ . This pattern defines the support of the channel vector

$$\boldsymbol{H}_{ab}[t] = \Big(H_1[t], \ H_2[t] \ \dots \ H_{L_{\max}}[t]\Big),$$

i.e.,  $H_{\ell}[t] = 0$  if and only if  $S_{ab,\ell}[t] = 0$ . The channel coefficients  $H_{\ell}[t]$  are independent  $\mathcal{CN}(0,\nu_{\ell}^2)$  variable where the variance  $\nu_{\ell}^2 = 0$  if  $S_{ab,\ell}[t] = 0$ . The channel has *unit* power, i.e.,  $\sum_{\ell} \nu_{\ell}^2 = 1$ . Later, we use "channel degrees-of-freedom" (DoF) to refer to the *weight* of the realization of the vector  $S_{ab}[t]$ . We also call  $S_{ab}[t]$  the *state* of  $H_{ab}[t]$ . A *rich* multipath channel corresponds to  $\delta \to 1$ .

The sparsity pattern  $S_{ab}[t]$  will, in general, be time-varying. However, in most case of interest,  $S_{ab}[t]$  will change much more slowly than the channel coefficients  $H_{ab}[t]$ . This is because the main reflectors, by which paths are resolved by different delay bins, move more slowly than the phase changes that influences the fading coefficients [36, 38, 39]. Because of this, most of the secret key rate will be generated by the randomness inherent to the channel coefficients rather

<sup>&</sup>lt;sup>2</sup>Here and in the following, we say  $g(x^n) \doteq g$  if  $g(x^n) \rightarrow g$  when n is sufficiently large.

than the sparsity pattern itself. Furthermore, there exists good techniques to estimate the sparsity pattern reliably based on few observations, e.g., [40]. Thus, we consider  $S_{ab}[t]$  known to Alice and Bob. Let n be the number of channel sounding periods during which the sparsity pattern remains constant. We term this the *sparsity coherence period*. Thus, the m-th sparsity coherent period extends from t = (m - 1)n + 1 to t = mn. In this interval  $S_{ab}[t]$  remains constant, i.e.,  $S_{ab}[t] = S_{ab}[mn]$  for all t,  $(m-1)n+1 \le t \le mn$ . We further assume that  $S_{ab}[t]$  is independent across periods.

Modeling the distribution of the state itself is a difficult task, so we consider a simple model

$$\Pr(S_{ab,\ell} = 1) = \frac{L}{L_{\max}} = (\tau_{\max}W)^{-(1-\delta)} \triangleq \rho$$
(4)

for all  $\ell$ . In other words, the  $S_{ab,\ell}$  is Bernoulli distribution with parameter  $\rho$  (denoted Bern ( $\rho$ )). 2) *Eavesdropper's correlation model:* Eve's channel output is similar to (1)<sup>3</sup>:

$$\boldsymbol{Z}[t] = \mathbf{D}\boldsymbol{H}_e[t] + \boldsymbol{W}_3[t] \qquad \text{(Eve)} , \qquad (5)$$

where the noise is  $\mathcal{CN}(\mathbf{0}, \sigma_e^2 \mathbf{I}_N)$ . The channel coefficient vector  $\mathbf{H}_e[t] = (H_{e,1}[t], \cdots, H_{e,L_{\max}}[t])^T$  is also  $\delta$ -sparse with state denoted by  $\mathbf{S}_e[t]$ , and each element  $\mathcal{CN}(0, v_\ell^2)$  distributed. We model the correlation between  $\mathbf{H}_e[t]$  and  $\mathbf{H}_{ab}[t]$  in a two-step process as follows:

• Correlation between  $S_e$  and  $S_{ab}$ : For each delay bin  $\ell$  for which  $S_{ab,\ell} = 1$ , the probability that Eve also has non-zero channel gain is  $\theta$ . i.e.,

$$\Pr(S_{e,\ell} = 1 | S_{ab,\ell} = 1) = \theta \tag{6}$$

for all  $1 \leq \ell \leq L_{\max}$ .

• Correlation between individual channel coefficient: For those channel coefficients in the "common support" delay bins, i.e., in the set  $\{\ell : S_{ab,\ell} = S_{e,\ell} = 1\}$ , the correlation coefficients are

$$\eta(H_{\ell}, H_{e,\ell}) \triangleq \frac{E[H_{\ell}H_{e,\ell}^*]}{\sqrt{E[|H_{\ell}|^2]E[|H_{e,\ell}|^2]}} = \eta \; .$$

The parameter  $\theta$  captures the fraction of DoF that the main channel and Eve's channel have in common. One can think of the relationship between the state (sparsity pattern) of the main channel and that of the eavesdropper's observation as a binary memoryless channel. However, because Eve's marginal channel has the same  $\delta$ -sparsity as the main channel (since the users are in the same environment), the channel is not symmetric. In other words, transition probability  $\Pr(S_{e,\ell} = 1|S_{ab,\ell} = 0) \neq \Pr(S_{e,\ell} = 0|S_{ab,\ell} = 1)$ . This is illustrated in Figure 1. Finally, the parameter  $\eta$  captures the effect that the paths (of both channel) located in the common delay bin shares the same physical scattering.

*Remark* 1: The parameter space  $\{(\theta, \eta), \delta\}$  of our model captures many scenarios of interest. From a physical aspect, there are two factors effecting Eve's channel correlation: the distance between Eve and Bob (which mainly impacts leakage to Eve), and richness/sparseness of the multipath (which impacts both leakage to Eve and the common randomness between Alice and Bob). When Eve gets close to Bob, generally, both  $\theta$  and  $\eta$  will increase (and vice versa); this will generally increase the leakage. The parameter  $\delta$  (and thus  $\rho$ ) controls the maximum

<sup>&</sup>lt;sup>3</sup>In order to get meaningful observations, we assume Eve is located close to one of the users. So only one of the two Eve's channel outputs during the two-way sounding correlates with the main channel. The other output is independent of the main channel due to fast spatial decorrelation.



Fig. 1. Transition probability  $Pr(S_{e,\ell}|S_{ab,\ell})$ .

number of DoF. When multipath is rich,  $\rho$  is high and  $\eta$  is closer to zero (i.e., high overlap but independent), resulting in the highest capacity and lowest leakage. For sparse multipath,  $\rho$  is lower (lower common randomness) and  $\eta$  could be large even at larger distances between Eve and Bob. In this case leakage will likely increase more slowly as Eve gets closer to Bob.

To generate a secret key, users repeat the channel sounding (1) (and (5)) nM times and generate a key based on a pair of super-block  $\{(\boldsymbol{X}[t], \boldsymbol{S}_{ab}[t]), (\boldsymbol{Y}[t], \boldsymbol{S}_{ab}[t])\}_{t=1}^{nM}$ . In the following section we abstract away the actual sounding process and specify a state-dependent source model where the state varies more slowly than the underlying source-realization process from which the key is generated. When we study the ergodic case, we will let both n and M go to infinity, while when we study the outage case, M = 1, and n can be large.

