

Smoothed L_p -Minimization for Green Cloud-RAN with User Admission Control

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Abstract—The cloud radio access network (Cloud-RAN) has recently been proposed as one cost-effective and energy-efficient technique for 5G wireless networks. By moving the signal processing functionality to a single baseband unit (BBU) pool, centralized signal processing and resource allocation are enabled in Cloud-RAN, thereby providing the promise of improving the energy efficiency via effective network adaptation and interference management. In this paper, we propose a holistic sparse optimization framework to design green Cloud-RAN by taking into consideration the power consumption of the fronthaul links, multicast services, as well as user admission control. Specifically, we first identify the sparsity structures in the solutions of both the network power minimization and user admission control problems, which call for adaptive remote radio head (RRH) selection and user admission. However, finding the optimal sparsity structures turns out to be NP-hard, with the coupled challenges of the ℓ_0 -norm based objective functions and the nonconvex quadratic QoS constraints due to multicast beamforming. In contrast to the previous works on convex but non-smooth sparsity inducing approaches, e.g., the group sparse beamforming algorithm based on the mixed ℓ_1/ℓ_2 -norm relaxation [1], we adopt the nonconvex but smoothed ℓ_p -minimization ($0 < p \leq 1$) approach to promote sparsity in the multicast setting, thereby enabling efficient algorithm design based on the principle of the majorization-minimization (MM) algorithm and the semidefinite relaxation (SDR) technique. In particular, an iterative reweighted- ℓ_2 algorithm is developed, which will converge to a Karush-Kuhn-Tucker (KKT) point of the relaxed smoothed ℓ_p -minimization problem from the SDR technique. We illustrate the effectiveness of the proposed algorithms with extensive simulations for network power minimization and user admission control in multicast Cloud-RAN.

Index Terms—5G networks, green communications, Cloud-RAN, multicast beamforming, sparse optimization, semidefinite relaxation, smoothed ℓ_p -minimization, and user admission control.

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I. INTRODUCTION

The great success of wireless industry is driving the proposal of new services and innovative applications, such as Internet of Things (IoT) and mobile Cyber-Physical applications, which yield an exponential growth of wireless traffic with billions of connected devices. To handle the enormous mobile data traffic, network densification and heterogeneity supported by various radio access technologies (e.g., massive MIMO [2] and millimeter-wave communications [3]) have become an irreversible trend in 5G wireless networks [4]. However, this will have a profound impact and bring formidable challenges to the design of 5G wireless communication systems in terms of energy efficiency, capital expenditure (CAPEX), operating expenditure (OPEX), and interference management [5]. In particular, the energy consumption will become prohibitively high in such dense wireless networks in the era of mobile data deluge. Therefore, to accommodate the upcoming diversified and high-volume data services in a cost-effective and energy-efficient way, a paradigm shift is required in the design of 5G wireless networks.

By leveraging the cloud computing technology [6], the cloud radio access network (Cloud-RAN) [7], [8] is a disruptive technology to address the key challenges of energy efficiency in 5G wireless networks. Specifically, by moving the baseband units (BBUs) into a single BBU pool (i.e., a cloud data center) with shared computation resources, scalable and parallel signal processing, coordinated resource allocation and cooperative interference management algorithms [9], [10] can be enabled among a large number of radio access points, thereby significantly improving the energy efficiency [1], [11] and spectral efficiency [12]. As the conventional compact base stations are replaced by low-cost and low-power remote radio heads (RRHs), which are connected to the BBU pool through high-capacity and low-latency fronthaul links, Cloud-RAN provides a cost-effective and energy-efficient way to densify the radio access networks [5].

While Cloud-RAN has a great potential to reduce the energy consumption of each RRH, with additional fronthaul link components and dense deployment of RRHs, new challenges arise for designing green Cloud-RAN. In particular, instead of only minimizing the total transmit power consumption via coordinated beamforming [13], the network power consumption consisting of the fronthaul link power consumption and the RRH power consumption should be adopted as the performance metric for designing green Cloud-RAN [1], [11], [14]. To minimize the network power consumption, a group sparse

beamforming framework was proposed in [1] to adaptively select the active RRHs and the corresponding fronthaul links via controlling the group sparsity structures of the beamforming vectors. Such an idea of exploiting sparsity structures in the solutions has also demonstrated its effectiveness in solving other mixed combinatorial optimization problems in Cloud-RAN, e.g., the data assignment problem [15] and the joint uplink and downlink network power minimization problem [11].

Although network adaption by selecting the active RRHs provides a promising way to minimize the network power consumption, it is critical to maximize the user capacity (i.e., the number of admitted users) when the network power minimization problem is infeasible [16], [17], [18]. This infeasibility issue may often occur in the scenarios with a large number of mobile devices requesting high data rates or some users with unfavorable channel conditions. Furthermore, exploiting the benefits of integrating diversified unicast and multicast services [19] has been well recognized as a promising way to improve the energy efficiency and user capacity, and thus multicast beamforming should be incorporated in Cloud-RAN. From the system design perspective, to design a green Cloud-RAN with multicast transmission, a holistic approach is needed to enable network adaptation for RRH selection and user admission in a unified way.

Unfortunately, such design problems fall into the category of highly complicated mixed combinatorial optimization problems. The key observation to address this challenge is that the network power minimization and user admission control can be achieved by adaptively selecting the active RRHs and admitting the mobile users (MUs) via controlling the sparsity structures in the corresponding solutions. Specifically, for the network power minimization problem, selecting active RRHs is equivalent to controlling the group sparsity structure in the aggregative multicast beamforming vector [1]. That is, all the beamforming coefficients of a particular RRH that is switched off need to be set to zeros simultaneously. For the user admission control problem that is needed when the network power minimization problem is infeasible, maximizing the number of admitted users is equivalent to minimizing the number of violated QoS constraints [20]. Mathematically, this is the same as minimizing the sparsity of the auxiliary vector indicating the violations of the QoS constraints. Based on these observations, we will thus formulate both design problems as sparse optimization problems based on the ℓ_0 -norm minimization in a unified framework, based on which efficient algorithms will be developed.

A. Contributions

Based on the above discussions, we propose a sparse optimization framework to design a multicast green Cloud-RAN as shown in Fig. 2, thereby enabling adaptive RRH selection and user admission. However, in contrast to the previous works on the multicast beamforming problem [21] with convex objectives but nonconvex QoS constraints and the group sparse beamforming problem [1] with convex QoS constraints but nonconvex objective functions in unicast Cloud-RAN, to design efficient algorithms for the resulting sparse

optimization problems in multicast Cloud-RAN, we need to address the following coupled challenges in a unified way:

- Nonconvex quadratic QoS constraints due to multicast transmission;
- Combinatorial objective functions for RRH selection and user admission.

In particular, we summarize the major contributions as follows:

- 1) A sparse optimization framework based on the ℓ_0 -norm minimization is proposed to design a multicast green Cloud-RAN by adaptive RRH selection and user admission via controlling the sparsity structures of the solutions.
- 2) To address the combinatorial challenges in the objective functions, we propose a nonconvex but smoothed ℓ_p -minimization approach to induce the sparsity structures in the solutions. The main advantage of this method is that it helps develop the group sparse inducing penalty with quadratic forms in the multicast beamforming vectors. Therefore, the objective function in the resulting group sparse inducing optimization problem is compatible with the nonconvex quadratic QoS constraints. The SDR technique can then be adopted to solve the resulting nonconvex quadratic group sparse inducing optimization problem.
- 3) To address the challenges of the nonconvex smoothed objective functions and the nonconvex quadratic QoS constraints, we propose an iterative reweighted- ℓ_2 algorithm to solve the resulting nonconvex smoothed ℓ_p -minimization problems based on the principle of the MM algorithm and the SDR technique. This algorithm is proven to converge to a KKT point of the relaxed smoothed ℓ_p -minimization problems over the convex constraint set using the SDR technique.
- 4) Simulation results will demonstrate the effectiveness of the proposed algorithms to minimize the network power minimization and maximize the user capacity, and their performance gains compared with the existing convex approximation approaches. In particular, the proposed algorithms can achieve near-optimal performance in the simulated settings.

B. Related Works

Sparse optimization by exploiting sparsity structures of the solutions has been proven to be very powerful to solve various hard optimization problems in machine learning, compressive sensing and high-dimensional statistics [22]. This approach has recently received enormous attentions in designing wireless networks, i.e., the group sparse beamforming approach for network adaption [1], [11], [23], and data assignment in wireless backhaul networks [15], [24]. In particular, the convex relaxation approaches, e.g., the ℓ_1 -minimization [17], the mixed ℓ_1/ℓ_2 -norm [1] and the mixed ℓ_1/ℓ_∞ -norm [25], have become popular due to the computational efficiency as well as performance guarantees in some scenarios [26].

To further improve the performance, there has been a great interest in applying nonconvex approaches in sparse optimization [27], [28], [29], [30] by enhancing sparsity. In particular,

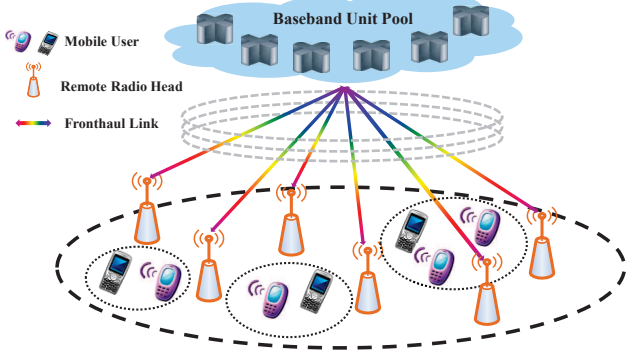


Fig. 1. The architecture of multicast Cloud-RAN, in which, all the RRHs are connected to a BBU pool through high-capacity and low-latency optical fronthaul links. All the MUs in the same dashed circle form a multicast group and request the same message. To enable full cooperation among all the RRHs, it is assumed that all the user data and channel state information (CSI) are available at the BBU pool.

it is observed that the nonconvex ℓ_p -minimization approach performs better than the traditional convex ℓ_1 -minimization especially when the underlying model is very sparse [28]. Motivated by this result, we adopt the ℓ_p -minimization approach to closely approximate the resulting ℓ_0 -norm based sparse optimization problems for multicast green Cloud-RAN. Furthermore, to deal with the unique challenges with the coupled nonconvex constraints and combinatorial objectives, thereby enabling efficient algorithm design, we will use a smoothed version of the ℓ_p -minimization approach to induce the sparsity structures in the solutions. Note that the existing work on the group sparse beamforming [1] can only handle problems with a combinatorial objective and convex second-order cone QoS constraints in unicast Cloud-RAN, and thus cannot be directly applied in the setting of multicast Cloud-RAN with nonconvex QoS constraints.

