

Energy-Efficient Small Cell with Spectrum-Power Trading

Qingqing Wu, *Student Member, IEEE*, Geoffrey Ye Li, *Fellow, IEEE*, Wen Chen, *Senior Member, IEEE*, and Derrick Wing Kwan Ng, *Member, IEEE*

Abstract

In this paper, we investigate spectrum-power trading between a small cell (SC) and a macro cell (MC), where the SC consumes power to serve the macro cell users (MUs) in exchange for some bandwidth from the MC. Our goal is to maximize the system energy efficiency (EE) of the SC while guaranteeing the quality of service of each MU as well as small cell users (SUs). Specifically, given the minimum data rate requirement and the bandwidth provided by the MC, the SC jointly optimizes MU selection, bandwidth allocation, and power allocation while guaranteeing its own minimum required system data rate. The problem is challenging due to the binary MU selection variables and the fractional-form objective function. We first show that the bandwidth of an MU is shared with at most one SU in the SC. Then, for a given MU selection, the optimal bandwidth and power allocation is obtained by exploiting the fractional programming. To perform MU selection, we first introduce the concept of the trading EE to characterize the data rate obtained as well as the power consumed for serving an MU. We then reveal a sufficient and necessary condition for serving an MU without considering the total power constraint and the minimum data rate constraint: the trading EE of the MU should be higher than the system EE of the SC. Based on this insight, we propose a low complexity MU selection method and also investigate the optimality condition. Simulation results verify our theoretical findings and demonstrate that the proposed resource allocation achieves near-optimal performance.

Index Terms

Qingqing Wu and Geoffrey Ye Li are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, USA, email: {qingqing.wu, liye}@ece.gatech.edu. Wen Chen is with Department of Electronic Engineering, Shanghai Jiao Tong University, China, email: wenchen@sjtu.edu.cn. Derrick Wing Kwan Ng is with the School of Electrical Engineering and Telecommunications, The University of New South Wales, Australia, email: wingn@ece.ubc.ca. This paper has been submitted in part to the IEEE Globecom 2016.

I. INTRODUCTION

The fifth generation (5G) mobile networks are expected to provide ubiquitous ultra-high data rate services and seamless user experience across the whole system [1]. The concept of small cell (SC) networks, such as femtocells, has been recognized as a key technology that can significantly enhance the performance of 5G networks. The underlaying SCs enable the macro cells (MCs) to offload huge volume of data and large numbers of users [2]. In particular, the SC could help to serve some macro cell users (MUs) with high data rate requirements, especially when these MUs are far away from the MC base station (BS) [3]. Although the MUs offloading reduces the power consumption of MCs, additional power consumption is imposed to SCs that may degrade the quality of services (QoSs) of small cell users (SUs). Therefore, motivating the SC to serve MUs is a critical problem, especially when the SC BS does not belong to the same mobile operator with the MC BS [4].

Meanwhile, the explosive growth of data hungry applications and various services has triggered a dramatic increase in energy consumption of wireless communications. Due to rapidly rising energy costs and tremendous carbon footprints [5]–[10], energy efficiency (EE), measured in bits-per-joule, has attracted considerable attention as a new performance metric in both academia and industry [11]–[23]. Energy-efficient resource allocation has been studied in [13] for a single cell with large numbers of base station antennas. Then, this work is extended into the context of physical layer security [14] and the multi-cell with limited backhaul capacity [15], respectively. Subsequently, similar EE maximization problems are further investigated for example for relay [16], [17], full duplex [18], heterogeneous [19], cognitive radio (CR) [20], coordinated multi-point (CoMP) transmission [21], and multi-input-multi-output orthogonal frequency division multiplexing (MIMO-OFDM) [22], [23] networks. Furthermore, the authors in [24] propose a BS switching on-off scheme for heterogeneous cellular networks under a stochastic geometry model. It has been shown that significant power consumption is reduced after adopting strategic sleeping.

However, all these previous works ignored spectrum sharing or energy cooperation between the SC and the MC, which are expected to enhance the performances of both networks simultaneously. The notion of spectrum or energy cooperation has been recently pursued in [25]–[28].

In [25], energy-efficient resource allocation has been studied for heterogeneous cognitive radio networks with femtocells, where a cognitive BS maximizes its profit by allocating the spectrum resource bought from the primary networks to the femtocells. However, only the spectrum disparity is exploited between these two communication networks. In [26], joint energy and spectrum cooperation between two neighbouring cellular networks are considered to minimize the total costs on the pre-priced bandwidth and power given the QoS requirement. However, the monetary based spectrum sharing and energy cooperation are unable to capture the instantaneous characteristics of wireless channels [27], [28].

In this paper, we study spectrum-power trading between an SC and an MC where the SC BS consumes additional power to serve MUs while the MC allows the SC BS to obtain additional bandwidth. Specifically, the SC BS splits the allocated bandwidth of an MU into two parts. One part is allocated to meet the QoS of the MU and the other part can be utilized to serve its own SUs. The spectrum-power trading is motivated by the following two observations. To serve the MUs that are far away from the MC BS, the transmit power consumption limits the system performance rather than the bandwidth, since the MC BS generally operates in low signal-to-noise ratio (SNR) regimes [29]. In contrast, for the SC, the bandwidth limits the system performance rather than the transmit power, since it generally works in high SNR regimes due to a small coverage [29]. Thus, the spectrum-power trading in this paper will exploit the disparities between the MC networks and the SC networks from both the spectrum and the power perspectives. Note that the spectrum-power trading is always beneficial to the MC by reducing its power consumption. Hence, we focus on how to enhance the performance of the SC. Although the spectrum-power trading enables the SC to have higher data rate via seeking more bandwidth from the MUs, it also causes additional power consumption to the SC in order to serve the MUs. Thus, this may leave less transmit power for the SUs such that the SC operates in the low SNR regime. As a result, the power consumption becomes a critical problem. In order to balance the power consumption and the achievable data rate, we adopt the system EE as the performance metric.

To ensure spectrum-power trading based EE, we need to address the following fundamental issues. First, when should the SC serve an MU? For example, if the required data rate of an MU is required too stringent but the bandwidth assigned to it is insufficient, it may not be beneficial for the SC to serve that MU. Second, how much bandwidth should be obtained and how much power should be utilized in order to achieve the maximum EE as well as guaranteeing the QoS

of the MUs? This question arises because if the SC desires to seek more bandwidth from the MU, it has to transmit with a higher transmit power for this MU. However, this may in turn leave a lower transmit power for its own SUs and thereby lead to a lower system data rate as well as a lower system EE. Thus, there exists a non-trivial spectrum and power tradeoff in the spectrum-power trading. These issues have never been investigated in previous works [11]–[23] and we will address them in this paper. The main contributions are summarized as follows:

- We study spectrum-power trading between an SC and an MC where the SC consumes additional power to serve MUs in exchange for additional bandwidth from the MC. We focus on enhancing the performance of the SC. In particular, MU selection, bandwidth allocation, and power allocation are jointly optimized with the objective of maximizing the system EE of the SC while guaranteeing the QoS of both networks.
- We first simplify the original optimization problem by showing that the bandwidth of an MU served by the SC is only shared with at most one SU. However, the simplified problem is still non-convex due to the binary MU selection variable and the fractional-form objective function. Given an MU selection, the problem can be further reduced to a joint bandwidth and power allocation problem, where the fractional form objective function is then transformed into a subtractive form by exploiting fractional programming theory. We then derive closed-form expressions of the bandwidth and power allocation based on the analysis of the transformed problem.
- For the MU selection, we first introduce the trading EE of an MU that involves both the data rate brought for the SC and the power consumed by the SC in the spectrum-power trading. Then, we investigate the relationships between the trading EE of an MU and the system EE of the SC subject to various constraints in the original problem formulation. In particular, we reveal that in the absence of the maximum power constraint and the minimum system data rate constraint, serving an MU can improve the system EE of the SC if and only if its trading EE is higher than the current system EE. Based on this observation, we develop a low computational complexity algorithm for the MU selection. Finally, we also study the optimality condition of the proposed algorithm for the original problem.

The rest of this paper is organized as follows. Section II introduces the spectrum-power trading model as well as the power consumption model. In Section III, we formulate and analyze the EE

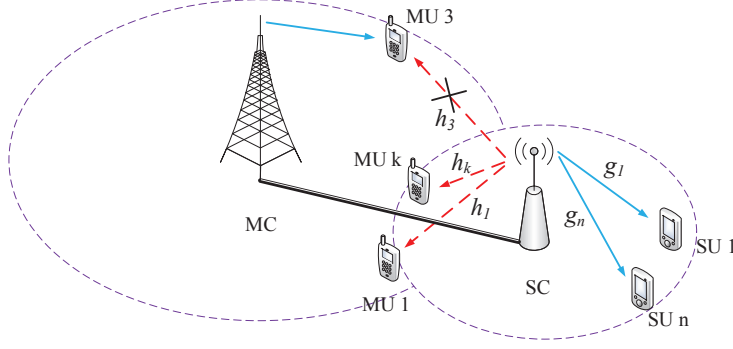


Fig. 1. The spectrum-power trading model between an SC and an MC. For example, the SC may agree to serve MU 1 but refuse to serve MU 3 in order to maximize its performance.

maximization problem. In Section IV, we study joint bandwidth and power allocation with a given MU selection. In Section V, we investigate the MU selection based on the proposed trading EE. Section VI provides simulation results to demonstrate the effectiveness of the proposed algorithm. Finally, Section VII concludes the paper.

II. SYSTEM MODEL

In this section, we first introduce the spectrum-power trading model between SC and MC networks. Then, we discuss the power consumption model of the SC BS under the context of spectrum-power trading.

A. Spectrum-Power Trading Model between SC and MC

We consider a spectrum-power trading scenario which consists of an MC and an SC, as depicted in Figure 1. The MC BS aims at offloading the data traffic of some cell edge MUs to the SC BS in order to reduce its own power consumption. The set of MUs who may be served by the SC is denoted by \mathcal{K} with $|\mathcal{K}| = K$ and the set of SUs in the SC is denoted by \mathcal{N} with $|\mathcal{N}| = N$, where $|\cdot|$ indicates the cardinality of a set. Each MU and SU have been assigned a licensed bandwidth by the MC and the SC, respectively, denoted as W_{MC}^k and B_{SC}^n . To incentivize the SC to serve MUs, the MC allows it to utilize some of the licensed bandwidth of MUs to enhance the QoS of SUs. Thus, for the SC, the bandwidth obtained from MUs can be viewed as a compensation of the power consumed for serving MUs. To simplify the problem, we assume that the SC BS as well as each user is equipped with a single antenna [26].

