

Human-Centric Resource Allocation in the Metaverse over Wireless Communications

Jun Zhao, Liangxin Qian, Wenhan Yu

Abstract—The Metaverse will provide numerous immersive applications for human users, by consolidating technologies like extended reality (XR), video streaming, and cellular networks. Optimizing wireless communications to enable the human-centric Metaverse is important to satisfy the demands of mobile users. In this paper, we formulate the optimization of the system utility-cost ratio (UCR) for the Metaverse over wireless networks. Our human-centric utility measure for virtual reality (VR) applications of the Metaverse represents users' perceptual assessment of the VR video quality as a function of the data rate and the video resolution and is learned from real datasets. The variables jointly optimized in our problem include the allocation of both communication and computation resources as well as VR video resolutions. The system cost in our problem comprises the energy consumption and delay and is non-convex with respect to the optimization variables. To solve the non-convex optimization, we develop a novel fractional programming technique, which contributes to optimization theory and has broad applicability beyond our paper. Our proposed algorithm for the system UCR optimization is computationally efficient and finds a stationary point to the constrained optimization. Through extensive simulations, our algorithm is demonstrated to outperform other approaches.

Index Terms—Metaverse, human-centric, resource allocation, virtual reality, wireless communications.

I. INTRODUCTION

The Metaverse is expected to offer a myriad of opportunities for mobile users to interact with the immersive virtual world [1]. In various Augmented/Virtual Reality (AR/VR) applications for the Metaverse, humans are at the core since users judge whether the AR/VR videos or games provide a satisfying Quality of Experience (QoE) [2]. Compared with the traditional Quality of Service (QoS) that measures the objective service performance (e.g., bit rate, data accuracy), QoE as a utility measure concerns the enjoyment of users [3]. Providing satisfying utilities to multiple users in a resource-constrained system requires allocating resources wisely. In this paper, we formulate and solve human-centric resource allocation for VR in the Metaverse over wireless communications. Our goal is to reduce the Metaverse system's cost in terms of delay and energy, as well as to enhance the human-centric utilities of mobile users accessing the Metaverse via wireless networks. Tackling this problem also motivates us to propose a new optimization technique.

Studied problem. Our researched system consists of one Metaverse Server (MS) and multiple VR Users (VUs). We consider downlink wireless communications, where the MS sends to each VU the corresponding VR video via frequency division multiple access (FDMA). The MS solves the system utility-cost ratio (UCR) optimization by allocating 1) communication resources (i.e., bandwidth and transmission power)

for the MS's communication with each VU, and 2) the MS's computation resources for processing the videos to be sent to VUs, as well as deciding 3) the video resolution for all VUs, and 4) the CPU frequencies for the VUs. Then, the MS uses the allocated computation resource to process each VR video with the selected resolution, and transmits the videos to the VUs with the decided bandwidth and transmission power. Each VU receives VR frames of a video and processes the frames with the arranged CPU frequency. To perform the UCR optimization, the MS knows the human-centric utilities of all VUs and how the energy or delay depends on the optimization variables. The system cost is a weighted sum of the energy consumption and delay. For energy, we take into account both the MS and VUs. The energy usage on the MS comprises those for video processing and transmission, whereas the energy of VUs is for video processing. The delay computation includes the processing on the MS, the wireless transmission, and the processing on each VU. Next, we discuss VUs' human-centric utilities.

Human-centric utility. Prior resource allocation studies [4]–[9] for wireless communications typically do not consider human-centric utilities. Incorporating subjective user perception into the design is critical for the development of VR and the Metaverse, as it provides valuable insights into how the technologies can be improved to deliver the best possible experiences for VUs. A recent work [2] also argues the importance of developing the Metaverse to be human-centric. In our paper, the human-centric utility for each VU is learned from the VU's perceptual assessment of the VR video quality as a function of the data rate and the video resolution, as illustrated by a recent dataset reporting users' evaluation of watching 360° VR videos [10]. Then UCR is the ratio of all VUs' sum human-centric utilities to the system cost.

Our **contributions** include the problem formulation, a novel fractional programming technique, and an efficient optimization algorithm, as listed below.

- To the best of our knowledge, our work is the first in the literature to consider the optimization of the system utility-cost ratio (UCR) for the Metaverse over wireless communications. Our work is also among pioneering studies that incorporate human-centric utility for Metaverse optimization.
- We propose a novel technique for fractional programming (FP), where the objective to be minimized is the sum of a convex function and a series of non-convex ratios with convex numerators and concave denominators. FP of the above kind cannot be addressed by prior work [11], [12] (viz., Section IV). Our technique contributes to optimization theory and is applicable to many other problems.
- Our UCR optimization is difficult to solve due to the following two aspects: 1) the objective function being the sum of a complicated function and a sequence of

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non-convex fractions, and 2) jointly deciding five vector variables (for bandwidth, transmission power, video resolution, allocation of the Metaverse server's computing resource, CPU frequencies of VR users, respectively). Despite the challenges, we propose an efficient algorithm by leveraging our novel FP technique above and carefully identifying a roadmap (on Pages 8 and 8) to solve the variables step-by-step.

- Simulations demonstrate the superiority of our algorithm over other baselines. The human-centric utilities used in the simulations are learnt from real-world data including a recent VR dataset [10].

Roadmap. This work is organized as follows. We review related work in Section II. The system model is presented in Section III. We propose a novel technique for fractional programming (FP) in Section IV. Using this FP technique, we analyze how to solve the UCR optimization for the Metaverse over wireless communications in Sections V and VI. Based on the analysis, Section VII presents our algorithm for the UCR optimization, as well as its performance including solution quality, convergence, and time complexity. We model human-centric utilities from real datasets in Section VIII, and use the obtained utility functions to provide simulation results in Section IX. We conclude the paper in Section X.

II. RELATED WORK

We discuss related work from the following aspects: optimization in wireless networks, the Metaverse over wireless communications, human-centric utility, and the fractional programming technique.

Optimization for wireless networks. Many studies have addressed optimization related to delay, energy, or utility for wireless networks, as discussed below. Optimizing the system cost, defined as the weighted sum of system delay and system energy consumption, is investigated in [4], [5] for wireless federated learning, in [7] for UAV-enabled mobile edge computing, in [8] for 5G networks. In [9], the difference between the utility and the energy consumption in a heterogeneous network is maximized. There are also papers on ratio optimization to improve the system's performance. The ratio is often energy efficiency (EE) [13], [14] or computation efficiency (CE) [15], [16], which denotes the ratio of the number of transmitted or computed bits to energy consumption. EE or CE above can be understood as $\frac{\text{utility}}{\text{energy}}$, but surprisingly there seems no existing work in communication/network publications on optimizing $\frac{\text{utility}}{\text{energy}+\text{delay}}$ like our paper, although we have conducted an extensive literature survey. Moreover, we consider human-centric utility for the Metaverse, which further increases the novelty of our studied problem.

Metaverse over wireless communications. Recently, researching the Metaverse over wireless communications and networks has become an emerging topic. Recent papers [1], [3] have surveyed Metaverse research from different aspects: [1] focusing on fundamental underlying technologies as well as security/privacy issues, [3] on how edge computing empowers the Metaverse. In addition to surveys [1], [3] above, we discuss representative technical work [17]–[21] below. In [17], sam-

pling, communication and prediction are co-designed to minimize the communication load for synchronizing a real-world device and its digital model in the Metaverse. Yu *et al.* [18] optimize the delay and reliability of wireless Metaverse using deep reinforcement learning. Contest theory is utilized in [19] for the Metaverse with semantic communications, while game theory is applied in [20], [21] for the vehicular Metaverse. In [14] led by the current paper's first author, fractional programming (FP) is leveraged for energy efficiency optimization of the Metaverse subject to physical-layer security of wireless communications. In addition to the difference in terms of problem formulation compared with ours, [14] allocates communication resources only without optimizing computing resources and video resolutions. Also [14] does not use our novel FP technique of Section IV.

Network Utility Maximization (NUM). Our work is related to the research on network utility maximization (NUM) [22]. For a network of users, NUM considers that all users act altruistically to maximize the total network utility [23], defined as the sum of all users' individual utilities. In the classical NUM problem by Kelly *et al.* [24], the goal is to allocate traffic rates to users in order to maximize the total network utility subject to resource constraints (e.g., link capacity limitations). Since then, various NUM problems have been investigated in the literature [25]–[27]. The total network utility is also referred to as the social welfare in [28], where game theory is adopted to solve the problem. Despite the relevance of NUM research to our work, we emphasize that our objective is optimizing the ratio $\frac{\mathcal{U}}{\mathcal{C}}$ of the total system utility \mathcal{U} to the total system cost \mathcal{C} , rather than just maximizing \mathcal{U} . The optimization of the fraction $\frac{\mathcal{U}}{\mathcal{C}}$ is more challenging than that of \mathcal{U} due to the non-convexity of the fraction.

Human-centric utility. When utility is referred to as video quality, it can be measured using objective or subjective assessment methods. The subjective quality assessment (SQA) results in human-centric perceptual utility since human subjects are asked for their opinions directly. Higher human-centric utility means better Quality of Experience (QoE), which is in contrast with the traditional notion of Quality of Service (QoS) that quantifies the objective performance of the system. The survey [29] covers human-centric utility for traditional 2D video applications. Human-centric design for Augmented/Virtual Reality (AR/VR) and the Metaverse has received much interest recently. A 2023 survey [30] systematically reviews human-centric mobile AR. Elwady *et al.* [10] report SQA of users watching 360° videos when wearing HTC Vive Pro VR headsets. In [2], the human-centric nature of the Metaverse and using it for personalized value creation are discussed. In the current paper on VR for the Metaverse, we model human-centric utility functions of VR users from SQA video datasets [10], [31], as elaborated on in Section VIII later. The logarithmic function form will be adopted, which has been used in [32] for crowdsourcing, in [33] for mobile edge computing, and in [34] for space-air-ground integrated networks.

Fractional programming (FP). In this paper, we present a novel FP technique and use it to transform a non-convex optimization problem into parametric convex optimization.

The detailed comparison between our work and other FP papers [11], [12] is deferred to Section IV.

III. SYSTEM MODEL

Our studied system consists of one Metaverse Server (MS) and N VR Users (VUs), indexed by $\mathcal{N} = \{1, 2, \dots, N\}$. In downlink wireless communications, the MS sends to each VU the corresponding VR video via frequency division multiple access (FDMA) so that communications do not interfere.

We have overviewed the system operation in the ‘‘Studied problem’’ paragraph on Page 1. As already stated, our goal is to optimize the system utility-cost ratio (UCR), by deciding communication resources (i.e., bandwidth and transmission power) and computation resources (i.e., the MS’s computation allocation and the VUs’ CPU frequencies), as well as VR video resolutions. Figure 1 illustrates the system model.

Note that before the video transmission, there are message exchanges between the MS and VUs for control purpose; e.g., each VU informs the MS of its maximum CPU frequency and utility function, and the MS notifies the obtained CPU frequency for each VU from the system utility-cost ratio (UCR) optimization. We ignore the overhead of the control information since it is much smaller than the video data sizes. Below we first introduce notations, which are used to define the system utility and cost in Sections III-A and III-B. Then we formalize the UCR optimization in Section III-C.

For communications via FDMA, we define $\mathbf{b} = [b_1, b_2, \dots, b_N]$, $\mathbf{p} = [p_1, p_2, \dots, p_N]$ as the bandwidths and transmission powers used for the MS to communicate with VUs. With g_n being the channel attenuation from MS to VU n , the achievable rate from MS to VU n is given by the function notation below:

$$r_n(b_n, p_n) = b_n \log_2(1 + \frac{g_n p_n}{\sigma^2 b_n}). \quad (1)$$

A. Modeling the human-centric utilities of VR users

Based on the subjective test in [10], for each VU n , we formulate the human-centric utility as $U_n(r_n, s_n)$, a function of the transmission rate r_n and resolution s_n satisfying Assumption 1 below.

Assumption 1. $U_n(r_n, s_n)$ is non-decreasing in r_n and s_n , concave in r_n , and concave in s_n .

The vector $\mathbf{s} = [s_1, r_2, \dots, s_N]$ gives the resolutions of VR frames for the VUs. The system utility, defined as the sum of all N VUs’ human-centric utilities, is given by

$$\mathcal{U}(\mathbf{b}, \mathbf{p}, \mathbf{s}) = \sum_{n \in \mathcal{N}} U_n(r_n(b_n, p_n), s_n). \quad (2)$$

Our analysis and algorithm use Assumption 1, and do not need $U_n(r_n, s_n)$ ’s joint concavity in r_n and s_n , though the expression of $U_n(r_n, s_n)$ in Section VIII from real datasets is jointly concave in r_n and s_n .

B. System cost comprising delay and energy consumption

We start with defining some notations. For each $n \in \mathcal{N}$, let f_n^{MS} be the MS’s computational resource allocated to process the frames for VU n . Such allocation of computing resources is also considered in [35] for edge computing. The CPU frequency of VU n is denoted by f_n^{VU} . Then $\mathbf{f}^{\text{MS}} := [f_1^{\text{MS}}, f_2^{\text{MS}}, \dots, f_N^{\text{MS}}]$ and $\mathbf{f}^{\text{VU}} := [f_1^{\text{VU}}, f_2^{\text{VU}}, \dots, f_N^{\text{VU}}]$. About the frames for VU n , let μ_n be the number of bits per pixel,

and $\nu_n > 1$ be the compression ratio. The MS will generate a VR video of Λ_n frames for XU n . Let $\mathcal{A}_n(s_n, \Lambda_n)$ (resp., $\mathcal{B}_n(s_n, \Lambda_n)$) be the number of CPU cycles on MS (resp., VU n) to process a part of those Λ_n frames before (resp., after) wireless transmission. While later frames of the VR video are yet to be generated, earlier frames can be transmitted from the MS to each VU n . Similarly, while later frames of the VR video are yet to be received, VU n can process earlier frames which have already been accepted. Hence, the following three stages partially overlap: processing at the MS, wireless transmission from the MS to VU n , and processing at VU n , as shown in Fig. 2.

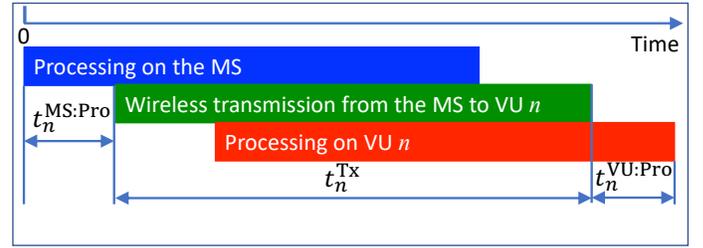


Fig. 2: The timeline.