C. Organization

The remainder of the paper is organized as follows. Section II presents the system model and problem formulations. Section III presents an algorithmic framework for network power minimization and user admission control based on the smoothed ℓ_p -minimization. The iterative reweighted- ℓ_2 algorithm is developed in Section IV. Simulation results will be demonstrated in Section V. Finally, conclusions and discussions are presented in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we first introduce the system model of the multicast Cloud-RAN. Then, the network power minimization problem is formulated. For the scenario when it is not feasible to serve all the MUs, the user admission control problem is formulated. It will be revealed that both problems are sparse optimization problems, for which unique challenges will be identified.

A. System Model

We consider a multicast Cloud-RAN with L multi-antenna RRHs and K single-antenna MUs, where the l -th RRH

is equipped with N_l antennas, as shown in Fig. 1. Let $\mathcal{L} = \{1, \dots, L\}$ and $\mathcal{N} = \{1, \dots, K\}$ denote the sets of all the RRHs and all the MUs, respectively. We assume that the K MUs form M non-overlapping and non-empty multicast groups, which are denoted as $\{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_M\}$ with $\cup_i \mathcal{G}_i = \mathcal{N}$ and $\mathcal{G}_j \cap \mathcal{G}_j = \emptyset$ with \mathcal{G}_m as the set of MUs in the m -th multicast group. Let $\mathcal{M} = \{1, \dots, M\}$ be the set of all the multicast groups. We consider the downlink transmission, and the centralized signal processing is performed at the BBU pool [1], [7].

The propagation channel from the l -th RRH to the k -th MU is denoted as $\mathbf{h}_{kl} \in \mathbb{C}^{N_l}$, $\forall k, l$. Let $\mathbf{v}_{lm} \in \mathbb{C}^{N_l}$ be the transmit beamforming vector from the l -th RRH to MUs in multicast group \mathcal{G}_m . Then the transmit signal at the l -th RRH is given by

$$\mathbf{x}_l = \sum_{m=1}^M \mathbf{v}_{lm} s_m, \forall l \in \mathcal{L}, \quad (1)$$

where $s_m \in \mathbb{C}$ is the encoded information symbol for the multicast group m with $\mathbb{E}[|s_m|^2] = 1$. Assume that all the RRHs have their own transmit power constraints, which form the following feasible set of the beamforming vectors \mathbf{v}_{lm} 's,

$$\mathcal{V} = \left\{ \mathbf{v}_{lm} \in \mathbb{C}^{N_l} : \sum_{m=1}^M \|\mathbf{v}_{lm}\|_2^2 \leq P_l, \forall l \in \mathcal{L} \right\}, \quad (2)$$

where $P_l > 0$ is the maximum transmit power of the l -th RRH.

The received signal $y_{km} \in \mathbb{C}$ at MU k in the m -th multicast group is given by

$$y_{km} = \sum_{l=1}^L \mathbf{h}_{kl}^H \mathbf{v}_{lm} s_m + \sum_{i \neq m} \sum_{l=1}^L \mathbf{h}_{kl}^H \mathbf{v}_{li} s_i + n_k, \forall k \in \mathcal{G}_m, \quad (3)$$

where $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the additive Gaussian noise at MU k . We assume that s_m 's and n_k 's are mutually independent and all the users apply single user detection. Therefore, the signal-to-interference-plus-noise ratio (SINR) of MU k in multicast group m is given by

$$\Gamma_{k,m}(\mathbf{v}) = \frac{\mathbf{v}_m^H \boldsymbol{\Theta}_k \mathbf{v}_m}{\sum_{i \neq m} \mathbf{v}_i^H \boldsymbol{\Theta}_k \mathbf{v}_i + \sigma_k^2}, \forall k \in \mathcal{G}_m, \quad (4)$$

where $\boldsymbol{\Theta}_k = \mathbf{h}_k \mathbf{h}_k^H \in \mathbb{C}^{N \times N}$ with $N = \sum_{l=1}^L N_l$ and $\mathbf{h}_k = [\mathbf{h}_{k1}^H, \mathbf{h}_{k2}^H, \dots, \mathbf{h}_{kL}^H]^H \in \mathbb{C}^N$ as the channel vector consisting of the channel coefficients from all the RRHs to MU k , $\mathbf{v}_m = [\mathbf{v}_{1m}^H, \mathbf{v}_{2m}^H, \dots, \mathbf{v}_{Lm}^H]^H \in \mathbb{C}^N$ is the beamforming vector consisting of the beamforming coefficients from all the RRHs to the m -th multicast group, and $\mathbf{v} = [\tilde{\mathbf{v}}_l]_{l=1}^L \in \mathbb{C}^{MN}$ is the aggregative beamforming vector with $\tilde{\mathbf{v}}_l = [\mathbf{v}_{lm}]_{m=1}^M \in \mathbb{C}^{MN_l}$ as the beamforming vector consisting of all the beamforming coefficients from the l -th RRH to all the multicast groups.

B. Network Power Minimization for Green Cloud-RAN

With densely deployed RRHs, it is critical to enable energy-efficient transmission via centralized signal processing at the BBU pool. Coordinated beamforming among RRHs will help

reduce the transmit power. Due to the mobile data traffic variations in temporal and spatial domains, it is also critical to enable network adaptation to switch off some RRHs and the corresponding fronthaul links to save power [1], [31]. Thus, we need to consider the network power consumption when designing green Cloud-RAN, which is defined as the following combinatorial composite function parameterized by the beamforming coefficients [1],

$$p(\mathbf{v}) = p_1(\mathbf{v}) + p_2(\mathbf{v}), \quad (5)$$

with the total transmit power consumption, denoted as $p_1(\mathbf{v})$, and the total relative fronthaul links power consumption, denoted as $p_2(\mathbf{v})$, given as

$$p_1(\mathbf{v}) = \sum_{l=1}^L \sum_{m=1}^M \frac{1}{\eta_l} \|\mathbf{v}_{lm}\|_2^2, \quad (6)$$

and

$$p_2(\mathbf{v}) = \sum_{l=1}^L P_l^c I(\text{Supp}(\mathbf{v}) \cap \mathcal{V}_l \neq \emptyset), \quad (7)$$

respectively. Here, $I(\text{Supp}(\mathbf{v}) \cap \mathcal{V}_l \neq \emptyset)$ is an indicator function that takes value zero if $\text{Supp}(\mathbf{v}) \cap \mathcal{V}_l = \emptyset$ (i.e., all the beamforming coefficients at the l -th RRH are zeros, indicating that the corresponding RRH is switched off) and one otherwise, where \mathcal{V}_l is defined as $\mathcal{V}_l := \{M \sum_{l=1}^{L-1} N_l + 1, \dots, M \sum_{l=1}^L N_l\}$, and $\text{Supp}(\mathbf{v})$ is the support of the vector \mathbf{v} . In (6), $\eta_l > 0$ is the drain inefficiency coefficient of the radio frequency power amplifier with the typical value as 25%, and $P_l^c \geq 0$ in (7) is the relative fronthaul link power consumption [1], which is the static power saving when both the RRH and the corresponding fronthaul link are switched off. For the passive optical fronthaul network [32], P_l^c is given by $(P_{a,l}^{\text{rrh}} + P_{a,l}^{\text{fn}}) - (P_{s,l}^{\text{rrh}} + P_{s,l}^{\text{fn}})$ with $P_{a,l}^{\text{rrh}}$ ($P_{s,l}^{\text{rrh}}$) and $P_{a,l}^{\text{fn}}$ ($P_{s,l}^{\text{fn}}$) as the power consumptions for the l -th RRH and the l -th fronthaul link in the active (sleep) mode, respectively. The typical values are $P_{a,l}^{\text{rrh}} = 6.8W$, $P_{s,l}^{\text{rrh}} = 4.3W$, $P_{a,l}^{\text{fn}} = 3.85W$, $P_{s,l}^{\text{fn}} = 0.75W$ and $P_l^c = 5.6W$ [1], [32]. *Note that the energy consumption of the optical fronthaul links should depend on the receiving periods and data transmission [32], which is a function of the beamforming vectors.* Given the target SINR requirements $(\gamma_1, \gamma_2, \dots, \gamma_K)$ for all the MUs, to design a multicast green Cloud-RAN, we propose to minimize the network power consumption subject to the QoS constraints and the RRH transmit power constraints. Specifically, we have the following QoS constraints

$$\Gamma_{k,m}(\mathbf{v}) \geq \gamma_k, \forall k \in \mathcal{G}_m, m \in \mathcal{M}, \quad (8)$$

which can be rewritten as the following quadratic constraints

$$F_{k,m}(\mathbf{v}) = \gamma_k \left(\sum_{i \neq m} \mathbf{v}_i^H \Theta_k \mathbf{v}_i + \sigma_k^2 \right) - \mathbf{v}_m^H \Theta_k \mathbf{v}_m \leq 0, \quad (9)$$

for any $k \in \mathcal{G}_m$ and $m \in \mathcal{M}$, which are nonconvex. Therefore, the network power minimization problem can be formulated as

$$\begin{aligned} & \underset{\mathbf{v} \in \mathcal{V}}{\text{minimize}} && p_1(\mathbf{v}) + p_2(\mathbf{v}) \\ & \text{subject to} && F_{k,m}(\mathbf{v}) \leq 0, \forall k \in \mathcal{G}_m, m \in \mathcal{M}, \end{aligned} \quad (10)$$

which is highly intractable due to the nonconvex combinatorial composite objective and the nonconvex quadratic QoS constraints (9). When there are some RRHs needed to be switched off to minimize the network power consumption, the solution of problem (10) has the *group sparsity* structure [1]. That is, all the beamforming coefficients in $\tilde{\mathbf{v}}_l$, which forms a group at the l -th RRH, are set to be zeros simultaneously if the l -th RRH needs to be switched off.

Therefore, inspired by the fact that the solution of problem (10) has the group sparsity structure in the aggregative beamforming vector \mathbf{v} , the weighted mixed ℓ_1/ℓ_2 -norm was proposed in [1] to relax the combinatorial composite function as the tightest convex surrogate to induce the group sparsity structure in the solution \mathbf{v} to guide the RRH selection, defined as

$$\mathcal{J}(\mathbf{v}) = \sum_{l=1}^L \rho_l \|\tilde{\mathbf{v}}_l\|_2, \quad (11)$$

where $\rho_l > 0$ is the weight for the beamforming coefficients group $\tilde{\mathbf{v}}_l$ at RRH l .