The channels between the SC and MUs as well as SUs are assumed to be quasi-static block fading, i.e., the channel coefficients remain constant during each block, but may vary from one block to another [30]. We also assume that SU n , $\forall n \in \mathcal{N}$, experiences frequency flat fading on its own licensed bandwidth B_{SC}^n and each MU k 's bandwidth W_{MC}^k , respectively. In addition, MU k , $\forall k \in \mathcal{K}$, also experiences frequency flat fading on its own licensed bandwidth W_{MC}^k . Note that the results in this paper can also be extended to the more general case when the bandwidth of each user (SU and MU) is modeled by the multiple orthogonal subcarriers. It is also assumed that the channel state information (CSI) of all users is perfectly known to the SC in order to explore the EE upper bound and extract useful design insights of the considered systems. In practice, the CSI can be estimated by each individual user and then fed back to the SC. Signaling overhead and imperfect CSI would result in performance loss and their impacts can be analyzed as in [25], which is beyond the scope of this paper.

For MU k , $\forall k \in \mathcal{K}$, the channel power gain between the SC and MU k on its own licensed bandwidth W_{MC}^k is denoted as h_k , cf. Figure 1. The corresponding transmit power and the bandwidth that are allocated to MU k by the SC are denoted as q_k and w_k , respectively. Thus, the achievable data rate of MU k can be expressed as

$$r_k = w_k \log_2 \left(1 + \frac{q_k h_k}{w_k N_0} \right), \quad (1)$$

where N_0 is the spectral density of the additive white Gaussian noise.

For SU n , $\forall n \in \mathcal{N}$, the channel power gain between the SC and SU n on its own licensed bandwidth B_{SC}^n is denoted as g_n , cf. Figure 1. The corresponding transmit power is denoted as p_n . Then, the achievable data rate of SU n on its own bandwidth can be expressed as

$$r_{SC}^n = B_{SC}^n \log_2 \left(1 + \frac{p_n g_n}{B_{SC}^n N_0} \right). \quad (2)$$

In addition to B_{SC}^n , each SU may obtain some additional bandwidth from MUs due to the spectrum-power trading between the SC and the MC. Denote the channel power gain between the SC and SU n on the bandwidth of MU k as $g_{k,n}$. The bandwidth that the SC allocates for SU n from W_{MC}^k is denoted as $b_{k,n}$ and the corresponding transmit power is denoted as $p_{k,n}$. Then, the achievable data rate of SU n on the bandwidth of MU k can be expressed as

$$r_{k,n} = b_{k,n} \log_2 \left(1 + \frac{p_{k,n} g_{k,n}}{b_{k,n} N_0} \right). \quad (3)$$

Thus, the total data rate of SU n in the context of the spectrum-power trading is given by

$$R_n = B_{SC}^n \log_2 \left(1 + \frac{p_n g_n}{B_{SC}^n N_0} \right) + \sum_{k=1}^K x_k b_{k,n} \log_2 \left(1 + \frac{p_{k,n} g_{k,n}}{b_{k,n} N_0} \right), \quad (4)$$

where x_k is the MU selection variable and defined as

$$x_k = \begin{cases} 1, & \text{if MU } k \text{ is served by the SC,} \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

Therefore, the overall system data rate of SUs can be expressed as

$$R_{\text{total}} = \sum_{n=1}^N R_n = \sum_{n=1}^N r_{SC}^n + \sum_{n=1}^N \sum_{k=1}^K x_k r_{k,n}. \quad (6)$$

B. Power Consumption Model for SC BS

Here, we adopt the power consumption model from [31] in which the overall energy consumption of the BS consists of two parts: the dynamic power consumed in the power amplifier for transmission, P_t , and the static power consumed for circuits, P_c .

The dynamic power consumption is modeled as a linear function of the transmit power that includes both the transmit power consumption for SUs and that for MUs, i.e.,

$$P_t = \sum_{n=1}^N \frac{p_n}{\xi} + \sum_{n=1}^N \sum_{k=1}^K x_k \frac{p_{k,n}}{\xi} + \sum_{k=1}^K x_k \frac{q_k}{\xi}, \quad (7)$$

where $\xi \in (0, 1]$ is a constant that accounts for the power amplifier (PA) efficiency and the value of ξ depends on the specific type of the BS. In general, the PA efficiency decreases for smaller BS types and a detailed discussion on it can be found in [32]. The static power consumption for circuits is denoted as P_c , which is caused by filters, frequency synthesizer, etc. Therefore, the overall power consumption of the SC BS can be expressed as

$$P_{\text{total}} = \sum_{n=1}^N \frac{p_n}{\xi} + \sum_{n=1}^N \sum_{k=1}^K x_k \frac{p_{k,n}}{\xi} + \sum_{k=1}^K x_k \frac{q_k}{\xi} + P_c. \quad (8)$$

III. PROBLEM FORMULATION AND ANALYSIS

Our goal is to enhance the system EE of the SC in the context of the spectrum-power trading while guaranteeing the QoS of the MUs as well as the SC network. Thus, the system EE of the SC is defined as the ratio of the total achievable data rate of SUs and the total power consumption

that includes not only the power consumed for providing services for SUs, but also the power consumed for spectrum-power trading, i.e.,

$$EE = \frac{R_{\text{total}}}{P_{\text{total}}} = \frac{\sum_{n=1}^N r_{SC}^n + \sum_{k=1}^K \sum_{n=1}^N x_k r_{k,n}}{\sum_{n=1}^N \frac{p_n}{\xi} + \sum_{n=1}^N \sum_{k=1}^K x_k \frac{p_{k,n}}{\xi} + \sum_{k=1}^K x_k \frac{q_k}{\xi} + P_c}. \quad (9)$$

Specifically, we aim to maximize the system EE of the SC via jointly optimizing MU selection, bandwidth allocation, and power allocation. The system EE maximization problem is formulated as

$$\begin{aligned} & \underset{\substack{\{p_n\}, \{p_{k,n}\}, \{b_{k,n}\}, \\ \{x_k\}, \{q_k\}, \{w_k\}}}{\text{maximize}} & & \frac{\sum_{n=1}^N r_{SC}^n + \sum_{k=1}^K \sum_{n=1}^N x_k r_{k,n}}{\sum_{n=1}^N \frac{p_n}{\xi} + \sum_{n=1}^N \sum_{k=1}^K x_k \frac{p_{k,n}}{\xi} + \sum_{k=1}^K x_k \frac{q_k}{\xi} + P_c} \\ & \text{s.t.} & & \text{C1: } \sum_{n=1}^N p_n + \sum_{n=1}^N \sum_{k=1}^K p_{k,n} + \sum_{k=1}^K q_k \leq P_{\text{max}}^{SC}, \\ & & & \text{C2: } \sum_{n=1}^N b_{k,n} + w_k \leq x_k W_{MC}^k, \quad \forall k \in \mathcal{K}, \\ & & & \text{C3: } w_k \log_2 \left(1 + \frac{q_k h_k}{w_k N_0} \right) \geq x_k R_{MC}^k, \quad \forall k \in \mathcal{K}, \\ & & & \text{C4: } \sum_{n=1}^N r_{SC}^n + \sum_{k=1}^K \sum_{n=1}^N x_k r_{k,n} \geq R_{\text{min}}^{SC}, \\ & & & \text{C5: } x_k \in \{0, 1\}, \quad \forall k \in \mathcal{K}, \\ & & & \text{C6: } b_{k,n} \geq 0, \quad w_k \geq 0, \quad \forall k \in \mathcal{K}, n \in \mathcal{N}, \\ & & & \text{C7: } p_n \geq 0, \quad p_{k,n} \geq 0, \quad q_k \geq 0, \quad \forall k \in \mathcal{K}, n \in \mathcal{N}. \end{aligned} \quad (10)$$

In problem (10), C1 limits the maximum transmit power of the SC BS to P_{max}^{SC} . C2 ensures that the bandwidth allocated to SUs and MU k does not exceed the available bandwidth, W_{MC}^k , that has been licensed to MU k by the MC. In C3, R_{MC}^k is the minimum data rate requirement of MU k . C4 guarantees the minimum required system data rate of the SC. C5 indicates whether to serve MU k or not. Note that if $x_k = 0$, then from C2 and C4, both $b_{k,n}$ and q_k will be forced to be zeros at the optimal solution of problem (10), which means that the SC does not obtain additional bandwidth from MU k and does not serve MU k either. C6 and C7 are non-negativity constraints on the bandwidth and power allocation variables, respectively. In general, different priorities and fairness among the SUs could be realized by adopting the weighted sum rate instead of the sum rate in problem (10) [13], [33]. Since the weights do not affect the algorithm

design, we assume that all the SUs are equally weighted in this paper without loss of generality.

Remark 1: Although we focus on improving the EE of the SC via spectrum-power trading, the EE of the MC as well as the system-wide EE will also be improved correspondingly, which can be explained as follows. Note that the MUs that the MC is willing to offload to the SC are in general those users with poor channel conditions or cell-edge users. This means that a large amount of transmit power will be consumed if these MUs are directly served by the MC. In fact, this is also the fundamental reason why the MC desires to offload them. That is, through offloading, the MC only needs to serve MUs with good channel conditions, which thus results in a higher system EE. In other words, if an MU can be served by the MC with a small amount of transmit power, there is no motivation for the MC to establish the offloading. Therefore, the EE of the MC will obviously increase via offloading. Therefore, the system-wide EE will increase due to the EE increases of both the SC and the MC.

Remark 2: It is worth noting that problem (10) generalizes several interesting special cases which are discussed as follows.

- If we set $x_k = 0, \forall k \in \mathcal{K}$, then problem (10) is reduced to a system EE maximization problem without spectrum-power trading.
- If we set $x_k = 1, \forall k \in \mathcal{K}$, it suggests that the SC helps to provide services for all of the MUs without considering its own performance, which usually happens when the SC BS and the MC BS belongs to the same operator.
- If we set $B_{SC}^n = 0, \forall n \in \mathcal{N}$, it implies that the SC does not have its own licensed bandwidth to assign to SUs and can only seek the bandwidth from the MC via the spectrum-power trading. In this case, the SC is reduced to a *cognitive (secondary) network* while the MC can be regarded as a *primary network* [27].