Then we define the following:

- the time used on the MS to generate and process frames for VU n before wireless transmission: $t_n^{\text{MS:Pro}}(s_n, f_n^{\text{MS}}) = \frac{\mathcal{A}_n(s_n, \Lambda_n)}{f_n^{\text{MS}}}$,
- the time expended to transmit all Λ_n VR frames from the MS to VU n : $t_n^{\text{Tx}}(b_n, p_n, s_n) = \frac{s_n \mu_n \Lambda_n}{r_n(b_n, p_n) \nu_n}$,
- the time cost on VU n for processing frames after wireless transmission: $t_n^{\text{VU:Pro}}(s_n, f_n^{\text{VU}}) = \frac{\mathcal{B}_n(s_n, \Lambda_n)}{f_n^{\text{VU}}}$,

We will set the expressions of $\mathcal{A}_n(\cdot)$, $\mathcal{B}_n(\cdot)$ in Section IX on simulations. Thus, the delay for VU n is

$$t_n(b_n, p_n, s_n, f_n^{\text{MS}}, f_n^{\text{VU}}) = t_n^{\text{MS:Pro}}(s_n, f_n^{\text{MS}}) + t_n^{\text{Tx}}(b_n, p_n, s_n) + t_n^{\text{VU:Pro}}(s_n, f_n^{\text{VU}}). \quad (3)$$

Then, we let the maximum of all VUs’ delays be the system delay:

$$\mathcal{T}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}) = \max_{n \in \mathcal{N}} t_n(b_n, p_n, s_n, f_n^{\text{MS}}, f_n^{\text{VU}}). \quad (4)$$

Let κ_n^{MS} , κ_n^{VU} be MS’s and VU n ’s effective switched capacitance. From the process of generating Λ_n frames at MS to rendering them at VU n , the following energy will be consumed:

- energy spent on MS to process Λ_n VR frames for VU n : $E_n^{\text{MS:Pro}}(s_n, f_n^{\text{MS}}) = \kappa_n^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n) (f_n^{\text{MS}})^2$,
- energy spent for transmitting Λ_n VR frames from MS to VU n : $E_n^{\text{Tx}}(b_n, p_n, s_n) = \frac{(p_n + p_n^{\text{cir}}) s_n \mu_n \Lambda_n}{r_n(b_n, p_n) \nu_n}$,
- energy spent on VU n to process Λ_n VR frames: $E_n^{\text{VU:Pro}}(s_n, f_n^{\text{VU}}) = \kappa_n^{\text{VU}} \mathcal{G}_n(s_n, \Lambda_n) (f_n^{\text{VU}})^2$,

where we note that $\mathcal{F}_n(s_n, \Lambda_n)$ (resp., $\mathcal{G}_n(s_n, \Lambda_n)$) is different from $\mathcal{A}_n(s_n, \Lambda_n)$ (resp., $\mathcal{B}_n(s_n, \Lambda_n)$) above, since the latter is only before (resp., after) wireless transmission as shown in Fig. 2, while the former considers CPU cycles to process Λ_n frames. The notations above highlight the dependence on Λ_n , but we do not write Λ_n in delay and energy functions as Λ_n

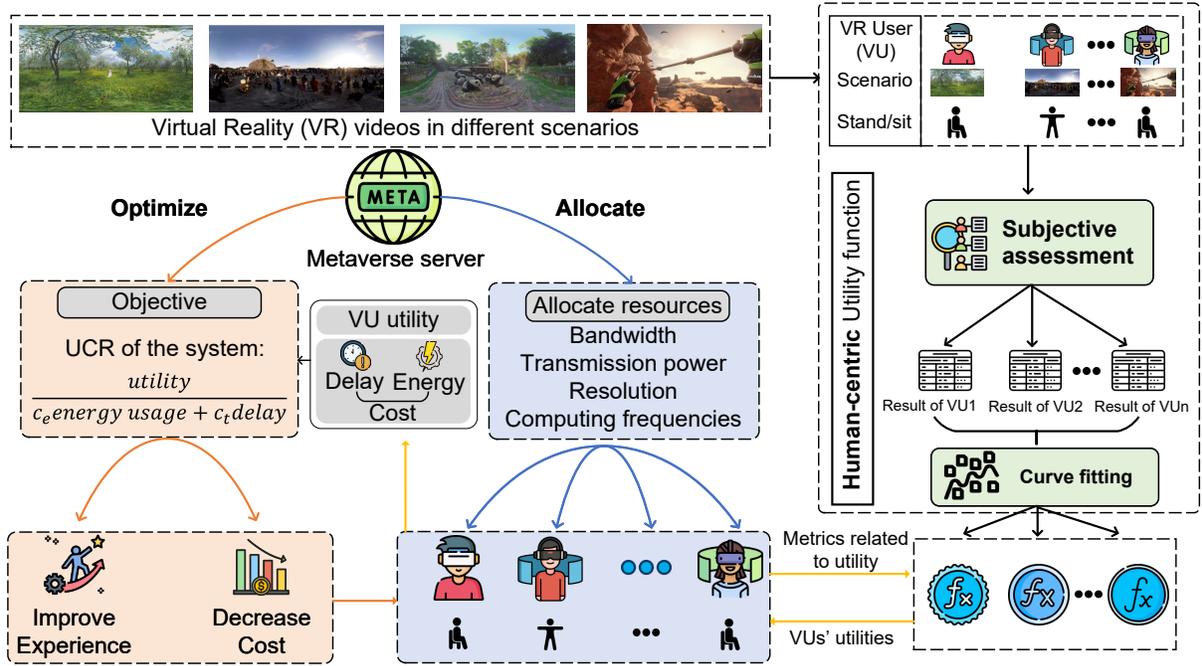


Fig. 1: Optimizing the system utility-cost ratio (UCR), in a system of one Metaverse Server (MS) and N VR Users (VUs), by deciding communication and computation resources as well as VR video resolutions.

is not optimized. The total consumed energy is:

$$\mathcal{E}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}) = \sum_{n \in \mathcal{N}} E_n^{\text{MS:Pro}}(s_n, f_n^{\text{MS}}) + \sum_{n \in \mathcal{N}} E_n^{\text{Tx}}(b_n, p_n, s_n) + \sum_{n \in \mathcal{N}} E_n^{\text{VU:Pro}}(s_n, f_n^{\text{VU}}). \quad (5)$$

The system cost is a weighted sum of the system delay in Eq. (4) and energy consumption in Eq. (5):

$$\mathcal{C}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}) = c_e \mathcal{E}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}) + c_t \mathcal{T}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}), \quad (6)$$

where c_e and c_t are the weight parameters for energy and delay, respectively. With the utility in Eq. (2) and the cost of the whole system in Eq. (6), we present the optimization problem in the next section.

C. Optimization problem

The aim is to maximize the utility-cost ratio (UCR) of the system as follows:

$$\text{Problem } \mathbb{P}_1 : \max_{\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}} \frac{u(\mathbf{b}, \mathbf{p}, \mathbf{s})}{\mathcal{C}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}})} \quad (7)$$

$$\text{s.t. } \sum_{n \in \mathcal{N}} b_n \leq b_{\max}, \quad (7a)$$

$$\sum_{n \in \mathcal{N}} p_n \leq p_{\max}, \quad (7b)$$

$$s_n \in \mathbb{S}_n, \quad \forall n \in \mathcal{N}, \quad \text{where the set } \mathbb{S}_n \text{ can be continuous or discrete,} \quad (7c)$$

$$\sum_{n \in \mathcal{N}} f_n^{\text{MS}} \leq f_{\max}^{\text{MS}}, \quad (7d)$$

$$f_n^{\text{VU}} \leq f_{n, \max}^{\text{VU}}, \quad \forall n \in \mathcal{N}. \quad (7e)$$

Constraints (7a), (7b), and (7d) mean the sum-limit of the bandwidth, power, and computing resources of the MS. Constraint (7c) gives the range of the resolution, and (7e) sets the CPU frequency limit of each VU. Our approach to solving \mathbb{P}_1 will use a fractional programming technique presented next.

IV. OUR PROPOSED TECHNIQUE FOR FRACTIONAL PROGRAMMING (FP)

In this section, we will first formulate the FP problem and then explain how our proposed FP technique differs from those in the state-of-the-art work [11], [12].

Fractional programming (FP) problem. Let $A_n(\mathbf{x}), B_n(\mathbf{x}), G(\mathbf{x})$ be functions of variable(s) \mathbf{x} , and these functions have definitions on a convex set \mathcal{S} , which is a subset of a real vector space. Also, for $\mathbf{x} \in \mathcal{S}$, we have $A_n(\mathbf{x}) \geq 0$ and $B_n(\mathbf{x}) > 0$. Then we consider:

FP-problem: optimizing $H(\mathbf{x}) := G(\mathbf{x}) + \sum_{n=1}^N \frac{A_n(\mathbf{x})}{B_n(\mathbf{x})}$ subject to $\mathbf{x} \in \mathcal{S}$. (8)

Two specific instances of FP-problem above are as follows:

FP-maximization: maximizing $H(\mathbf{x})$ subject to $\mathbf{x} \in \mathcal{S}$, for concave $A_n(\mathbf{x})$ and convex $B_n(\mathbf{x})$, (9)

FP-minimization: minimizing $H(\mathbf{x})$ subject to $\mathbf{x} \in \mathcal{S}$, for convex $A_n(\mathbf{x})$ and concave $B_n(\mathbf{x})$. (10)

Note that the above problem formulations (8) (9) (10) cover constrained optimization where the constraints are convex so that we can incorporate the constraints into defining \mathcal{S} .

An overview of our contribution in FP technique. With problems defined in (8) (9) (10) above, Table II compares our novel FP technique and those in [11], [12]. We present the details in the paragraphs below, describing [11], [12] and our work, respectively.

Prior work [11] on FP. In case of $G(\mathbf{x}) \equiv 0$, [11] optimizes $\sum_{n=1}^N \frac{A_n(\mathbf{x})}{B_n(\mathbf{x})}$, which is referred to as the sum of ratios (SoR). Then FP-maximization (resp., FP-minimization) under $G(\mathbf{x}) \equiv 0$ can be referred to SoR-maximization (resp., SoR-minimization). Via a transform into parametric convex optimization problems, [11] obtains a global optimum for SoR-maximization (i.e., maximizing $\sum_{n=1}^N \frac{A_n(\mathbf{x})}{B_n(\mathbf{x})}$ for concave $A_n(\mathbf{x})$ and convex $B_n(\mathbf{x})$) and SoR-minimization (i.e., maximizing $\sum_{n=1}^N \frac{A_n(\mathbf{x})}{B_n(\mathbf{x})}$ for convex $A_n(\mathbf{x})$ and concave $B_n(\mathbf{x})$). However, the approach of [11] is only applicable to the case of $G(\mathbf{x}) \equiv 0$. The

TABLE I: A comparison of our paper and other work [11], [12] on fractional programming (FP).

Paper	Contribution to fractional programming
Jong [11]	Find the global solution to FP-maximization in (9) and FP-minimization in (10), both only in the special case of $G(\mathbf{x}) \equiv 0$, but fail to tackle any problem in the case of $G(\mathbf{x}) \neq 0$.
Shen and Yu [12]	Find a stationary point for FP-maximization in (9) for both cases of $G(\mathbf{x}) \equiv 0$ and $G(\mathbf{x}) \neq 0$, but fail to tackle FP-minimization.
Our current work	Find a stationary point for FP-minimization in (10) for both cases of $G(\mathbf{x}) \equiv 0$ and $G(\mathbf{x}) \neq 0$.

reason is that although the original SoR optimization and the transformed problem find the same optimal solution for the variable(s), the optimal objective-function values of the two problems are different.

Prior work [12] on FP. To address cases of $G(\mathbf{x}) \equiv 0$ and $G(\mathbf{x}) \neq 0$, in the breakthrough work [12], Shen and Yu transform each $\frac{A_n(\mathbf{x})}{B_n(\mathbf{x})}$ into $J_n(\mathbf{x}, y_n) := 2y_n\sqrt{A_n(\mathbf{x})} - y_n^2 B_n(\mathbf{x})$, and prove that for concave $A_n(\mathbf{x})$ and convex $B_n(\mathbf{x})$, FP-maximization in (9) is the same as maximizing $V(\mathbf{x}, \mathbf{y}) := G(\mathbf{x}) + \sum_{n=1}^N J_n(\mathbf{x}, y_n)$ subject to $\mathbf{x} \in \mathcal{S}$ and $y_n \in \mathbb{R}$ (the set of real numbers). Then alternating optimization (AO) is adopted to optimize \mathbf{x} and $\mathbf{y} := [y_1, \dots, y_N]$ in an alternating manner, since $V(\mathbf{x}, \mathbf{y})$ is concave in \mathbf{x} and concave in \mathbf{y} , despite not being jointly concave in them. This AO algorithm leads to a stationary point for FP-maximization. Note that [12] tackles only FP-maximization and does not address FP-minimization. $J_n(\mathbf{x}, y_n)$ above can not be used for FP-minimization, since the minimum of $J_n(\mathbf{x}, y_n)$ is $-\infty$.

Our new technique for FP. Based on the above discussions, [11], [12] do not cover FP-minimization in (10) with $G(\mathbf{x}) \neq 0$. To fill this gap, our paper proposes the following technique for FP-minimization in both cases of $G(\mathbf{x}) \equiv 0$ and $G(\mathbf{x}) \neq 0$. Specifically, we transform each $\frac{A_n(\mathbf{x})}{B_n(\mathbf{x})}$ into $K_n(\mathbf{x}, y_n) := [A_n(\mathbf{x})]^2 y_n + \frac{1}{4[B_n(\mathbf{x})]^2 y_n}$, and prove that for convex $A_n(\mathbf{x})$ and concave $B_n(\mathbf{x})$, FP-minimization in (10) is the same as minimizing $W(\mathbf{x}, \mathbf{y}) := G(\mathbf{x}) + \sum_{n=1}^N K_n(\mathbf{x}, y_n)$ subject to $\mathbf{x} \in \mathcal{S}$ and $y_n \in \mathbb{R}^+$. The above holds because with $\mathbf{y}^\#(\mathbf{x})$ denoting $\mathbf{y} \in (\mathbb{R}^+)^N$ which minimizes $W(\mathbf{x}, \mathbf{y})$ given \mathbf{x} (i.e., $y_n^\#(\mathbf{x}) := \frac{1}{2A_n(\mathbf{x})B_n(\mathbf{x})}$), the partial derivative of $W(\mathbf{x}, \mathbf{y})$ with respect to \mathbf{x} at \mathbf{y} being $\mathbf{y}^\#(\mathbf{x})$ is the same as the derivative of $H(\mathbf{x})$ with respect to \mathbf{x} , where the computations are straightforward and shown in the Appendix of our full version [36]. Then FP-minimization in (10) can be tackled by optimizing \mathbf{x} and \mathbf{y} in an alternating manner to minimize $W(\mathbf{x}, \mathbf{y})$, since $W(\mathbf{x}, \mathbf{y})$ is convex in \mathbf{x} (for convex $G(\mathbf{x})$) and convex in \mathbf{y} (for any $G(\mathbf{x})$), despite not being jointly convex in them. For non-convex $G(\mathbf{x})$, optimizing $W(\mathbf{x}, \mathbf{y})$ with respect to \mathbf{x} can employ techniques such as difference-of-convex programming or successive convex approximation [37]. The above alternating optimization finds a stationary point for FP-minimization in (10). Finally, we note that while our proposed FP technique will be used to solve the current paper's Problem \mathbb{P}_1 of Section III-C, the technique can also be applied to many other FP problems [12]

beyond our paper.

V. SOLVE THE OPTIMIZATION PROBLEM \mathbb{P}_1

We present our method of solving \mathbb{P}_1 of (7) below. The denominator in the objective function of (7), i.e., the system cost, is given by (6) and involves a “maximize” term from the system delay in (4). We add an auxiliary variable T to circumvent that “maximize” so that \mathbb{P}_1 is transformed into Problem \mathbb{P}_2 :

$$\text{Problem } \mathbb{P}_2 : \max_{\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T} \frac{U(\mathbf{b}, \mathbf{p}, \mathbf{s})}{c_c \mathcal{E}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}) + c_t T} \quad (11)$$

$$\text{s.t. } t_n(b_n, p_n, s_n, \mathbf{f}_n^{\text{MS}}, \mathbf{f}_n^{\text{VU}}) \leq T, \quad \forall n \in \mathcal{N}, \quad (11a)$$

$$(7a), (7b), (7c), (7d), (7e). \quad (11b)$$

The subsections below present our steps for solving \mathbb{P}_1 . These steps together induce our Algorithm A1 on Page 6, which can be better understood after readers have finished all subsections below.

A. Dinkelbach's transform for the ratio optimization

For \mathbb{P}_2 maximizing a ratio (i.e., $\frac{\text{numerator of (11)}}{\text{denominator of (11)}}$), we use Dinkelbach's transform [12] to transform \mathbb{P}_2 into a series of parametric optimization $\mathbb{P}_3(y)$ which maximizes “numerator of (11) – $y \cdot$ denominator of (11)” subject to \mathbb{P}_2 's constraints, where solving the current $\mathbb{P}_3(y)$ decides “ y ” used in the next $\mathbb{P}_3(y)$. For an optimization problem \mathbb{P}_i , let $H_{\mathbb{P}_i}$ denote its objective function. Then $H_{\mathbb{P}_3}$ and \mathbb{P}_3 are as follows:

$$\begin{aligned} H_{\mathbb{P}_3}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T | y) : \\ = \text{numerator of (11)} - y \cdot \text{denominator of (11)} \\ = U(\mathbf{b}, \mathbf{p}, \mathbf{s}) - y \cdot (c_c \mathcal{E}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}) + c_t T), \end{aligned} \quad (12)$$

Problem $\mathbb{P}_3(y)$:

$$\begin{aligned} \max_{\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T} H_{\mathbb{P}_3}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T | y) \quad (13) \\ \text{s.t. } (7a), (7b), (7c), (7d), (7e), (11a). \end{aligned}$$

The process of using \mathbb{P}_3 to solve \mathbb{P}_2 is as follows. For ease of explanation, we denote $[\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T]$ by \mathbf{x} , and write the objective function of $\mathbb{P}_3(y)$ as $U(\mathbf{x}) - y \cdot C(\mathbf{x})$. Then starting from a feasible $\mathbf{x}^{(0)}$ at initialization, we set $\mathbf{y}^{(0)}$ as $\frac{U(\mathbf{x}^{(0)})}{C(\mathbf{x}^{(0)})}$. Then we solve $\mathbb{P}_3(\mathbf{y}^{(0)})$, denote the obtained solution as $\mathbf{x}^{(1)}$, set $\mathbf{y}^{(1)}$ as $\frac{U(\mathbf{x}^{(1)})}{C(\mathbf{x}^{(1)})}$. This process continues iteratively: in the $(i+1)$ th iteration, $\mathbf{y}^{(i)}$ is set as $\frac{U(\mathbf{x}^{(i)})}{C(\mathbf{x}^{(i)})}$ (given by Line 5 of Algorithm A1 on Page 6), and $\mathbf{x}^{(i+1)}$ is obtained from solving $\mathbb{P}_3(\mathbf{y}^{(i)})$. As stated in [12], the above process converges and does not lose optimality; i.e., under global optimization of each $\mathbb{P}_3(y)$ (not achieved in our current

Algorithm A1: Solve Problem \mathbb{P}_2 and hence Problem \mathbb{P}_1 for the system UCR optimization.