## B. State-dependent discrete memoryless multiple source

To leverage results on information theoretic security, we consider a state-dependent (SD) DMMS model depicted in Figure 2. The observation triple  $(X^{nM}, Y^{nM}, Z^{nM}) \in \mathcal{X}^{nM} \times \mathcal{Y}^{nM} \times \mathcal{Z}^{nM}$  is generated according to  $p(x^{nM}, y^{nM}, z^{nM} | s^M_{ab}, s^M_e)$ , conditioning on the pair of length-M sequences:  $(s^M_{ab}, s^M_e) \in \mathcal{S}^M \times \mathcal{S}^M$ .

As discussed in Section II-A,  $S_{ab}^{M}$  is the state sequence of Alice and Bob's correlated source  $X^{nM}, Y^{nM}$  and  $S_e^{M}$  is the state sequence of Eve's observation  $Z^{nM}$ . The states have joint distribution  $p(s_{ab}^{M}, s_e^{M})$ . Since the states vary more slowly than the conditonally-generated sources, there is a length of time n during which the states remain constant. This corresponds to the sparsity coherence period discussed earlier. A large n means that the states are changing slowly. We assume that the states are available to the corresponding observers but not to other users. In other words, Alice and Bob both know  $S_{ab}$  but not  $S_e$  while Eve knows  $S_e$  but not  $S_{ab}$ . This is depicted in Figure 2. We call the state *memoryless* if

$$p(s_{ab}^{M}, s_{e}^{M}) = \prod_{m=1}^{M} p(s_{ab,m}, s_{e,m}) .$$
(7)

Similarly, the source is *memoryless* if

$$p(x^{nM}, y^{nM}, z^{nM} | s^M_{ab}, s^M_e) = \prod_{m=1}^M \prod_{i=(m-1)n+1}^{mn} p(x_i, y_i, z_i | s_{ab,m}, s_{e,m}) .$$
(8)



Fig. 2. State-dependent DMMS model

Note that in (8) one see the effect of the sparsity coherence period. The triplet of source samples  $(X_i, Y_i, Z_i)$  is conditionally and independently generated from the same state pair  $(S_{ab,m}, S_{e,m})$  for all i,  $(m-1)n < i \leq mn$ . Each of  $(X_i, Y_i, Z_i)$  stands for the vector of channel output in II-A.

In the one-way discussion protocol (which will be detailed next in II-C), Alice sends a message  $\Phi$  over a public channel. Bob recovers Alice's key based on his observation  $(Y^{nM}, S^M_{ab})$  and  $\Phi$ . Eve's source Z is a *degraded* version of Y if

$$p(x, y, z|s_{ab}, s_e) = p(x, y|s_{ab})p(z|y, s_{ab}, s_e) .$$
(9)

In other words, for given states  $(s_{ab}, s_e)$ , Eve's output is a cascade of the Bob's output and a channel represented by  $p(z|y, s_{ab}, s_e)$ .

## C. One-way discussion key generation protocol

Let  $\mathcal{K} = [1:2^{nR}]$  be the key space. There is an authenticated public channel available to users to exchange error-free public messages in the set  $\Phi = [1:2^{nR_{\phi}}]$ . The one-way public discussion secret key generation protocol consists of three functions:

$$f_1: \mathcal{X}^{nM} \times \mathcal{S}^M \to \mathcal{K} , \qquad (10a)$$

$$q: \mathcal{X}^{nM} \times \mathcal{S}^M \to \Phi , \qquad (10b)$$

$$f_2: \mathcal{Y}^{nM} \times \mathcal{S}^M \times \Phi \to \mathcal{K} , \qquad (10c)$$

which define Alice's key, public message, and Bob's key, respectively. Namely,

$$K = f_1(X^{nM}, S^M_{ab}) , (11a)$$

$$\phi = g(X^{nM}, S^M_{ab}) , \qquad (11b)$$

$$\hat{K} = f_2(Y^{nM}, S^M_{ab}, \phi)$$
 (11c)

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**Definition 1** (Achievability). A secret key rate R is (weakly) achievable if for any  $\epsilon > 0$ , there is a secret key generation system defined in (10) such that for sufficient large n and M,

$$\Pr(K \neq \tilde{K}) < \epsilon , \tag{12}$$

$$\frac{1}{nM}I(K;Z^{nM},S^M_e,\Phi) < \epsilon , \qquad (13)$$

$$\frac{1}{nM}H(K) > R - \epsilon . (14)$$

Condition (14) means the key is almost uniformly distributed over the set  $\mathcal{K}$ . System secrecy is measured in terms of the mutual information defined in (13) which says that the information about the key leaked to eavesdropper is negligible. The supremum of achievable secret key rates is called the *secret key capacity*.

# III. ERGODIC SECRET KEY CAPACITY

For applications that can tolerate longer delays, the key generation protocol can operate across a large number of independent state realizations. In this setting n and M can both be arbitrary large. The secret key capacity in the Shannon sense is well-defined and is termed the *ergodic* secret key capacity,  $C_{\rm er}$ .

## A. Ergodic Capacity of SD-DMMS

The theorems developed by Ahlswede, Csiszár [1, Theorem 1] and Maurer [2, Theorem 1,2] can be applied to the ergodic case of the source model in Figure 2 to get the following lemma.

## Lemma 1.

$$C_{\rm er}^- \le C_{\rm er} \le C_{\rm er}^+ , \qquad (15)$$

where

$$C_{\rm er}^{-} = I(X; Y|S_{ab}) - I(X; Z, S_e|S_{ab}) + \frac{1}{n}H(S_{ab}|S_e)$$
(16)

$$C_{\rm er}^{+} = I(X; Y|Z, S_{ab}, S_e) + \frac{1}{n} H(S_{ab}|S_e) .$$
(17)

*Proof:* The proof is given in Appendix A.

The important observation about (16) and (17) is that they both consist of two types of terms: mutual information terms and entropy terms. The latter quantifies the amount of uncertainty in the sparsity pattern of the main (Alice-to-Bob) channel given Eve's observation  $S_e$ . The former quantifies the conditional secret key capacity given the latter. The following lemma says that the upper and lower bound equal one another when the eavesdropper's observation is degraded.

**Corollary 2.** For the situation in which the eavesdropper's source is degraded per (9), the ergodic secret key capacity is

$$C_{\rm er} = I(X; Y|S_{ab}) - I(X; Z, S_e|S_{ab}) + \frac{1}{n}H(S_{ab}|S_e) .$$
<sup>(18)</sup>

*Proof:* It can be verified by examining (17) that

$$C_{\rm er}^{+} = I(X; Y|Z, S_{ab}, S_e) + \frac{1}{n} H(S_{ab}|S_e)$$
  
=  $I(X; Y, Z, S_e|S_{ab}) - I(X; Z, S_e|S_{ab}) + \frac{1}{n} H(S_{ab}|S_e)$   
=  $I(X; Y|S_{ab}) - I(X; Z, S_e|S_{ab}) + \frac{1}{n} H(S_{ab}|S_e)$   
=  $C_{\rm er}^{-}$ ,

where the third equality is due to the fact that given  $S_{ab}$  we have Markov chain  $X - Y - (Z, S_e)$ . This holds since the eavesdropper is degraded.