Therefore, problem (10) is more challenging and more interesting than the previous work [28].

Note that problem (10) is neither a concave nor a quasi-concave optimization problem due to the fractional-form objective function and the binary optimization variables $x_k, \forall k$. Nevertheless, in the following theorem proved in Appendix A, we first transform the energy-efficient optimization problem into a simplified one based on its special structure.

Theorem 1: The optimal solution of problem (10) is equivalent to that of the following

problem

$$\begin{aligned}
& \underset{\substack{\{p_n\}, \{p_{k,k'}\}, \{b_{k,k'}\}, \\ \{x_k\}, \{q_k\}, \{w_k\}}}{\text{maximize}} & & \frac{\sum_{n=1}^N r_{SC}^n + \sum_{k=1}^K x_k r_{k,k'}}{\sum_{n=1}^N \frac{p_n}{\xi} + \sum_{k=1}^K x_k \frac{p_{k,k'}}{\xi} + \sum_{k=1}^K x_k \frac{q_k}{\xi} + P_c} \\
& \text{s.t.} & & \text{C5, C6, C7,} \\
& & & \text{C1: } \sum_{n=1}^N p_n + \sum_{k=1}^K p_{k,k'} + \sum_{k=1}^K q_k \leq P_{\max}^{SC}, \\
& & & \text{C2: } b_{k,k'} + w_k = x_k W_{MC}^k, \forall k \in \mathcal{K}, \\
& & & \text{C3: } w_k \log_2 \left(1 + \frac{q_k h_k}{w_k N_0} \right) = x_k R_{MC}^k, \forall k \in \mathcal{K}, \\
& & & \text{C4: } \sum_{n=1}^N r_{SC}^n + \sum_{k=1}^K x_k r_{k,k'} \geq R_{\min}^{SC}, \tag{11}
\end{aligned}$$

where $k' = \arg \max_{n \in \mathcal{N}} g_{k,n}$.

Theorem 1 suggests that if the SC decides to serve MU k , the most energy-efficient strategy is only to share the bandwidth of MU k with one SU who has the largest channel power gain on the traded bandwidth, W_{MC}^k . In addition, constraints C2 and C3 are also met with equalities at the optimal solution since it is always beneficial for the SC to seek as much as bandwidth while consuming as less as transmit power in the spectrum-power trading with the MC. With Theorem 1, we only need to focus on solving problem (11) in the rest of the paper. Although problem (11) is more tractable than problem (10), it is still a combinatorial non-convex optimization problem. In general, there is no efficient method for this problem and the exhaustive search among all the possible cases leads to an exponential computational complexity, which is prohibitive in practice. Thus, we aim to develop a low complexity approach via exploiting the special structure of the problem.

IV. ENERGY-EFFICIENT RESOURCE ALLOCATION FOR GIVEN MU SELECTION

Denote Ψ as a set of MUs that are scheduled by the SC, i.e., $\Psi \triangleq \{k \mid x_k = 1, k \in \mathcal{K}\}$, and denote EE_{Ψ} as the maximum system EE of problem (11) based on set Ψ , i.e., $EE = EE_{\Psi}$. For a given Ψ , problem (11) is reduced to a joint bandwidth and power allocation problem. However, the reduced problem is still non-convex due to the fractional-form objective function. In the following, we show that the optimal solution of the reduced problem can be efficiently obtained by exploiting the fractional structure of the objective function in (11).

A. Problem Transformation

According to the nonlinear fractional programming theory [34], for a problem of the form,

$$q^* = \underset{S \in \mathcal{F}}{\text{maximize}} \frac{R_{\text{total}}(S)}{P_{\text{total}}(S)}, \quad (12)$$

where S is a feasible solution and \mathcal{F} is the corresponding feasible set, there exists an equivalent problem in subtractive form that satisfies

$$T(q^*) = \underset{S \in \mathcal{F}}{\text{maximize}} \left\{ R_{\text{total}}(S) - q^* P_{\text{total}}(S) \right\} = 0. \quad (13)$$

The equivalence between (12) and (13) can be easily verified with the corresponding maximum value q^* that is also the maximum system EE. Besides, Dinkelbach provides an iterative method in [34] to obtain q^* . Specifically, for a given q , we solve a maximization problem with the subtractive-form objective function as (13). The value of q is then updated and problem (13) is solved again in the next iteration until convergence. By applying this transformation to (11) with $b_{k,k'} = W_{MC}^k - w_k$ and $q_k = \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{w_k N_0}{h_k}, \forall k \in \Psi$, we obtain the following optimization problem for a given q in each iteration

$$\begin{aligned} & \underset{\{p_n\}, \{p_{k,k'}\}, \{w_k\}}{\text{maximize}} && \sum_{n=1}^N B_{SC}^n \log_2 \left(1 + \frac{p_n g_n}{B_{SC}^n N_0} \right) + \sum_{k \in \Psi} (W_{MC}^k - w_k) \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{(W_{MC}^k - w_k) N_0} \right) \\ & && - q \left(\sum_{n=1}^N \frac{p_n}{\xi} + \sum_{k \in \Psi} \frac{p_{k,k'}}{\xi} + \sum_{k \in \Psi} \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{w_k N_0}{\xi h_k} + P_c \right) \\ \text{s.t.} \quad & \text{C1:} && \sum_{n=1}^N p_n + \sum_{k \in \Psi} p_{k,k'} + \sum_{k \in \Psi} \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{w_k N_0}{h_k} \leq P_{\text{max}}^{SC}, \\ & \text{C4:} && \sum_{n=1}^N r_{SC}^n + \sum_{k \in \Psi} r_{k,k'} \geq R_{\text{min}}^{SC}, \quad \text{C6:} \quad W_{MC}^k \geq w_k \geq 0, \quad \forall k \in \Psi, \\ & \text{C7:} && p_n \geq 0, \quad p_{k,k'} \geq 0, \quad \forall k \in \Psi, n \in \mathcal{N}. \end{aligned} \quad (14)$$

After the transformation, it can be verified that problem (14) is jointly concave with respect to all optimization variables and also satisfies Slater's constraint qualification [35]. As a result, the duality gap between problem (14) and its dual problem is zero, which means that the optimal solution of problem (14) can be obtained by applying the Lagrange duality theory [14]. In the next section, we will derive the optimal bandwidth and power allocation via exploiting the Karush-Kuhn-Tucker (KKT) conditions of problem (14) that leads to a computationally efficient algorithm.

B. Joint Bandwidth and Power Allocation

The partial Lagrangian function of problem (14) can be written as

$$\begin{aligned}
& \mathcal{L}(p_n, p_{k,k'}, w_k, \lambda, \mu) \\
&= \sum_{n=1}^N B_{SC}^n \log_2 \left(1 + \frac{p_n g_n}{B_{SC}^n N_0} \right) + \sum_{k \in \Psi} (W_{MC}^k - w_k) \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{(W_{MC}^k - w_k) N_0} \right) \\
&- q \left(\sum_{n=1}^N \frac{p_n}{\xi} + \sum_{k \in \Psi} \frac{p_{k,k'}}{\xi} + \sum_{k \in \Psi} \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{w_k N_0}{\xi h_k} + P_c \right) \\
&+ \lambda \left(P_{\max}^{SC} - \sum_{n=1}^N p_n - \sum_{k \in \Psi} p_{k,k'} - \sum_{k \in \Psi} \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{w_k N_0}{h_k} \right) \\
&+ \mu \left(\sum_{n=1}^N B_{SC}^n \log_2 \left(1 + \frac{p_n g_n}{B_{SC}^n N_0} \right) + \sum_{k \in \Psi} (W_{MC}^k - w_k) \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{(W_{MC}^k - w_k) N_0} \right) - R_{\min}^{SC} \right),
\end{aligned} \tag{15}$$

where λ and μ are the non-negative Lagrange multipliers associated with constraints C1 and C4, respectively. The boundary constraints C6 and C7 are absorbed into the optimal solution in the following. Then, from Appendix B, the optimal solution can be obtained as in Theorem 2.

Theorem 2: Given λ and μ , the optimal bandwidth and power allocation of maximizing the Lagrangian function, \mathcal{L} , is given by

$$w_k = \min \left(\frac{R_{MC}^k \ln 2}{\mathcal{W} \left(\frac{1}{e} \left(\frac{C h_k}{(q+\lambda) N_0} - 1 \right) \right) + 1}, W_{MC}^k \right), \forall k \in \Psi, \tag{16}$$

$$p_{k,k'} = (W_{MC}^k - w_k) \left[\frac{(1+\mu)\xi}{(q+\lambda\xi) \ln 2} - \frac{N_0}{g_{k,k'}} \right]^+, \forall k \in \Psi, \tag{17}$$

$$p_n = B_{SC}^n \left[\frac{(1+\mu)\xi}{(q+\lambda\xi) \ln 2} - \frac{N_0}{g_n} \right]^+, \forall n \in \mathcal{N}, \tag{18}$$

where $[x]^+ \triangleq \max\{x, 0\}$ and $\mathcal{W}(x)$ is the Lambert \mathcal{W} function [36], i.e., $x = \mathcal{W}(x)e^{\mathcal{W}(x)}$. In addition, $C = (1+\mu) \log_2 \left(1 + \tilde{p}_{k,k'} \frac{g_{k,k'}}{N_0} \right) - \left(\frac{q}{\xi} + \lambda \right) \tilde{p}_k$, and $\tilde{p}_{k,k'} = \left[\frac{(1+\mu)\xi}{(q+\lambda\xi) \ln 2} - \frac{N_0}{g_{k,k'}} \right]^+$.

From (16), it is easy to show that the bandwidth allocated to MU k by the SC, i.e., w_k , increases with its minimum required data rate by the MC, R_{MC}^k , while decreasing with its channel power gain, h_k . This implies that the SC is able to seek more bandwidth from the MUs who require lower user data rates but are closer to the SC BS, which also coincides with the intuition discussed previously. Furthermore, we also observe that the optimal transmit

TABLE I
ENERGY-EFFICIENT JOINT BANDWIDTH AND POWER ALLOCATION

Algorithm 1 Energy-Efficient Joint Bandwidth and Power Allocation Algorithm

- 1: **Initialize** the maximum accuracy ϵ and set $q = 1$ with given MU k and SU k' ;
 - 2: **repeat**
 - 3: Initialize λ and μ ;
 - 4: **repeat**
 - 5: Obtain w_k , $p_{k,k'}$, and p_n from (16)-(18);
 - 6: Update dual variables λ and μ from (19) and (20);
 - 7: **until** λ and μ converge;
 - 8: Update q from (12);
 - 9: **until** $\left(R_{\text{tot}}(S) - qP_{\text{tot}}(S)\right) \leq \epsilon$.
-

power allocations, $p_{k,k'}$ and p_n , follow the conventional multi-level water-filling structure due to different bandwidth allocations. In contrast, the optimal transmit power densities, $\frac{p_{k,k'}}{W_{MC}^k - w_k}$ and $\frac{p_n}{B_{SC}^n}$, follow the single-level water-filling structure [14]. Given Lagrange multipliers λ and μ , the optimal bandwidth and power allocation can be obtained immediately from Theorem 2.