To handle the nonconvex QoS constraints in (9), we propose to lift the problem to higher dimensions with variables $\mathbf{Q}_m = \mathbf{v}_m \mathbf{v}_m^H \in \mathbb{C}^{N \times N}, \forall m$, which will help to apply the semidefinite relaxation (SDR) technique. However, we cannot extract the variables \mathbf{Q}_m 's from the non-smooth mixed ℓ_1/ℓ_2 -norm (11). Therefore, this convex relaxation approach cannot be directly applied to solve the network power minimization problem in multicast Cloud-RAN. Instead, to leverage the SDR technique, we need to develop a new group sparsity inducing approach with quadratic forms of the beamforming vectors, which will be presented in Section III and form one major contribution of this paper.

C. User Admission Control

With QoS constraints for potentially a large number of MUs in the serving area, it may happen that the network power minimization problem (10) is infeasible from time to time. In such scenarios, the design problem will be to maximize the user capacity (i.e., the maximum number of MUs that can be supported) via user admission control [16], [17], while serving these MUs with the minimum transmit power. While this aspect is ignored in [1], it is critical for practical applications. Mathematically, by adding auxiliary variables x_k 's to the right-hand side of the corresponding inequalities in (9), to maximize the number of admitted MUs is equivalent to minimize the number of non-zero x_k 's [20, Section 11.4]. Therefore, the user admission control problem can be formulated as the following sparsity minimization problem,

$$\begin{aligned} & \underset{\mathbf{v} \in \mathcal{V}, \mathbf{x} \in \mathbb{R}_+^K}{\text{minimize}} && \|\mathbf{x}\|_0 \\ & \text{subject to} && F_{k,m}(\mathbf{v}) \leq x_k, \forall k \in \mathcal{G}_m, m \in \mathcal{M}, \end{aligned} \quad (12)$$

where \mathbb{R}_+^K represents the K -dimensional nonnegative real vector. Once the admitted MUs are determined, coordinated beamforming can then be applied to minimize the total transmit power. The solution $\mathbf{x} = [x_k]$ of problem (12) has the *individual sparsity* structure, i.e., $x_k = 0$ indicates that the

k -th MU can be admitted. Therefore, the sparsity level will be increased if more MUs can be admitted.

Observing that both the sparse optimization problems (10) and (12) possess the same structure with nonconvex quadratic constraints and combinatorial objectives, in this paper, we will propose a unified way to handle them based on a smoothed ℓ_p -minimization approach.

D. Problem Analysis

In this subsection, we analyze the unique challenges of the network power minimization problem (10) and the user admission control problem (12) in the context of multicast Cloud-RAN. In particular, the differences from the previous works [1], [11] will be highlighted.

1) *Nonconvex Quadratic Constraints*: The physical-layer multicast beamforming problem [21] yields nonconvex quadratic QoS constraints (9), while only unicast services were considered in [1], [11]. The SDR technique [33] proves to be an effective way to obtain good approximate solutions to these problems by lifting the quadratic constraints into higher dimensions, which will also be adopted in this paper.

2) *Combinatorial Objective Functions*: Although the SDR technique provides an efficient way to convexify the non-convex quadratic QoS constraints in problems (10) and (12), the inherent combinatorial objective functions still make the resulting problems highly intractable. While the ℓ_1 -norm can be adopted to relax the ℓ_0 -norm in problem (12) after SDR, which is also known as the sum-of-infeasibilities relaxation in optimization theory [17], [20], the convex relaxation approach based on the non-smooth mixed ℓ_1/ℓ_2 -norm [1] cannot be applied to problem (10), as we cannot extract the variables \mathbf{Q}_m 's after SDR.

Therefore, in this paper, we propose a new powerful approach to induce the sparsity structures in the solutions for both problems (10) and (12), which is based on a smoothed ℓ_p -norm [27]. The main advantage of this method is that it can help develop group sparsity inducing penalties with quadratic forms in the beamforming vectors, thereby leveraging the SDR technique to relax the nonconvex quadratic QoS constraints for problem (10). Furthermore, by adjusting the parameter p , this approach has the potential to yield a better approximation for the original ℓ_0 -norm based objectives, thereby providing improved solutions for problems (10) and (12). The smoothed ℓ_p -minimization framework will be presented in Section III, while the iterative reweighted- ℓ_2 algorithm will be developed in Section IV to solve the smoothed ℓ_p -minimization problem.

III. A SMOOTHED ℓ_p -MINIMIZATION FRAMEWORK FOR NETWORK POWER MINIMIZATION WITH USER ADMISSION CONTROL

In this section, we first present the smoothed ℓ_p -minimization method as a unified way to induce sparsity structures in the solutions of problems (10) and (12), thereby providing guidelines for RRH selection and user admission. After obtaining the active RRHs and admitted MUs by performing the corresponding selection procedure, we will

minimize the total transmit power consumption for the size-reduced network. The algorithmic advantages and performance improvement of the proposed smoothed ℓ_p -minimization based framework will be revealed in the sequel.

A. Smoothed ℓ_p -Minimization for Sparsity Inducing

To promote sparse solutions, instead of applying the convex ℓ_1 -minimization approach, we adopt a nonconvex ℓ_p -minimization ($0 < p \leq 1$) approach to seek a tighter approximation of the ℓ_0 -norm in the objective functions in problems (10) and (12) [27]. This is motivated by the fact that the ℓ_0 -norm $\|\mathbf{z}\|_0$ is the limit as $p \rightarrow 0$ of $\|\mathbf{z}\|_p^p$ in the sense of $\|\mathbf{z}\|_0 = \lim_{p \rightarrow 0} \|\mathbf{z}\|_p^p = \lim_{p \rightarrow 0} \sum |z_i|^p$. We thus adopt $\|\mathbf{x}\|_p^p$ as the optimization objective function to seek sparser solutions. Furthermore, to enable efficient algorithm design as well as induce the quadratic forms in the resulting approximation problems, we instead adopt the following smoothed version of $\|\mathbf{z}\|_p^p$ to induce sparsity:

$$f_p(\mathbf{z}; \epsilon) := \sum_{i=1}^m (z_i^2 + \epsilon^2)^{p/2}, \quad (13)$$

for $\mathbf{z} \in \mathbb{R}^m$ and some small fixed regularizing parameter $\epsilon > 0$. Based on the smoothed ℓ_p -norm (13), we will present the algorithmic advantages of the smoothed ℓ_p -minimization approach in Section IV.

1) *Smoothed ℓ_p -Minimization for Group Sparsity Inducing*: For network power minimization, to seek quadratic forms of beamforming vectors in the objective functions to leverage the SDR technique for the non-convex quadratic QoS constraints, we adopt the smoothed ℓ_p -norm $f_p(\mathbf{z}; \epsilon)$ (13) to induce group sparsity in the aggregative beamforming vector \mathbf{v} for problem (10), resulting the following optimization problem:

$$\begin{aligned} & \underset{\mathbf{v} \in \mathcal{V}}{\text{minimize}} && \sum_{l=1}^L \rho_l (\|\tilde{\mathbf{v}}_l\|_2^2 + \epsilon^2)^{p/2} \\ & \text{subject to} && F_{k,m}(\mathbf{v}) \leq 0, \forall k \in \mathcal{G}_m, m \in \mathcal{M}. \end{aligned} \quad (14)$$

The induced (approximated) group sparse beamformers will guide the RRH selection. The resulting problem (14) thus becomes a quadratic optimization problem and enjoys the algorithmic advantages.

2) *Smoothed ℓ_p -Minimization for User Admission Control*: For user admission control, we adopt the smoothed ℓ_p -norm (13) to approximate the objective function in problem (12), yielding the following optimization problem:

$$\begin{aligned} & \underset{\mathbf{v} \in \mathcal{V}, \mathbf{x} \in \mathbb{R}_+^K}{\text{minimize}} && \sum_{k=1}^K (x_k^2 + \epsilon^2)^{p/2} \\ & \text{subject to} && F_{k,m}(\mathbf{v}) \leq x_k, \forall k \in \mathcal{G}_m, m \in \mathcal{M}. \end{aligned} \quad (15)$$

This will help to induce individual sparsity in the auxiliary variables \mathbf{x} , thereby guiding the user admission.

Although the resulting optimization problems (14) and (15) are still nonconvex, they can readily be solved by the SDR technique and the MM algorithm. Specifically, we will demonstrate that the nonconvex quadratic QoS constraints can be convexified by the SDR technique in the next subsection.

The resulting convex constrained smoothed ℓ_p -minimization problem will be solved by the MM algorithm, yielding an iterative reweighted- ℓ_2 algorithm, as will be presented in Section IV.

B. SDR for Nonconvex Quadratic Constraints

In this part, we will demonstrate how to apply the SDR technique to resolve the challenge of the nonconvex quadratic QoS constraints in both problems (14) and (15). Specifically, let $\mathbf{Q}_m = \mathbf{v}_m^H \mathbf{v}_m \in \mathbb{C}^{N \times N}$ with $\text{rank}(\mathbf{Q}_m) = 1, \forall m \in \mathcal{M}$. Therefore, the QoS constraint (9) can be rewritten as

$$L_{k,m}(\mathbf{Q}) \leq 0, \forall k \in \mathcal{G}_m, m \in \mathcal{M}, \quad (16)$$

with $L_{k,m}(\mathbf{Q})$ given by

$$L_{k,m}(\mathbf{Q}) = \gamma_k \left(\sum_{i \neq m} \text{Tr}(\Theta_k \mathbf{Q}_i) + \sigma_k^2 \right) - \text{Tr}(\Theta_k \mathbf{Q}_m), \quad (17)$$

where $\mathbf{Q} = [\mathbf{Q}_m]_{m=1}^M$ and all \mathbf{Q}_m 's are rank-one constrained. The per-RRH transmit power constraints (2) can be rewritten as

$$\mathcal{Q} = \left\{ \mathbf{Q}_m \in \mathbb{C}^N : \sum_{m=1}^M \text{Tr}(\mathbf{C}_{lm} \mathbf{Q}_m) \leq P_l, \forall l \in \mathcal{L} \right\}, \quad (18)$$

where $\mathbf{C}_{lm} \in \mathbb{R}^{n \times n}$ is a block diagonal matrix with the identity matrix \mathbf{I}_{N_l} as the l -th main diagonal block square matrix and zeros elsewhere.