- 1 Initialize $i \leftarrow -1$ and for all $n \in \mathcal{N}$: $b_n^{(0)} = \frac{b_{\max}}{N}$, $p_n^{(0)} = \frac{p_{\max}}{N}$, $s_n^{(0)} = \frac{s_{\min} + s_{\max}}{2}$, $(f_n^{\text{MS}})^{(0)} = \frac{f_{\max}^{\text{MS}}}{N}$, $(f_n^{\text{VU}})^{(0)} = f_{n,\max}^{\text{VU}}$;
 - 2 $T^{(0)} \leftarrow \max_{n \in \mathcal{N}} t_n(b_n^{(0)}, p_n^{(0)}, s_n^{(0)}, (f_n^{\text{MS}})^{(0)}, (f_n^{\text{VU}})^{(0)})$;
 - 3 **repeat**
 - 4 Let $i \leftarrow i + 1$;
 - 5 $y^{(i)} \leftarrow \frac{\mathcal{U}(\mathbf{b}^{(i)}, \mathbf{p}^{(i)}, \mathbf{s}^{(i)})}{c_e \mathcal{E}(\mathbf{b}^{(i)}, \mathbf{p}^{(i)}, \mathbf{s}^{(i)}, (\mathbf{f}^{\text{MS}})^{(i)}, (\mathbf{f}^{\text{VU}})^{(i)}) + c_t T^{(i)}}$;
 - 6 //Lines 8–25 solve $\mathbb{P}_3(y^{(i)})$
 - 7 Initialize $j \leftarrow -1$,
 - 8 $[\mathbf{b}^{(i,0)}, \mathbf{p}^{(i,0)}, \mathbf{s}^{(i,0)}, (\mathbf{f}^{\text{MS}})^{(i,0)}, (\mathbf{f}^{\text{VU}})^{(i,0)}, T^{(i,0)}] \leftarrow [\mathbf{b}^{(i)}, \mathbf{p}^{(i)}, \mathbf{s}^{(i)}, (\mathbf{f}^{\text{MS}})^{(i)}, (\mathbf{f}^{\text{VU}})^{(i)}, T^{(i)}]$;
 - 9 **repeat**
 - 10 //Lines 13–22 solve $\mathbb{P}_4(y^{(i)}, \mathbf{s}^{(i,j)})$
 - 11 Let $j \leftarrow j + 1$.
 - 12 Initialize $\ell \leftarrow -1$;
 - 13 $[\mathbf{b}^{(i,j,0)}, \mathbf{p}^{(i,j,0)}] \leftarrow [\mathbf{b}^{(i,j)}, \mathbf{p}^{(i,j)}]$;
 - 14 **repeat**
 - 15 Let $\ell \leftarrow \ell + 1$.
 - 16 Set $z_n^{(i,j,\ell)} \leftarrow \frac{1}{2 \cdot (p_n^{(i,j,\ell)} + p_n^{\text{cir}}) s_n^{(i)} \mu_n \Lambda_n \cdot r_n(b_n^{(i,j,\ell)}, p_n^{(i,j,\ell)}) \nu_n}$
 - 17 Obtain $[\mathbf{b}^{(i,j,\ell)}, \mathbf{p}^{(i,j,\ell)}, (\mathbf{f}^{\text{MS}})^{(i,j,\ell)}, (\mathbf{f}^{\text{VU}})^{(i,j,\ell)}, T^{(i,j,\ell+1)}]$ through solving Problem $\mathbb{P}_5(\mathbf{z}^{(i,j,\ell)}, y^{(i)}, \mathbf{s}^{(i,j)})$ according to Algorithm A2, and denote the resulting optimal objective-function value of $\mathbb{P}_5(\mathbf{z}^{(i,j,\ell)}, y^{(i)}, \mathbf{s}^{(i,j)})$ by $V_{\mathbb{P}_5}^*(\mathbf{z}^{(i,j,\ell)}, y^{(i)}, \mathbf{s}^{(i,j)})$;
 - 18 **until** $\ell \geq 1$ and the relative difference between the optimal objective-function values for Problem $\mathbb{P}_5(\mathbf{z}^{(i,j,\ell)}, y^{(i)}, \mathbf{s}^{(i,j)})$ and Problem $\mathbb{P}_5(\mathbf{z}^{(i,j,\ell-1)}, y^{(i)}, \mathbf{s}^{(i,j)})$ is no greater than ϵ_3 for a small positive number ϵ_3 (i.e., $\frac{V_{\mathbb{P}_5}^*(\mathbf{z}^{(i,j,\ell)}, y^{(i)}, \mathbf{s}^{(i,j)})}{V_{\mathbb{P}_5}^*(\mathbf{z}^{(i,j,\ell-1)}, y^{(i)}, \mathbf{s}^{(i,j)})} - 1 \leq \epsilon_3$);
 - 19 Set $[\mathbf{b}^{(i,j+1)}, \mathbf{p}^{(i,j+1)}, (\mathbf{f}^{\text{MS}})^{(i,j+1)}, (\mathbf{f}^{\text{VU}})^{(i,j+1)}, T^{(i,j+1)}] \leftarrow [\mathbf{b}^{(i,j,\ell+1)}, \mathbf{p}^{(i,j,\ell+1)}, (\mathbf{f}^{\text{MS}})^{(i,j,\ell+1)}, (\mathbf{f}^{\text{VU}})^{(i,j,\ell+1)}, T^{(i,j,\ell+1)}]$, which we consider as a solution to Problem $\mathbb{P}_4(y^{(i)}, \mathbf{s}^{(i,j)})$;
 - 20 Set \mathbf{s} as $\mathbf{s}^{(i,j+1)}$ denoting the optimal solution of Problem $\mathbb{P}_6(\mathbf{b}^{(i,j+1)}, \mathbf{p}^{(i,j+1)}, (\mathbf{f}^{\text{MS}})^{(i,j+1)}, (\mathbf{f}^{\text{VU}})^{(i,j+1)}, T^{(i,j+1)}, y^{(i)})$, which is obtained from (19).
 - 21 **until** the relative difference between $H_{\mathbb{P}_3}(\mathbf{b}^{(i,j+1)}, \mathbf{p}^{(i,j+1)}, \mathbf{s}^{(i,j+1)}, (\mathbf{f}^{\text{MS}})^{(i,j+1)}, (\mathbf{f}^{\text{VU}})^{(i,j+1)}, T^{(i,j+1)} | y^{(i)})$ and $H_{\mathbb{P}_3}(\mathbf{b}^{(i,j)}, \mathbf{p}^{(i,j)}, \mathbf{s}^{(i,j)}, (\mathbf{f}^{\text{MS}})^{(i,j)}, (\mathbf{f}^{\text{VU}})^{(i,j)}, T^{(i,j)} | y^{(i)})$ is no greater than ϵ_2 for a small positive number ϵ_2 (i.e., $\frac{H_{\mathbb{P}_3}(\mathbf{b}^{(i,j+1)}, \mathbf{p}^{(i,j+1)}, \mathbf{s}^{(i,j+1)}, (\mathbf{f}^{\text{MS}})^{(i,j+1)}, (\mathbf{f}^{\text{VU}})^{(i,j+1)}, T^{(i,j+1)} | y^{(i)})}{H_{\mathbb{P}_3}(\mathbf{b}^{(i,j)}, \mathbf{p}^{(i,j)}, \mathbf{s}^{(i,j)}, (\mathbf{f}^{\text{MS}})^{(i,j)}, (\mathbf{f}^{\text{VU}})^{(i,j)}, T^{(i,j)} | y^{(i)})} - 1 \leq \epsilon_2$);
 - 22 Set $[\mathbf{b}^{(i+1)}, \mathbf{p}^{(i+1)}, \mathbf{s}^{(i+1)}, (\mathbf{f}^{\text{MS}})^{(i+1)}, (\mathbf{f}^{\text{VU}})^{(i+1)}, T^{(i+1)}] \leftarrow [\mathbf{b}^{(i,j+1)}, \mathbf{p}^{(i,j+1)}, \mathbf{s}^{(i,j+1)}, (\mathbf{f}^{\text{MS}})^{(i,j+1)}, (\mathbf{f}^{\text{VU}})^{(i,j+1)}, T^{(i,j+1)}]$, which we consider as a solution to Problem $\mathbb{P}_3(y^{(i)})$;
 - 23 **until** the relative difference between $y^{(i+1)}$ (i.e., $\frac{\mathcal{U}(\mathbf{b}^{(i+1)}, \mathbf{p}^{(i+1)}, \mathbf{s}^{(i+1)})}{c_e \mathcal{E}(\mathbf{b}^{(i+1)}, \mathbf{p}^{(i+1)}, \mathbf{s}^{(i+1)}, (\mathbf{f}^{\text{MS}})^{(i+1)}, (\mathbf{f}^{\text{VU}})^{(i+1)}) + c_t T^{(i+1)}}$) and $y^{(i)}$ is no greater than ϵ_1 for a small positive number ϵ_1 (i.e., $\frac{y^{(i+1)}}{y^{(i)}} - 1 \leq \epsilon_1$);
 - 24 Return $[\mathbf{b}^{(i+1)}, \mathbf{p}^{(i+1)}, \mathbf{s}^{(i+1)}, (\mathbf{f}^{\text{MS}})^{(i+1)}, (\mathbf{f}^{\text{VU}})^{(i+1)}, T^{(i+1)}]$ as a solution to Problem \mathbb{P}_2 , which means $[\mathbf{b}^{(i+1)}, \mathbf{p}^{(i+1)}, \mathbf{s}^{(i+1)}, (\mathbf{f}^{\text{MS}})^{(i+1)}, (\mathbf{f}^{\text{VU}})^{(i+1)}]$ is a solution to Problem \mathbb{P}_1 ;
-

paper, as discussed later), global optimization of P_2 is also achieved.

We will solve each $\mathbb{P}_3(y)$ by alternating optimizing (AO) \mathbf{s} and $\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T$. Section V-B optimizes $\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T$ given \mathbf{s} , while Section V-D optimizes \mathbf{s} given $\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T$. AO means looping through these two steps until convergence; i.e., the relative difference between the objective-function values of consecutive iterations is no more than the error tolerance, as shown in Line 21 of Algorithm A1.

B. Optimizing $\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T$ given \mathbf{s} for Problem $\mathbb{P}_3(y)$

For Problem $\mathbb{P}_3(y)$, given \mathbf{s} , optimizing $\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T$ means the following optimization:

Problem $\mathbb{P}_4(y, \mathbf{s})$:

$$\max_{\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T} F(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}} | \mathbf{s}, y) - y c_e \cdot E^{\text{Tx}}(\mathbf{b}, \mathbf{p}, \mathbf{s}) \quad (14)$$

s.t. (7a), (7b), (7d), (7e), (11a).

where $F(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T | \mathbf{s}, y) = \mathcal{U}(\mathbf{b}, \mathbf{p}, \mathbf{s})$

$$- y \cdot [c_e \cdot (\sum_{n \in \mathcal{N}} E_n^{\text{MS:Pro}}(s_n, f_n^{\text{MS}}) + \sum_{n \in \mathcal{N}} E_n^{\text{VU:Pro}}(s_n, f_n^{\text{VU}})) + c_t T],$$

and $E^{\text{Tx}}(\mathbf{b}, \mathbf{p}, \mathbf{s}) := \sum_{n \in \mathcal{N}} E_n^{\text{Tx}}(b_n, p_n, s_n)$

$$= \sum_{n \in \mathcal{N}} \frac{(p_n + p_n^{\text{cir}}) s_n \mu_n \Lambda_n}{r_n(b_n, p_n) \nu_n}. \quad (15)$$

Eq. (15) has a summation of non-convex ratios, which we address using our fractional programming technique of Section IV, as detailed soon. Note that we cannot use the sum-of-ratios approach in [12], since the objective function in (14) includes not just the sum of ratios, but also $F(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T | \mathbf{s}, y)$.

C. Leveraging our fractional programming technique to solve $\mathbb{P}_4(y, \mathbf{s})$

We utilize our fractional programming technique of Section IV to transform \mathbb{P}_4 into a series of \mathbb{P}_5 :

$$\mathbb{P}_5(\mathbf{z}, y, \mathbf{s}) : \max_{\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T} F(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T | \mathbf{s}, y) - y c_e \cdot \sum_{n \in \mathcal{N}} \left\{ [(p_n + p_n^{\text{cir}}) s_n \mu_n \Lambda_n]^2 z_n + \frac{1}{4(r_n(b_n, p_n) \nu_n)^2 z_n} \right\} \quad (16)$$

s.t. (7a), (7b), (7d), (7e), (11a), (17)

where we introduce the auxiliary $\mathbf{z} := [z_1, z_2, \dots, z_N]$ with $z_n > 0$. We solve $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$ in Section VI.

The process of using $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$ to solve $\mathbb{P}_4(y, \mathbf{s})$ is as follows. For ease of explanation, we denote $[\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T]$ by $\boldsymbol{\chi}$, and write the objective function of $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$ as $A(\boldsymbol{\chi}) - \sum_{n \in \mathcal{N}} (B_n(\boldsymbol{\chi}) z_n + \frac{C_n(\boldsymbol{\chi})}{z_n})$. Then starting from a feasible $\boldsymbol{\chi}^{(0)}$ at initialization, we set $z_n^{(0)}$ as $\sqrt{\frac{C_n(\boldsymbol{\chi}^{(0)})}{B_n(\boldsymbol{\chi}^{(0)})}}$ (i.e., optimizing the above objective function with respect to z given $\boldsymbol{\chi} = \boldsymbol{\chi}^{(0)}$). Then we solve $\mathbb{P}_5(\mathbf{z}^{(0)}, y, \mathbf{s})$, denote the obtained solution as $\boldsymbol{\chi}^{(1)}$, set $z_n^{(1)}$ as $\sqrt{\frac{C_n(\boldsymbol{\chi}^{(1)})}{B_n(\boldsymbol{\chi}^{(1)})}}$. This process continues iteratively: in the $(\ell + 1)$ th iteration, $z_n^{(\ell)}$ is set as $\sqrt{\frac{C_n(\boldsymbol{\chi}^{(\ell)})}{B_n(\boldsymbol{\chi}^{(\ell)})}}$, and $\boldsymbol{\chi}^{(\ell+1)}$ is obtained from solving $\mathbb{P}_5(\mathbf{z}^{(\ell)}, y, \mathbf{s})$. As explained in Section V-C, the above process is alternating optimization and thus converges. We will discuss its performance in Section VI.

D. Optimizing \mathbf{s} given $\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T$ for Problem $\mathbb{P}_3(y)$

When \mathbb{S}_n in (7c) is $[s_n^{\min}, s_n^{\max}]$, given $\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T$, optimizing \mathbf{s} for $\mathbb{P}_3(y)$ means the following:

$$\begin{aligned} & \text{Problem } \mathbb{P}_6(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T, y) : \\ & \max_{\mathbf{s}} \mathcal{U}(\mathbf{b}, \mathbf{p}, \mathbf{s}) - y \cdot (c_e \mathcal{E}(\mathbf{b}, \mathbf{p}, \mathbf{s}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}) + c_t T) \\ & \text{s.t. } s_n^{\min} \leq s_n \leq \min\{s_n^{\max}, v_n(b_n, p_n, f_n^{\text{MS}}, f_n^{\text{VU}}, T)\}, \\ & \quad \forall n \in \mathcal{N}, \end{aligned} \quad (18)$$

where $v_n(b_n, p_n, f_n^{\text{MS}}, f_n^{\text{VU}}, T)$ is defined as s_n which makes $t_n(b_n, p_n, s_n, f_n^{\text{MS}}, f_n^{\text{VU}})$ equal T .