After computing the primal variables $(p_n, p_{k,k'}, w_k)$, we now proceed to solve the dual problem, i.e., minimize $\mathcal{G}(\lambda, \mu)$, where $\mathcal{G}(\lambda, \mu) = \underset{\lambda \geq 0, \mu \geq 0}{\text{maximize}} \underset{p_n, p_{k,k'}, w_k}{\mathcal{L}(p_n, p_{k,k'}, w_k, \lambda, \mu)}$. Since a dual function is always convex by definition, the commonly used ellipsoid method can be employed for updating (λ, μ) toward the optimal solution with guaranteed convergence [35]. In addition, it has been pointed in [37] that the ellipsoid method is able to converge faster and more stable across a wide variety of situations. Thus, in this paper, we adopt the ellipsoid method to update the Lagrange multipliers and the subgradients that will be used are given by

$$\Delta\lambda = P_{\max}^{SC} - \sum_{n=1}^N \frac{p_n}{\xi} - \sum_{k \in \Psi} \frac{p_{k,k'}}{\xi} - \sum_{k \in \Psi} \frac{q_k}{\xi}, \quad (19)$$

$$\Delta\mu = \sum_{n=1}^N r_{SC}^n + \sum_{k \in \Psi} r_{k,k'} - R_{\min}^{SC}. \quad (20)$$

A discussion regarding the choice of the initial ellipsoid, the updating of the ellipsoid, and the stopping criterion for the ellipsoid method can be found in [37] (Section IV-B) and is thus omitted here for brevity. The updated Lagrange multipliers in (19) and (20) can be used to obtain the bandwidth and power allocation variables in the primary variable optimization. Due to the concavity of primary problem (14), the iterative optimization between $(p_n, p_{k,k'}, w_k)$ and (λ, μ) is guaranteed to converge to the optimal solution of (14). The details of the bandwidth and power allocation for a given MU selection are summarized in Algorithm 1 in Table I.

V. ENERGY-EFFICIENT MU SELECTION

In this section, we investigate the MU selection problem, i.e., to find the MU set Ψ where $x_k = 1, \forall k \in \Psi$. We first introduce the concept of the trading EE that plays a key role in the algorithm development. Then, we study the MU selection condition under different cases and propose a low computational complexity algorithm based on the trading EE. Finally, we analyze the computational complexity of the proposed algorithm.

A. Trading EE

The Trading EE of MU k , $\forall k \in \mathcal{K}$, is defined as the total data rate of MU k brought for the SC over the total power consumed by the SC in the spectrum-power trading, i.e.,

$$EE_k = \frac{b_{k,k'} \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{b_{k,k'} N_0} \right)}{\frac{p_{k,k'}}{\xi} + \frac{q_k}{\xi}}, \quad (21)$$

where the numerator, $b_{k,k'} \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{b_{k,k'} N_0} \right)$, is the additional data rate that the SC obtains via serving MU k and the denominator, $\frac{p_{k,k'}}{\xi} + \frac{q_k}{\xi}$, is the total power consumed for both supporting SU k' and meeting the QoS of MU k . As a result, the trading EE is in fact an evaluation of an MU in terms of the power utilization efficiency and can be regarded as a profit of the SC in the spectrum-power trading. Then, the trading EE maximization problem of MU k can be

formulated as

$$\begin{aligned}
& \underset{p_{k,k'}, b_{k,k'}, q_k, w_k}{\text{maximize}} & EE_k &= \frac{b_{k,k'} \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{b_{k,k'} N_0} \right)}{\frac{p_{k,k'}}{\xi} + \frac{q_k}{\xi}} \\
& \text{s.t.} & \text{C2: } & b_{k,k'} + w_k \leq W_{MC}^k, \\
& & \text{C3: } & w_k \log_2 \left(1 + \frac{q_k h_k}{w_k N_0} \right) \geq R_{MC}^k, \\
& & \text{C7: } & b_{k,k'} \geq 0, \quad w_k \geq 0.
\end{aligned} \tag{22}$$

It is worth noting that problem (22) can be regarded as a special case of problem (10) where there is only one MU and one SU. Therefore, problem (22) can be solved similarly by the algorithm proposed in Section III. However, in order to provide more insight, we show that the optimal solution can be solved more efficiently in the following theorem that is proved in Appendix C.

Theorem 3: Problem (22) is equivalent to the following quasi-concave maximization problem

$$\underset{p_{k,k'} \geq 0, w_k \geq 0}{\text{maximize}} \quad EE_k = \frac{(W_{MC}^k - w_k) \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{(W_{MC}^k - w_k) N_0} \right)}{\frac{p_{k,k'}}{\xi} + \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{w_k N_0}{h_k \xi}}, \tag{23}$$

where EE_k is strictly and jointly quasi-concave over $p_{k,k'}$ and w_k .

Since EE_k is strictly and jointly quasi-concave over $p_{k,k'}$ and w_k under a convex feasible set, the optimal solutions of $p_{k,k'}$ and w_k are both unique. This suggests that the alternating method, also known as coordinated descent method [38], can be employed to obtain the optimal $p_{k,k'}$ and w_k efficiently [39]. Specifically, for a given w_k or $p_{k,k'}$, problem (23) is simplified into a univariate quasi-concave maximization with respect to $p_{k,k'}$ or w_k , where the optimal values can be easily obtained by the bisection method [35]. For example, for a given w_k , it has been shown in [39] that by judging the derivative of EE_k with respect to $p_{k,k'}$ is zero or not, we can obtain the optimal $p_{k,k'}$.

B. Trading EE based MU Selection

The key observation of the user trading EE is that both $b_{k,k'} \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{b_{k,k'} N_0} \right)$ and $\frac{p_{k,k'}}{\xi} + \frac{q_k}{\xi}$ will be removed respectively from the numerator and the denominator of the objective function in problem (11) if MU k is not served by the SC. With the user trading EE defined in Section III-A, we now investigate the MU selection conditions for different cases. Recall that Ψ denotes

an arbitrary set of MUs that are scheduled by the SC, i.e., $\Psi \triangleq \{k \mid x_k = 1, k \in \mathcal{K}\}$, and EE_{Ψ}^* denotes the maximum system EE of problem (11), which can be obtained by Algorithm 1 based on set Ψ . Then, we have the following theorem, proved in Appendix D, to facilitate the algorithm development.

Theorem 4: For any unscheduled MU m , i.e., $m \in \mathcal{K}, m \notin \Psi$:

- 1) in the absence of constraints C1 and C4 in problem (11), serving MU m improves the EE of the SC if and only if $EE_m^* > EE_{\Psi}^*$;
- 2) in the absence of constraint C1 in problem (11), serving MU m improves the EE of the SC if $EE_m^* > EE_{\Psi}^*$;
- 3) in the absence of constraint C4 in problem (11), serving MU m improves the EE of the SC only if $EE_m^* > EE_{\Psi}^*$.

Theorem 4 reveals the relationship between the inequality $EE_k^* > EE_{\Psi}^*$ and the MU selection under different constraints in problem (11). Specifically, without considering both the maximum power constraint and the system minimum data rate constraint, $EE_k^* > EE_{\Psi}^*$ is the sufficient and necessary condition for serving MU k . Besides, without considering the maximum power constraint, $EE_k^* > EE_{\Psi}^*$ is reduced to a sufficient condition for serving MU k . In contrast, without considering the minimum system data rate constraint, $EE_k^* > EE_{\Psi}^*$ is reduced to a necessary condition for serving MU k . Since these two constraints, i.e., C1 and C4, may not be met with equalities simultaneously in most cases, it means that $EE_k^* > EE_{\Psi}^*$ is either sufficient or necessary for serving MU k in practice. It is also interesting to mention that $EE_k^* > EE_{\Psi}^*$ has an important practical interpretation: the trading EE of MU k that is selected by the SC should be higher than the current EE of the SC. In other words, the spectrum-power trading on this MU enables the SC to have a better utilization of the power. Otherwise, the spectrum-power trading is only beneficial to the MC and does not bring any benefit for the SC.

The main implication of Theorem 4 is that an MU with higher user trading EE is more likely to be scheduled by the SC. Based on this insight, a computationally efficient MU selection scheme is designed as follows. First, sort all the MUs in the descending order according to the user trading EE. Second, for MU k satisfying the condition $EE_k^* > EE_{\Psi}^*$ in Theorem 4, set $x_k = 1$ and maximize the system EE in problem (11) by Algorithm 1. Third, by comparing the updated system EE with previous system EE where $x_k = 0$ holds, decide whether to schedule MU k . The details of the MU selection procedure is summarized in Algorithm 2 in Table II. To

TABLE II
ENERGY-EFFICIENT SPECTRUM-POWER TRADING ALGORITHM

Algorithm 2 Energy-Efficient Spectrum-Power Trading Algorithm

- 1: Obtain EE_k , $\forall k$, by solving problem (23) in Theorem 3;
 - 2: Sort all MUs in the descending order of trading EE, i.e., $EE_1^* > EE_2^* > \dots > EE_K^*$;
 - 3: Set $\Psi = \emptyset$ and obtain EE_Ψ^* by Algorithm 1;
 - 4: **for** $k = 1 : K$
 - 5: Obtain $EE_{\Psi \cup \{k\}}^*$ by Algorithm 1;
 - 6: **if** $EE_{\Psi \cup \{k\}}^* > EE_\Psi^*$
 - 7: $\Psi = \Psi \cup \{k\}$;
 - 8: **end**
 - 9: **end**
-

understand Algorithm 2 better, we provide the following corollary to characterize the optimality condition that has been proved in Appendix E.

Corollary 1: Algorithm 2 is optimal for problem (11) in the absence of constraints C1 and C4.