Note that when the state changes slowly, which is equivalent to when n is large,  $\frac{1}{n}H(S_{ab}|S_e) \rightarrow 0$ . That is, the contribution to the secret key capacity due to the sparsity pattern  $S_{ab}$  is very small. As discussed in Section II-A this will be the common situation. Thus, in following, we focus on the non-vanishing term of (18), which we denote as  $R_{er}$ , i.e.,  $R_{er} = I(X; Y|S_{ab}) - I(X; Z, S_e|S_{ab})$ .

## B. Ergodic secret key rate of sparse wireless channel

We now first apply Lemma 1 to the sparse channel model specified in II-A. In Section III-B1 we first examine the expressions for mutual information  $I(X; Y|S_{ab})$  and  $I(X; Z, S_e|S_{ab})$  for the vector channel described by (1) and (5). Then, in Section III-B2 we identify conditions under which the eavesdropper's observation is degraded. Finally, in Sections III-B3 and III-B4 we focus in on the randomness due to the sparity patterns and analyze the wideband limit.

1) Mutual information: Define  $Q_{\ell}$  to be the product  $S_{ab,\ell} \times S_{e,\ell}$  so  $Q_{\ell} \in \{0,1\}$ . Thus  $Q_{\ell} = 1$  if and only if the support (the sparsity pattern) of  $H_{ab}$  and of  $H_e$  are both non-zero in the  $\ell$ -th delay bin. Also define two functions:

$$I_{ab}(\gamma_a, \gamma_b) = \log\left(\frac{(1+\gamma_a)(1+\gamma_b)}{1+\gamma_a+\gamma_b}\right) , \qquad (19a)$$

$$I_e(\gamma_a, \gamma_e) = \log\left(\frac{(1+\gamma_a)(1+\gamma_e)}{1+\gamma_a\gamma_e(1-|\eta|^2)+\gamma_a+\gamma_e}\right) .$$
(19b)

and  $\gamma_a = \frac{P}{\sigma_a^2}$ ,  $\gamma_b = \frac{P}{\sigma_b^2}$  and  $\gamma_e = \frac{P}{\sigma_e^2}$ . We show in Appendix B that

$$I(\boldsymbol{X};\boldsymbol{Y}|\boldsymbol{S}_{ab}) = E\left[\sum_{\ell=1}^{L_{\max}} S_{ab,\ell} I_{ab}(\nu_{\ell}^{2}\gamma_{a},\nu_{\ell}^{2}\gamma_{b})\right]$$
(20a)

$$I(\boldsymbol{X}; \boldsymbol{Z}, \boldsymbol{S}_{e} | \boldsymbol{S}_{ab}) = E\left[\sum_{\ell=1}^{L_{\max}} Q_{\ell} I_{e}(\nu_{\ell}^{2} \gamma_{a}, \upsilon_{\ell}^{2} \gamma_{e})\right]$$
(20b)

In the above expressions, the expectation is taken over the random sparsity patterns  $S_{ab}$  and  $S_e$ . Note the factor  $Q_\ell$  in (20b). When  $S_{ab,\ell} = 1$  but  $S_{e,\ell} = 0$  the eavesdropper has no measurement of that channel coefficient ( $Q_\ell = 0$ ). Thus, the eavesdropper has no observation of that common randomness and the negative mutual information term in (16) is zero.

It is clear in (20) that channel sparsity patterns ( $S_{ab}$  and  $S_e$ ) effect the mutual information via the channel DoF (the number of terms in the summation) and the correlation coefficient  $\eta$  effects the information leakage via  $I_e(\cdot)$  in each delay bin observed by the eavesdropper.

2) Degraded condition: Because the Eve's channel is correlated to the main channel, she may get a good estimation of  $H_{ab}$  if she has a higher SNR than Alice and Bob. To guarantee the positivity of the secret key rate, we need to characterize the conditions under which the eavesdropper has a worse observation than Alice and Bob. To develop such conditions we first consider a delay bin where  $Q_{\ell} = 1^4$ . Project the channel outputs onto  $d_{\ell}$ , the  $\ell$ -th column of D, we get

$$X_{\ell} = \mathbf{d}_{\ell}^{H} \mathbf{X} \doteq P H_{\ell} + W_{1,\ell}$$
(21a)

$$Y_{\ell} = \mathbf{d}_{\ell}^{H} \boldsymbol{Y} \doteq P H_{\ell} + W_{2,\ell} .$$
(21b)

Because the sounding signal is an (asymptotically) white sequence,  $X_{\ell}$  (and  $Y_{\ell}$ ) are sufficient statistic for estimating  $H_{\ell}$ . The noise  $W_{1,\ell}$  (resp.  $W_{2,\ell}$ ) is a zero mean complex Gaussian with variance  $P\sigma_a^2$  (resp.  $P\sigma_b^2$ ). Similarly, Eve's sufficient static is

$$Z_{\ell} = \mathbf{d}_{\ell}^{H} \mathbf{Z} \doteq P H_{e,\ell} + W_{3,\ell}$$
$$\equiv P \left( \frac{\nu_{\ell}}{\nu_{\ell}} \eta H_{\ell} + \sqrt{1 - |\eta|^2} H_{\ell}' \right) + W_{3,\ell} .$$
(22)

Because of  $\eta(H_{\ell}, H_{e,\ell}) = \eta$ , we have equivalently written  $H_{e,\ell}$  as a sum of two terms. The first term is a scaled version of  $H_{\ell}$ . The second,  $H'_{\ell}$ , is a  $\mathcal{CN}(0, v^2_{\ell})$  random variable that is independent of  $H_{\ell}$ . We see from (22) that Eve's observation  $Z_{\ell}$  contains two types of noise. The first is the receiver noise  $W_{3,\ell}$ . The second is due to the uncorrelated  $H'_{\ell}$ .

Eve's observation  $Z_{\ell}$  will be a degraded version of  $Y_{\ell}$  if Eve has a smaller SNR than Bob. This occurs if

$$\frac{\nu_{\ell}^2 P}{\sigma_b^2} > \frac{|\eta|^2 v_{\ell}^2 P}{(1-|\eta|^2) v_{\ell}^2 P + \sigma_e^2} .$$
(23)

Otherwise,  $Y_{\ell}$  is a degraded version of  $Z_{\ell}$ . If the sounding signal power is small and Eve has a suitably smaller noise variance  $\sigma_e^2$ , in particular, when

$$(1 - |\eta|^2) v_\ell^2 P < |\eta|^2 \frac{v_\ell^2}{\nu_\ell^2} \sigma_b^2 - \sigma_e^2 , \qquad (24)$$

then Bob's output is noisier and no secret key can be extracted at a positive rate from the  $\ell$ -th delay bin. This is because when P is small, Eve's independent noise (due to  $H'_{\ell}$ ) is decreased. It is observed in [41] that there is a cutoff SNR below which the secret key capacity is zero. If  $\nu_{\ell}^2 = \nu_{\ell}^2$  and all the users (Alice, Bob and Eve) are with the same SNR, i.e.,  $\sigma_a^2 = \sigma_b^2 = \sigma_e^2 = \sigma^2$ , the secret key capacity will be positive because Eve has an extra noise (due to the uncorrelated  $H'_{\ell}$ ).