Therefore, by dropping the rank-one constraints for all the matrices \mathbf{Q}_m 's based on the principle of the SDR technique, problem (14) can be relaxed as

$$\begin{aligned} \mathcal{P} : \text{minimize}_{\mathbf{Q} \in \mathcal{Q}} \quad & \sum_{l=1}^L \rho_l \left(\sum_{m=1}^M \text{Tr}(\mathbf{C}_{lm} \mathbf{Q}_m) + \epsilon^2 \right)^{p/2} \\ \text{subject to} \quad & L_{k,m}(\mathbf{Q}) \leq 0, \forall k \in \mathcal{G}_m, \\ & \mathbf{Q}_m \succeq \mathbf{0}, \forall m \in \mathcal{M}. \end{aligned} \quad (19)$$

Similarly, by dropping the rank-one constraints of all the matrices \mathbf{Q}_m 's, problem (15) can be relaxed as

$$\begin{aligned} \mathcal{D} : \text{minimize}_{\mathbf{Q} \in \mathcal{Q}, \mathbf{x} \in \mathbb{R}_+^K} \quad & \sum_{k=1}^K (x_k^2 + \epsilon^2)^{p/2} \\ \text{subject to} \quad & L_{k,m}(\mathbf{Q}) \leq x_k, \forall k \in \mathcal{G}_m, \\ & \mathbf{Q}_m \succeq \mathbf{0}, \forall m \in \mathcal{M}. \end{aligned} \quad (20)$$

Although problems \mathcal{P} and \mathcal{D} are still nonconvex due to the nonconvex objective functions, the resulting smoothed ℓ_p -minimization problems preserve the algorithmic advantages, as will be presented in Section IV. In particular, an iterative reweighted- ℓ_2 algorithm will be developed in Section IV based on the principle of the MM algorithm to find a stationary point to the non-convex smoothed ℓ_p -minimization problems \mathcal{P} and \mathcal{D} .

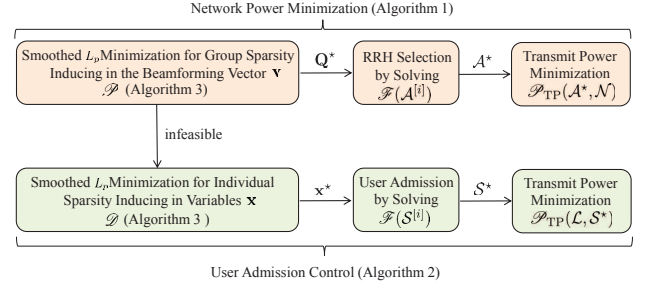


Fig. 2. Sparse optimization for network power minimization and user admission control in multicast Cloud-RAN.

C. A Sparse Optimization Framework for Network Power Minimization with User Admission Control

Denote the solutions of problems \mathcal{P} and \mathcal{D} as \mathbf{Q}^* and \mathbf{x}^* , respectively. Based on these induced (approximated) sparse solutions, we propose a sparse optimization framework for network power minimization and user admission control in multicast Cloud-RAN. The main idea is illustrated in Fig. 2 and details will be explained in the following. In particular, Algorithm 3 will be developed in Section IV, which yields a KKT point for problems \mathcal{P} and \mathcal{D} .

1) *Network Power Minimization*: If problem \mathcal{P} is feasible, once obtaining its solution \mathbf{Q}^* , we can extract the group sparsity structure information in the beamforming vector \mathbf{v} based on the relation: $\|\tilde{\mathbf{v}}_l\|_2 = \sqrt{\sum_{m=1}^M \text{Tr}(\mathbf{C}_{lm} \mathbf{Q}_m)}$, which will be zero if all the beamforming coefficients in $\tilde{\mathbf{v}}_l$ are zeros simultaneously. By further incorporating system parameters to improve the performance [1], we adopt the following RRH ordering criteria to determine the priorities of the RRHs that should be switched off to minimize the network power consumption,

$$\theta_l = \sqrt{\frac{\eta_l \kappa_l}{P_l^c}} \left(\sum_{m=1}^M \text{Tr}(\mathbf{C}_{lm} \mathbf{Q}_m^*) \right)^{1/2}, \forall l \in \mathcal{L}, \quad (21)$$

where $\kappa_l = \sum_{k=1}^K \|\mathbf{h}_{kl}\|_2^2$ is the channel gain from the l -th RRH to all the MUs. The RRHs with a smaller parameter θ_l will have a higher priority to be switched off. Intuitively, the RRH with a lower channel power gain κ_l , lower drain inefficiency efficiency η_l , lower beamforming gain $\|\tilde{\mathbf{v}}_l\|_2$, and higher relative fronthaul link power consumption P_l^c , should have a higher priority to be switched off.

In this paper, we adopt a simple RRH selection procedure, i.e., the bi-section method, to switch off RRHs. This method was shown to provide good performance in [1]. Specifically, based on the RRH ordering rule in (21), we sort the coefficients in the ascending order: $\theta_{\pi_1} \leq \theta_{\pi_2} \leq \dots \leq \theta_{\pi_L}$ to determine the active RRHs. Let J_0 be the maximum number of RRHs that can be switched off such that the remaining RRHs can support the QoS requirements for all the MUs. To find J_0 , in each bi-section search iteration, we need to solve the following size-reduced convex feasibility problems based on the SDR

technique,

$$\begin{aligned} \mathcal{F}(\mathcal{A}^{[i]}) : \text{find } & \mathbf{Q}_1^{[i]}, \dots, \mathbf{Q}_M^{[i]} \\ \text{subject to } & L_{k,m}(\{\mathbf{Q}_m^{[i]}\}_{m \in \mathcal{M}}) \leq 0, \forall k \in \mathcal{G}_m, \\ & \mathbf{Q}_m^{[i]} \succeq \mathbf{0}, \mathbf{Q}_m^{[i]} \in \mathcal{Q}^{[i]}, \forall m \in \mathcal{M}, \end{aligned} \quad (22)$$

where $\mathbf{Q}_m^{[i]} \in \mathbb{C}^{(\sum_{l \in \mathcal{A}^{[i]}} N_l) \times (\sum_{l \in \mathcal{A}^{[i]}} N_l)}$ with $\mathcal{A}^{[i]} = \{\pi_{i+1}, \dots, \pi_L\}$ as the active RRH set, $\mathcal{Q}^{[i]}$ represents the per-RRH transmit power constraints for the active RRHs in $\mathcal{A}^{[i]}$ and the QoS constraints are obtained after omitting the channel coefficients corresponding to the left-out RRHs. If problem $\mathcal{F}(\mathcal{A}^{[i]})$ is feasible, it implies that a feasible solution exists to $\mathcal{F}(\mathcal{A}^{[J]})$ for all $J < i$. Likewise, if problem $\mathcal{F}(\mathcal{A}^{[i]})$ is infeasible, it implies that no feasible solution exists for any $J > i$. Therefore, determining the largest $J = J_0$ that results in a feasible solution to problem $\mathcal{F}(\mathcal{A}^{[J]})$ can be accomplished by solving no more than $(1 + \lceil \log(1 + L) \rceil)$ such feasibility problems (22) via bi-section search [20]. Specifically, the set $\{0, 1, \dots, L\}$ is guaranteed to contain J_0 , i.e., $J_0 \in \{0, 1, \dots, L\}$ at each step. In each iteration, the set is divided in two sets, i.e., bisected, so the length of the set after k iterations is $2^{-k}(L+1)$ with $(L+1)$ as the length of the initial set. It follows that exactly $(1 + \lceil \log_2(1 + L) \rceil)$ iterations are required before the bi-section algorithm terminates. This procedure mainly reduces the relative fronthaul link power consumption by switching off RRHs and the corresponding fronthaul links.

Finally, denote the set of active RRHs as $\mathcal{A}^* = \{\pi_{J_0+1}, \dots, \pi_L\}$. To further reduce the network power consumption, we need to solve the following size-reduced transmit power minimization problem with $|\mathcal{A}^*|$ RRHs and $|\mathcal{N}|$ MUs based on the SDR technique,

$$\begin{aligned} \mathcal{P}_{\text{TP}}(\mathcal{A}^*, \mathcal{N}) : \text{minimize } & \sum_{l \in \mathcal{A}^*} \sum_{m=1}^M \frac{1}{\eta_l} \text{Tr}(\mathbf{C}_{lm} \mathbf{Q}_m^{[J_0]}) \\ \text{subject to } & L_{k,m}(\mathbf{Q}^{[J_0]}) \leq 0, \forall k \in \mathcal{G}_m, \\ & \mathbf{Q}_m^{[J_0]} \succeq \mathbf{0}, \forall m \in \mathcal{M}, \end{aligned} \quad (23)$$

which is a semidefinite programming (SDP) problem and can be solved in polynomial time using the interior-point method.

The algorithm for solving the network power minimization problem is presented in Algorithm 1.

2) *User Admission Control*: When problem \mathcal{P} is infeasible, we need to perform user admission control to maximize the user capacity. Specifically, let \mathbf{x}^* be the solution to the individual sparsity inducing optimization problem \mathcal{D} . Observe that x_k represents the gap between the target SINR and the achievable SINR for MU k . We thus propose to admit the MUs with the smallest x_i 's [17], [18]. We order the coefficients in the descending order: $x_{\pi_1} \geq x_{\pi_2} \geq \dots \geq x_N$. The bi-section search procedure will be adopted to find the maximum number of admitted MUs. Let N_0 be the minimum number of MUs to be removed such that all the RRHs can support the QoS requirements for all the remaining MUs. To determine the value of N_0 , a sequence of the following convex sized-reduced

Algorithm 1: Network Power Minimization

Step 0: Solve the group sparse inducing optimization problem \mathcal{P} (19) using Algorithm 3 in Section IV.

- 1) **If** it is infeasible, **go to Algorithm 2** for user admission control.
- 2) **If** it is feasible, obtain the solutions \mathbf{Q}_m^* 's, calculate the ordering criterion (21), and sort them in the ascending order: $\theta_{\pi_1} \leq \dots \leq \theta_{\pi_L}$, **go to Step 1**.

Step 1: Initialize $J_{\text{low}} = 0$, $J_{\text{up}} = L$, $i = 0$.

Step 2: Repeat

- 1) Set $i \leftarrow \lfloor \frac{J_{\text{low}} + J_{\text{up}}}{2} \rfloor$.
- 2) Solve problem $\mathcal{F}(\mathcal{A}^{[i]})$ (22): if it is infeasible, set $J_{\text{up}} = i$; otherwise, set $J_{\text{low}} = i$.

Step 3: Until $J_{\text{up}} - J_{\text{low}} = 1$, obtain $J_0 = J_{\text{low}}$ and obtain the optimal active RRH set $\mathcal{A}^* = \{\pi_{J_0+1}, \dots, \pi_L\}$.