Corollary 1 reveals that Algorithm 2 achieves the maximum system EE of the SC if constraints C1 and C4 are not considered. This can be interpreted as follows. Without considering C1 and C4, Theorem 4 points out that an MU with trading EE higher than the current system EE is sufficient and necessary to be scheduled. In addition, the updated system EE after scheduling the MU is still lower than the trading EE of this MU. This indicates that if MU k is scheduled in the optimal solution, then the MUs with higher trading EE than MU k should also be scheduled.

C. Computational Complexity Analysis

The computational complexity of Algorithm 2 can be evaluated as follows. First, the complexity for obtaining bandwidth and power allocation variables in Algorithm 1 linearly increases with the number of MUs and the number of SUs, i.e., $\mathcal{O}(K + N)$. Second, the complexities of the ellipsoid method for updating dual variables [37] and the Dinkelbach method for updating q [13], [35], [40], [41] are both independent of K and N . Finally, the complexity of performing

the MU selection linearly increases with K . Therefore, the total complexity of Algorithm 2 is $\mathcal{O}(K(K + N))$ [42]¹.

D. Discussion on Arbitrary Weights

As mentioned in Section III, assigning different weights to different SUs in problem (10) does not affect the proposed optimization framework. Now, we show how to tackle problem (10) with arbitrary weights of different SUs. From (6), we know that the weighted system data rate of SUs can be expressed as

$$\begin{aligned}
 R_{\text{total}} &= \sum_{n=1}^N \alpha_n R_n = \sum_{n=1}^N \alpha_n \left(r_{SC}^n + \sum_{k=1}^K x_k r_{k,n} \right) \\
 &= \sum_{n=1}^N \alpha_n r_{SC}^n + \sum_{n=1}^N \alpha_n \sum_{k=1}^K x_k r_{k,n} \\
 &= \sum_{n=1}^N \alpha_n r_{SC}^n + \sum_{k=1}^K x_k \sum_{n=1}^N \alpha_n r_{k,n},
 \end{aligned} \tag{24}$$

where α_n denotes the weight of SU n , $\forall n$. Due to joint effects of weights and channel conditions, Theorem 1 does not hold any more. More specifically, the bandwidth obtained by the SC via spectrum-power trading from a MU may be shared with multiple SUs rather than one SU in the case of equal weights for SUs. However, the resource allocation algorithm and the MU selection scheme proposed in Section IV and Section V can be readily extended, which are shown as follows.

For given MU selection variables x_k , $\forall k$, the problem transformation between (12) and (13) can still be applied and it is also easy to verify that the resulting problem in subtractive form is also a concave maximization problem as problem (14). Thus, the joint bandwidth and power allocation can be similarly obtained via exploiting the KKT conditions of the transformed problem. Now, we show how to modify the defined trading EE to tackle the case of arbitrary weights. Recall that the main characteristic of the trading EE in Section V-A is to characterize the obtained throughput and the power consumption in the spectrum-power trading on an MU.

¹Note that big $\mathcal{O}(\cdot)$ notation is a mathematical notation that is used to illustrate algorithms by how they respond to the changes of the problem size [39]. Thus, factors that are independent of the problem size K and N are omitted in its formal expression.

TABLE III
SIMULATION PARAMETERS

Parameter	Description
Maximum allowed transmit power of the SC, P_{\max}^{SC}	30 dBm [3]
Licensed bandwidth of each MU, W_{MC}^k	360 kHz [43]
Licensed bandwidth of each SU, B_{SC}^n	180 kHz [43]
Static circuit power of the SC, P_c	2 W [3]
Power spectral density of thermal noise	-174 dBm/Hz
Power amplifier efficiency, ξ	0.38
Path loss model	$(128.1 + 37.6 \log_{10} d/1000)$ dB
Lognormal Shadowing	8 dB
Penetration loss	20 dB
Fading	Rayleigh fading

Thus, the trading EE of MU k , $\forall k \in \mathcal{K}$, in the case of arbitrary weights can be modified as

$$EE_k = \frac{\sum_{n=1}^N \alpha_n r_{k,n}}{\sum_{n=1}^N \frac{p_{k,n}}{\xi} + \frac{q_k}{\xi}} = \frac{\sum_{n=1}^N \alpha_n b_{k,n} \log_2 \left(1 + \frac{p_{k,n} g_{k,n}}{b_{k,n} N_0} \right)}{\sum_{n=1}^N \frac{p_{k,n}}{\xi} + \frac{q_k}{\xi}}, \quad (25)$$

where the numerator, $\sum_{n=1}^N \alpha_n r_{k,n}$, is the additional data rate that the SC obtains via serving MU k and the denominator, $\sum_{n=1}^N \frac{p_{k,n}}{\xi} + \frac{q_k}{\xi}$, is the total power consumed for both supporting SUs and meeting the QoS of MU k . Then, the maximum trading EE can be still readily obtained by solving a counterpart of problem (22). It is worth noting that the introduction of the weights α_n , $\forall n$, will not affect the structural properties of the relationship between the system EE of SC and the trading EE, i.e., Lemma 1, (33), and (34) in Appendix D still hold with the modified trading EE. Therefore, Theorem 4 and Corollary 1 can be similarly extended.

VI. NUMERICAL RESULTS

In this section, we provide numerical results to demonstrate the effectiveness of the proposed spectrum-power trading based resource allocation algorithm. The main parameters adopted in this work are from relevant works [3], [43]–[46]. We consider a two-tier heterogeneous network where there exist an MC and an SC with the coverage radii of 500 m and 50 m , respectively. Five SUs are uniformly distributed within the coverage of the SC BS while five MUs are uniformly distributed within the distances of [20 200] m away from the SC BS. The distance between the SC BS and the MC BS is set to 500 m . Without loss of generality, we assume that all MUs have identical parameters, i.e., the same amount of available bandwidth, W_{MC}^k , and minimum

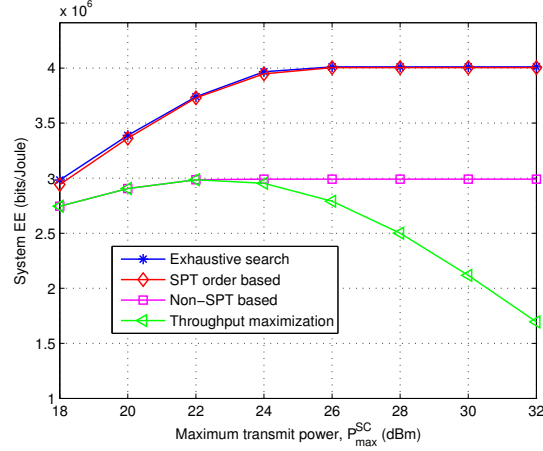


Fig. 2. System EE versus the maximum allowed transmit power of the SC.

data rate requirement, R_{MC}^k . In addition, all SUs have the identical licensed bandwidth, B_{SC}^n . Unless specified otherwise, the major parameters are listed in Table III and R_{SC} and R_{MC}^k are set to be 1000 Kbits and 700 Kbits, respectively.

A. System EE versus Maximum Transmit Power of SC, P_{\max}^{SC}

In Figure 2, we compare the achieved system EE of the following schemes: 1) Exhaustive search [35]; 2) SPT order based: Algorithm 2 in Section V; 3) Non-SPT based: the EE maximization without spectrum-power trading [15]; 4) Throughput Maximization: conventional spectral efficiency maximization [44]. It is observed that the proposed Algorithm 2 achieves near-optimal performance and outperforms all other suboptimal schemes, which demonstrates the effectiveness of the proposed scheme. We also observe that the EEs of the SPT order based scheme and the non-SPT based scheme first increases and then remain constants as P_{\max}^{SC} increases. In contrast, the EE of the throughput maximization scheme first increases and then decreases with increasing P_{\max}^{SC} , which is due to its greedy use of the transmit power. In addition, it is also seen that the performance gap between the SPT order based scheme and the non-SPT based scheme first increases and then approaches a constant. This is because when the transmit power of the SC is limited, such as $P_{\max}^{SC} = 12$ dBm, the SC may not have sufficient transmit power freedom to serve many MUs and thereby the spectrum-power trading is less likely realized, which in return limits its own performance improvement. As P_{\max}^{SC} increases, compared with the non-SPT

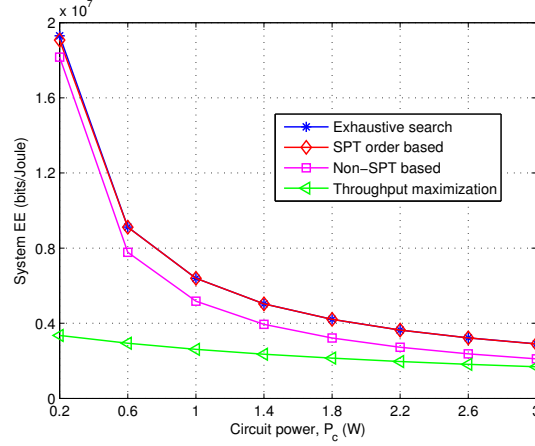


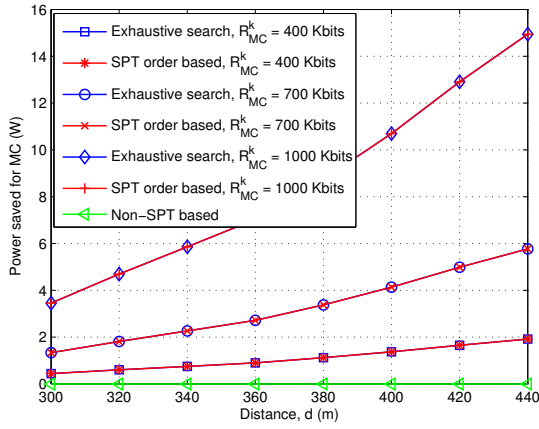
Fig. 3. System EE versus the circuit power of the SC.

based scheme, the SC not only has more transmit power to improve its EE via serving its own SUs, but also has more transmit power freedom to obtain additional bandwidth from the MC via spectrum-power trading, which thereby strengthens the effect of performance improvement. Finally, when all the ‘good’ MUs with higher trading EE are being scheduled by the SC, then the system EE improves with P_{\max}^{SC} with diminishing return and eventually approaches a constant due to the same reason as that of the non-SPT based scheme.