3) Achievable secret key rate: In order to see the effect of channel sparsity when the bandwidth is large (but finite), we focus on the equal-SNR case and consider a uniform delay profile, thus, having a degraded eavesdropper. Define the random number of non-zero channel coefficients in the main Alice-to-Bob and in Eve's channel to be, respectively,

$$B_{ab} = \sum_{\ell=1}^{L_{\text{max}}} S_{ab,\ell} \quad , \tag{25a}$$

$$B_e = \sum_{\ell=1}^{L_{\max}} S_{e,\ell} , \qquad (25b)$$

<sup>4</sup>In subspaces such that  $Q_{\ell} = 0$  either  $S_{ab,\ell} = 0$  or Eve has no observation of  $H_{ab,\ell}$ , so there is no need to consider those subspaces.

Note that  $B_{ab}$  and  $B_e$  are binomial  $\text{Bino}(L_{\max}, \rho)$  distributed random variables. Consider a uniform delay profile, i.e.,  $\nu_{\ell}^2 = \frac{1}{B_{ab}}$  for all  $\ell$  for which  $S_{ab,\ell} = 1$ ; similarly,  $v_{\ell}^2 = \frac{1}{B_e}$  for all  $\ell$  for which  $S_{e,\ell} = 1$ .

Let  $I_s(P)$  be the instantaneous key rate  $I(\mathbf{X}; \mathbf{Y} | \mathbf{S}_{ab}) - I(\mathbf{X}; \mathbf{Z}, \mathbf{S}_e | \mathbf{S}_{ab})$  for fixed  $\mathbf{S}_{ab}$  and  $\mathbf{S}_e$ . i.e.,

$$I_{\rm s}(\gamma) = B_{ab}I_{ab}\left(\frac{\gamma}{B_{ab}}, \frac{\gamma}{B_{ab}}\right) - B_qI_e\left(\frac{\gamma}{B_{ab}}, \frac{\gamma}{B_e}\right) \ , \tag{26}$$

where  $\gamma \triangleq \frac{P}{\sigma^2}$  and

$$B_q = \sum_{\ell=1}^{L_{\text{max}}} Q_\ell \quad \text{(the number of overlap delay bins)} . \tag{27}$$

From (20) and Corollary 2 the achievable secret key rate is

$$I_{\rm er}(\gamma) = E\left[I_{\rm s}(\gamma)\right] \ . \tag{28}$$

As we will see later in III-B4,  $I_s(\gamma)$  is convex in low SNR (and so is  $I_{er}(\gamma)$ ). Thus, a uniform sounding strategy using a sounding signal with constant power P is not optimal. Let  $\mathcal{P}$  denote all sounding policies that satisfy average power constraint  $E[\mathbf{d}^H\mathbf{d}] \leq P$ , we can achieve

$$R_{\rm er}(\gamma) = \max_{\mathcal{P}} I_{\rm er}(\gamma) .$$
<sup>(29)</sup>

Note that from Corollary 2 and the discussion thereafter,  $R_{\rm er}(P)$  approaches  $C_{\rm er}(P)$  from the below as  $n \to \infty$ .

Theorem 3 (An on-off sounding achieves capacity).

$$R_{\rm er}(\gamma) = \max_{0 < \lambda \le 1} \lambda I_{\rm er}\left(\frac{\gamma}{\lambda}\right) . \tag{30}$$

*Proof:* The proof is provided in Appendix C.

The physical interpretation of the auxiliary variable  $\lambda$  is that key rate  $\lambda I_{\text{er}}\left(\frac{\gamma}{\lambda}\right)$  can be achieved by an on-off sounding strategy that sounds the channel during  $\lambda$  ( $0 < \lambda \leq 1$ ) fraction of the time, each with power  $\frac{P}{\lambda}$ , and does not sound the channel (i.e., is turned off) during the rest of the time (i.e., a time-sharing scheme). Theorem 3 says that the ergodic secret key capacity can be achieved by an  $\lambda^*$  on-off sounding strategy where  $\lambda^*$  is the argumment maximizing (30). As we will discuss in III-B4, an optimal on-off signal is sparse in time (i.e.,  $\lambda^* \to 0$ ) in a low SNR ( $\gamma \to 0$ ) and is dense (i.e.,  $\lambda^* \to 1$ ) in a high SNR.

4) Wideband regime: One way to increase the secret key capacity is to increase the bandwidth W of the wireless channel. However, the channel DoF do not grow linearly in W. To see how W effects the secret key rate, we examine  $R_{er}(P)$  from (30) in the wideband regime.

In this case, each channel DoF is sounded at a low SNR. At low SNR we can approximate (19) as

$$I_{ab}(x,x) \approx \frac{x^2}{\ln 2} , \qquad (31a)$$

$$I_e(x,y) \approx \frac{|\eta|^2 xy}{\ln 2} . \tag{31b}$$



Fig. 3. Achievable secret key rate  $I_{\rm er}(\gamma)$  plotted versus SNR ( $\gamma$ ). The bandwidth is W = 100MHz, the maximum delay spread  $\tau_{\rm max} = 10\mu$ s, the conditional probability of overlap in  $S_{ab}$  and  $S_e$  is  $\theta = 0.5$  and the correlation between channel coefficients is  $\eta = 0.1$ . The sparsity parameter  $\delta \in [0.5, 1]$ .

for x and y small. The ergodic key rate

$$I_{\rm er}(\gamma) \approx \frac{1}{\ln 2} E \left[ B_{ab} \left( \frac{\gamma}{B_{ab}} \right)^2 - B_q |\eta|^2 \frac{\gamma}{B_{ab}} \frac{\gamma}{B_e} \right]$$
$$= \frac{\gamma^2}{\ln 2} E \left[ \frac{1}{B_{ab}} - |\eta|^2 \frac{B_q}{B_{ab}} \frac{1}{B_e} \right]$$
$$\stackrel{(a)}{\approx} \frac{\gamma^2}{\ln 2} \frac{(1 - \theta |\eta|^2)}{L} = \frac{\gamma^2}{\ln 2} \frac{(1 - \theta |\eta|^2)}{(\tau_{\rm max} W)^{\delta}} .$$
(32)

The approximation (a) is accurate when  $L \gg 1$  [42, eq.(5)]. The right hand side of (32) is a quadratic function of  $\gamma$ , thus  $I_{\rm er}(\gamma)$  is convex in low SNR.