Step 4: Solve problem $\mathcal{P}_{\text{TP}}(\mathcal{A}^*, \mathcal{N})$ (23) to obtain the multicast beamforming vectors for the active RRHs.

End

feasibility problems need to be solved,

$$\begin{aligned} \mathcal{F}(\mathcal{S}^{[i]}) : \text{find } & \{\mathbf{Q}_m\}_{m \in \mathcal{M}^{[i]}} \\ \text{subject to } & L_{k,m}(\{\mathbf{Q}_m\}_{m \in \mathcal{M}^{[i]}}) \leq 0, \forall k \in \mathcal{G}_m, \\ & \mathbf{Q}_m \succeq \mathbf{0}, \mathbf{Q}_m \in \mathcal{Q}^{[i]}, \forall m \in \mathcal{M}^{[i]}, \end{aligned} \quad (24)$$

where $\mathcal{S}^{[i]} = \{\pi_{i+1}, \dots, \pi_K\}$ denotes the set of admitted MUs, $\mathcal{M}^{[i]} = \{m : \mathcal{G}_m \cap \mathcal{S}^{[i]} \neq \emptyset\}$ is the set of multicast groups, and $\mathcal{Q}^{[i]}$ represents the per-RRH transmit power constraints with the served multicast groups $\mathcal{M}^{[i]}$. In this way, the QoS constraints of the admitted MUs will be satisfied.

Finally, let $\mathcal{S} = \{\pi_{N_0+1}, \dots, \pi_K\}$ be the admitted MUs. We need to solve the same type of size-reduced transmit power minimization problem (23) with $|\mathcal{L}|$ RRHs and $|\mathcal{S}^*|$ MUs to find the multicast transmit beamforming vectors for all the admitted MUs. We denote this problem as $\mathcal{P}_{\text{TP}}(\mathcal{L}, \mathcal{S}^*)$.

The proposed user admission control algorithm is presented in Algorithm 2.

Algorithm 2: User Admission Control

Step 0: Solve the individual sparse inducing optimization problem \mathcal{D} (20) using Algorithm 3 in Section IV. Obtain the solution \mathbf{x}^* and sort the entries in the descending order: $x_{\pi_1} \geq \dots \geq x_N$, **go to Step 1**.

Step 1: Initialize $N_{\text{low}} = 0$, $N_{\text{up}} = K$, $i = 0$.

Step 2: Repeat

- 1) Set $i \leftarrow \lfloor \frac{N_{\text{low}} + N_{\text{up}}}{2} \rfloor$.
- 2) Solve problem $\mathcal{F}(\mathcal{S}^{[i]})$ (24): if it is feasible, set $N_{\text{up}} = i$; otherwise, set $N_{\text{low}} = i$.

Step 3: Until $N_{\text{up}} - N_{\text{low}} = 1$, obtain $N_0 = N_{\text{up}}$ and obtain the admitted MU set $\mathcal{S}^* = \{\pi_{N_0+1}, \dots, \pi_K\}$.

Step 4: Solve problem $\mathcal{P}_{\text{TP}}(\mathcal{L}, \mathcal{S}^*)$ (23) to obtain the multicast beamforming vectors for the admitted MUs.

End

Remark 1 (Rank-One Approximation After SDR): The solutions for the SDR based optimization problems \mathcal{P} , \mathcal{D} , $\mathcal{F}(\mathcal{A}^{[i]})$, $\mathcal{F}(\mathcal{S}^{[i]})$ and $\mathcal{P}_{\text{TP}}(\mathcal{A}, \mathcal{S})$ may not be rank-one. If the rank-one solutions are failed to be obtained, the Gaussian randomization method [33] will be employed to obtain the feasible rank-one approximate solution. Specifically, the candidate multicast beamforming vectors are generated from the solution of the SDR problems, and one is picked yielding a feasible solution to the original problem with the minimum value of the objective function. The feasibility for the original problem is guaranteed by solving a sequence of multigroup multicast power control problems with the fixed beamforming directions via linear programming [33]. Please refer to [33, Section IV] for more details on the Gaussian randomization method and we adopt this method in our simulations to find approximate rank-one feasible solutions. While the optimality of this randomization method for general problems remains unknown, it has been widely applied and shown to provide good performance [21], [25].

3) *Complexity Analysis and Discussions:* To implement Algorithm 1 and Algorithm 2, a sequence of SDP optimization or feasibility problems (e.g., \mathcal{P} , \mathcal{D} , $\mathcal{F}(\mathcal{A}^{[i]})$, $\mathcal{F}(\mathcal{S}^{[i]})$ and $\mathcal{P}_{\text{TP}}(\mathcal{A}, \mathcal{S})$) need to be solved. In particular, to find the active RRH set \mathcal{A}^* and admitted MU set \mathcal{S}^* , we need to solve no more than $(1 + \lceil \log(1 + L) \rceil)$ and $(1 + \lceil \log(1 + K) \rceil)$ SDP feasibility problems $\mathcal{F}(\mathcal{A}^{[i]})$ and $\mathcal{F}(\mathcal{S}^{[i]})$, respectively. In addition, to solve the SDP problem $\mathcal{P}_{\text{TP}}(\mathcal{L}, \mathcal{N})$ (23) with M matrix optimization variables of size $N \times N$ and $(K + L)$ linear constraints, the interior-point method [20] will take $\mathcal{O}(\sqrt{MN} \log(1/\epsilon))$ iterations and cost $\mathcal{O}(M^3 N^6 + (K + L)MN^2)$ floating point operations to achieve an optimal solution with accuracy $\epsilon > 0$. Therefore, this makes the proposed network power minimization and user admission algorithms difficult to scale to large problem sizes with a large number of RRHs and/or MUs. To further improve the computational efficiency of the proposed SDP based algorithms, one promising approach is to apply the alternating direction method of multipliers (ADMM) algorithm [34] by leveraging parallelism in the cloud computing environment in the BBU pool [12]. This is, however, an on-going research topic, and we will leave it as our future work.

IV. ITERATIVE REWEIGHTED- ℓ_2 ALGORITHM FOR SMOOTHED ℓ_p -MINIMIZATION

In this section, we first develop an iterative reweighted- ℓ_2 algorithm to solve a general non-convex smoothed ℓ_p -minimization problem based on the principle of the MM algorithm. We then present how to apply this algorithm to solve the problems \mathcal{P} and \mathcal{D} to induce sparsity structures in the solutions, thereby guiding the RRH selection and user admission.

A. Iterative Reweighted- ℓ_2 Algorithm

Consider the following smoothed ℓ_p -minimization problem,

$$\mathcal{P}_{\text{sm}}(\epsilon) : \underset{\mathbf{z} \in \mathcal{C}}{\text{minimize}} \quad f_p(\mathbf{z}; \epsilon) := \sum_{i=1}^m (z_i^2 + \epsilon^2)^{p/2}, \quad (25)$$

where \mathcal{C} is an arbitrary convex set, $\mathbf{z} \in \mathbb{R}^m$ and $\epsilon > 0$ is some fixed regularizing parameter. In the following, we first prove that the optimal solution of the smoothed ℓ_p -minimization problem $\mathcal{P}_{\text{sm}}(\epsilon)$ is also optimal for the original non-smooth ℓ_p -minimization problem (i.e., $\mathcal{P}_{\text{sm}}(0)$) when ϵ is small. We then demonstrate the algorithmic advantages of the smoothness in the procedure of developing the iterative reweighted- ℓ_2 algorithm.

1) *Optimality of Smoothing the ℓ_p -Norm:* The set of KKT points of problem $\mathcal{P}_{\text{sm}}(\epsilon)$ is given as

$$\Omega(\epsilon) = \{\mathbf{z} \in \mathcal{C} : 0 \in \nabla_{\mathbf{z}} f_p(\mathbf{z}; \epsilon) + \mathcal{N}_{\mathcal{C}}(\mathbf{z})\}, \quad (26)$$

where $\mathcal{N}_{\mathcal{C}}(\mathbf{z})$ is the normal cone of a convex set \mathcal{C} at point \mathbf{z} consisting of the outward normals to all hyperplanes that support \mathcal{C} at \mathbf{z} , i.e.,

$$\mathcal{N}_{\mathcal{C}}(\mathbf{z}) := \{\mathbf{s} : \langle \mathbf{s}, \mathbf{x} - \mathbf{z} \rangle \leq 0, \forall \mathbf{x} \in \mathcal{C}\}. \quad (27)$$

Define the deviation of a given set \mathcal{Z}_1 from another set \mathcal{Z}_2 as [35],

$$\mathbb{D}(\mathcal{Z}_1, \mathcal{Z}_2) = \sup_{\mathbf{z}_1 \in \mathcal{Z}_1} \left(\inf_{\mathbf{z}_2 \in \mathcal{Z}_2} \|\mathbf{z}_1 - \mathbf{z}_2\| \right). \quad (28)$$

We then have the following theorem on the relationship between the smoothed ℓ_p -minimization problem $\mathcal{P}_{\text{sm}}(\epsilon)$ and the original non-smooth ℓ_p -minimization problem $\mathcal{P}_{\text{sm}}(0)$.

Theorem 1: Let Ω_ϵ be the set of KKT points of problem $\mathcal{P}_{\text{sm}}(\epsilon)$. Then, we have

$$\lim_{\epsilon \searrow 0} \mathbb{D}(\Omega(\epsilon), \Omega(0)) = 0. \quad (29)$$

Proof: Please refer to Appendix A for details. ■

This theorem indicates that any limit of the sequence of KKT points of problem $\mathcal{P}_{\text{sm}}(\epsilon)$ is a KKT pair of problem $\mathcal{P}_{\text{sm}}(0)$ when ϵ is small enough. That is, at least a local optimal solution can be achieved. In the sequel, we will focus on finding a KKT point of problem $\mathcal{P}_{\text{sm}}(\epsilon)$ with a small ϵ , yielding good approximations to the KKT points of the ℓ_p -minimization problem $\mathcal{P}_{\text{sm}}(0)$ to induce sparsity in the solutions.

2) *The MM Algorithm for the Smoothed ℓ_p -Minimization:* With the established asymptotic optimality, we then leverage the principle of the MM algorithm to solve problem (25). Basically, this algorithm generates the iterates $\{\mathbf{z}_n\}_{n=1}^\infty$ by successively minimizing upper bounds $Q(\mathbf{z}; \mathbf{z}^{[n]})$ of the objective function $f_p(\mathbf{z}; \epsilon)$. The quality of the upper bounds will control the convergence (rate) and optimality of the resulting algorithms. Inspired by the results in the expectation-maximization (EM) algorithm [36], [37], we adopt the upper bounds in the following proposition to approximate the smoothed ℓ_p -norm.