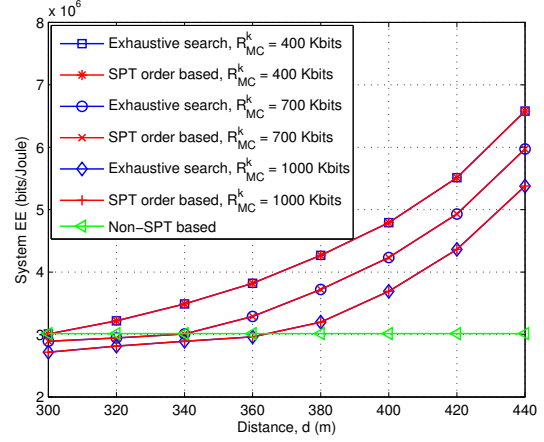
B. System EE versus Circuit Power of SC, P_c

Figure 3 illustrates the performance of all schemes as a function of the circuit power consumption of the SC. We can observe that the system EE of all schemes decreases with increasing P_c since the circuit power consumption is always detrimental to the system EE. Also, the proposed Algorithm 2 performs almost the same as the exhaustive search. In addition, the performance gap between the non-SPT scheme and the throughput maximization scheme decreases with increasing P_c . This is because as P_c increases, the circuit power consumption dominates the total power consumption rather than the transmit power consumption. Thus, improving the system EE is almost equivalent to improving the system data rate, which only results in marginal performance gap.

However, it is interesting to note that the performance gap between the SPT order based scheme and the non-SPT based scheme does not decrease but increases when P_c is in a relatively small



(a) Power saved for MC versus distance.



(b) System EE of the SC versus distance.

Fig. 4. Effect of the distance between MUs and the MC BS on the power saved for MC and the system EE of the SC.

regime, such as $P_c \in [0.2 \ 1]$ W. This is because when P_c is very small, the SC system itself enjoys a high system EE which leaves it a less incentive to perform spectrum-power trading with the MC. Thus, the system EE of the SPT order based scheme decreases with the similar slope as that of the non-SPT based scheme. As P_c increases, the system EE of the SC further decreases, which would motivate the SC to perform spectrum-power trading. As a result, the performance degradation caused by an increasing P_c is relieved for the SPT order based scheme, which thereby yields an increased performance gap between these two schemes in small P_c regime. Furthermore, when P_c is sufficiently large such that all the ‘good’ MUs are being selected, the performance gap between these two schemes decreases again due to the the domination of the circuit power in the total power consumption.

C. Effect of Distance between MUs and MC BS

In Figure 4, we evaluate the performances of the exhaustive search scheme, SPT order based scheme, and the non-SPT based scheme versus the distance between MUs and the MC BS. Without loss of generality, we assume that all MUs are located at the same distance from the SC BS and the SC BS helps to serve all MUs from the MC, i.e., $x_k = 1, \forall k$, where the MU selection is not performed. Thus, the SPT order based scheme performs the same as the exhaustive search. In Figure 4 (a), we can see that under the fixed minimum data rate requirements, more transmit power consumption is saved via the proposed spectrum-power trading when MUs are farther

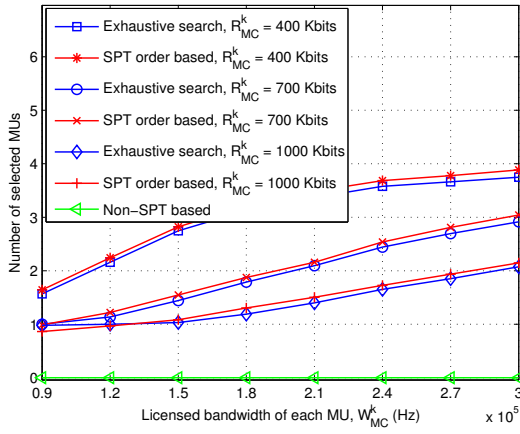
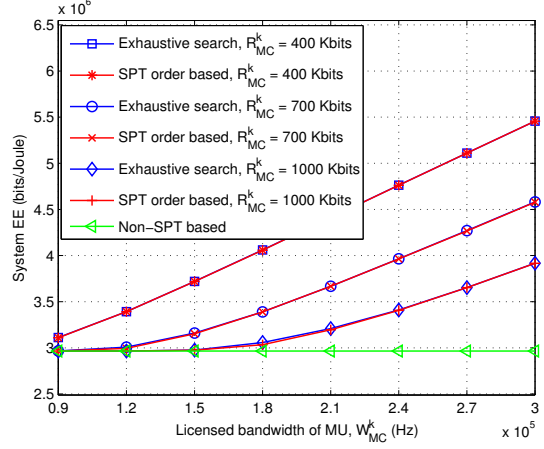
(a) Number of selected MUs versus W_{MC}^k .(b) System EE of the SC versus W_{MC}^k .

Fig. 5. Effect of licensed bandwidth of MUs on the number of selected MUs and the system EE of the SC.

away from the MC BS. In addition, when the MU data rate requirements are higher, it also saves more transmit power consumption for the MC BS. These implies that the proposed spectrum-power trading is effective by offloading the MUs to the SC BS, especially when the MUs are located in the cell edge area while requiring high user data rates. In contrast, in Figure 4 (b), we illustrate the system EE of the SC BS versus the distance between MUs and the MC BS. Basically, when the distance is larger, the proposed scheme enables higher system EE gain for the SC BS. However, as the MU data rate requirements, R_{MC}^k , increase, the system EE of the SC BS decreases since it either obtains less bandwidth or costs more transmit power by serving these MUs. In particular, when the MUs are farther away from the SC BS and also require higher user data rates, the achieved system EE may even be lower than the system EE without MU offloading. This means that although the spectrum-power trading benefits the MC, the MU selection is necessary to improve the system EE from the perspective of the SC.

D. Effect of Licensed Bandwidth of MUs, W_{MC}^k

In Figure 5, we evaluate the effect of licensed bandwidth on the number of MUs selected by the SC as well as on the system EE of the SC. Specifically, in Figure 5 (a), it is observed that the proposed SPT order based scheme still achieves an excellent performance which further demonstrates the effectiveness of exploiting the trading EE for MU selection. In addition, we can also find that the number of MUs selected by the SC increases with an increasing W_{MC}^k .

under a fixed user data rate requirement. This also coincides with our theoretical analysis for trading EE in Section V: due to the monotonically increasing characteristic of the trading EE with respect to W_{MC}^k , an MU with more bandwidth provided by the MC achieves a higher trading EE such that this MU is more likely to satisfy the MU selection condition. In contrast, for a given W_{MC}^k , requiring higher user data rates provides less incentives for the SC to serve MUs and thus the number of MUs selected by the SC decreases with a more stringent R_{MC}^k . In Figure 5 (b), it is clear to see that the system EE increases with an increasing W_{MC}^k . The performance improvement comes from two aspects. First, given a fixed minimum data rate requirement of the MU, the more the bandwidth provided by the MC, the less the power consumed by the SC via spectrum-power trading, which thus helps to improve the system EE of the SC. Second, as mentioned, a larger bandwidth will motivate the SC to serve more MUs and in return, to allow the SC to obtain more additional bandwidth via spectrum-power trading.

VII. CONCLUSIONS

In this paper, we investigated the spectrum-power trading between an SC and an MC to improve the system EE of the SC as well as reducing the power consumption of the MC. Specifically, MU selection, bandwidth allocation, and power allocation were jointly optimized while guaranteeing the QoS of both networks. The system EE maximization problem was first simplified by showing that the bandwidth from each MU is only shared with at most one SU in the SC. Given the MU selection, we transformed the fractional-form optimization problem into a subtractive one that can be solved efficiently with optimality. Then, we proposed a trading EE based MU selection scheme by studying the intrinsic relationship between the trading EE of an MU and the system EE. Simulation results showed that the proposed algorithm obtains close-to-optimal performance and also demonstrated the performance gains achieved by the proposed spectrum-power trading scheme for both the SC and the MC, especially when MUs are far away from the MC BS. For future work, although different weights can be assigned to different SUs to achieve a notion of fairness, it is still worth investigating the EE maximization problem with individual QoS constraints for SUs explicitly.

APPENDIX A

PROOF OF THEOREM 1

We first prove that the bandwidth of MU m is shared with at most one SU who has the largest channel power gain on the bandwidth of MU m . Here, we use “at most one” instead of “only one” is because this MU may also be rejected by the SC in terms of spectrum-power trading, and thus the bandwidth of this MU may not be shared with any SU. The proof is shown by contradiction as follows. Assume that $S = \{\{x_k^*\}, \{p_n^*\}, \{p_{k,n}^*\}, \{b_{k,n}^*\}, \{q_k^*\}, \{w_k^*\}\}$ achieves the optimal solution of problem (10) and there exist an MU m whose bandwidth has been shared with two SUs, SU m' and SU ℓ , $\ell \neq m'$, in the SC, i.e., $b_{m,m'}^* > 0$, $p_{m,m'}^* > 0$ and $b_{m,\ell}^* > 0$, $p_{m,\ell}^* > 0$. Denote SU m' is the SU that has the largest channel power gain on the bandwidth of MU m , i.e., $g_{m,m'} > g_{m,n}$, $\forall n \neq m$. Note that the probability of two SUs that have the same channel power gain is zero due to the continuity and the randomness of the channel fading. Then, we construct a different solution $\hat{S} = \{\{\hat{x}_k\}, \{\hat{p}_n\}, \{\hat{p}_{k,n}\}, \{\hat{b}_{k,n}\}, \{\hat{q}_k\}, \{\hat{w}_k\}\}$ where $\{\hat{x}_k\} = \{x_k^*\}$, $\{\hat{p}_n\} = \{p_n^*\}$, $\{\hat{q}_k\} = \{q_k^*\}$, $\{\hat{w}_k\} = \{w_k^*\}$, and

$$\hat{b}_{k,n} = \begin{cases} b_{m,m'}^* + b_{m,\ell}^*, & k = m, n = m', \\ 0, & k = m, n \neq m', \\ b_{k,n}^*, & k \neq m, n \in \mathcal{N}. \end{cases} \quad (26)$$

$$\hat{p}_{k,n} = \begin{cases} p_{m,m'}^* + p_{m,\ell}^*, & k = m, n = m', \\ 0, & k = m, n \neq m', \\ p_{k,n}^*, & k \neq m, n \in \mathcal{N}. \end{cases} \quad (27)$$