Assuming $\mathcal{A}_n(s_n, \Lambda_n)$, $\mathcal{B}_n(s_n, \Lambda_n)$, $\mathcal{F}_n(s_n, \Lambda_n)$, and $\mathcal{G}_n(s_n, \Lambda_n)$ of Section III-B to be convex in s_n (which hold in our simulations in Section IX), $\mathbb{P}_6(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T, y)$ is convex optimization for \mathbf{s} , for which the Karush–Kuhn–Tucker (KKT) conditions give a global optimum. Using the KKT conditions, we obtain that with $s_n^\#$ denoting the maximum point of the function $V_n(s_n) := U_n(r_n(b_n, p_n), s_n) - y c_e \times (\kappa^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n) (f_n^{\text{MS}})^2 + \frac{(p_n + p_n^{\text{cir}}) s_n \mu_n \Lambda_n}{r_n(b_n, p_n) \nu_n} + \kappa_n^{\text{VU}} \mathcal{G}_n(s_n, \Lambda_n) (f_n^{\text{VU}})^2)$ that is concave

with respect to $s_n \in (0, \infty)$, the optimal solution of s_n to $\mathbb{P}_6(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T, y)$ is

$$\begin{cases} \text{for continuous } \mathbb{S}_n = [s_n^{\min}, s_n^{\max}]: \\ \tilde{s}_n := \max\{s_n^{\min}, \min\{s_n^\#, s_n^{\max}, v_n(b_n, p_n, f_n^{\text{MS}}, f_n^{\text{VU}}, T)\}\}, \\ \text{for discrete } \mathbb{S}_n: \text{ one of } s_n^{\text{up}} \text{ and } s_n^{\text{low}} \text{ which gives a higher } \\ V_n(\cdot), \text{ for } s_n^{\text{up}} \text{ (resp., } s_n^{\text{low}}) \text{ denoting the} \\ \text{smallest (resp., largest) } s_n \text{ in } \mathbb{S}_n \text{ greater} \\ \text{(resp., less) than } \tilde{s}_n. \end{cases}$$

E. Putting the above together: Our Algorithm A1 on Page 6

Based on the above, we present Algorithm A1 on Page 6 to solve \mathbb{P}_1 . Algorithm A1 consists of three levels of iterations: the outermost iteration based on Dinkelbach's transform in Section V-A, the mid-level iteration for alternating optimization based on Sections V-B and V-D, and the innermost iteration using our fractional programming technique as discussed in Section V-C. In Algorithm A1's pseudocode, Line 3 represents the outermost iteration based on Dinkelbach's transform, which solves a series of Problem $\mathbb{P}_3(y)$ for iteratively-updated y , in order to solve Problem \mathbb{P}_2 at convergence. Line 9 corresponding to the mid-level iteration is to alternating solve Problem $\mathbb{P}_4(y, \mathbf{s})$ and Problem $\mathbb{P}_6(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T, y)$, in order to resolve Problem $\mathbb{P}_3(y)$ at convergence. In Line 14, the innermost iteration is executed to solve Problem $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$ for iteratively-updated \mathbf{z} , in order to settle Problem $\mathbb{P}_4(y, \mathbf{s})$ at convergence.

We defer the solution quality and time complexity of Algorithm A1 to Section VII after explaining in Section VI below how each $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$ in Line 17 of Algorithm A1 is solved.

VI. GLOBAL OPTIMIZATION OF PROBLEM $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$

IN (16)

The transmission rate $r_n(b_n, p_n)$ is jointly concave in b_n and p_n [6]. Then from the composition rule in Eq. (3.11) of [38] and our Assumption 1 on Page 3, $\mathcal{U}(\mathbf{b}, \mathbf{p}, \mathbf{s})$ in Eq. (2) is jointly concave in \mathbf{b} and \mathbf{p} . Thus, $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$ belongs to convex optimization. The CVX tool [38] can be used to solve it. However, the worst-case complexity of global convex optimization grows exponentially with the problem size N from Section 1.4.2 of [38]. Below we analyze the KKT conditions [38] to globally optimize $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$.

The Lagrange function of $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$ is given below, where $\alpha, \beta, \gamma, \delta, \zeta$ denote the multipliers:

$$\begin{aligned} & L_{\mathbb{P}_5}(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T, \alpha, \beta, \gamma, \delta, \zeta | \mathbf{z}, y, \mathbf{s}) \\ & = -F(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T | y, \mathbf{s}) \\ & + y c_e \cdot \sum_{n \in \mathcal{N}} \left\{ [(p_n + p_n^{\text{cir}}) s_n \mu_n \Lambda_n]^2 z_n + \frac{1}{4(r_n(b_n, p_n) \nu_n)^2 z_n} \right\} \\ & + \alpha \cdot (\sum_{n \in \mathcal{N}} b_n - b_{\max}) + \beta \cdot (\sum_{n \in \mathcal{N}} p_n - p_{\max}) \\ & + \gamma \cdot (\sum_{n \in \mathcal{N}} f_n^{\text{MS}} - f_{\max}^{\text{MS}}) + \sum_{n \in \mathcal{N}} [\delta_n \cdot (f_n^{\text{VU}} - f_{n, \max}^{\text{VU}})] \\ & + \sum_{n \in \mathcal{N}} [\zeta_n \cdot (t_n(b_n, p_n, s_n, f_n^{\text{MS}}, f_n^{\text{VU}}) - T)]. \end{aligned} \quad (19)$$

Abbreviating $L_{\mathbb{P}_5}(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T, \alpha, \beta, \gamma, \delta, \zeta | \mathbf{z}, y, \mathbf{s}, \mathbf{f}^{\text{VU}}, T | \mathbf{s}, y)$, $U_n(r_n(b_n, p_n), s_n)$ and $r_n(b_n, p_n)$ as $L_{\mathbb{P}_5}$, U_n and r_n for simplicity, we present the KKT conditions of $\mathbb{P}_5(\mathbf{z}, y, \mathbf{s})$ as (20)-(31) below.

• **Stationarity:**

$$\forall n \in \mathcal{N} : \frac{\partial L_{\mathbb{P}_5}}{\partial b_n} = 0, \text{ meaning}$$

$$-\frac{\partial U_n}{\partial r_n} \cdot \frac{\partial r_n}{\partial b_n} - \frac{y c_e}{2r_n^3 \nu_n^2 z_n} \frac{\partial r_n}{\partial b_n} + \alpha - \zeta_n \frac{s_n \mu_n \Lambda_n}{r_n^2 \nu_n} \frac{\partial r_n}{\partial b_n} = 0; \quad (20)$$

$$\forall n \in \mathcal{N} : \frac{\partial L_{\mathbb{P}_5}}{\partial p_n} = 0, \text{ meaning}$$

$$-\frac{\partial U_n}{\partial r_n} \cdot \frac{\partial r_n}{\partial p_n} + 2y c_e z_n (s_n \mu_n \Lambda_n)^2 (p_n + p_n^{\text{cir}}) - \frac{y c_e}{2r_n^3 \nu_n^2 z_n} \frac{\partial r_n}{\partial p_n} + \beta - \zeta_n \frac{s_n \mu_n \Lambda_n}{r_n^2 \nu_n} \frac{\partial r_n}{\partial p_n} = 0; \quad (21)$$

$$\forall n \in \mathcal{N} : \frac{\partial L_{\mathbb{P}_5}}{\partial f_n^{\text{MS}}} = 0, \text{ meaning}$$

$$y c_e \cdot 2\kappa_n^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n) f_n^{\text{MS}} + \gamma - \zeta_n \frac{A_n(s_n, \Lambda_n)}{(f_n^{\text{MS}})^2} = 0; \quad (22)$$

$$\forall n \in \mathcal{N} : \frac{\partial L_{\mathbb{P}_5}}{\partial f_n^{\text{VU}}} = 0, \text{ meaning}$$

$$y c_e \cdot 2\kappa_n^{\text{VU}} \mathcal{G}_n(s_n, \Lambda_n) f_n^{\text{VU}} + \delta_n - \zeta_n \frac{B_n(s_n, \Lambda_n)}{(f_n^{\text{VU}})^2} = 0; \quad (23)$$

$$\frac{\partial L_{\mathbb{P}_5}}{\partial T} = 0, \text{ meaning } \sum_{n \in \mathcal{N}} \zeta_n = y c_t. \quad (24)$$

• **Complementary slackness:**

$$\alpha \cdot (\sum_{n \in \mathcal{N}} b_n - b_{\max}) = 0; \quad (25)$$

$$\beta \cdot (\sum_{n \in \mathcal{N}} p_n - p_{\max}) = 0; \quad (26)$$

$$\gamma \cdot (\sum_{n \in \mathcal{N}} f_n^{\text{MS}} - f_{\max}^{\text{MS}}) = 0; \quad (27)$$

$$\delta_n \cdot (f_n^{\text{VU}} - f_{n, \max}^{\text{VU}}) = 0 \text{ for all } n \in \mathcal{N}; \quad (28)$$

$$\zeta_n \cdot (t_n(b_n, p_n, s_n, f_n^{\text{MS}}, f_n^{\text{VU}}) - T) = 0 \text{ for all } n \in \mathcal{N}. \quad (29)$$

• **Primal feasibility:** (7a), (7b), (7d), (7e), (11a). (30)

• **Dual feasibility:**

$$(31a): \alpha \geq 0; \quad (31b): \beta \geq 0; \quad (31c): \gamma \geq 0;$$

$$(31d): \delta_n \geq 0 \text{ for all } n \in \mathcal{N}; (31e): \zeta_n \geq 0 \text{ for all } n \in \mathcal{N}. \quad (31)$$

We now analyze the KKT conditions of (20)–(31), to solve $\mathbb{P}_5(\star)$, where “ \star ” denotes “ z, y, s ” from now on for notation simplicity. Among the Lagrange multipliers, it is clear from (20) that

$$\alpha > 0. \quad (32)$$

Then using (32) in (25), we obtain

$$\sum_{n \in \mathcal{N}} b_n = b_{\max}. \quad (33)$$

Thus, (25) (7a) (31a) can be replaced by (32) (33). Hence, the KKT conditions of Problem $\mathbb{P}_5(\star)$, which include (20)–(29), (30) (i.e., (7a), (7b), (7d), (7e), (11a)), and (31) (i.e., (31a), (31b), (31c), (31d), (31e)), can be expressed as \mathcal{S}_{KKT} , which denotes the collection of (20)–(24), (26)–(29), (7b), (7d), (7e), (11a), (31b), (31c), (31d), (31e), (32), and (33).

$$(34)$$

Identifying a roadmap to compute the variables step-by-step. Given “ z, y, s ” (denoted by “ \star ” below), we will find $\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T, \alpha, \beta, \gamma, \delta, \zeta$ to satisfy the KKT conditions \mathcal{S}_{KKT} defined above.

We will partition the KKT conditions given in \mathcal{S}_{KKT} of (34) for $\mathbb{P}_5(\star)$ into the sets $\mathcal{S}_{1.1}, \mathcal{S}_{1.2.1}, \mathcal{S}_{1.2.2.1}, \mathcal{S}_{1.2.2.2}, \mathcal{S}_{2.1}, \mathcal{S}_{2.2}$ defined below to enable a step-by-step approach, in order to solve for the variables:

Step 1: Considering ζ as a parameter, find $[\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \alpha, \beta, \gamma, \delta]$ satisfying $\mathcal{S}_{1.1} \cup \mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ as functions of $[\zeta, \star]$ through Steps 1.1 and 1.2 below, where $\mathcal{S}_{1.1} \cup \mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2} =$

$$\left\{ \begin{array}{l} (20), (21), (22), (23), (26), (27), (28), \\ (7b), (7d), (7e), (31b), (31c), (31d), (32), (33) \end{array} \right\}, \quad (35)$$

for $\mathcal{S}_{1.1}, \mathcal{S}_{1.2.1}, \mathcal{S}_{1.2.2.1}$ and $\mathcal{S}_{1.2.2.2}$ defined in (36), (38), (40) and (41) below,

Step 1.1: Find $\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \gamma, \delta$ satisfying $\mathcal{S}_{1.1}$ as functions of $[\zeta, \star]$:
 $\mathcal{S}_{1.1} := \{(22), (23), (27), (28), (7d), (7e), (31c), (31d)\},$
(36)

Step 1.2: Find $\mathbf{b}, \mathbf{p}, \alpha, \beta$ satisfying

$\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ as functions of $[\zeta, \star]$ through Steps 1.2.1, 1.2.2.1, and 1.2.2.2 below, where

$$\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2} = \{(20), (21), (26), (7b), (31b), (32), (33)\}, \quad (37)$$

for $\mathcal{S}_{1.2.1}, \mathcal{S}_{1.2.2.1}$ and $\mathcal{S}_{1.2.2.2}$ defined in (38), (40) and (41) below,

Step 1.2.1: Considering $[\alpha, \beta]$ as parameters, find \mathbf{b}, \mathbf{p} satisfying $\mathcal{S}_{1.2.1}$ as functions of $[\alpha, \beta, \zeta, \star]$, for $\mathcal{S}_{1.2.1} := \{(20), (21)\},$
(38)

Step 1.2.2: Using results of Step 1.2.1, find $[\alpha, \beta]$ satisfying $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ as functions of $[\zeta, \star]$ through Steps 1.2.2.1 and 1.2.2.2 below, where $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2} =$

$$\{(26), (7b), (31b), (32), (33)\}, \quad (39)$$

for $\mathcal{S}_{1.2.2.1}$ and $\mathcal{S}_{1.2.2.2}$ defined in (40) and (41) below,

Step 1.2.2.1: considering β as a parameter, find α satisfying $\mathcal{S}_{1.2.2.1}$ as a function of $[\beta, \zeta, \star]$, for $\mathcal{S}_{1.2.2.1} := \{(32), (33)\},$
(40)

Step 1.2.2.2: using results of Step 1.2.2.1, find β satisfying $\mathcal{S}_{1.2.2.2}$ as a function of $[\zeta, \star]$, for $\mathcal{S}_{1.2.2.2} := \{(26), (7b), (31b)\},$
(41)

Step 2: Using results of Steps 1.1 and 1.2, find $[T, \zeta]$ satisfying $\mathcal{S}_{2.1} \cup \mathcal{S}_{2.2}$ as a function of “ \star ” through Steps 2.1 and 2.2 below, where $\mathcal{S}_{2.1} \cup \mathcal{S}_{2.2} :=$
 $\{(24), (29), (11a), (31e)\},$
(42)

for $\mathcal{S}_{2.1}$ and $\mathcal{S}_{2.2}$ defined in (43) and (44) below,

Step 2.1: considering T as a parameter, find ζ satisfying $\mathcal{S}_{2.1}$ as a function of $[T, \star]$, for $\mathcal{S}_{2.1} := \{(29), (11a), (31e)\},$
(43)

Step 2.2: using results of Steps 1.1 and 1.2, find T satisfying $\mathcal{S}_{2.2}$ as a function of “ \star ”, for $\mathcal{S}_{2.2} := \{(24)\},$
(44)

$$(45)$$

Steps 1.1, 1.2.1, 1.2.2.1, 1.2.2.2, 2.1, and 2.2 use $\mathcal{S}_{1.1}, \mathcal{S}_{1.2.1}, \mathcal{S}_{1.2.2.1}, \mathcal{S}_{1.2.2.2}, \mathcal{S}_{2.1}$, and $\mathcal{S}_{2.2}$, respectively. Each

step uses a subset of the KKT conditions \mathcal{S}_{KKT} (defined in (34)) of Problem $\mathbb{P}_5(\star)$, and all steps combined together utilize all the conditions, since it holds from (36), (38), (40), (41), (43) and (44) that

$$\begin{aligned} & \mathcal{S}_{1.1} \cup \mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2} \cup \mathcal{S}_{2.1} \cup \mathcal{S}_{2.2} \\ & = \text{KKT conditions } \mathcal{S}_{KKT} \text{ in (34)}. \end{aligned} \quad (46)$$

We introduce notations to denote the computed results in the steps. The goal is to obtain $[\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), (\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), T^\#(\star), \alpha^\#(\star), \beta^\#(\star), \gamma^\#(\star), \delta^\#(\star), \zeta^\#(\star)]$, denoting a solution of $[\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T, \alpha, \beta, \gamma, \delta, \zeta]$ to the KKT conditions \mathcal{S}_{KKT} in (34); i.e., $[\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), (\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), T^\#(\star)]$ denotes a global optimum to $\mathbb{P}_5(\star)$.

(47)

For X being 1.1, 1.2, 1.2.1, 1.2.2.1, 1.2.2.2, 2.1, or 2.2, Proposition X below is a formal presentation of Step X above. For better clarity, we also present the following table to help understand notations.

Proposition 1.1. *We have the following results which formally explain Step 1.1 of Page 8.*

(i) Given “ \star ”, if in $\mathcal{S}_{1.1}$ defined in (36), we substitute ζ with $\zeta^\#(\star)$ defined in (47), then

$[\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \gamma, \delta]$ satisfying $\mathcal{S}_{1.1}$ is $[(\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), \gamma^\#(\star), \delta^\#(\star)]$ defined in (47).

(ii) Given “ \star ” and ζ , let the solution of $[\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \gamma, \delta]$ to $\mathcal{S}_{1.1}$ be $[\mathbf{f}^{\text{MS}}(\zeta | \star), \mathbf{f}^{\text{VU}}(\zeta | \star), \gamma(\zeta | \star), \delta(\zeta | \star)]$. Then

$$\begin{aligned} & [\mathbf{f}^{\text{MS}}(\zeta^\#(\star) | \star), \mathbf{f}^{\text{VU}}(\zeta^\#(\star) | \star), \gamma(\zeta^\#(\star) | \star), \\ & \delta(\zeta^\#(\star) | \star)] = [(\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), \gamma^\#(\star), \delta^\#(\star)]. \end{aligned} \quad (48)$$

Proof of Proposition 1.1: Given “ \star ” and ζ , the conditions in $\mathcal{S}_{1.1}$ of (36) are necessary and sufficient to decide $[\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \gamma, \delta]$. Since setting $[\zeta, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \gamma, \delta]$ as $[\zeta^\#(\star), (\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), \gamma^\#(\star), \delta^\#(\star)]$ satisfies $\mathcal{S}_{1.1}$ due to (47), Results (i) and (ii) of Proposition 1.1 clearly hold. \square

Proposition 1.2. *We have the following results, which formally explain Step 1.2 of Page 8.*

(i) Given “ \star ”, if in $\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ defined in (37), we substitute ζ with $\zeta^\#(\star)$ defined in (47), then $[\mathbf{b}, \mathbf{p}, \alpha, \beta]$ satisfying $\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ is $[\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), \alpha^\#(\star), \beta^\#(\star)]$ defined in (47).