Proposition 1: Given the iterate $\mathbf{z}^{[n]}$ at the n -th iteration, an upper bound for the objective function of the smoothed ℓ_p -norm $f_p(\mathbf{z}; \epsilon)$ can be constructed as follows,

$$Q(\mathbf{z}; \omega^{[n]}) := \sum_{i=1}^m \omega_i^{[n]} z_i^2, \quad (30)$$

where

$$\omega_i^{[n]} = \frac{p}{2} \left[\left(z_i^{[n]} \right)^2 + \epsilon^2 \right]^{\frac{p}{2}-1}, \forall i = 1, \dots, m. \quad (31)$$

From the weights given in (31), it is clear that, by adding the regularizer parameter $\epsilon > 0$, we can avoid yielding infinite values when some z_i 's become zeros in the iterations.

Proof: Define the approximation error as

$$f_p(\mathbf{z}; \epsilon) - Q(\mathbf{z}; \boldsymbol{\omega}^{[n]}) = \sum_{i=1}^m [\kappa(z_i^2) - \kappa'((z_i^{[n]})^2) z_i^2], \quad (32)$$

where $\kappa(s) = (s + \epsilon^2)^{p/2}$ with $s \geq 0$. The sound property of the Q -function (30) is that the approximation error (32) attains its maximum at $\mathbf{z} = \mathbf{z}^{[n]}$. In particular, we only need to prove that the function $g(s) = \kappa(s) - \kappa'(s^{[n]})s$ with $s \geq 0$ attains the maximum at $s = s^{[n]}$. This is true based on the facts that $g'(s^{[n]}) = 0$ and $\kappa(s)$ is strictly concave. ■

Let $\mathbf{z}^{[n+1]}$ be the minimizer of the upper bound function $Q(\mathbf{z}; \boldsymbol{\omega}^{[n]})$ at the n -th iteration, i.e.,

$$\mathbf{z}^{[n+1]} := \arg \min_{\mathbf{z} \in \mathcal{C}} Q(\mathbf{z}; \boldsymbol{\omega}^{[n]}). \quad (33)$$

Based on Proposition 1 and (33), we have

$$\begin{aligned} f_p(\mathbf{z}^{[n+1]}; \epsilon) &= Q(\mathbf{z}^{[n+1]}; \boldsymbol{\omega}^{[n]}) + f_p(\mathbf{z}^{[n+1]}; \epsilon) \\ &\quad - Q(\mathbf{z}^{[n+1]}; \boldsymbol{\omega}^{[n]}) \\ &\leq Q(\mathbf{z}^{[n+1]}; \boldsymbol{\omega}^{[n]}) + f_p(\mathbf{z}^{[n]}; \epsilon) - Q(\mathbf{z}^{[n]}; \boldsymbol{\omega}^{[n]}) \\ &\leq Q(\mathbf{z}^{[n]}; \boldsymbol{\omega}^{[n]}) + f_p(\mathbf{z}^{[n]}; \epsilon) - Q(\mathbf{z}^{[n]}; \boldsymbol{\omega}^{[n]}) \\ &= f_p(\mathbf{z}^{[n]}; \epsilon), \end{aligned} \quad (34)$$

where the first inequality is based on the fact that function $(f_p(\mathbf{z}; \epsilon) - Q(\mathbf{z}; \boldsymbol{\omega}^{[n]}))$ attains its maximum at $\mathbf{z} = \mathbf{z}^{[n]}$, and the second inequality follows from (33). Therefore, minimizing the upper bound, i.e., the Q -function in (30), can reduce the objective function $f_p(\mathbf{z}; \epsilon)$ successively.

Remark 2: In the context of the EM algorithm [38] for computing the maximum likelihood estimator of latent variable models, the functions $-f_p(\mathbf{z}; \epsilon)$ and $-Q(\mathbf{z}; \boldsymbol{\omega}^{[n]})$ can be regarded as the log-likelihood and comparison functions (i.e., the lower bound of the log-likelihood), respectively [36].

The MM algorithm for the smoothed ℓ_p -minimization problem is presented in Algorithm 3.

Algorithm 3: Iterative Reweighted- ℓ_2 Algorithm

input: Initialize $\boldsymbol{\omega}^{[0]} = (1, \dots, 1)$; I (the maximum number of iterations)

Repeat

1) Solve problem

$$\mathbf{z}^{[n+1]} := \arg \min_{\mathbf{z} \in \mathcal{C}} \sum_{i=1}^m \omega_i^{[n]} z_i^2. \quad (35)$$

If it is feasible, **go to 2**); otherwise, **stop** and return **output 2**.

2) Update the weights as

$$\omega_i^{[n+1]} = \frac{p}{2} \left[\left(z_i^{[n+1]} \right)^2 + \epsilon^2 \right]^{\frac{p}{2}-1}, \forall i = 1, \dots, m. \quad (36)$$

Until convergence or attain the maximum iterations and return **output 1**.

output 1: \mathbf{z}^* ; **output 2:** Infeasible.

The convergence of the iterates $\{\mathbf{z}^{[n]}\}_{n=1}^{\infty}$ (35) is presented in the following theorem.

Theorem 2: Let $\{\mathbf{z}^{[n]}\}_{n=1}^{\infty}$ be the sequence generated by the iterative reweighted- ℓ_2 algorithm (35). Then, every limit point $\bar{\mathbf{z}}$ of $\{\mathbf{z}^{[n]}\}_{n=1}^{\infty}$ has the following properties

- 1) $\bar{\mathbf{z}}$ is a KKT point of problem $\mathcal{P}_{\text{sm}}(\epsilon)$ (25);
- 2) $f_p(\mathbf{z}^{[n]}; \epsilon)$ converges monotonically to $f_p(\mathbf{z}^*; \epsilon)$ for some KKT point \mathbf{z}^* .

Proof: Please refer to Appendix B for details. ■

As noted in [39], without the convexity of $f_p(\mathbf{z}; \epsilon)$, the KKT point may be a local minimum or other point (e.g., a saddle point). We also refer to these points as stationary points [39, Page 194].

Remark 3: The algorithm consisting of the iterate (35) accompanied with weights (36) is known as the *iterative reweighted least squares* [29], [38], [40] in the fields of statistics, machine learning and compressive sensing. In particular, with a simple constraint \mathcal{C} , the iterates often yield closed-forms with better computational efficiency. For instance, for the noiseless compressive sensing problem [29], the iterates have closed-form solutions [29, (1.9)]. Therefore, this method has a higher computational efficiency compared with the conventional ℓ_1 -minimization approach for compressive sensing [26], wherein a linear programming problem needs to be solved via algorithms such as interior-point or barrier methods. Furthermore, empirically, it was observed that the iterative reweighted least squares method can improve the signal recovery capability by enhancing the sparsity for compressive sensing over the ℓ_1 -minimization method [29], [40].

In contrast to the existing works on the iterative reweighted least squares methods, we provide a new perspective to develop the iterative reweighted- ℓ_2 algorithm to solve the smoothed ℓ_p -minimization problem with convergence guarantees based on the principle of the MM algorithm. Furthermore, the main motivation and advantages for developing the iterates (35) is to induce the quadratic forms in the objective function in problem \mathcal{P} (19) to make it compliant with the SDR technique, thereby inducing the group sparsity structure in the multicast beamforming vectors via convex programming.

Remark 4: The advantages of the iterative reweighted- ℓ_2 algorithm include the capability of enhancing the sparsity, as well as inducing the quadratic forms for the multicast beamforming vectors by inducing the quadratic formulations (35). Note that the reweighted ℓ_1 -minimization algorithm in [30] can also induce the quadratic forms in the beamforming vectors $\tilde{\mathbf{v}}_i$'s by rewriting the indicator function (7) as the ℓ_0 -norm of the squared ℓ_2 -norm of the vectors $\tilde{\mathbf{v}}_i$'s, i.e., $\|\tilde{\mathbf{v}}_i\|_2^2$. Furthermore, the key ideas of the convergence proof of the iterative reweighted- ℓ_2 algorithm (i.e., Algorithm 3), by leveraging the EM theory to establish upper bounds in the iterates of the MM algorithm, should be useful for other iterative algorithms, e.g., [41].

B. Sparsity Inducing for RRH Selection and User Admission

In this subsection, we demonstrate how to apply the developed iterative reweighted- ℓ_2 algorithm to solve the nonconvex sparse optimization problems \mathcal{P} and \mathcal{D} for RRH selection and

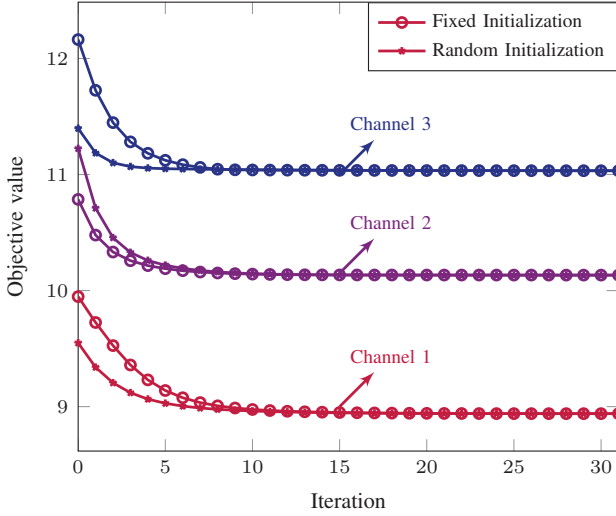


Fig. 3. Convergence of the iterative reweighted- ℓ_2 algorithm for the smoothed ℓ_p -minimization problem \mathcal{P} with different channel realizations and initial points.

user admission, respectively. In this way, we can find a KKT point for the nonconvex smoothed ℓ_p -minimization problems \mathcal{P} and \mathcal{D} with convex constraints. Specifically, let $\Omega := \{\mathbf{Q} \in \mathcal{Q} : L_{k,m}(\mathbf{Q}) \leq 0, \mathbf{Q}_m \succeq \mathbf{0}, \forall k \in \mathcal{G}_m, m \in \mathcal{M}\}$ be the set of constraints in problem \mathcal{P} . The iterative reweighted- ℓ_2 algorithm for problem \mathcal{P} generates a sequence $\{\mathbf{Q}^{[n]}\}_{n=1}^{\infty}$ as follows:

$$\mathcal{P}_{\text{SDP}}^{[n]} : \underset{\mathbf{Q} \in \Omega}{\text{minimize}} \sum_{l=1}^L \omega_l^{[n]} \left(\sum_{m=1}^M \text{Tr}(\mathbf{C}_{lm} \mathbf{Q}_m) \right), \quad (37)$$

with the weights as

$$\omega_l^{[n]} = \frac{\rho_l p}{2} \left[\sum_{m=1}^M \text{Tr}(\mathbf{C}_{lm} \mathbf{Q}_m^{[n]}) + \epsilon^2 \right]^{\frac{p}{2}-1}, \forall l \in \mathcal{L}. \quad (38)$$

Applying Algorithm 3 to problem \mathcal{D} is straightforward.