Figure 3 plots  $I_{er}(\gamma)$  versus  $\gamma$ , for a bandwidth of W = 100MHz,  $\tau_{max} = 10\mu$ s, and for various values of the sparsity parameter  $\delta \in [0.5, 1]$ . We see that a sparser channel (small  $\delta$ ) achieves a higher key rate at low SNRs. We can also observe this from (32). According to Theorem 3 and notice that (32) is quadratic in  $\gamma$ , we need a sparser signal in time (an on-off signal) to get a higher key rate. In other words, in the wideband (power-limited) regime, fewer DoF (either in channel or in time domain) can achieve a higher key rate. This occurs because the key generation problem is a combined channel sounding and channel coding problem. By focusing energy on fewer DoF we raise their SNR, enabling key generation to occur at a higher rate. In contrast, a richer channel (large  $\delta$ ) results in a higher key rate at a high SNR since that is a DoF-limited (and not a power-limited) regime.

Figure 4(a) and 4(b) plots  $I_{\rm er}$  as a function of W. This provides another view of the tradeoff between power and DoF. First, let  $\gamma$  be fixed at 10dB. Then, Figure 4(a) plots  $I_{\rm er}$  for different values of channel sparsity  $\delta$  in the range [0.5, 1]. In the wideband (low-SNR) regime, larger  $\delta$ results in a smaller key rate. In Figure 4(b),  $\delta = 0.5$  is fixed and  $\gamma$  is varied from 10dB to 30dB. We see that for each SNR, there is a unique  $W^*$  that achieves the highest key rate.



Fig. 4. Achievable secret key rate  $I_{er}(\gamma)$  for SNR fixed at  $\gamma = 10dB$  plotted versus bandwidth W. In subfigure (a) the tradeoff is plotted for for values of the sparsity parameter  $\delta \in [0.5, 1)$ . In subfigure (b) the sparsity parameter is fixed at  $\delta = 0.5$  and the tradeoff is plotted for four SNRs,  $\gamma$  (dB)  $\in [15, 16]$ .

## **IV. SECRECY OUTAGE**

In contrast to Section III when an application has a stringent delay requirement or when the state (i.e., the sparsity pattern) changes so slowly that it is roughly constant during the secret key generation process, the secret key capacity in th Shannon sense is not well-defined. To study this setting, in this section we set M = 1 while allowing n to be arbitrary large. Since users only know their state but not Eve's state, they cannot adapt the key generation rate to Eve's state. Thus, satisfying the secrecy condition (13) can be problematic. In this section, we consider an "outage" setting with a degraded eavesdropper. For any (M = 1) realization  $(S_{ab}, S_e) = (s_{ab}, s_e)$ ,

we say that a secrecy outage occurs if

$$\frac{1}{n}I(K;Z^n,\Phi|s_{ab},s_e) > R_e \tag{33}$$

for some  $R_e > 0$ . Namely, there is a non-vanishing information rate leaked to Eve. Let

$$C_s(s_{ab}, s_e) = I(X; Y|s_{ab}) - I(X; Z|s_{ab}, s_e)$$
(34)

be the conditional secret key capacity for state  $(s_{ab}, s_e)$ . Theorem 4 shows that the event  $R > C_s(s_{ab}, s_e)$  is a necessary and sufficient condition for the outage event (33).

**Theorem 4.** For any rate-R secret key generation systems for which Bob can reliability recover  $X^n$  (i.e.,  $\Pr(X^n \neq f_2(Y^n, s_{ab}, \phi)) \rightarrow 0$  for some  $f_2(\cdot)$ ), and let  $R_e(s_{ab}, s_e) = R - C_s(s_{ab}, s_e) > 0$ , then

(i) the information leaked to Eve is lower bounded as

$$\frac{1}{n}I(K;Z^n,\Phi|s_{ab},s_e) \ge R_e(s_{ab},s_e) - 2\epsilon , \qquad (35)$$

and

(ii) there exist a coding scheme (cf., (11)) that satisfies (12), (14) and

$$\frac{1}{n}I(K;Z^n,\Phi|s_{ab},s_e) \le R_e(s_{ab},s_e) + 2\epsilon .$$
(36)

*Proof:* The proof can be found in Appendix D.

#### A. Wideband sparse channel

In the reciprocal wireless channel case, we know from III-B that  $C_s(S_{ab}, S_e) = I_s(\gamma)$  given in (26). Using the approximations from (31), we have in the wideband regime that

$$C_s(\boldsymbol{S}_{ab}, \boldsymbol{S}_e) \approx \frac{\gamma^2}{\ln 2} \left( \frac{1}{B_{ab}} - |\eta|^2 \frac{B_q}{B_{ab}} \frac{1}{B_e} \right) .$$
(37)

Recall that  $B_{ab}$  (resp.  $B_e$ ) is the weight of vector  $S_{ab}$  (resp.  $S_e$ ) (cf., (25)) and  $B_q$  is the weight of support common to  $S_{ab}$  and  $S_e$  (cf., (27)). Also note that  $B_{ab}$ ,  $B_e$ ,  $B_q$  are random variables so that the overall quantity in (37) is a random variable. Unfortunately, there is no simple expression for the distribution of  $C_s(S_{ab}, S_e)$  in (37). Since the users are assumed to know  $S_{ab}$ (and, therefore,  $B_{ab}$ ), in order to understand how the channel sparsity effects the probability of outage, we consider the case where  $B_{ab} = L$  (i.e., its mean which is its most likely value) and  $B_e = L$ . The only uncertainty at users is  $B_q$ , the number of delay bins from which Eve can learn the key. Conditioned on  $B_{ab} = L$  and according to the correlation model in (6),  $B_q$  has a Binomial distribution Bino  $(L, \theta)$ .

From Theorem 4, the outage probability is

$$P_{\text{out}} = \Pr\left(R > C_s(\boldsymbol{S}_{ab}, \boldsymbol{S}_e)\right)$$
  

$$\approx \Pr\left(R > \frac{\gamma^2}{L \ln 2} \left(1 - |\eta|^2 \frac{B_q}{L}\right)\right)$$
  

$$= \Pr\left(B_q > \frac{1}{|\eta|^2} \left(1 - \ln 2 \frac{LR}{\gamma^2}\right)\right) , \qquad (38)$$

where  $L = (W\tau_{\text{max}})^{\delta}$ . We see that a larger  $\gamma$  (SNR), a larger W, or a smaller  $\eta$  will decrease the outage probability. However, the sparsity  $\delta$  changes both the distribution of  $B_q$  and the right hand side of the argument in (38) via L. Thus, it is still not clear how  $\delta$  impact  $P_{\text{out}}$ . When users don't know the instantaneous secret key capacity, a conservative strategy is to generate a key at a smaller rate.

Consider a strategy in which the key is generated at rate  $R = \alpha I_{er}(\gamma)$  ( $0 < \alpha < 1$ ), i.e., a backoff from the ergodic key rate (32). We refer to this strategy as the " $\alpha$ -backoff" strategy. The outage probability (38) can now be simplified to be

P<sub>out</sub> 
$$\approx \Pr(B_q \ge aL)$$
 (39)  
where  $a = (1 - \alpha) \frac{1}{|\eta|^2} + \alpha \theta$ .