(ii) Given “ \star ” and ζ , let the solution of $[\mathbf{b}, \mathbf{p}, \alpha, \beta]$ to $\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ be $[\mathbf{b}(\zeta | \star), \mathbf{p}(\zeta | \star), \alpha(\zeta | \star), \beta(\zeta | \star)]$. Then $[\mathbf{b}(\zeta^\#(\star) | \star), \mathbf{p}(\zeta^\#(\star) | \star), \alpha(\zeta^\#(\star) | \star), \beta(\zeta^\#(\star) | \star)] = [\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), \alpha^\#(\star), \beta^\#(\star)]$. \square

Proof of Proposition 1.2: Given “ \star ” and ζ , the conditions in $\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ of (37) are necessary and sufficient to decide $[\mathbf{b}, \mathbf{p}, \alpha, \beta]$. Since setting $[\zeta, \mathbf{b}, \mathbf{p}, \alpha, \beta]$ as $[\zeta^\#(\star), \mathbf{b}^\#(\star), \mathbf{p}^\#(\star), \alpha^\#(\star), \beta^\#(\star)]$ satisfies $\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ due to (47), Results (i) and (ii) of Proposition 1.2 clearly hold. \square

Proposition 1.2.1. *We have the following result which formally explains Step 1.2.1 of Page 8.*

(i) Given “ \star ” and ζ , if in $\mathcal{S}_{1.2.1}$ defined in (38), we substitute $[\alpha, \beta]$ with $[\alpha(\zeta | \star), \beta(\zeta | \star)]$ defined in Proposition 1.2,

then $[\mathbf{b}, \mathbf{p}]$ satisfying $\mathcal{S}_{1.2.1}$ is $[\mathbf{b}(\zeta | \star), \mathbf{p}(\zeta | \star)]$ defined in Proposition 1.2.

(ii) Given “ \star ” and $[\alpha, \beta, \zeta]$, let the solution of $[\mathbf{b}, \mathbf{p}]$ to $\mathcal{S}_{1.2.1}$ be $[\mathbf{b}(\alpha, \beta, \zeta | \star), \mathbf{p}(\alpha, \beta, \zeta | \star)]$. Then

$$\begin{aligned} & [\mathbf{b}(\alpha(\zeta | \star), \beta(\zeta | \star), \zeta | \star), \mathbf{p}(\alpha(\zeta | \star), \beta(\zeta | \star), \zeta | \star))] \\ & = [\mathbf{b}(\zeta | \star), \mathbf{p}(\zeta | \star)]. \end{aligned} \quad (50)$$

Proof of Proposition 1.2.1: Given “ \star ” and $[\alpha, \beta, \zeta]$, the conditions in $\mathcal{S}_{1.2.1}$ of (38) are necessary and sufficient to decide $[\mathbf{b}, \mathbf{p}]$. Since setting $[\mathbf{b}, \mathbf{p}, \alpha, \beta]$ as $[\mathbf{b}(\zeta | \star), \mathbf{p}(\zeta | \star), \alpha(\zeta | \star), \beta(\zeta | \star)]$ satisfies $\mathcal{S}_{1.2.1}$ by the $\blacksquare(\cdot)$ notations in Proposition 1.2, Results (i) and (ii) of Proposition 1.2.1 clearly hold. \square

Proposition 1.2.2. *We have the following result which formally explains Step 1.2.2 of Page 8.*

Given “ \star ” and $[\alpha, \beta, \zeta]$, if in $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ given in (39), we substitute $[\mathbf{b}, \mathbf{p}]$

with $[\mathbf{b}(\alpha, \beta, \zeta | \star), \mathbf{p}(\alpha, \beta, \zeta | \star)]$ defined in Proposition 1.2.1, then $[\alpha, \beta]$ satisfying $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ is $[\alpha(\zeta | \star), \beta(\zeta | \star)]$ defined in Proposition 1.2.

Proof of Proposition 1.2.2: Given “ \star ” and $[\alpha, \beta, \zeta]$, if in $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ of (39), we substitute $[\mathbf{b}, \mathbf{p}]$ with $[\mathbf{b}(\alpha, \beta, \zeta | \star), \mathbf{p}(\alpha, \beta, \zeta | \star)]$ defined in Proposition 1.2.1, the conditions in $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ of (39) are necessary and sufficient to decide $[\alpha, \beta]$. Since setting $[\mathbf{b}, \mathbf{p}, \alpha, \beta]$ as

$$[\mathbf{b}(\alpha(\zeta | \star), \beta(\zeta | \star), \zeta | \star), \mathbf{p}(\alpha(\zeta | \star), \beta(\zeta | \star), \zeta | \star), \alpha(\zeta | \star), \beta(\zeta | \star))]$$

(i.e., $[\mathbf{b}(\zeta | \star), \mathbf{p}(\zeta | \star), \alpha(\zeta | \star), \beta(\zeta | \star)]$ according to (50)) satisfies $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ by the definition of the $\blacksquare(\cdot)$ notations in Proposition 1.2, Proposition 1.2.2 clearly follows. \square

Despite Proposition 1.2.2, simultaneously solving for $[\alpha, \beta]$ to $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ is challenging. Instead, we solve for α as a function of β first and then decide β in Propositions 1.2.2.1 and 1.2.2.2 below.

Proposition 1.2.2.1. *We have the following results which formally explain Step 1.2.2.1 of Page 8.*

(i) Given “ \star ” and ζ , if in $\mathcal{S}_{1.2.2.1}$ defined in (40), we substitute \mathbf{b} with $\mathbf{b}(\alpha, \beta(\zeta | \star), \zeta | \star)$, then α satisfying $\mathcal{S}_{1.2.2.1}$ is $\alpha(\zeta | \star)$, where the $\blacksquare(\cdot)$ and $\blacksquare(\cdot)$ notations are defined in Propositions 1.2 and 1.2.1.

(ii) Given “ \star ” and ζ , if in $\mathcal{S}_{1.2.2.1}$, we substitute \mathbf{b} with $\mathbf{b}(\alpha, \beta, \zeta | \star)$ defined in Proposition 1.2.1, let the solution of α to $\mathcal{S}_{1.2.2.1}$ be $\check{\alpha}(\beta, \zeta | \star)$. Then

$$\check{\alpha}(\beta(\zeta | \star), \zeta | \star) \text{ equals } \alpha(\zeta | \star), \text{ and} \quad (51)$$

$$\begin{aligned} & [\mathbf{b}(\check{\alpha}(\beta(\zeta | \star), \zeta | \star), \beta(\zeta | \star), \zeta | \star), \mathbf{p}(\check{\alpha}(\beta(\zeta | \star), \zeta | \star), \beta(\zeta | \star), \zeta | \star))] \\ & \text{ equals } [\mathbf{b}(\zeta | \star), \mathbf{p}(\zeta | \star)]. \end{aligned} \quad (52)$$

Proof of Proposition 1.2.2.1: Given “ \star ” and ζ , if in $\mathcal{S}_{1.2.2.1}$ defined in (40), we substitute \mathbf{b} with $\mathbf{b}(\alpha, \beta(\zeta | \star), \zeta | \star)$, then the conditions in $\mathcal{S}_{1.2.2.1}$ of (40) are necessary and sufficient to decide α . Since setting α as $\alpha(\zeta | \star)$ and setting \mathbf{b} as $\mathbf{b}(\alpha(\zeta | \star), \beta(\zeta | \star), \zeta | \star)$ (i.e., $\mathbf{b}(\zeta | \star)$ according to (50)) satisfies $\mathcal{S}_{1.2.2.1}$ by the definition of the $\blacksquare(\cdot)$ notations in Proposition 1.2, Proposition 1.2.2.1 clearly follows. In particular, after we have (51), we further obtain (52) from (50) and (51). \square

Proposition 1.2.2.2. *We have the following result which for-*

TABLE II: Notes for notations, where “■” denotes a wildcard symbol hereinafter for convenience.

Notations	Notes
★	Represent “ z, y, s ”
■(·) notations	Defined in Propositions 1.1 and 1.2
■ [#] (·), ■ [~] (·), ■ [∨] (·), ■ [∩] (·)	Defined in (47), and Propositions 1.2.1, 1.2.2.1, and 2.1, respectively

ally explains Step 1.2.2.2 of Page 8.

Given “★”, if in $\mathcal{S}_{1.2.2.2}$ defined in (41), we substitute \mathbf{p} with $\tilde{\mathbf{p}}(\tilde{\alpha}(\beta, \zeta | \star), \beta, \zeta | \star)$, then β satisfying $\mathcal{S}_{1.2.2.2}$ is $\hat{\beta}(\zeta | \star)$, where ■(·), ■[~](·), and ■[∨](·) notations are defined in Propositions 1.2.2.1, 1.2.1, and 1.2.

Proof of Proposition 1.2.2.2: Given “★”, if in $\mathcal{S}_{1.2.2.2}$ defined in (41), we substitute \mathbf{p} with $\tilde{\mathbf{p}}(\tilde{\alpha}(\beta, \zeta | \star), \beta, \zeta | \star)$, then the conditions in $\mathcal{S}_{1.2.2.2}$ of (41) are necessary and sufficient to decide β . Since setting β as $\hat{\beta}(\zeta | \star)$ and setting \mathbf{p} as $\tilde{\mathbf{p}}(\tilde{\alpha}(\hat{\beta}(\zeta | \star), \zeta | \star), \hat{\beta}(\zeta | \star), \zeta | \star)$ (i.e., $\tilde{\mathbf{p}}(\zeta | \star)$ based on (51) and (52)) satisfies $\mathcal{S}_{1.2.2.2}$ by the ■(·) notations in Proposition 1.2, Proposition 1.2.2.2 clearly follows. □

Proposition 2. We have the following result which formally explains Step 2 of Page 8.

Given “★”, if in $\mathcal{S}_{2.1} \cup \mathcal{S}_{2.2}$ defined in (42), we substitute $[\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \alpha, \beta, \gamma, \delta]$ with $[\hat{\mathbf{b}}(\zeta | \star), \hat{\mathbf{p}}(\zeta | \star), \hat{\mathbf{f}}^{\text{MS}}(\zeta | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta | \star), \hat{\alpha}(\zeta | \star), \hat{\beta}(\zeta | \star), \hat{\gamma}(\zeta | \star), \hat{\delta}(\zeta | \star)]$ defined in Propositions 1.1 and 1.2, then $[\zeta, T]$ satisfying $\mathcal{S}_{2.1} \cup \mathcal{S}_{2.2}$ is $[\zeta^{\#}(\star), T^{\#}(\star)]$ defined in (47).

Despite Proposition 2, simultaneously solving for $[\zeta, T]$ to $\mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2}$ is challenging. Instead, we solve for ζ as a function of T first and then decide T in Propositions 2.1 and 2.2 below.

Proposition 2.1. We have the following result which formally explains Step 2.1 of Page 8.

(i) Given “★”, if in $\mathcal{S}_{2.1}$ defined in (43), we substitute $[T, \mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}]$ with $[T^{\#}(\star), \hat{\mathbf{b}}(\zeta | \star), \hat{\mathbf{p}}(\zeta | \star), \hat{\mathbf{f}}^{\text{MS}}(\zeta | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta | \star)]$, then ζ satisfying $\mathcal{S}_{2.1}$ is $\zeta^{\#}(\star)$, where the ■[#](·) and ■(·) notations are defined in (47) and Proposition 1.2 respectively.

(ii) Given “★”, if in $\mathcal{S}_{2.1}$ defined in (43), we substitute $[\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}]$ with $[\hat{\mathbf{b}}(\zeta | \star), \hat{\mathbf{p}}(\zeta | \star), \hat{\mathbf{f}}^{\text{MS}}(\zeta | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta | \star)]$, let the solution of ζ to $\mathcal{S}_{2.1}$ be $\hat{\zeta}(T | \star)$. Then

$$\hat{\zeta}(T^{\#}(\star) | \star) \text{ equals } \zeta^{\#}(\star), \text{ and} \quad (53)$$

$$[\hat{\mathbf{b}}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\mathbf{p}}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\mathbf{f}}^{\text{MS}}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\mathbf{f}}^{\text{VU}}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\alpha}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\beta}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\gamma}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\delta}(\hat{\zeta}(T^{\#}(\star) | \star) | \star)]$$

$$\text{equals } [\mathbf{b}^{\#}(\star), \mathbf{p}^{\#}(\star), (\mathbf{f}^{\text{MS}})^{\#}(\star), (\mathbf{f}^{\text{VU}})^{\#}(\star),$$

$$\alpha^{\#}(\star), \beta^{\#}(\star), \gamma^{\#}(\star), \delta^{\#}(\star)]. \quad (54)$$

Proof of Proposition 2.1: Given “★”, if in $\mathcal{S}_{2.1}$ defined in (43), we substitute $[T, \mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}]$ with $[T^{\#}(\star), \hat{\mathbf{b}}(\zeta | \star), \hat{\mathbf{p}}(\zeta | \star), \hat{\mathbf{f}}^{\text{MS}}(\zeta | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta | \star)]$, then the conditions in $\mathcal{S}_{2.1}$ of (43) are necessary and sufficient to decide ζ . Since setting $[\zeta, T, \mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}]$ as $[\zeta^{\#}(\star), T^{\#}(\star), \hat{\mathbf{b}}(\zeta^{\#}(\star) | \star), \hat{\mathbf{p}}(\zeta^{\#}(\star) | \star), \hat{\mathbf{f}}^{\text{MS}}(\zeta^{\#}(\star) | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{\#}(\star) | \star)]$ (i.e., $[\zeta^{\#}(\star), T^{\#}(\star), \mathbf{b}^{\#}(\star), \mathbf{p}^{\#}(\star), (\mathbf{f}^{\text{MS}})^{\#}(\star), (\mathbf{f}^{\text{VU}})^{\#}(\star)]$ according to (48) and (49)) satisfies $\mathcal{S}_{2.1}$ by the ■[#](·)

notations in (47), Proposition 2.1 clearly follows. □

Proposition 2.2. We have the following result which formally explains Step 2.2 of Page 8.

Given “★”, if in $\mathcal{S}_{2.2}$ defined in (44), we substitute $[\zeta, \mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}]$ with $[\hat{\zeta}(T | \star), \hat{\mathbf{b}}(\hat{\zeta}(T | \star) | \star), \hat{\mathbf{p}}(\hat{\zeta}(T | \star) | \star), \hat{\mathbf{f}}^{\text{MS}}(\hat{\zeta}(T | \star) | \star), \hat{\mathbf{f}}^{\text{VU}}(\hat{\zeta}(T | \star) | \star)]$, then T satisfying $\mathcal{S}_{2.2}$ is $T^{\#}(\star)$, where ■(·), ■[~](·), and ■[#](·) notations are in Proposition 1.2, Proposition 2.1, and (47).

Proof of Proposition 2.2: Given “★”, if in $\mathcal{S}_{2.2}$ defined in (44), we substitute $[\zeta, \mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}]$ with

$$[\hat{\zeta}(T | \star), \hat{\mathbf{b}}(\hat{\zeta}(T | \star) | \star), \hat{\mathbf{p}}(\hat{\zeta}(T | \star) | \star), \hat{\mathbf{f}}^{\text{MS}}(\hat{\zeta}(T | \star) | \star), \hat{\mathbf{f}}^{\text{VU}}(\hat{\zeta}(T | \star) | \star)],$$

then the conditions in $\mathcal{S}_{2.2}$ of (44) are necessary and sufficient to decide T . Since setting $[\zeta, T, \mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}]$ as $[\hat{\zeta}(T^{\#}(\star) | \star), \hat{\mathbf{b}}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\mathbf{p}}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\mathbf{f}}^{\text{MS}}(\hat{\zeta}(T^{\#}(\star) | \star) | \star), \hat{\mathbf{f}}^{\text{VU}}(\hat{\zeta}(T^{\#}(\star) | \star) | \star)]$ (i.e., $[\zeta^{\#}(\star), T^{\#}(\star), \mathbf{b}^{\#}(\star), \mathbf{p}^{\#}(\star), (\mathbf{f}^{\text{MS}})^{\#}(\star), (\mathbf{f}^{\text{VU}})^{\#}(\star)]$ according to (48) (49) and (53)) satisfies $\mathcal{S}_{2.2}$ by the definition of the ■[#](·) notations in (47), Proposition 2.2 clearly follows. □

Based on the above propositions, Fig. 3 and Algorithm A2 present our procedure to solve $\mathbb{P}_5(\star)$. For clarity, the function “Alg-Solve- $x(\cdot)$ ” is to compute $x(\cdot)$; e.g., Alg-Solve- $T^{\#}(\star)$ obtains $T^{\#}(\star)$.

Algorithm A2: Given z, y, s (written as “★” below), find a globally optimal solution to Problem $\mathbb{P}_5(\star)$, denoted as $[\mathbf{b}^{\#}(\star), \mathbf{p}^{\#}(\star), (\mathbf{f}^{\text{MS}})^{\#}(\star), (\mathbf{f}^{\text{VU}})^{\#}(\star), T^{\#}(\star)]$.