We also note that the constructed solution satisfies all the constraints in problem (10) and is thereby a feasible solution. Since the case when $x_k^* = 0$ is obviously satisfied, we only discuss the case when $x_k^* = 1$ in the following. Then, the data rate of MU m brought for the SC can be expressed as

$$\begin{aligned} \hat{b}_{m,m'} \log_2 \left(1 + \frac{\hat{p}_{m,m'} g_{m,m'}}{\hat{b}_{m,m'} N_0} \right) &= (b_{m,m'}^* + b_{m,\ell}^*) \log_2 \left(1 + \frac{(p_{m,m'}^* + p_{m,\ell}^*) g_{m,m'}}{(b_{m,m'}^* + b_{m,\ell}^*) N_0} \right) \\ &\stackrel{(a)}{\geq} b_{m,n}^* \log_2 \left(1 + \frac{p_{m,m'}^* g_{m,m'}}{b_{m,m'}^* N_0} \right) + b_{m,\ell}^* \log_2 \left(1 + \frac{p_{m,\ell}^* g_{m,m'}}{b_{m,\ell}^* N_0} \right) \\ &\stackrel{(b)}{>} b_{m,n}^* \log_2 \left(1 + \frac{p_{m,m'}^* g_{m,m'}}{b_{m,m'}^* N_0} \right) + b_{m,\ell}^* \log_2 \left(1 + \frac{p_{m,\ell}^* g_{m,\ell}}{b_{m,\ell}^* N_0} \right), \end{aligned} \quad (28)$$

where inequality (a) holds due to the concavity of $f \log_2(1 + \frac{y}{f})$ and strict inequality (b) holds due to $g_{m,m'} > g_{m,\ell}$, $\ell \neq m'$. This means that the constructed solution \hat{S} achieves higher system data rate with the same total power consumption and thus yields higher system EE than S^* which contradicts the assumption that S^* is optimal.

Now, we show that constraints C2 and C3 are met with equalities and the proof is summarized as follows. 1) if $x_k = 0$, then from C2, any feasible solution of problem (10) must satisfy $b_{k,k'} + w_k \leq 0$. Since $b_{k,k'} \geq 0$ and $w_k \geq 0$, it follows that $b_{k,k'} = 0$ and $w_k = 0$. Thus, C2 and C3 are met with equalities. 2) if $x_k = 1$ and $b_{k,n} + w_k < W_{MC}^k$ and (or) $w_k \log_2 \left(1 + \frac{q_k h_k}{w_k N_0}\right) < R_{MC}^k$ holds in the optimal solution, we can always construct another solution by increasing $b_{k,k'}$ and (or) decreasing p_k such that C2 and C3 are met with equalities while achieving a larger system EE, which contradicts that the optimal solution is achieved under strict inequality constraints C2 and C3. Theorem 1 is thus proved.

APPENDIX B

PROOF OF THEOREM 2

Taking the partial derivative of \mathcal{L} with respect to p_n , $p_{k,k'}$, and w_k , respectively, yields

$$\frac{\partial \mathcal{L}}{\partial p_n} = \frac{(1 + \mu) B_{SC}^n g_n}{(B_{SC}^n N_0 + p_n g_n) \ln 2} - \left(\frac{q}{\xi} + \lambda \right), \forall n \in \mathcal{N}, \quad (29)$$

$$\frac{\partial \mathcal{L}}{\partial p_{k,k'}} = \frac{(1 + \mu)(W_{MC}^k - w_k) g_{k,k'}}{((W_{MC}^k - w_k) N_0 + p_{k,k'} g_{k,k'}) \ln 2} - \left(\frac{q}{\xi} + \lambda \right), \forall k \in \Psi, \quad (30)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w_k} = & -(1 + \mu) \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{(W_{MC}^k - w_k) N_0} \right) + \frac{(1 + \mu) p_{k,k'} g_{k,k'}}{((W_{MC}^k - w_k) N_0 + p_{k,k'} g_{k,k'}) \ln 2} \\ & - \left(\frac{q}{\xi} + \lambda \right) \left(\left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{N_0}{h_k} - 2^{\frac{R_{MC}^k}{w_k}} \frac{R_{MC}^k N_0}{w_k h_k} \ln 2 \right), \forall k \in \Psi. \end{aligned} \quad (31)$$

Setting $\frac{\partial \mathcal{L}}{\partial p_{k,k'}} = 0$ and $\frac{\partial \mathcal{L}}{\partial p_n} = 0$, the optimal transmit power $p_{k,k'}$ and p_n can be obtained as (17) and (18), respectively. Substituting (17) into $\frac{\partial \mathcal{L}}{\partial w_k}$ yields

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w_k} = & -(1 + \mu) \log_2 \left(1 + \tilde{p}_{k,k'} \frac{g_{k,k'}}{N_0} \right) + \left(\frac{q}{\xi} + \lambda \right) \tilde{p}_k \\ & - \left(\frac{q}{\xi} + \lambda \right) \left(\left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{N_0}{h_k} - 2^{\frac{R_{MC}^k}{w_k}} \frac{R_{MC}^k N_0}{w_k h_k} \ln 2 \right), \forall k \in \Psi, \end{aligned} \quad (32)$$

where $\tilde{p}_{k,k'} = \left[\frac{(1+\mu)\xi}{(q+\lambda\xi)\ln 2} - \frac{N_0}{g_{k,k'}} \right]^+$. Note that $\frac{\partial \mathcal{L}}{\partial w_k}$ now only involves the optimization variable w_k . Setting $\frac{\partial \mathcal{L}}{\partial w_k} = 0$, we have

$$\left(2^{\frac{R_{MC}^k}{w_k}} \frac{R_{MC}^k N_0}{w_k h_k} \ln 2 - \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{N_0}{h_k} \right) = \frac{\mathcal{C}}{\frac{q}{\xi} + \lambda}, \forall k \in \Psi, \quad (33)$$

where $\mathcal{C} = (1+\mu) \log_2 \left(1 + \tilde{p}_{k,k'} \frac{g_{k,k'}}{N_0} \right) - \left(\frac{q}{\xi} + \lambda \right) \tilde{p}_k$ and (16) is obtained from (33). In addition, it is easy to verify that the left hand side of (33) is a monotonically decreasing function of w_k , which implies that there exists a unique w_k that satisfies (33). Thus, the value of w_k can be efficiently obtained by the bisection method.

APPENDIX C

PROOF OF THEOREM 3

Given w_k in problem (22), it is easy to see that EE_k increases with $b_{k,k'}$ and decreases with q_k . Thus, it can be verified that C2 and C4 are met with equalities at the optimal solution. Substituting $b_{k,k'} = W_{MC}^k - w_k$ and $q_k = \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{w_k N_0}{h_k}$ into problem (22) results in (23). In addition, since $(W_{MC}^k - w_k) \log_2 \left(1 + \frac{p_{k,k'} g_{k,k'}}{(W_{MC}^k - w_k) N_0} \right)$ is strictly concave over w_k and $\frac{p_{k,k'}}{\xi} + \left(2^{\frac{R_{MC}^k}{w_k}} - 1 \right) \frac{w_k N_0}{h_k \xi}$ are jointly convex over $p_{k,k'}$ and w_k , then it follows that the objective function of problem (23), EE_k , is jointly quasi-concave with respect to $p_{k,k'}$ and w_k [35], [39], which completes the proof of Theorem 3.

APPENDIX D

PROOF OF THEOREM 4

We first introduce a lemma [11] to facilitate the proof.

Lemma 1: Assume that a , b , c , and d are arbitrary positive numbers. Then, we have

$$\min \left\{ \frac{a}{b}, \frac{c}{d} \right\} \leq \frac{a+c}{b+d} \leq \max \left\{ \frac{a}{b}, \frac{c}{d} \right\}, \quad (34)$$

where “=” holds if and only if $\frac{a}{b} = \frac{c}{d}$.

Based on Lemma 1, we first prove 1) in Theorem 4. Let $\mathcal{S}^* = \left\{ \{p_n^*\}, \{p_{k,k'}^*\}, \{q_k^*\}, \{b_{k,k'}^*\}, \{w_k^*\} \right\}$ denote the optimal solution of problem (22) and its corresponding user EE is denoted as EE_k . Let $\hat{\mathcal{S}} = \left\{ \{p_n^*\}, \{\hat{p}_{k,k'}\}, \{\hat{q}_k\}, \{\hat{b}_{k,k'}\}, \{\hat{w}_k\} \right\}$ and $\tilde{\mathcal{S}} = \left\{ \{\tilde{p}_n\}, \{\tilde{p}_{k,k'}\}, \{\tilde{q}_k\}, \{\tilde{b}_{k,k'}\}, \{\tilde{w}_k\} \right\}$ denote the optimal solutions of problem (10) with $x_k = 1$ for $k \in \Psi$ and $k \in \Psi \cup \{m\}$, respectively,

where $m \notin \Psi$. The corresponding system EEs are denoted as EE_Ψ^* and $EE_{\Psi \cup \{m\}}^*$, respectively. Then, we have the following

$$\begin{aligned}
EE_{\Psi \cup \{m\}}^* &= \frac{\sum_{n=1}^N r_{SC}^n(\tilde{p}_n) + \sum_{k \neq m} r_{k,k'}(\tilde{b}_{k,k'}, \tilde{p}_{k,k'}) + r_{m,m'}(\tilde{b}_{m,m'}, \tilde{p}_{m,m'})}{\sum_{n=1}^N \frac{\tilde{p}_n}{\xi} + \sum_{k \neq m} \frac{\tilde{p}_{k,k'}}{\xi} + \sum_{k \neq m} \frac{\tilde{q}_k}{\xi} + P_c + \frac{\tilde{p}_{m,m'}}{\xi} + \frac{\tilde{q}_m}{\xi}} \\
&\stackrel{(a)}{\geq} \frac{\sum_{n=1}^N r_{SC}^n(\hat{p}_n) + \sum_{k \neq m} r_{k,k'}(\hat{b}_{k,k'}, \hat{p}_{k,k'}) + r_{m,m'}(b_{m,m'}^*, p_{m,m'}^*)}{\sum_{n=1}^N \frac{\hat{p}_n}{\xi} + \sum_{k \neq m} \frac{\hat{p}_{k,k'}}{\xi} + \sum_{k \neq m} \frac{\hat{q}_k}{\xi} + P_c + \frac{p_{m,m'}^*}{\xi} + \frac{q_m^*}{\xi}} \\
&\stackrel{(b)}{\geq} \min \left\{ \frac{\sum_{n=1}^N r_{SC}^n(\hat{p}_n) + \sum_{k \neq m} r_{k,k'}(\hat{b}_{k,k'}, \hat{p}_{k,k'})}{\sum_{n=1}^N \frac{\hat{p}_n}{\xi} + \sum_{k \neq m} \frac{\hat{p}_{k,k'}}{\xi} + \sum_{k \neq m} \frac{\hat{q}_k}{\xi} + P_c}, \frac{r_{m,m'}(b_{m,m'}^*, p_{m,m'}^*)}{\frac{p_{m,m'}^*}{\xi} + \frac{q_m^*}{\xi}} \right\} \\
&= \min \{EE_\Psi^*, EE_m^*\}, \tag{35}
\end{aligned}$$

where inequality (a) holds due to the fact that \tilde{S} is the optimal solution of problem (10) with $x_k = 1$ for $k \in \Psi \cup \{m\}$. Inequality (b) holds due to Lemma 1 and the equality “=” holds only when $EE_\Psi^* = EE_m^*$. Thus, we can conclude $EE_m^* > EE_\Psi^* \implies EE_{\Psi \cup \{m\}}^* > EE_\Psi^*$, which completes the proof of the “if” part. In the next, we prove $EE_{\Psi \cup \{m\}}^* > EE_\Psi^* \implies EE_m^* > EE_\Psi^*$, which is equivalent to its contrapositive proposition, i.e., $EE_m^* \leq EE_\Psi^* \implies EE_{\Psi \cup \{m\}}^* \leq EE_\Psi^*$.