Since  $B_q$  approximates Bino  $(L, \theta)$ , the sparsity  $\delta$  (and therefore the actual DoF L) determines the distribution of Eve's DoF  $B_q$  to observe the main channel. From (39), we can see that when the  $\alpha$ -backoff strategy is used, the correlation coefficient  $\eta$  determines how fast the threshold aLdeviates from  $\theta L$  (the mean of  $B_q$ ) as  $\alpha$  decreases. Note that in (39), the SNR (or equivalently, power P) does not appear in the formula. This is because the key rate is proportional to ergodic key rate  $I_{er}(\gamma)$ , which is a quadratic function of  $\gamma$  in the wideband regime (cf. (32)), and thus cancels the  $\gamma^2$  in (38).

We next use results from large deviation theory [43] to upper bound the tail probability of binomial distribution.

**Lemma 5.** [43, Theorem 1] Let  $S_n$  be a binomial random variable Bino(n, p). For p < a < 1, and for  $n = 1, 2, 3, \dots$ , then

$$\Pr(S_n \ge an) \le 2^{-nD(a\|p)} \tag{40}$$

where

$$D(a||p) \equiv a \log \frac{a}{p} + (1-a) \log \frac{(1-a)}{1-p}$$
(41)

is the Kullback-Leibler divergence between the probability distributions Bern(a) and Bern(p).

By this lemma, the outage probability is upper bounded as

$$\mathsf{P}_{\text{out}} < 2^{-LD(a\|\theta)} \ . \tag{42}$$

Figure 5 plots the numerical results of the secrecy outage exponent  $LD(a||\theta)$  in the wideband regime. It shows that when the  $\alpha$ -backoff strategy is used, the mechanism through which the channel sparsity impacts the outage probability differs from how the channel sparsity impacts the ergodic secret key rate. A richer channel (larger  $\delta$ ) always has larger exponent than a sparser channel. In contrast, Figure 3 demonstrates a sparser channel yields a higher ergodic secret key rate in the wideband regime.

# V. CONCLUSIONS

In this paper we study a setting in which two users desire to distill a common secret key based on the inherent randomness of a reciprocal wireless channel. Our particular interest is the effect of channel sparsity (e.g., in delay), which scales sub-linearly in signal bandwidth, on secret key generation. Channel sparsity affects the inherent randomness of the main channel and increases eavesdropper's observability of the main channel. Since channel sparsity is an important



Fig. 5. Plot of outage exponent  $LD(a||\theta)$  vs. bandwidth W. Other parameters include the maximum delay spread  $\tau_{max} = 100$ ns, the conditional probability of overlap in  $S_{ab}$  and  $S_e$  is  $\theta = 0.5$  and the correlation between channel coefficients is  $\eta = 0.1$ . The sparsity parameter  $\delta$  is plotted for various values between  $0.5 \le \delta \le 1$ .

characteristic of many real-world wireless channels and since it has such a large impact on secret key capacity, it is crucial to understand this interplay fully. This will help us to deliver secure communication systems with robust guarantees.

We first consider the ergodic setting. In this setting, at each SNR there is an adequate bandwidth to maximizes the ergodic secret key rate. Moreover, when a white sounding sequence is adopted in the wideband (low-SNR) regime, a higher secret key rate can be achieved by a sparser channel.

For channels whose sparsity changes relatively slowly, a secrecy outage measure of performance is adopted. If the key rate is a fraction  $\alpha$  of the ergodic capacity, we show that richer channels always have larger exponents characterizing the decay of the outage probability. This result illustrates that a larger number of DoF can smooth out of the detrimental effects of an unknown eavesdropper state.

## APPENDIX

## A. Proof of Lemma 1

Apply the results in [1], [2] where, respectively,  $(X^n, S_{ab})$  are Alice's,  $(Y^n, S_{ab})$  are Bob's, and  $(Z^n, S_e)$  are Eve's observations. A lower bound on the ergodic capacity is (see [1, Theorem 1], [2, Theorem 3])

$$C_{\rm er}^{-} = \frac{1}{n} \Big[ I(X^n, S_{ab}; Y^n, S_{ab}) - I(X^n, S_{ab}; Z^n, S_e) \Big].$$

$$I(X^{n}, S_{ab}; Y^{n}, S_{ab})$$
  
=  $I(S_{ab}; Y^{n}, S_{ab}) + I(X^{n}; Y^{n}, S_{ab}|S_{ab})$   
=  $H(S_{ab}) + \sum_{i=1}^{n} I(X_{i}; Y^{n}|X^{i-1}, S_{ab})$   
 $\stackrel{(a)}{=} H(S_{ab}) + \sum_{i=1}^{n} I(X_{i}; Y_{i}|S_{ab})$   
=  $H(S_{ab}) + nI(X; Y|S_{ab})$ ,

where (a) follows by applying the memoryless property to source  $(X_i, Y_i)$ . Similarly, the second term reduces to

$$I(X^{n}, S_{ab}; Z^{n}, S_{e})$$

$$= I(S_{ab}; Z^{n}, S_{e}) + I(X^{n}; Z^{n}, S_{e}|S_{ab})$$

$$\stackrel{(b)}{=} I(S_{ab}; S_{e}) + \sum_{i=1}^{n} I(X_{i}; Z^{n}, S_{e}|X^{i-1}, S_{ab})$$

$$= I(S_{ab}; S_{e}) + \sum_{i=1}^{n} I(X_{i}; Z_{i}, S_{e}|S_{ab})$$

$$= I(S_{ab}; S_{e}) + nI(X; Z, S_{e}|S_{ab}) ,$$

where (b) is due to the Markov chain  $S_{ab} - S_e - Z^n$  and applying chain rule on the second term. Thus, the lower bound is

$$C_{\rm er}^{-} = I(X; Y|S_{ab}) - I(X; Z, S_e|S_{ab}) + \frac{1}{n}H(S_{ab}|S_e) .$$
(43)

The upper bound is the conditional mutual information (see [1, Theorem 1], [2, Corollary 1]):

$$C_{er}^{+} = \frac{1}{n} I(X^{n}, S_{ab}; Y^{n}, S_{ab} | Z^{n}, S_{e})$$
  
=  $\frac{1}{n} \Big( I(S_{ab}; Y^{n}, S_{ab} | Z^{n}, S_{e}) + I(X^{n}; Y^{n}, S_{ab} | Z^{n}, S_{e}, S_{ab}) \Big)$   
 $\leq \frac{1}{n} \Big( H(S_{ab} | Z^{n}, S_{e}) + \sum_{i=1}^{n} I(X_{i}; Y^{n} | X^{i-1}, Z^{n}, S_{e}, S_{ab}) \Big)$   
 $\stackrel{(c)}{=} \frac{1}{n} \Big( H(S_{ab} | S_{e}) + \sum_{i=1}^{n} I(X_{i}; Y_{i} | Z_{i}, S_{e}, S_{ab}) \Big)$   
 $= I(X; Y | Z, S_{ab}, S_{e}) + \frac{1}{n} H(S_{ab} | S_{e}) ,$ 

where (c) follows by Markov condition  $S_{ab} - S_e - Z^n$  and the memoryless property.