- 1 Use Alg-Solve- $T^{\#}(\star)$ of Algorithm 2.2 on Page 14 to obtain $T^{\#}(\star)$;
 - 2 Use Algorithm 2.1 on Page 13 to obtain $\hat{\zeta}(T^{\#}(\star) | \star)$, which is $\zeta^{\#}(\star)$ from (53);
 - 3 Use Algorithm 1.2 on Page 11 to obtain $[\hat{\mathbf{b}}(\zeta^{\#}(\star) | \star), \hat{\mathbf{p}}(\zeta^{\#}(\star) | \star), \hat{\alpha}(\zeta^{\#}(\star) | \star)]$, which equals $[\mathbf{b}^{\#}(\star), \mathbf{p}^{\#}(\star)]$ from (49); //Comment: Algorithm 1.2 uses Algorithms 1.2.1, 1.2.2.1, and 1.2.2.2.
 - 4 Use Algorithm 1.1 on Page 12 to obtain $[\hat{\mathbf{f}}^{\text{MS}}(\zeta^{\#}(\star) | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{\#}(\star) | \star)]$, which equals $[(\mathbf{f}^{\text{MS}})^{\#}(\star), (\mathbf{f}^{\text{VU}})^{\#}(\star)]$ from (48);
-

For X being 1.1, 1.2, 1.2.1, 1.2.2.1, 1.2.2.2, 2.1, or 2.2, Algorithm X will be presented for the computation in Proposition X, as shown in Fig. 3 and explained in detail below.

Algorithm 1.1: Computing $[\hat{\mathbf{f}}^{\text{MS}}(\zeta | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta | \star), \hat{\gamma}(\zeta | \star), \hat{\delta}(\zeta | \star)]$ defined in Proposition 1.1 using $\mathcal{S}_{1.1}$ of (36). Among $\mathcal{S}_{1.1}$, $[\hat{\mathbf{f}}^{\text{MS}}(\zeta | \star), \hat{\gamma}(\zeta | \star)]$ is $[\mathbf{f}^{\text{MS}}, \gamma]$ satisfying (22) (27) (7d) (31c), while $[\hat{\mathbf{f}}^{\text{VU}}(\zeta | \star), \hat{\delta}(\zeta | \star)]$

- Computing $[b^\#(\star), p^\#(\star), (f^{\text{MS}})^\#(\star), (f^{\text{VU}})^\#(\star), T^\#(\star)]$

which denotes a globally optimal solution to Problem $\mathbb{P}_5(\star)$ as defined in (47)

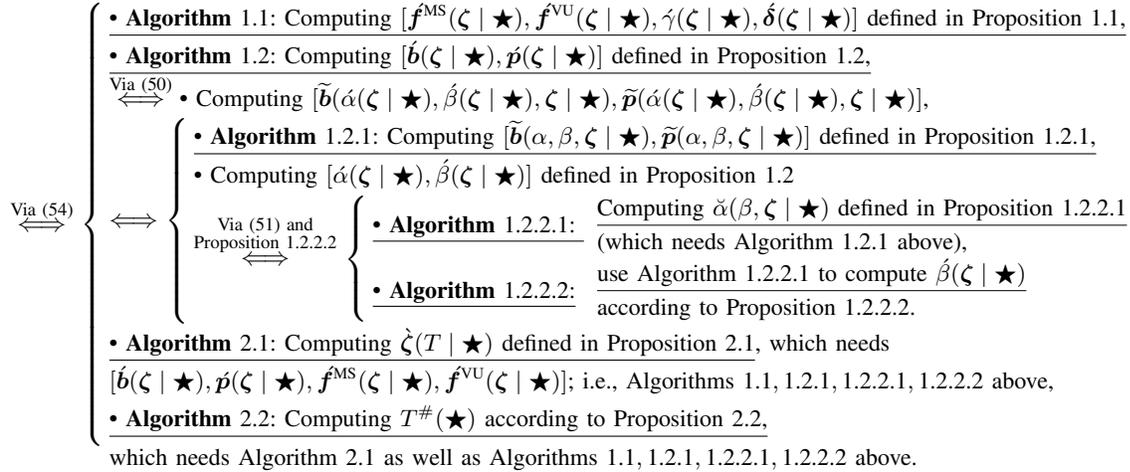


Fig. 3: Our procedure to solve Problem $\mathbb{P}_5(\star)$.

is $[f^{\text{VU}}, \delta]$ satisfying (23) (28) (7e) (31d). From (22) and (23), with PositiveRoot(\mathbb{E}) for an equation \mathbb{E} denoting the positive root of \mathbb{E} , we have

$$f_n^{\text{MS}} = \text{PositiveRoot}(\zeta_n \frac{A_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n)x = \gamma), \quad (55)$$

$$f_n^{\text{VU}} = \text{PositiveRoot}(\zeta_n \frac{B_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{VU}} \mathcal{G}_n(s_n, \Lambda_n)x = \delta_n). \quad (56)$$

To obtain $[f^{\text{VU}}, \delta]$ satisfying (23) (28) (7e) (31d), we have the following two cases:

- If $\text{PositiveRoot}(\zeta_n \frac{B_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{VU}} \mathcal{G}_n(s_n, \Lambda_n)x = 0) > f_{n, \max}^{\text{VU}}$; i.e., if $\sqrt[3]{\frac{B_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{G}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{VU}}}} > f_{n, \max}^{\text{VU}}$, setting δ_n as 0 violates (7e). This with (31d) means $\delta_n > 0$, which with (28) induces $f_n^{\text{VU}} = f_{n, \max}^{\text{VU}}$;
- If $\text{PositiveRoot}(\zeta_n \frac{B_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{VU}} \mathcal{G}_n(s_n, \Lambda_n)x = 0) \leq f_{n, \max}^{\text{VU}}$; i.e., if $\sqrt[3]{\frac{B_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{G}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{VU}}}} \leq f_{n, \max}^{\text{VU}}$, then setting δ_n as 0 and $f_n^{\text{VU}} = \sqrt[3]{\frac{B_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{G}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{VU}}}}$ satisfies (23) (28) (7e) (31d).

To obtain $[f^{\text{MS}}, \gamma]$ satisfying (22) (27) (7d) (31c), we discuss the following two cases:

- 1 If $\text{PositiveRoot}(\zeta_n \frac{A_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n)x = 0) > f_{\max}^{\text{MS}}$; i.e., if $\sum_{n \in \mathcal{N}} \sqrt[3]{\frac{A_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{F}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{MS}}}} > f_{\max}^{\text{MS}}$, then setting γ as 0 violates (7d). This with (31c) means $\gamma > 0$, which is used in (27) to induce
$$\sum_{n \in \mathcal{N}} \text{PositiveRoot}(\zeta_n \frac{A_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n)x = \gamma) = f_{\max}^{\text{MS}}. \quad (57)$$

After obtaining the desired γ , we use it in (55) to get f_n^{MS} .

- 2 If $\text{PositiveRoot}(\zeta_n \frac{A_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n)x = 0) \leq f_{\max}^{\text{MS}}$; i.e., if $\sum_{n \in \mathcal{N}} \sqrt[3]{\frac{A_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{F}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{MS}}}} \leq f_{\max}^{\text{MS}}$, then setting γ as 0 and $f_n^{\text{MS}} = \sqrt[3]{\frac{A_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{F}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{MS}}}}$ satisfies (22) (27) (7d) (31c).

Algorithm 1.1 explained above:

Alg-Solve- $f^{\text{MS}}(\zeta | \star)$ -and- $f^{\text{VU}}(\zeta | \star)$.

- 1 Set $f_n^{\text{VU}}(\zeta | \star)$ as $\min\{f_{n, \max}^{\text{VU}}, \sqrt[3]{\frac{B_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{G}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{VU}}}}\}$;
- 2 if $\sum_{n \in \mathcal{N}} \sqrt[3]{\frac{A_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{F}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{MS}}}} \leq f_{\max}^{\text{MS}}$, then set $f_n^{\text{MS}}(\zeta | \star)$ as $\sqrt[3]{\frac{A_n(s_n, \Lambda_n)\zeta_n}{2\mathcal{F}_n(s_n, \Lambda_n)y_{c_e}\kappa_n^{\text{MS}}}}$;
- 3 else with function $C(\cdot)$ being set as $\sum_{n \in \mathcal{N}} \text{PositiveRoot}(\zeta_n \frac{A_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n)x = \gamma)$, and C_{target} being set as f_{\max}^{MS} , run Standard-Bisection-Search($[\zeta, \star]$, function $C(\cdot)$, $C_{\text{target}}, 0, f_{\max}^{\text{MS}}$) based on Algorithm A4 on Page 14 to obtain the desired γ satisfying Eq. (57),
- 4 let the obtained γ be $\hat{\gamma}$, and set $f_n^{\text{MS}}(\zeta | \star)$ as $\text{PositiveRoot}(\zeta_n \frac{A_n(s_n, \Lambda_n)}{x^2} - y_{c_e} \cdot 2\kappa_n^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n)x = \hat{\gamma})$ for each $n \in \mathcal{N}$;
- 5 Return $f^{\text{VU}}(\zeta | \star)$ (resp., $f^{\text{MS}}(\zeta | \star)$) as $[f_n^{\text{VU}}(\zeta | \star)]_{n \in \mathcal{N}}$ (resp., $[f_n^{\text{MS}}(\zeta | \star)]_{n \in \mathcal{N}}$);

Algorithm 1.2: Computing $[\hat{b}(\zeta | \star), \hat{p}(\zeta | \star)]$ defined in Proposition 1.2. As shown in the pseudocode, Algorithm 1.2 calls Algorithms 1.2.1, 1.2.2.1, and 1.2.2.2 detailed below.

Algorithm 1.2.1: Computing $[\tilde{b}(\alpha, \beta, \zeta | \star), \tilde{p}(\alpha, \beta, \zeta | \star)]$ defined in Proposition 1.2.1 using $\mathcal{S}_{1.2.1}$ of (38). Recall that $\mathcal{S}_{1.2.1}$ includes (20) (21). From (20) (21) and (1), with ϑ_n defined by

$$\vartheta_n := \frac{g_n p_n}{\sigma_n^2 b_n}, \quad (58)$$

we obtain that $[\tilde{b}_n(\alpha, \beta, \zeta | \star), \tilde{p}_n(\alpha, \beta, \zeta | \star)]$ is the solution of $[b_n, p_n]$ to

$$\left(\frac{\partial U_n(r_n, s_n)}{\partial r_n} + \frac{y_{c_e}}{2z_n r_n^3 \nu_n^2} + \frac{\zeta_n s_n \mu_n \Lambda_n}{r_n^2 \nu_n}\right) \times (\log_2(1 + \vartheta_n) - \frac{\vartheta_n}{(1 + \vartheta_n) \ln 2}) = \alpha, \quad \text{and} \quad (59)$$

$$\left(\frac{\partial U_n(r_n, s_n)}{\partial r_n} + \frac{y_{c_e}}{2z_n r_n^3 \nu_n^2} + \frac{\zeta_n s_n \mu_n \Lambda_n}{r_n^2 \nu_n}\right) \cdot \frac{g_n}{\sigma_n^2 (1 + \vartheta_n) \ln 2} = \beta + 2(p_n + p_n^{\text{cir}}) y_{c_e} z_n (s_n \mu_n \Lambda_n)^2. \quad (60)$$

Algorithm 1.2 explained above:Alg-Solve- $\hat{b}(\zeta \mid \star)$ -and- $\hat{p}(\zeta \mid \star)$.

- 1 Run Alg-Solve- $\hat{\beta}(\zeta \mid \star)$ of Algorithm 1.2.2.2 to obtain $\hat{\beta}(\zeta \mid \star)$;
- 2 Use the just obtained $\hat{\beta}(\zeta \mid \star)$ as an input to Alg-Solve- $\check{\alpha}(\beta, \zeta \mid \star)$ of Algorithm 1.2.2.1 to get $\check{\alpha}(\hat{\beta}(\zeta \mid \star), \zeta \mid \star)$, which equals $\check{\alpha}(\zeta \mid \star)$ according to (51);
- 3 For each $n \in \mathcal{N}$, use Lines 2 and 1's obtained $\check{\alpha}(\zeta \mid \star)$ and $\hat{\beta}(\zeta \mid \star)$ as inputs to Alg-Solve- $\tilde{b}_n(\alpha, \beta, \zeta \mid \star)$ (resp., Alg-Solve- $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$) of Algorithm 1.2.1 to get $\tilde{b}_n(\check{\alpha}(\zeta \mid \star), \hat{\beta}(\zeta \mid \star), \zeta \mid \star)$ (resp., $\tilde{p}_n(\check{\alpha}(\zeta \mid \star), \hat{\beta}(\zeta \mid \star), \zeta \mid \star)$), which equals $\hat{b}_n(\zeta \mid \star)$ (resp., $\hat{p}_n(\zeta \mid \star)$) according to (50);
- 4 Return $\hat{b}(\zeta \mid \star)$ (resp., $\hat{p}(\zeta \mid \star)$) as $[\hat{b}_n(\zeta \mid \star)]_{n \in \mathcal{N}}$ (resp., $[\hat{p}_n(\zeta \mid \star)]_{n \in \mathcal{N}}$);

With (59) divided by (60), it holds that

$$\frac{\log_2(1+\vartheta_n) - \frac{\vartheta_n}{(1+\vartheta_n)\ln 2}}{\frac{\vartheta_n}{\sigma_n^2(1+\vartheta_n)\ln 2}} = \frac{\alpha}{\beta+2(p_n+p_n^{\text{cfr}})y_{\text{ce}}z_n(s_n\mu_n\Lambda_n)^2}. \quad (61)$$

Based on (61), we get

$$\begin{aligned} \vartheta_n &= \psi_n(p_n, \alpha, \beta \mid \star), \text{ for } \psi_n(p_n, \alpha, \beta \mid \star) \\ &:= \exp \left\{ 1 + W \left(\frac{1}{e} \left(\frac{g_n \alpha}{[\beta+2(p_n+p_n^{\text{cfr}})y_{\text{ce}}z_n(s_n\mu_n\Lambda_n)^2]\sigma_n^2} - 1 \right) \right) \right\} - 1. \end{aligned} \quad (62)$$

Then we can substitute ϑ_n from (62) into (59) and (60), to decide $[b, p]$, as shown in Lemma 1 below.

Lemma 1. *Given $[\alpha, \beta, \zeta, \star]$, we know from Proposition 1.2.1 that $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$ and $\tilde{b}_n(\alpha, \beta, \zeta \mid \star)$ denote the values of p_n and b_n satisfying (20) and (21). We define $\psi_n(p_n, \alpha, \beta \mid \star)$ in (62), and define*

$$\bar{r}_n(p_n, \alpha, \beta \mid \star) := \frac{g_n p_n \log_2(1+\psi_n(p_n, \alpha, \beta \mid \star))}{\sigma_n^2 \psi_n(p_n, \alpha, \beta \mid \star)}. \quad (63)$$

Then $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$ is a solution of p_n to

$$\begin{aligned} & \left[\frac{\left(\frac{\partial U_n(r_n, s_n)}{\partial r_n} \right) \Big|_{r_n = \bar{r}_n(p_n, \alpha, \beta \mid \star)}}{+ \frac{y_{\text{ce}}}{2z_n \cdot [\bar{r}_n(p_n, \alpha, \beta \mid \star)]^3 \nu_n^2} + \frac{\zeta_n s_n \mu_n \Lambda_n}{[\bar{r}_n(p_n, \alpha, \beta \mid \star)]^2 \nu_n}} \right] \times \\ & \left[\log_2(1+\psi_n(p_n, \alpha, \beta \mid \star)) - \frac{\psi_n(p_n, \alpha, \beta \mid \star)}{(1+\psi_n(p_n, \alpha, \beta \mid \star))\ln 2} \right] = \alpha. \end{aligned} \quad (64)$$

With $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$ decided according to (64) above, the corresponding b_n satisfying (20) and (21) is

$$\tilde{b}_n(\alpha, \beta, \zeta \mid \star) = \frac{g_n \tilde{p}_n(\alpha, \beta, \zeta \mid \star)}{\sigma_n^2 \psi_n(\tilde{p}_n(\alpha, \beta, \zeta \mid \star), \alpha, \beta \mid \star)}. \quad (65)$$

Proof of Lemma 1: First, for p_n and b_n satisfying (20) and (21), we have already explained above that $\frac{g_n p_n}{\sigma_n^2 b_n}$ (i.e., ϑ_n defined in (58)) equals $\psi_n(p_n, \alpha, \beta \mid \star)$ defined in (62). Then b_n equals $\frac{g_n p_n}{\sigma_n^2 \psi_n(p_n, \alpha, \beta \mid \star)}$, and $r_n(b_n, p_n)$ denoting $b_n \log_2(1 + \frac{g_n p_n}{\sigma_n^2 b_n})$ equals the right hand side of (63), which we denote as $\bar{r}_n(p_n, \alpha, \beta \mid \star)$ in (63) for notation simplicity. Then letting r_n be $\bar{r}_n(p_n, \alpha, \beta \mid \star)$ in (59), we obtain (64) which will be used to solve p_n . With $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$ denoting the obtained p_n , the corresponding b_n is given by (65). Hence, we have proved that $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$ and $\tilde{b}_n(\alpha, \beta, \zeta \mid \star)$, denoting p_n and b_n satisfying (20) and (21), are given by (64) and (65), respectively. \square

Then we use (64) and (65) of Lemma 1 to solve $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$

and $\tilde{b}_n(\alpha, \beta, \zeta \mid \star)$, respectively. They are considered as two subprocedures of Algorithm 1.2.1, as described below.