V. SIMULATION RESULTS

In this section, we will simulate the proposed algorithms based on the iterative reweighted- ℓ_2 algorithm (IR2A) for network power minimization and user admission control in multicast Cloud-RAN. We set the parameters as follows: $P_l = 1W, \forall l$; $P_l^c = [5.6 + l - 1]W, \forall l$; $\eta_l = 1/4, \forall l$; $\sigma_k = 1, \forall k$. Denote the channel propagation from the l -th RRH and the k -th MU as $\mathbf{h}_{kl} = D_{kl} \mathbf{g}_{kl}$ with D_{kl} as the large-scale fading coefficients and $\mathbf{g}_{kl} \in \mathcal{CN}(\mathbf{0}, \mathbf{I})$ as the small-scale fading coefficients. We set $\epsilon = 10^{-3}$ in the iterative reweighted- ℓ_2 algorithm and the algorithm will terminate if either the number of iterations exceeds 30 or the difference between the objective values of consecutive iterations is less than 10^{-3} . The pre-determined number of randomization is set to be 50 in the Gaussian randomization method.

A. Convergence of the Iterative Reweighted- ℓ_2 Algorithm

Consider a network with $L = 6$ 2-antenna RRHs and 2 multicast groups with 2 single-antenna MUs in each group.

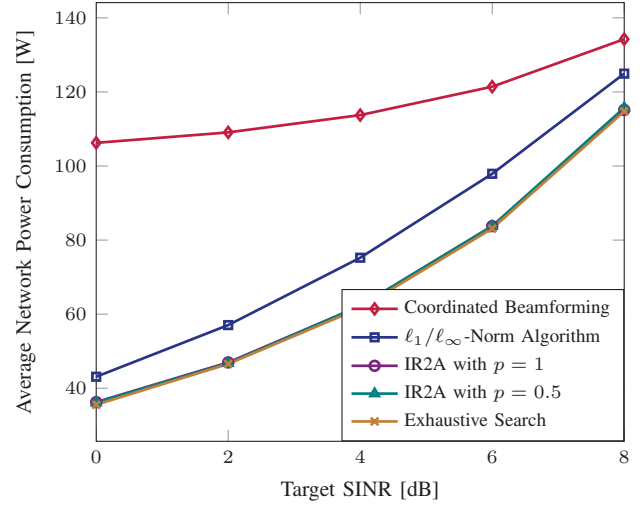


Fig. 4. Average network power consumption versus target SINR with different algorithms.

The channels are spatially uncorrelated. For each MU k , we set $D_{lk} = 1, \forall l \in \Omega_1$ with $|\Omega_1| = 2$; $D_{lk} = 0.7, \forall l \in \Omega_2$ with $|\Omega_2| = 2$; $D_{lk} = 0.5, \forall l \in \Omega_3$ with $|\Omega_3| = 2$. All the sets Ω_i 's are uniformly drawn from the RRH set $\mathcal{L} = \{1, \dots, L\}$ with $\cup \Omega_i = \Omega$. Fig. 3 illustrates the convergence of the iterative reweighted- ℓ_2 algorithm for the smoothed ℓ_p -minimization problem \mathcal{P} with different initial points and different channel realizations. Specifically, we set $p = 1$. The three different channel realizations are generated uniformly and independently. For each channel realization, we simulate Algorithm 3 with the fixed initial point as $\omega^{[0]} = (1, \dots, 1)$ and randomly generated initial point $\omega^{[0]}$. The random initialization instances for the three channel realizations are given as $\omega_{\text{ch1}}^{[0]} = [0.1389, 0.2028, 0.1987, 0.6038, 0.2722, 0.1988]$, $\omega_{\text{ch2}}^{[0]} = [0.5657, 0.7165, 0.5113, 0.7764, 0.4893, 0.1859]$, $\omega_{\text{ch3}}^{[0]} = [0.4093, 0.4635, 0.6109, 0.0712, 0.3143, 0.6084]$, respectively. In particular, this figure shows that the sequence of the objective functions converges monotonically, which confirms the convergence results in Theorem 2. In addition, it illustrates the robustness of the convergence of the reweighted- ℓ_2 algorithm with different initial points and different problem parameters. Furthermore, it also demonstrates the fast convergence rate of the proposed algorithm in the simulated setting. Empirically, the iterative reweighted- ℓ_2 algorithm converges in 20 iterations on average in all the simulated channels in this paper.

B. Network Power Minimization

Consider a network with $L = 12$ 2-antenna RRHs and 5 multicast groups with 2 single-antenna MUs in each group. The channels are spatially uncorrelated. For each MU k , we set $D_{lk} = 1, \forall l \in \Omega_1$ with $|\Omega_1| = 4$; $D_{lk} = 0.7, \forall l \in \Omega_2$ with $|\Omega_2| = 4$; $D_{lk} = 0.5, \forall l \in \Omega_3$ with $|\Omega_3| = 4$. All the sets Ω_i 's are uniformly drawn from the RRH set $\mathcal{L} = \{1, \dots, L\}$ with $\cup \Omega_i = \Omega$. The proposed iterative reweighted- ℓ_2 algorithm is compared with the reweighted ℓ_1/ℓ_∞ -norm based algorithm [25], in which, the objective function in problem \mathcal{P} is

TABLE I
THE AVERAGE NUMBER OF ACTIVE RRHS WITH DIFFERENT ALGORITHMS

Target SINR [dB]	0	2	4	6	8
Coordinated Beamforming	12.00	12.00	12.00	12.00	12.00
ℓ_1/ℓ_∞ -Norm Algorithm	4.39	5.43	6.70	8.35	10.35
IR2A with $p = 1$	4.74	5.43	6.65	8.22	10.13
IR2A with $p = 0.5$	4.61	5.30	6.65	8.22	10.17
Exhaustive Search	4.43	5.30	6.39	8.04	10.09

TABLE II
THE AVERAGE RELATIVE FRONTHAUL LINKS POWER CONSUMPTION WITH DIFFERENT ALGORITHMS

Target SINR [dB]	0	2	4	6	8
Coordinated Beamforming	102.0	102.0	102.0	102.0	102.0
ℓ_1/ℓ_∞ -Norm Algorithm	30.65	40.52	53.74	69.70	87.48
IR2A with $p = 1$	24.57	29.91	40.26	56.30	78.04
IR2A with $p = 0.5$	24.13	29.39	40.43	56.30	78.87
Exhaustive Search	22.96	29.65	39.00	54.78	77.22

TABLE III
THE AVERAGE TOTAL TRANSMIT POWER CONSUMPTION WITH DIFFERENT ALGORITHMS

Target SINR [dB]	0	2	4	6	8
Coordinated Beamforming	4.26	7.07	11.74	19.42	32.24
ℓ_1/ℓ_∞ -Norm Algorithm	12.45	16.53	21.49	28.20	37.46
IR2A with $p = 1$	11.69	17.05	21.94	27.46	37.10
IR2A with $p = 0.5$	11.97	17.36	21.79	27.46	36.85
Exhaustive Search	12.59	16.91	22.62	28.33	37.53

replaced by $\sum_{l_1=1}^L \sum_{l_2=1}^L \max_m \max_{n_{l_1}} \max_{n_{l_2}} |\mathbf{Q}_m(n_{l_1}, n_{l_2})|$ with $\mathbf{Q}_m(i, j)$ as the (i, j) -th entry in \mathbf{Q}_m . And the RRHs with smaller beamforming coefficients (measured by ℓ_∞ -norm) have higher priorities to be switched off. Fig. 4 demonstrates the average network power consumption with different target SINRs using different algorithms. Each point of the simulation results is averaged over 50 randomly generated channel realizations for which problem \mathcal{P} is feasible. From this figure, we can see that the proposed iterative reweighted- ℓ_2 algorithm can achieve near-optimal performance compared with the exhaustive search algorithm (i.e., solving problem (10) with convexified QoS constraints based on the SDR technique). It yields lower network power consumption compared with the existing ℓ_1/ℓ_∞ -norm based algorithm [25], while the coordinated multicast beamforming algorithm [13], [21] with all the RRHs active has the highest network power consumption. Note that the number of SDP problems $\mathcal{F}(\mathcal{A}^{[i]})$ (22) needed to be solved grows *logarithmically* with L for the proposed network power minimization algorithm and the previous ℓ_1/ℓ_∞ -norm based algorithm [25]. But the number of SDP problems (22) to be solved for exhaustive search grows exponentially with L . Although the coordinated beamforming algorithm has the lowest computational complexity by only solving the total transmit power minimization problem $\mathcal{P}_{\text{TP}}(\mathcal{L}, \mathcal{N})$ (23) with all the RRHs active, it yields the highest network power consumption. Our proposed algorithm thus

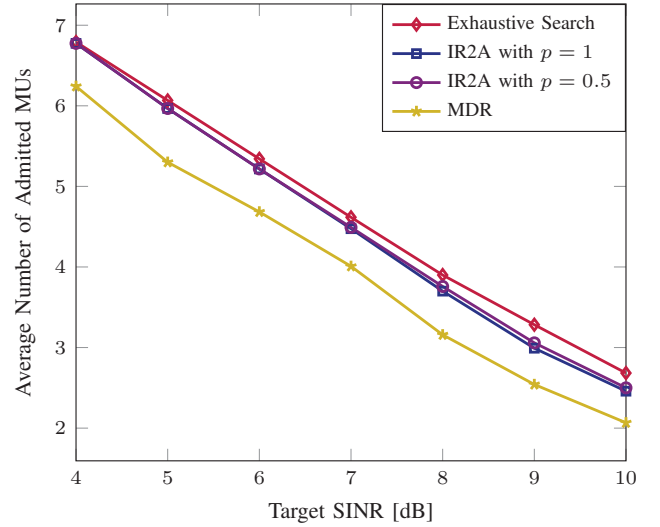


Fig. 5. Average number of admitted MUs versus target SINR with different algorithms.

achieves a good trade-off between computational complexity and performance.