# B. Derivation of mutual information (20)

First consider  $I(\mathbf{X}; \mathbf{Y} | \mathbf{S}_{ab})$ :

$$I(\boldsymbol{X}; \boldsymbol{Y} | \boldsymbol{S}_{ab}) = E[h(\boldsymbol{X} | \boldsymbol{S}_{ab}) - h(\boldsymbol{Y} | \boldsymbol{S}_{ab}) - h(\boldsymbol{X}, \boldsymbol{Y} | \boldsymbol{S}_{ab})]$$
$$= E\left[\log\left(\frac{\det(\mathbf{R}_{\boldsymbol{X}}) \cdot \det(\mathbf{R}_{\boldsymbol{Y}})}{\det(\mathbf{R}_{\boldsymbol{X}\boldsymbol{Y}})}\right)\right],$$
(44)

where  $h(\mathbf{X})$  is the differential entropy [44] of  $\mathbf{X}$ , and the expectation is taken over the distribution of  $\mathbf{S}_{ab}$ . Let  $\mathbf{R}_{\mathbf{X}}$  denote the covariance matrix of  $\mathbf{X}$  when the input  $\mathbf{S}_{ab} = \mathbf{S}$  is fixed, i.e.,

$$\mathbf{R}_{\boldsymbol{X}} = E[\boldsymbol{X}\boldsymbol{X}^{H}|\mathbf{S}] = \mathbf{D}\mathbf{R}_{\mathbf{h}}\mathbf{D}^{H} + \sigma_{1}^{2}\mathbf{I}_{K} ,$$

where  $\mathbf{R_h} = \operatorname{diag}(\nu_1^2, \cdots, \nu_L^2)$  and  $\nu_\ell^2 = 0$  if  $S_\ell = 0$ . Similarly,

$$\mathbf{R}_{Y} = \mathbf{D}\mathbf{R}_{\mathbf{h}}\mathbf{D}^{H} + \sigma_{2}^{2}\mathbf{I}_{N}$$
$$\mathbf{R}_{XY} = \left[\begin{array}{c|c} \mathbf{R}_{X} & \mathbf{D}\mathbf{R}_{\mathbf{h}}\mathbf{D}^{H} \\ \hline \mathbf{D}\mathbf{R}_{\mathbf{h}}\mathbf{D}^{H} & \mathbf{R}_{Y} \end{array}\right]$$

\*\*

We simplify the determinants as follows,

$$\det(\mathbf{R}_{\mathbf{X}}) = \det(\mathbf{D}\mathbf{R}_{\mathbf{h}}\mathbf{D}^{H} + \sigma_{1}^{2}\mathbf{I}_{N})$$

$$= (\sigma_{1}^{2})^{N} \det\left(I_{N} + \frac{\mathbf{D}\mathbf{R}_{\mathbf{h}}\mathbf{D}^{H}}{\sigma_{1}^{2}}\right)$$

$$\stackrel{(a)}{=} (\sigma_{1}^{2})^{N} \det\left(I_{L} + \frac{\Lambda\mathbf{D}^{H}\mathbf{D}\Lambda}{\sigma_{1}^{2}}\right)$$

$$\stackrel{(b)}{=} (\sigma_{1}^{2})^{N} \prod_{\ell=1}^{L} \left(1 + \frac{P}{\sigma_{1}^{2}}\nu_{\ell}^{2}\right)$$

$$= (\sigma_{1}^{2})^{N} \prod_{\ell:S_{\ell}=1} \left(1 + \frac{P}{\sigma_{1}^{2}}\nu_{\ell}^{2}\right),$$

where (a) follows by defining  $\Lambda = \sqrt{\mathbf{R}_{\mathbf{h}}}$  and applying Sylvester's determinant formula: det( $\mathbf{I}_m + \mathbf{AB}$ ) = det( $\mathbf{I}_n + \mathbf{BA}$ ) where  $\mathbf{A}$  is an *m*-by-*n* matrix, and  $\mathbf{B}$  is an *n*-by-*m* matrix. Step (b) is due to (2). Similarly, we find that

$$\det(\mathbf{R}_{\boldsymbol{X}}) \doteq (\sigma_2^2)^N \prod_{\ell:S_\ell=1} \left( 1 + \frac{P}{\sigma_2^2} \nu_\ell^2 \right)$$
$$\det(\mathbf{R}_{\boldsymbol{X}\boldsymbol{Y}}) \doteq (\sigma_1^2 \sigma_2^2)^N \prod_{\ell:S_\ell=1} \left( 1 + \frac{(\sigma_1^2 + \sigma_2^2)P}{\sigma_1^2 \sigma_2^2} \nu_\ell^2 \right).$$

Substituting into (44), we get (20a).

Follow a similar calculation, we get  $I(X; Z, S_e | S_{ab})$  in (20b) by noting that

$$\mathbf{R}_{\boldsymbol{X}\boldsymbol{Z}} = \begin{bmatrix} \mathbf{R}_{\boldsymbol{X}} & \mathbf{D}\mathbf{R}_{\mathbf{h}\tilde{\mathbf{h}}}\mathbf{D}^{H} \\ \hline \mathbf{D}\mathbf{R}_{\mathbf{h}\tilde{\mathbf{h}}}\mathbf{D}^{H} & \mathbf{R}_{\boldsymbol{Z}} \end{bmatrix} \ .$$

where  $\mathbf{R}_{\mathbf{h}\tilde{\mathbf{h}}}$  is a diagonal matrix and its  $\ell$ -th diagonal element is equal to  $\eta\nu_{\ell}^2$  if  $Q_{\ell} = 1$  or is equal to zero if  $Q_{\ell} = 0$ .

## C. Proof of Theorem 3

The proof is similar to [10, Theorem 4]. First note that  $I_{\rm s}(\gamma)$  is non-decreasing in  $\gamma$  and so is  $I_{\rm er}(\gamma)$ . This can be verified by evaluating  $\frac{\partial I_{\rm s}(\gamma)}{\partial \gamma}$ , which is non-negative. Define  $\bar{I}_{\rm er}(\gamma) = \max_{0 < \lambda \leq 1} \lambda I_{\rm er}\left(\frac{\gamma}{\lambda}\right)$ . Note that  $\bar{I}_{\rm er}(\gamma)$  is a concave and non-decreasing function of  $\gamma$ . We are going to show  $\bar{I}_{\rm er}\left(\frac{P}{\sigma^2}\right)$  is equal to  $R_{\rm er}\left(\frac{P}{\sigma^2}\right)$  defined in (29) over the average power constraint P. Let  $\mathcal{P}$  be the set of all sounding policies satisfying average power constraint P. Specifically, let the sounding policy in  $\mathcal{P}$  allocate power  $P_s$  to sounding signals with probability p(s) such that  $E[P_s] = \sum_s p(s)P_s \leq P$ . Note that  $R_{\rm er}\left(\frac{P}{\sigma^2}\right) \geq \bar{I}_{\rm er}\left(\frac{P}{\sigma^2}\right)$ . We can also upper bound

$$R_{\rm er}\left(\frac{P}{\sigma^2}\right) = \max_{\mathcal{P}} \sum_{s} p(s) I_{\rm er}\left(\frac{P_s}{\sigma^2}\right)$$
$$\leq \max_{\mathcal{P}} \sum_{s} p(s) \left[\max_{0<\lambda\leq 1} \lambda I_{\rm er}\left(\frac{P_s}{\lambda\sigma^2}\right)\right]$$
$$= \max_{\mathcal{P}} \sum_{s} p(s) \bar{I}_{\rm er}\left(\frac{P_s}{\sigma^2}\right)$$
$$\stackrel{(a)}{\leq} \max_{\mathcal{P}} \bar{I}_{\rm er}\left(\frac{\sum_{s} p(s) P_s}{\sigma^2}\right)$$
$$\stackrel{(b)}{\leq} \bar{I}_{\rm er}\left(\frac{P}{\sigma^2}\right) ,$$

where (a) and (b) are due to the concavity and non-decreasing function of  $I_{\rm er}(\gamma)$ .