Algorithm 1.2.1's Subprocedure 1 via Eq. (64) for transmission power: Alg-Solve- $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$.

- 1 With function $C(\cdot)$ being set as the left hand side of (64), and C_{target} being set as α , run Standard-Bisection-Search($[\alpha, \beta, \zeta, \star]$, function $C(\cdot)$, $C_{\text{target}}, 0, p_{\text{max}}$) based on Algorithm A4 on Page 14 to obtain the desired p_n satisfying Eq. (64), and set this p_n as $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$;

Algorithm 1.2.1's Subprocedure 2 via Eq. (65) for bandwidth: Alg-Solve- $\tilde{b}_n(\alpha, \beta, \zeta, \star)$.

- 1 Run Alg-Solve- $\tilde{p}_n(\alpha, \beta, \zeta, \star)$ (i.e., Algorithm 1.2.1's Subprocedure 1 above) to obtain $\tilde{p}_n(\alpha, \beta, \zeta \mid \star)$;
- 2 Compute $\tilde{b}_n(\alpha, \beta, \zeta \mid \star)$ according to Eq. (65);

Algorithm 1.2.2.1: Computing $\check{\alpha}(\beta, \zeta \mid \star)$ defined in Proposition 1.2.2.1 using $\mathcal{S}_{1.2.2.1}$ of (40). Recall that $\mathcal{S}_{1.2.2.1}$ includes (32) and (33). Proposition 1.2.2.1 defines $\check{\alpha}(\beta, \zeta \mid \star)$ as the solution of α to

$$\sum_{n \in \mathcal{N}} \tilde{b}_n(\alpha, \beta, \zeta \mid \star) = b_{\text{max}} \text{ and } \alpha > 0. \quad (66)$$

Algorithm 1.2.2.1 explained above:Alg-Solve- $\check{\alpha}(\beta, \zeta \mid \star)$.

- 1 With function $C(\cdot)$ being set as $\sum_{n \in \mathcal{N}} \text{Alg-Solve-}\tilde{b}_n(\alpha, \beta, \zeta, \star)$ (whose computation requires Algorithm 1.2.1's Subprocedure 2), and C_{target} being set as b_{max} , run Bisection-Search-with-No-Known-Upper-Bound($[\beta, z, y, s]$, function $C(\cdot)$, C_{target}) based on Algorithm A3 on Page 14 to obtain the desired α satisfying Eq. (66), and set this α as $\check{\alpha}(\beta \mid z, y, s)$;

Algorithm 1.2.2.2 (Pseudocode on Page 13 based on the following analysis): Using Algorithm 1.2.2.1 to compute $\hat{\beta}(\zeta \mid \star)$ according to Proposition 1.2.2.2 using $\mathcal{S}_{1.2.2.2}$ of (41). Recall from (41) that $\mathcal{S}_{1.2.2.2}$ includes (26), (7b), and (31b). Then $\hat{\beta}(\zeta \mid \star)$ is the solution of β to

- $$\begin{cases} \text{based on (26), we ensure} \\ \beta \cdot (\sum_{n \in \mathcal{N}} \tilde{p}_n(\check{\alpha}(\beta, \zeta \mid \star), \beta, \zeta \mid \star) - p_{\text{max}}) = 0; & (67a) \\ \text{based on (7b), we ensure} \\ \sum_{n \in \mathcal{N}} \tilde{p}_n(\check{\alpha}(\beta, \zeta \mid \star), \beta, \zeta \mid \star) \leq p_{\text{max}}; & (67b) \\ \text{based on (31b), we ensure } \beta \geq 0. & (67c) \end{cases}$$
- We have the following two cases:

- If $\sum_{n \in \mathcal{N}} \tilde{p}_n(\check{\alpha}(0, \zeta \mid \star), 0, \zeta \mid \star) > p_{\text{max}}$, then the solution of β to (67a)–(67c) cannot be 0 (which along with (67c) means $\beta > 0$), since setting β as 0 violates (67b). We use $\beta > 0$ in (67a) to get
$$\sum_{n \in \mathcal{N}} \tilde{p}_n(\check{\alpha}(\beta, \zeta \mid \star), \beta, \zeta \mid \star) = p_{\text{max}}. \quad (68)$$

- If $\sum_{n \in \mathcal{N}} \tilde{p}_n(\check{\alpha}(0, \zeta | \star), 0, \zeta | \star) \leq p_{\max}$, then 0 is a solution of β to (67a)–(67c).

Algorithm 1.2.2.2 explained above: Alg-Solve- $\hat{\beta}(\zeta | \star)$.

- 1 **if** $\sum_{n \in \mathcal{N}} \tilde{p}_n(\check{\alpha}(0, \zeta | \star), 0, \zeta | \star) \leq p_{\max}$, then set $\hat{\beta}(\zeta | \star)$ as 0;
 - 2 **else** with function $C(\cdot)$ being set as $\sum_{n \in \mathcal{N}} \tilde{p}_n(\check{\alpha}(\beta, \zeta | \star), \beta, \zeta | \star)$ (whose computation requires Algorithm 1.2.2.1 and Algorithm 1.2.1's Subprocedure 1), and C_{target} being set as p_{\max} , run Bisection-Search-with-No-Known-Upper-Bound ($[\zeta, \star]$, function $C(\cdot)$, C_{target}) based on Algorithm A3 on Page 14 to obtain the desired β satisfying Eq. (68), and set this β as $\hat{\beta}(\zeta | \star)$;
-

Algorithm 2.1: Computing $\zeta(T | \star)$ defined in Proposition 2.1 using $\mathcal{S}_{2.1}$ defined in (43). Recall from (43) that $\mathcal{S}_{2.1}$ includes (29), (11a), and (31e). Then for each $n \in \mathcal{N}$, Proposition 2.1 means $\zeta_n(T | \star)$ is ζ_n which satisfies the following:

$$\left\{ \begin{array}{l} \text{based on (29), we ensure} \\ \zeta_n \cdot (t_n(\hat{b}_n(\zeta | \star), \hat{p}_n(\zeta | \star), s_n, f_n^{\text{MS}}(\zeta | \star), \\ f_n^{\text{VU}}(\zeta | \star)) - T) = 0; \end{array} \right. \quad (69a)$$

$$\left\{ \begin{array}{l} \text{based on (11a), we ensure} \\ t_n(\hat{b}_n(\zeta | \star), \hat{p}_n(\zeta | \star), s_n, f_n^{\text{MS}}(\zeta | \star), \\ f_n^{\text{VU}}(\zeta | \star)) \leq T; \end{array} \right. \quad (69b)$$

$$\left\{ \begin{array}{l} \text{based on (31e), we ensure } \zeta_n \geq 0. \end{array} \right. \quad (69c)$$

Given $n \in \mathcal{N}$, we can discuss two cases for ζ_n as follows:

Case 1: If setting ζ_n to 0 violates (69b), then ζ_n must be strictly positive, which is used in (69a) to show that the inequality in (69b) actually becomes equality in this case.

Case 2: If setting ζ_n to 0 satisfies (69b), then we can just set ζ_n as 0.

Summarizing the above two cases, we know that after defining $\check{\zeta}^{(n)} := [\zeta_1, \zeta_2, \dots, \zeta_{n-1}, 0, \zeta_{n+1}, \dots, \zeta_N]$, and

$$h_n(\zeta | T) := \left\{ \begin{array}{l} -\zeta_n, \text{ if } t_n(\hat{b}_n(\check{\zeta}^{(n)} | \star), \hat{p}_n(\check{\zeta}^{(n)} | \star), s_n, \\ f_n^{\text{MS}}(\check{\zeta}^{(n)} | \star), f_n^{\text{VU}}(\check{\zeta}^{(n)} | \star)) \leq T, \\ t_n(\hat{b}_n(\zeta | \star), \hat{p}_n(\zeta | \star), s_n, f_n^{\text{MS}}(\zeta | \star), \\ f_n^{\text{VU}}(\zeta | \star)) - T, \text{ otherwise,} \end{array} \right. \quad (71a)$$

$$(71b)$$

$$(71)$$

setting ζ as $\check{\zeta}(T | \star)$ always ensures that $h_n(\zeta | T) = 0$. Letting n iterate through \mathcal{N} and defining

$$\mathbf{h}(\zeta | T) := [h_n(\zeta | T)]_{n \in \mathcal{N}}, \quad (72)$$

we know that setting ζ as $\check{\zeta}(T | \star)$ ensures that

$$\mathbf{h}(\zeta | T) \text{ equals the } N\text{-dimensional zero vector } \mathbf{0}. \quad (73)$$

We define ζ_n^{upper} such that when ζ is $[0^{n-1}, \zeta_n^{\text{upper}}, 0^{N-n}]$,

$$h_n(\zeta | T) = 0. \quad (74)$$

We will prove the following results and Lemma 2:

- $\check{\zeta}_n(T | \star) \leq \zeta_n^{\text{upper}}$, which along with $\zeta_n \geq 0$ means

$$\zeta_n \in [0, \zeta_n^{\text{upper}}]; \quad (75)$$

- Given any $n \in \mathcal{N}$, for any $\zeta_1, \zeta_2, \dots, \zeta_{n-1}, \zeta_{n+1}, \dots, \zeta_N$, we have $h_n([\zeta_1, \zeta_2, \dots, \zeta_{n-1}, \zeta_n^{\text{upper}}, \zeta_{n+1}, \dots, \zeta_N] | T) \leq 0$; (76)

- Given any $n \in \mathcal{N}$, for any $\zeta_1, \zeta_2, \dots, \zeta_{n-1}, \zeta_{n+1}, \dots, \zeta_N$, we have $h_n([\zeta_1, \zeta_2, \dots, \zeta_{n-1}, 0, \zeta_{n+1}, \dots, \zeta_N] | T) \geq 0$; i.e., $h_n(\check{\zeta}^{(n)}) \geq 0$ for $\check{\zeta}^{(n)}$ defined in (70). (77)

Lemma 2 (Proved in the Appendix of our full version [36]). *Given ζ and T , given $n \in \mathcal{N}$, given $\zeta_1, \zeta_2, \dots, \zeta_{n-1}, \zeta_{n+1}, \dots, \zeta_N$, then $h_n(\zeta | T)$ is non-increasing as ζ_n increases.*

We prove the above Result (75). From Lemma 2 and (74), it holds that

$$\begin{aligned} h_n([0^{n-1}, \zeta_n^{\text{upper}}, 0^{N-n}] | T) &= 0 = h_n(\zeta | T) \\ &\leq h_n([0^{n-1}, \zeta_n, 0^{N-n}] | T) \leq 0, \end{aligned} \quad (78)$$

which with Lemma 2 means $\check{\zeta}_n(T | \star) \leq \zeta_n^{\text{upper}}$.

The above Result (76) clearly follows from Result (75) and Lemma 2.

We prove the above Result (77). From (71), it holds that

$$h_n(\check{\zeta}^{(n)}) := \left\{ \begin{array}{l} 0, \text{ if } t_n(\hat{b}_n(\check{\zeta}^{(n)} | \star), \hat{p}_n(\check{\zeta}^{(n)} | \star), \\ s_n, f_n^{\text{MS}}(\check{\zeta}^{(n)} | \star), f_n^{\text{VU}}(\check{\zeta}^{(n)} | \star)) \leq T, \\ t_n(\hat{b}_n(\check{\zeta}^{(n)} | \star), \hat{p}_n(\check{\zeta}^{(n)} | \star), \\ s_n, f_n^{\text{MS}}(\check{\zeta}^{(n)} | \star), f_n^{\text{VU}}(\check{\zeta}^{(n)} | \star)) - T, \\ \text{otherwise,} \end{array} \right\} \geq 0.$$

With the above Results (75) (76) (77), we apply the Poincaré–Miranda theorem [39] and solve (73) to obtain $\check{\zeta}(T | \star)$ using the multivariate bisection algorithm of [40]. The pseudocode is given as Algorithm 2.1 below. Readers may wonder why the multivariate bisection is not used to jointly solve $[\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \alpha, \beta, \gamma, \delta]$. The reason is that the conditions to use the multivariate bisection are quite strict; e.g., Results (75) (76) (77) are **for any** $\zeta_1, \zeta_2, \dots, \zeta_{n-1}, \zeta_{n+1}, \dots, \zeta_N$ given any $n \in \mathcal{N}$. We do not have such strong conditions if we try to solve $[\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \alpha, \beta, \gamma, \delta]$ together.

Algorithm 2.1 explained above: Alg-Solve- $\check{\zeta}(T | \star)$.

- 1 For $\zeta := [\zeta_1, \zeta_2, \dots, \zeta_N] \in \prod_{n \in \mathcal{N}} [0, \zeta_n^{\text{upper}}]$ for ζ_n^{upper} defined in (74), given Results (75) (76) (77), use the multivariate bisection algorithm proposed by [40] to obtain ζ which induces $\mathbf{h}(\zeta | T)$ to be the N -dimensional zero vector $\mathbf{0}$, to achieve the tolerance level of $\|\mathbf{h}(\zeta | T)\|_2 \leq \epsilon_5$, where computing $\mathbf{h}(\zeta | T)$ defined in (71) and (72) will call Algorithm 1.1 (resp., Algorithm 1.2) to compute $f_n^{\text{MS}}(\check{\zeta}^{(n)} | \star)$ and $f_n^{\text{VU}}(\check{\zeta}^{(n)} | \star)$ as well as $f_n^{\text{MS}}(\zeta | \star)$ and $f_n^{\text{VU}}(\zeta | \star)$ (resp., $\hat{b}_n(\check{\zeta}^{(n)} | \star)$ and $\hat{p}_n(\check{\zeta}^{(n)} | \star)$ as well as $\hat{b}_n(\zeta | \star)$ and $\hat{p}_n(\zeta | \star)$) as inputs to the function $t_n(\cdot)$ defined in (3), where $\check{\zeta}^{(n)}$ is defined in (70);
 - 2 Return the obtained ζ as $\check{\zeta}(T | \star)$;
-

Algorithm 2.2: Computing $T^\#(\star)$ according to Proposition 2.2 using $\mathcal{S}_{2.2}$ defined in (44). Recall from (44) that $\mathcal{S}_{2.2}$ includes (24). Then Proposition 2.2 defines $T^\#(\star)$ as the solution of T to

$$\sum_{n \in \mathcal{N}} \hat{\zeta}_n(T | \star) = y_{c_t}. \quad (79)$$

Algorithm 2.2 explained above: Alg-Solve- $T^\#(\star)$.

- 1 Run Bisection-Search-with-No-Known-Upper-Bound ($\star, \sum_{n \in \mathcal{N}} \hat{\zeta}_n(T | \star), y_{c_t}$) based on Algorithm A3 on Page 14 to obtain the desired T satisfying Eq. (79), and set this T as $T^\#(\star)$, where computing $\sum_{n \in \mathcal{N}} \hat{\zeta}_n(T | \star)$ will call Algorithm 2.1;
-

The bisection method is used repeatedly in the algorithms above. The pseudocodes are given below.

Algorithm A3:

Bisection-Search-with-No-Known-Upper-Bound (\mathbf{v} , function $C(u, \mathbf{v}), C_{\text{target}}$), which returns $u \in [0, \infty)$ such that $C(u, \mathbf{v})$ equals (or is arbitrarily close to) C_{target} given \mathbf{v} , where $C(u, \mathbf{v})$ is non-increasing in u given \mathbf{v} .

- 1 Randomly pick $u^{(0)}$ from $(0, \infty)$;
 - 2 **if** $C(u^{(0)}, \mathbf{v}) = C_{\text{target}}$: return $u^{(0)}$ as the desired u ;
 - 3 **if** $C(u^{(0)}, \mathbf{v}) < C_{\text{target}}$
//In this case, the solution u is in $[0, u^{(0)})$
 - 4 Use Standard-Bisection-Search(\mathbf{v} , function $C(u, \mathbf{v}), C_{\text{target}}, 0, u^{(0)}$) based on Algorithm A4 to find the result, and return it as the desired u ;
 - 5 **if** $C(u^{(0)}, \mathbf{v}) > C_{\text{target}}$
//In this case, the solution u is in $(u^{(0)}, \infty)$
 - 6 Find $i \geq 0$ such that $C(u^{(0)} \cdot 2^i, \mathbf{v}) > C_{\text{target}}$ but $C(u^{(0)} \cdot 2^{i+1}, \mathbf{v}) \leq C_{\text{target}}$
 - 7 **if** $C(u^{(0)} \cdot 2^{i+1}, \mathbf{v}) = C_{\text{target}}$: return $u^{(0)} \cdot 2^{i+1}$ as the desired u ;
 - 8 **elseif** $C(u^{(0)} \cdot 2^{i+1}, \mathbf{v}) < C_{\text{target}}$
 - 9 Use Standard-Bisection-Search(\mathbf{v} , function $C(u, \mathbf{v}), C_{\text{target}}, u^{(0)} \cdot 2^i, u^{(0)} \cdot 2^{i+1}$) based on Algorithm A4 to find the result, and return it as the desired u ;
 - 10 **endif**
 - 11 **endif**
-

To better understand bisection search in our algorithms above, we prove in the Appendix of our full paper [36] that the left-hand side of (57) (resp., (64), (66), (68), (79)) is non-increasing with respect to γ (resp., p_n, α, β, T). In simulations, the above often decreases so that there is a unique solution.