Tables I-III show the corresponding average number of active RRHs, average fronthaul network power consumption and average total transmit power consumption, respectively. Specifically, Table I confirms the existence of the group sparsity pattern in the aggregative multicast beamforming vector. That is, the switched off RRHs indicate that all the beamforming coefficients at the RRHs are set to be zeros simultaneously. It also shows that the proposed iterative reweighted- ℓ_2 algorithm has the capability of enhancing sparsity, thereby switching off more RRHs compared with the ℓ_1/ℓ_∞ -norm based algorithm except for very low SNR. Table II shows that the proposed iterative reweighted- ℓ_2 algorithm can achieve much lower fronthaul network power consumption and attain similar values with exhaustive search. With all the RRHs being active, the coordinated multicast beamforming algorithm yields the highest fronthaul network power consumption and the lowest transmit power consumption as shown in Table II and Table III, respectively. This indicates that it is critical to take the relative fronthaul link power consumption into consideration when designing a green Cloud-RAN. Although the exhaustive search may yield higher transmit power consumption in some scenarios as shown in Table III, overall it achieves the lowest network power consumption as much lower relative fronthaul network power consumption can be attained as indicated in Table II. Furthermore, we can see that, in the simulated settings, different values of p in the proposed iterative reweighted- ℓ_2 algorithm yield similar performance, while all achieve near-optimal performance compared with the exhaustive search.

C. User Admission Control

Consider a network with $L = 6$ 2-antenna RRHs and 4 multicast groups with 2 single-antenna MUs in each group. The channel model is the same as Section V-A. The proposed

iterative reweighted- ℓ_2 algorithm based user admission control algorithm is compared with the existing convex relaxation approach (i.e., the multicast membership deflation by relaxation (MDR) [16]) and the exhaustive search. The simulation results are illustrated in Fig. 5 for the average number of admitted MUs. Each point of the simulation results is averaged over 200 randomly generated channel realizations for which problem \mathcal{P} is infeasible. From Fig. 5, we can see that the proposed iterative reweighted- ℓ_2 algorithm outperforms the existing MDR approach [16]. In particular, the performance of the iterative reweighted- ℓ_2 algorithm is almost the same with different values of p and achieves near-optimal performance compared to the exhaustive search via solving problem (12) with convexified QoS constraints based on the SDR technique. Although all the simulated results demonstrate that the performances are robust to the parameter p , it is very interesting to theoretically identify the typical scenarios, where smaller values of p will yield much better performances.

VI. CONCLUSIONS AND FUTURE WORKS

This paper developed a sparse optimization framework for network power minimization and user admission in green Cloud-RAN with multicast beamforming. A smoothed ℓ_p -minimization method was proposed to induce the sparsity structures in the solutions, thereby guiding the RRH selection and user admission. This approach has the advantages in terms of promoting sparsity and assisting algorithmic design by introducing the quadratic forms in the group sparse inducing penalty. In particular, by leveraging the MM algorithm and the SDR technique, we developed an iterative reweighted- ℓ_2 algorithm with convergence and optimality guarantees (i.e., KKT points) to solve the resulting smoothed ℓ_p -minimization problems \mathcal{P} and \mathcal{D} with convex constraints. The effectiveness of the proposed algorithms was demonstrated via simulations for network power minimization and user admission control.

Several future directions are listed as follows:

- Although the proposed methods only need to solve a sequence of convex optimization problems, the complexity of solving the large-scale SDP problem using the interior-point method is prohibitively high. One promising option is to use the first-order method (e.g., the operator splitting method [42], [12]) by leveraging the parallel computing platform in the BBU pool [5], [43], which will require further investigation.
- It is desirable to establish the optimality and perform probabilistic analysis for the iterative reweighted- ℓ_2 algorithm in the context of inducing group sparsity in the multicast beamforming vectors. However, with the complicated constraints set, this becomes much more challenging compared with the compressive sensing problems with simple constraints (e.g., affine constraints) [29]. It is also interesting to apply this algorithm to solve other mixed combinatorial optimization problems in wireless networks, e.g., wireless caching problems [44].
- It is interesting to apply the proposed smoothed ℓ_p -minimization approach with the iterative reweighted- ℓ_2 algorithm in the scenarios with CSI uncertainty. The only

requirement is that the resulting non-convex constraints due to the CSI uncertainty can be convexified [45], [46]. It is also interesting but challenging to characterize the performance degradations due to CSI acquisition errors in multicast Cloud-RANs [47], [48].

APPENDIX A PROOF OF THEOREM 1

We first need to prove that

$$\limsup_{\epsilon \searrow 0} \Omega(\epsilon) \subset \Omega(0), \quad (39)$$

where $\limsup_{\epsilon \searrow 0} \Omega(\epsilon)$ is defined as [49, Page 152]

$$\begin{aligned} \limsup_{\epsilon \searrow 0} \Omega(\epsilon) &:= \bigcup_{\epsilon^{[n]} \searrow 0} \limsup_{n \rightarrow \infty} \Omega(\epsilon^{[n]}) \\ &= \{\bar{z} | \exists \epsilon^{[n]} \searrow 0, \exists z^{[n]} \rightarrow \bar{z}\}, \end{aligned} \quad (40)$$

where $z^{[n]} \in \Omega(\epsilon^{[n]})$. Therefore, for any $\bar{z} \in \limsup_{\epsilon \searrow 0} \Omega(\epsilon)$, there exists $z^{[n]} \in \Omega(\epsilon^{[n]})$ such that $z^{[n]} \rightarrow \bar{z}$ and $\epsilon^{[n]} \searrow 0$. To prove (39), we only need to prove that $\bar{z} \in \Omega(0)$. Specifically, $z^{[n]} \in \Omega(\epsilon^{[n]})$ indicates that

$$0 \in \nabla_z f_p(z^{[n]}; \epsilon^{[n]}) + \mathcal{N}_{\mathcal{C}}(z^{[n]}). \quad (41)$$

As $f_p(z; \epsilon)$ is continuously differentiable in both z and ϵ , we have

$$\begin{aligned} \lim_{n \rightarrow \infty} \nabla_z f_p(z^{[n]}; \epsilon^{[n]}) &= \lim_{z^{[n]} \rightarrow \bar{z}} \lim_{\epsilon^{[n]} \searrow 0} \nabla_z f_p(z^{[n]}; \epsilon^{[n]}) \\ &= \nabla_z f_p(\bar{z}; 0). \end{aligned} \quad (42)$$

Furthermore, based on results for the limits of normal vectors in [49, Proposition 6.6], we have

$$\limsup_{z^{[n]} \rightarrow \bar{z}} \mathcal{N}_{\mathcal{C}}(z^{[n]}) = \mathcal{N}_{\mathcal{C}}(\bar{z}). \quad (43)$$

Specifically, if $z^{[n]} \rightarrow \bar{z}$, $s^{[n]} \in \mathcal{N}_{\mathcal{C}}(z^{[n]})$ and $s^{[n]} \rightarrow s$, then $s \in \mathcal{N}_{\mathcal{C}}(\bar{z})$. That is, the set $\{(z, s) | s \in \mathcal{N}_{\mathcal{C}}(z)\}$ is closed relative to $\mathcal{C} \times \mathbb{R}^m$. Based on (42) and (43) and taking $n \rightarrow \infty$ in (41), we thus prove (39). Based on [50, Theorem 4], we complete the proof for (29).

APPENDIX B CONVERGENCE OF THE ITERATIVE REWEIGHTED- ℓ_2 ALGORITHM

1) We will show that any convergent subsequence $\{z^{[n_k]}\}_{k=1}^{\infty}$ of $\{z^{[n]}\}_{n=1}^{\infty}$ satisfies the definition of the KKT points of problem $\mathcal{P}_{\text{sm}}(\epsilon)$ (26). Specifically, let $z^{[n_k]} \rightarrow \bar{z}$ be one such convergent subsequence with

$$\lim_{k \rightarrow \infty} z^{[n_k+1]} = \lim_{k \rightarrow \infty} z^{[n_k]} = \bar{z}. \quad (44)$$

As

$$z^{[n_k+1]} := \arg \min_{z \in \mathcal{C}} Q(z; \omega^{[n_k]}), \quad (45)$$

which is a convex optimization problem, the KKT condition holds at $z^{[n_k+1]}$, i.e.,

$$0 \in \nabla_z Q(z^{[n_k+1]}; \omega^{[n_k]}) + \mathcal{N}_{\mathcal{C}}(z^{[n_k+1]}). \quad (46)$$

Based on [49, Proposition 6.6] and (44), we have

$$\limsup_{\mathbf{z}^{[n_k+1]} \rightarrow \bar{\mathbf{z}}} \mathcal{N}_C(\mathbf{z}^{[n_k+1]}) = \mathcal{N}_C(\bar{\mathbf{z}}). \quad (47)$$

Furthermore, based on (44), we also have

$$\begin{aligned} \lim_{k \rightarrow \infty} \nabla_{\mathbf{z}} Q(\mathbf{z}^{[n_k+1]}; \boldsymbol{\omega}^{[n_k]}) &= \lim_{k \rightarrow \infty} 2 \sum_{i=1}^m \omega^{[n_k]} z_i^{n_k+1} \\ &= \lim_{k \rightarrow \infty} \sum_{i=1}^m \frac{p z_i^{[n_k+1]}}{\left[\left(z_i^{[n_k]} \right)^2 + \epsilon^2 \right]^{1-\frac{p}{2}}} \\ &= \nabla_{\mathbf{z}} f_p(\bar{\mathbf{z}}; \epsilon). \end{aligned} \quad (48)$$

Therefore, taking $k \rightarrow \infty$ in (46), we have

$$0 \in \nabla_{\mathbf{z}} Q(\bar{\mathbf{z}}; \bar{\boldsymbol{\omega}}) + \mathcal{N}_C(\bar{\mathbf{z}}), \quad (49)$$

which indicates that $\bar{\mathbf{z}}$ is a KKT point of problem $\mathcal{P}_{\text{sm}}(\epsilon)$. We thus complete the proof.

2) As $f_p(\mathbf{z}; \epsilon)$ is continuous and \mathcal{C} is compact, we have the fact that the limit of the sequence $f_p(\mathbf{z}^{[n]}; \epsilon)$ is finite. Furthermore, we have $f_p(\mathbf{z}^{[n+1]}; \epsilon) \leq f_p(\mathbf{z}^{[n]}; \epsilon)$ according to (34). Based on the results in 1), we complete the proof. Note that a similar result was presented in [37] by leveraging the results in the EM algorithm theory.

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