## D. Proof of Theorem 4

We first show the lower bound and then the upper bound.

*1) Lower bound* (35):

$$\begin{split} I(K; Z^{n}, \Phi | s_{ab}, s_{e}) &= H(K | s_{ab}, s_{e}) - H(K | Z^{n}, \Phi, s_{ab}, s_{e}) \\ &= H(K | s_{ab}, s_{e}) - [H(K, \Phi | Z^{n}, s_{ab}, s_{e}) - H(\Phi | Z^{n}, s_{ab}, s_{e}) \\ &\stackrel{(a)}{\geq} H(K | s_{ab}, s_{e}) + H(\Phi | Y^{n}, s_{ab}, s_{e}) \\ &- [H(X^{n}, K, \Phi | Z^{n}, s_{ab}, s_{e}) - H(X^{n} | K, \Phi, Z^{n}, s_{ab}, s_{e})] \\ &\stackrel{(b)}{\geq} n(R - \epsilon) + H(\Phi | Y^{n}, s_{ab}, s_{e}) - H(X^{n} | Z^{n}, s_{ab}, s_{e}) \\ &= n(R - \epsilon) + H(\Phi | Y^{n}, s_{ab}, s_{e}) - H(X^{n} | Y^{n}, s_{ab}, s_{e}) \\ &+ H(X^{n} | Y^{n}, s_{ab}, s_{e}) - H(X^{n} | Z^{n}, s_{ab}, s_{e}) \\ &\stackrel{(c)}{=} n(R - \epsilon) - H(X^{n} | \Phi, Y^{n}, s_{ab}, s_{e}) - nC_{s}(s_{ab}, s_{e}) \\ &\stackrel{(d)}{\geq} n(R - C_{s}(s_{ab}, s_{e}) - 2\epsilon) , \end{split}$$

where (a) is due to the fact that given  $(s_{ab}, s_e)$ ,  $\Phi - X^n - Y^n - Z^n$  form a Markov chain. Thus  $H(\Phi|Z^n, s_{ab}, s_e) \ge H(\Phi|Y^n, s_{ab}, s_e)$ . (b) holds because entropy is non-negative and  $(K, \Phi)$  is function of  $(X^n, s_{ab})$ , we can take  $K, \Phi$  away from  $H(X^n, K, \Phi|Z^n, s_{ab}, s_e)$ . For the same reason, we can add  $\Phi$  in  $H(X^n|Y^n, s_{ab}, s_e)$  and use chain rule to get (c). (d) is due to the reliable condition  $\Pr(X^n \neq f_2(Y^n, s_{ab}, \Phi)) \to 0$  by applying Fano's inequality [44].

2) Upper bound (36):

$$I(K; Z^{n}, \Phi | s_{ab}, s_{e})$$

$$= H(K | s_{ab}, s_{e}) - H(K | Z^{n}, \Phi, s_{ab}, s_{e})$$

$$\stackrel{(a)}{\leq} nR - H(K | Z^{n}, \Phi, s_{ab}, s_{e})$$

because  $K \in [1:2^{nR}]$ . We need to show there exist a coding scheme such that

$$\frac{1}{n}H(K|Z^{n}, \Phi, s_{ab}, s_{e}) \ge C_{s}(s_{ab}, s_{e}) - 2\epsilon .$$
(45)

We will use the following lemma in the proof.

**Lemma 6.** (cf. [45, eq.(25)] [11, eq.(16)]) Any  $\epsilon > 0$ , if  $\frac{1}{n}H(\Phi|s_{ab}, s_e) \ge H(X|Y, s_{ab}) + \epsilon$ , there exists a coding scheme where  $f_2(\cdot)$  and  $f_3(\cdot)$  are the decoding functions at Bob and Eve, respectively, such that for sufficiently large n,

(i)  $\Pr(X^n \neq f_2(Y^n, s_{ab}, \Phi)) \rightarrow 0$ , (ii)  $\frac{1}{n}H(X^n|K, \Phi, Z^n, s_{ab}, s_e) \leq \epsilon$ .

The proof of Lemma 6 uses a random coding technique to show existence. The first statement is exactly the Slepian-Wolf theorem [46]. The second statement is the standard equivocation analysis which says if Eve knows K and  $\Phi$  along with her observation  $Z^n$ , she can recover sequence  $X^n$ . We refer to [11, 45] for the details.

Adopting the coding scheme in Lemma 6 where the public message  $\Phi \in [1 : 2^{n(H(X|Y,s_{ab})+\epsilon)}]$ . We prove (45) through a sequence of (in)equalities:

$$\begin{split} H(K|\Phi, Z^{n}, s_{ab}, s_{e}) \\ &= H(X^{n}, K|\Phi, Z^{n}, s_{ab}, s_{e}) - H(X^{n}|K, \Phi, Z^{n}, s_{ab}, s_{e}) \\ &\stackrel{(a)}{\geq} H(X^{n}, K|\Phi, Z^{n}, s_{ab}, s_{e}) - n\epsilon \\ &= H(X^{n}, K, \Phi|Z^{n}, s_{ab}, s_{e}) - H(\Phi|Z^{n}, s_{ab}, s_{e}) - n\epsilon \\ &\stackrel{(b)}{\equiv} H(X^{n}|Z^{n}, s_{ab}, s_{e}) - H(\Phi|Z^{n}, s_{ab}, s_{e}) - n\epsilon \\ &\geq H(X^{n}|Z^{n}, s_{ab}, s_{e}) - H(\Phi) - n\epsilon \\ &\stackrel{(c)}{\geq} H(X^{n}|Z^{n}, s_{ab}, s_{e}) - nH(X|Y, s_{ab}) - 2n\epsilon \\ &= n(H(X|Z, s_{ab}, s_{e}) - H(X|Y, s_{ab}) - 2\epsilon) \\ &= n(C_{s}(s_{ab}, s_{e}) - 2\epsilon) , \end{split}$$

where (a) is due to (ii) of Lemma 6. (b) follows because  $(K, \Phi)$  is a function of  $(X^n, s_{ab})$ . (c) comes from the fact that  $\Phi \in [1 : 2^{n(H(X|Y,s_{ab})+\epsilon)}]$  and the entropy is upper bounded by a uniform distribution. This completes the proof.

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