Then, we have the following

$$\begin{aligned}
EE_{\Psi \cup \{m\}}^* &= \frac{\sum_{n=1}^N r_{SC}^n(\tilde{p}_n) + \sum_{k \neq m} r_{k,k'}(\tilde{b}_{k,k'}, \tilde{p}_{k,k'}) + r_{m,m'}(\tilde{b}_{m,m'}, \tilde{p}_{m,m'})}{\sum_{n=1}^N \frac{\tilde{p}_n}{\xi} + \sum_{k \neq m} \frac{\tilde{p}_{k,k'}}{\xi} + \sum_{k \neq m} \frac{\tilde{q}_k}{\xi} + P_c + \frac{\tilde{p}_{m,m'}}{\xi} + \frac{\tilde{q}_m}{\xi}} \\
&\stackrel{(c)}{\leq} \max \left\{ \frac{\sum_{n=1}^N r_{SC}^n(\tilde{p}_n) + \sum_{k \neq m} r_{k,k'}(\tilde{b}_{k,k'}, \tilde{p}_{k,k'})}{\sum_{n=1}^N \frac{\tilde{p}_n}{\xi} + \sum_{k \neq m} \frac{\tilde{p}_{k,k'}}{\xi} + \sum_{k \neq m} \frac{\tilde{q}_k}{\xi} + P_c}, \frac{r_{m,m'}(\tilde{b}_{m,m'}, \tilde{p}_{m,m'})}{\frac{\tilde{p}_{m,m'}}{\xi} + \frac{\tilde{q}_m}{\xi}} \right\} \\
&\stackrel{(d)}{\leq} \max \left\{ \frac{\sum_{n=1}^N r_{SC}^n(\hat{p}_n) + \sum_{k \neq m} r_{k,k'}(\hat{b}_{k,k'}, \hat{p}_{k,k'})}{\sum_{n=1}^N \frac{\hat{p}_n}{\xi} + \sum_{k \neq m} \frac{\hat{p}_{k,k'}}{\xi} + \sum_{k \neq m} \frac{\hat{q}_k}{\xi} + P_c}, \frac{r_{m,m'}(b_{m,m'}^*, p_{m,m'}^*)}{\frac{p_{m,m'}^*}{\xi} + \frac{q_m^*}{\xi}} \right\} \\
&= \max \{EE_\Psi^*, EE_m^*\}, \tag{36}
\end{aligned}$$

where inequality (c) holds due to Lemma 1 and the equality “=” represents the special case when the current SC EE is the same as trading EE of MU k . Inequality (d) holds due to the fact that both \hat{S} and S^* are optimal solutions of problem (10) with $x_k = 1$ for $k \in \Psi$ and problem (22), respectively. Thus, if $EE_m^* \leq EE_\Psi^*$, then we can conclude $EE_{\Psi \cup \{m\}}^* \leq EE_\Psi^*$ from (36), which completes the proof of the “only if” part.

Based on 1), we next prove 2) in Theorem 4. When the minimum system data rate constraint C4 instead of C1 is considered in problem (10), the inequality (d) may not hold in (36). This

is because \tilde{S} only needs to satisfy

$$\begin{aligned} & \sum_{n=1}^N r_{SC}^n(\tilde{p}_n) + \sum_{k=1}^K r_{k,k'}(\tilde{b}_{k,k'}, \tilde{p}_{k,k'}) \\ &= \sum_{n=1}^N r_{SC}^n(\tilde{p}_n) + \sum_{k \neq m} r_{k,k'}(\tilde{b}_{k,k'}, \tilde{p}_{k,k'}) + r_{m,m'}(\tilde{b}_{m,m'}, \tilde{p}_{m,m'}) \geq R_{\min}^{SC}, \end{aligned} \quad (37)$$

while \hat{S} has to satisfy

$$\sum_{n=1}^N r_{SC}^n(\hat{p}_n) + \sum_{k \neq m} r_{k,k'}(\hat{b}_{k,k'}, \hat{p}_{k,k'}) \geq R_{\min}^{SC}. \quad (38)$$

From (37) and (38), we note that the feasible transmit power region of \tilde{S} is larger than the feasible region composed of S^* and \hat{S} , which leads that the inequality (d) may not hold in (36). However, based on this, it is straightforward to show that inequality (a) still holds in (35).

Based on 1), we next prove 3) in Theorem 4. When the total power constraint C1 instead of C4 is considered in problem (10), the inequality (a) may not hold in (35). This is because the solutions S^* and \hat{S} are restricted to individual total power constraints, i.e.,

$$\sum_{n=1}^N \hat{p}_n + \sum_{k \neq m} \hat{p}_{k,k'} + \sum_{k \neq m} \hat{q}_k \leq P_{\max}^{SC}, \quad (39)$$

$$p_{m,m'}^* \geq 0, q_m^* \geq 0, \quad (40)$$

while \tilde{S} only have one total power constraint,

$$\sum_{n=1}^N \tilde{p}_n + \sum_{k=1}^K \tilde{p}_{k,k'} + \sum_{k=1}^K \tilde{q}_k = \sum_{n=1}^N \tilde{p}_n + \sum_{k \neq m} \tilde{p}_{k,k'} + \sum_{k \neq m} \tilde{q}_k + \tilde{p}_{m,m'} + \tilde{q}_m \leq P_{\max}^{SC}. \quad (41)$$

From (39), (40), and (41), we note that the feasible transmit power region composed of S^* and \hat{S} is larger than the feasible region of \tilde{S} , which leads that the inequality (a) may not hold in (35). However, based on this, it is straightforward to show that inequality (d) still holds in (36).

APPENDIX E

PROOF OF COROLLARY 1

Since the MUs are sorted in the descending order in terms of the trading EE, i.e., $EE_1^* > EE_2^* > \dots > EE_K^*$, we have the following lemma in the absence of constraints C1 and C4.

Lemma 2: 1) If selecting MU k increases the system EE of the SC, i.e., $EE_{\Psi} < EE_k$, then selecting MU ℓ , $\forall \ell \leq k$, also increases the system EE of the SC. 2) If selecting MU k decreases

the system EE of the SC, i.e., $EE_\Psi > EE_k$, then selecting MU ℓ , $\forall \ell \geq k$, also decreases the system EE of the SC.

Proof: If $EE_k > EE_\Psi$, then we have $EE_\ell \geq EE_k > EE_\Psi$, $\forall \ell \leq k$, due to the descending order of MUs. Since $EE_\ell > EE_\Psi$ has been proved as the sufficient and necessary for selecting MU ℓ in Theorem 4, we have the first statement. If $EE_k < EE_\Psi$, then we have $EE_\ell \leq EE_k < EE_\Psi$, $\forall \ell \geq k$, which results the second statement. ■

From Lemma 2, it is easy to prove that there exists an MU k^* , for $0 \leq k^* \leq K$ such that the system EE of SC increases with k for $0 \leq k \leq k^*$ and decreases with k for $k^* \leq k \leq K$, respectively. As special cases, $k^* = 0$ or $k^* = K$ means the system EE without spectrum-power trading or with spectrum-power trading for all MUs. Thus, we have the following corollary which can be easily proved based on previous the above discussion.

Corollary 2: MUs only with order index $0 \leq k \leq k^*$, $\forall 0 \leq k^* \leq K$, are selected by Algorithm 2.

For the purpose of illustration, we denote $\Psi = \{0\}$ as the case when no MU is selected by the SC for spectrum-power trading. With Corollary 2, we only need to prove that the system EE of the SC achieved based on MU set $\Psi^* = \{0, 1, \dots, k^*\}$ is larger than that of any other set Ψ , which is shown by contradiction. Without loss of generality, we assume that $\hat{\Psi}$ is the optimal MU set but there exists an MU m for $m \leq k^*$ that does not belong to $\hat{\Psi}$ and an MU n for $n > k^*$ that belongs to $\hat{\Psi}$, i.e., $\hat{\Psi} = \{0, \dots, m-1, m+1, \dots, k^*, n\}$. All other cases can be directly extended from the study of this assumption. Thus, we only need to show $EE_{\Psi^*}^* > EE_{\hat{\Psi}}^*$. We introduce an auxiliary MU set $\tilde{\Psi} = \{0, 1, \dots, k^*, n\}$. Since MU k^* and $(k^* + 1)$ is selected and not selected by Algorithm 2, respectively, from Theorem 4, we have $EE_{\Psi^*}^* < EE_{k^*}^* \leq EE_m^*$ and $EE_{\Psi^*}^* > EE_{k^*+1}^* \geq EE_n^*$, respectively, due to the descending order. With $EE_{\Psi^*}^* > EE_n^*$, it follows that $EE_{\Psi^* \cup \{n\}}^* = EE_{\tilde{\Psi}}^* < EE_{\Psi^*}^* < EE_{k^*}^* \leq EE_m^*$. Then, with $EE_{\tilde{\Psi}}^* < EE_m^*$, it follows that $EE_{\tilde{\Psi} \setminus m}^* = EE_{\hat{\Psi}}^* < EE_{\tilde{\Psi}}^*$, which contradicts the assumption that $\hat{\Psi}$ is the optimal MU set. Corollary 1 is thus proved.

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