VII. OUR ALGORITHM TO SOLVE PROBLEM \mathbb{P}_1

Algorithm A1 has been presented on Page 6 to solve the system UCR optimization \mathbb{P}_1 . In Section V-E, we have

Algorithm A4:

Standard-Bisection-Search

(\mathbf{v} , function $C(u, \mathbf{v}), C_{\text{target}}$, lower bound L_0 , upper bound U_0), which returns $u \in [L_0, U_0]$ such that $C(u, \mathbf{v})$ equals (or is arbitrarily close to) C_{target} given \mathbf{v} , where $C(u, \mathbf{v})$ is non-increasing in u given \mathbf{v} .

- 1 Initialize $B_{\text{lower}} \leftarrow L_0, B_{\text{upper}} \leftarrow U_0$;
 - 2 **repeat**
 - 3 $u \leftarrow \frac{B_{\text{lower}} + B_{\text{upper}}}{2}$;
 - 4 **if** $C(u, \mathbf{v}) = C_{\text{target}}$: return u ;
 - 5 **if** $C(u, \mathbf{v}) > C_{\text{target}}$: $B_{\text{lower}} \leftarrow u$;
 - 6 **else**: $B_{\text{upper}} \leftarrow u$;
 - 7 **endif**
 - 8 **until** $B_{\text{upper}} - B_{\text{lower}}$ is no greater than ϵ_4 for a small positive number ϵ_4 ;
-

also explained how the different building blocks in Sections V-A, V-B, V-C, and V-D are combined together to produce Algorithm A1's pseudocode. We now discuss the performance of Algorithm A1.

Solution quality and convergence. Algorithm A1 comprises three levels of iterations, with the innermost iteration from Line 17 containing Algorithm A2 in Line 20. Algorithm A2 obtains a global optimum for Problem $\mathbb{P}_5(\star)$. The outermost iteration from Line 4 is based on Dinkelbach's transform and does not lose optimality. However, the mid-level iteration from Line 11 is based on alternating optimization and cannot guarantee local/global optimality. Hence, Algorithm A1 cannot guarantee local/global optimality for \mathbb{P}_1 . Yet, using the terminology of stationary points in [12] for constrained optimization, Algorithm A1 finds a stationary point for \mathbb{P}_1 . The convergence of Algorithm A1 is also clear from the above analysis.

Time Complexity. In Algorithm A1 and its subroutine Algorithm A2, the bisection search is repeatedly used. Then the complexity of Algorithm A1 is polylogarithmic in the error tolerance's reciprocal in various calls of the bisection search. Below we analyze the complexities of Algorithms A1 and A2 with respect to the number N of users. Line 1 of Algorithm A2 calls Algorithm 2.2, which calls Algorithm 2.1. Algorithm 2.1 called in the above and in Line 2 of Algorithm A2 calls Algorithms 1.1 and 1.2. Algorithm 1.2 called in the above and in Line 3 of Algorithm A2 calls Algorithms 1.2.2.2, 1.2.2.1, and 1.2.1. For the multivariate bisection search in Algorithm 2.1, from Theorem 2.3 of [40], to achieve the tolerance level of $\|\mathbf{h}(\zeta | T)\|_2 \leq \epsilon_5$, the number of iterations required is $\log_2 \frac{\sum_{n \in \mathcal{N}} \zeta_n^{\text{upper}}}{\epsilon_5}$ for ζ_n^{upper} define in (74); i.e., logarithmic in N . Computing $\tilde{\mathbf{b}}(\alpha, \beta, \zeta | \star)$ and $\tilde{\mathbf{p}}(\alpha, \beta, \zeta | \star)$ takes $\mathcal{O}(N)$. Then calculating $\tilde{\alpha}(\beta, \zeta | \star)$ costs $\mathcal{O}(N)$. Thus, obtaining $\tilde{\beta}(\zeta | \star)$ takes $\mathcal{O}(N)$. Finally, computing $\mathbf{h}(\zeta | T)$ costs $\mathcal{O}(N)$. Line 1 of Alg. A2 takes $\mathcal{O}(N \log N)$. We analyze other lines of Alg. A2 similarly. Then Alg. A2 and each innermost iteration of Alg. A1 cost $\mathcal{O}(N \log N)$. In the outermost and mid-level iterations, each computation of the utility $\mathcal{U}()$ in (2) and the cost in (6) requires $\mathcal{O}(N)$. To summarize,

Algorithm A1 takes $\mathcal{O}(N^2 \log N)$.

VIII. MODELING THE HUMAN-CENTRIC UTILITY FROM REAL DATA

We now model users' human-centric utilities in the Metaverse over wireless communications using two datasets [10], [31] explained below, which are both based on real experiments of humans assessing videos.

SSV360 dataset. This dataset of [10] captures users' evaluation of 360° videos when wearing HTC Vive Pro Virtual Reality (VR) headsets. Each data point exhibits a user's perceptual quality assessment of a 360° scene of a given bitrate and a given video resolution, under standing or seated viewing (SSV).

Netflix dataset. This dataset is a part of Netflix's Emmy Award-winning VMAF project [31]. Each data point represents users' mean opinion score for a video at a given bitrate and a given resolution.

The wireless data rate needs to be large enough for users' smooth watching experience at the given video bitrate [41]. We consider the bitrate as a constant fraction (say θ) of the wireless rate. Then substituting the bitrate r_{bitrate} with the wireless rate r_{wireless} just involves replacing r_{bitrate} with $r_{\text{wireless}}/\theta$. Hence, for both datasets above, we perform curve-fitting with the bitrate and the resolution to obtain the utility functions.

Modeling human-centric utilities. Based on the two datasets above, the human-centric utility of each user n , denoted by $U_n(r_n, s_n)$, is modeled as a function of the bitrate r_n and the video resolution s_n . We adopt the logarithmic utility function, which is used in [32]–[34] for various communication/network systems. The logarithmic function reflects users' diminishing marginal gain as the bitrate and the resolution increase. Formally, we have $U_n(r_n, s_n) = \kappa_n \ln(1 + l_n^s s_n + l_n^r r_n)$ for coefficients κ_n, l_n^s, l_n^r , which are decided by fitting data. Since using a three-dimensional plot to show the two-variable function $U_n(r_n, s_n)$ is difficult for visual interpretation, we use the following transform to obtain a two-dimensional plot. Let r_n^{\max} (resp., s_n^{\max}) be the maximum r_n (resp., s_n) from the dataset. After defining $\alpha_n := l_n^r r_n^{\max} + l_n^s s_n^{\max}$, we let $\frac{l_n^r}{\alpha_n} r_n + \frac{l_n^s}{\alpha_n} s_n$ be the x -axis coordinate, and plot $\kappa_n \ln(1 + \alpha_n x)$ as the y -axis coordinate, since it holds that $U_n(r_n, s_n) = \kappa_n \ln(1 + \alpha_n \cdot (\frac{l_n^r}{\alpha_n} r_n + \frac{l_n^s}{\alpha_n} s_n)) = \kappa_n \ln(1 + \alpha_n x)$. With the above transformation, each data point's x -coordinate is between 0 and 1.

In Fig. 4(a) for the SSV360 dataset, the data and curves are about two users watching a 360° video ("Alcatraz" or "FormationPace" [10]) under seated or standing view. The score is an integer from 1 to 5 based on the well-known Absolute Category Rating. In Fig. 4(b) for the Netflix dataset, the data and curves present users' average assessment (from 0 to 100) of different videos (BirdsInCage, BigBuckBunny, ElFuente1, or CrowdRun [31]). Both subfigures demonstrate that the curves of the logarithmic human-centric utility functions fit the data. The specific expressions of the functions are provided in the legends.

IX. SIMULATIONS

In this section on simulations, we first describe the default settings and then report various results.

Default settings. We consider a macro-cell wireless channel model for urban areas. With d_n denoting the distance between the Metaverse server (MS) and a virtual-reality user (VU) indexed by n , the path loss between them is $128.1 + 37.6 \log d_n$ along with 8 decibels (dB) for the standard deviation of shadow fading [6], where the unit of d_n is kilometer. The power spectral density of Gaussian noise σ_n^2 is -174 dBm/Hz (i.e., the thermal noise amount at 20°C room temperature). VUs are randomly located in a circle of radius 500m centered at the MU. The default total bandwidth b_{\max} is 20GHz, and the total transmission power p_{\max} is 30W. The effective switched capacitance κ^{MS} and κ^{VU} are set as 10^{-27} . The number μ_n of bits per pixel is 16, and the compression rate ν_n is 100. The maximum CPU frequencies at the MS and VUs, f_{\max}^{MS} and $f_{n,\max}^{\text{VU}}$, are 300GHz and 50GHz, respectively. The default weights for energy and delay are $c_e = 0.5$ and $c_t = 0.5$. The default VU number is 5. Based on measurements, Section V of [41] quantifies the computational complexity of processing a video frame of resolution s_n as $w(s_n) = (7 \times 10^{-10} \times s_n^{3/2} + 0.083)$ tera (i.e., trillion) floating-point operations (FLOPs). From Fig. 2 in Section III-B, we know that $\mathcal{A}_n(s_n, \Lambda_n)$ (resp., $\mathcal{B}_n(s_n, \Lambda_n)$) of Page 3 for delay is less than $\mathcal{F}_n(s_n, \Lambda_n)$ (resp., $\mathcal{G}_n(s_n, \Lambda_n)$) of Page 3 for energy. In the simulations, we set both $\mathcal{A}_n(s_n, \Lambda_n)$ and $\mathcal{B}_n(s_n, \Lambda_n)$ as $\Lambda_n w(s_n)/30$, and set both $\mathcal{F}_n(s_n, \Lambda_n)$ and $\mathcal{G}_n(s_n, \Lambda_n)$ as $\Lambda_n w(s_n)$, which make all of them convex in s_n . For all VU n , we let Λ_n be the same Λ . Then the optimization objective becomes a multiple of Λ , which thus has no impact. The above avoids considering the impact of heterogeneous Λ_n for simplicity. Possible values for the resolution s_n are 4096×2160 , 3072×1620 , 2048×1080 , 1920×1080 , and 1280×720 pixels, which are also referred to as 4k, 3k, 2k, 1080p, and 720p. The SSV360 dataset [10] in Section VIII includes the perceptual assessment of users watching VR videos. We use those data for curve-fitting different logarithmic utility functions, and assign the functions to users in the simulations: one function for one user.

Comparison with baselines. For the simulation results, we first compare our algorithm with baselines:

- **average allocation**, which sets each b_n as $\frac{b_{\max}}{N}$, each p_n as $\frac{p_{\max}}{N}$, each s_n as 2048×1080 (i.e., 2k resolution), each f_n^{MS} as $\frac{f_{\max}^{\text{MS}}}{N}$, and each f_n^{VU} as $f_{n,\max}^{\text{VU}}$;
- **optimize b, p, and s only**, while setting each f_n^{MS} as $\frac{f_{\max}^{\text{MS}}}{N}$ and each f_n^{VU} as $f_{n,\max}^{\text{VU}}$,
- **optimize f^{MS} and f^{VU} only**, while setting each b_n as $\frac{b_{\max}}{N}$, each p_n as $\frac{p_{\max}}{N}$, and each s_n as 2048×1080 .

Various simulation results are plotted in the subfigures of Fig. 5 for a detailed comparison and examining the impact of different parameters on the system utility-cost ratio (UCR). We discuss the results below.

- **UCR versus the total bandwidth.** Here we vary the total bandwidth from 1GHz to 20GHz. In Fig. 5(a), larger bandwidth induces higher data rates, which reduce latency and energy consumption, thus increasing the system UCR. In addition, the difference in UCR between the proposed algorithm and

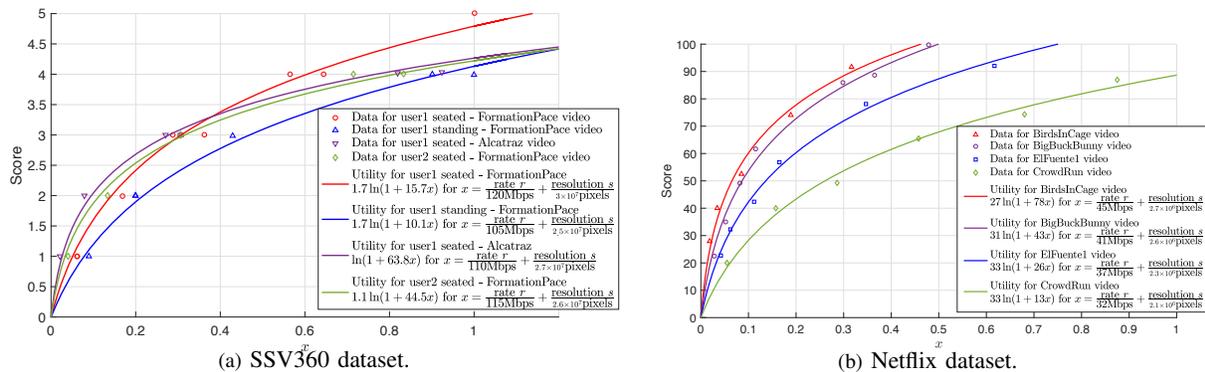


Fig. 4: Modeling the logarithmic human-centric utility functions from the SSV360 and Netflix datasets.

the average allocation baseline rises from 522.3% to 630.4% as the total bandwidth increases.

- UCR versus minimum resolution.** We fix the maximum resolution as 4096×2160 (4k) and change the minimum resolution from 1280×720 (720P) to 4096×2160 . From Fig. 5(b), the UCR performance of all the algorithms improves as the minimum resolution decreases, with our proposed algorithm showing a significant improvement. The reason for this is that the high data volumes associated with high resolution can lead to higher energy consumption and system delay. The UCR of the proposed algorithm gradually plateaus when the minimum resolution reaches below 1920×1080 (1080p).
- UCR versus transmission power.** Here we configure the maximum downlink transmission power from 0.03W to 100W. From Fig. 5(c), the UCR of all algorithms increases as the transmission power grows, since raising the transmission power widens the search space for the optimization. When the transmission power is very small (e.g., 0.03W or 0.3W), the UCR of the proposed algorithm is slightly higher than other algorithms, but as the transmission power increases, the performance of the proposed algorithm far exceeds others. The UCR of all methods plateaus when the transmission power reaches 50W.
- UCR versus computation resource.** We vary the maximum server CPU frequency from 0.5GHz to 60GHz and all VU's maximum CPU frequencies from 10MHz to 10GHz. In Fig. 5(d) and Fig. 5(e), the system UCR increases as the maximum CPU frequency grows, since the optimization problem has a wider search space. The proposed algorithm outperforms the average allocation, reaching a difference of 660.9% and 653.2% for server CPU frequency at 60GHz and VU CPU frequencies at 10GHz, respectively.
- Impact of user number on UCR.** We now amplify the bandwidth, transmission power, and server's CPU frequency by 10 times and fix other parameters to see the impact of user number N on UCR. In Fig. 6, as the user number increases from 10 to 160, the average UCR decreases. This is because the server allocates fewer resources to each VU and induces decreasing utility. In general, the comfortable frame rate for VR applications is at least 90 [42], i.e., at most 11ms for one frame. Note that the system delay in Fig. 6 is to complete each user's all frames. In all user scenarios shown in Fig. 6, the delay for one frame (i.e., $\frac{\text{delay}}{\text{frame number}}$) is less than 7 ms, which satisfies the comfortable frame rate requirement.

All the above simulations compare our algorithm with the baselines. Below we provide additional simulation results to show the impact of other settings to our algorithm.

Impact of cost weights on UCR. We configure different cost weights of energy and delay (c_e, c_t) to see the effect on the system UCR, where we enforce $c_e + c_t = 1$. In Fig. 5(f), as c_t rises to 0.8, the system UCR also increases, reflecting the importance of delay optimization for the whole system. However, as c_t increases to 0.9, the system UCR instead drops significantly, since emphasizing the latency overwhelmingly while undervaluing the energy may enlarge the system cost.

Reviewing different users' allocated results. In Fig. 7(a), we present the optimized bandwidth b_n , transmit power p_n , resolution s_n , MS resource allocation f_n^{MS} , and user CPU frequency f_n^{VU} of five users to visualize the resource allocation. Fig. 7(b) shows the impact of user scenarios on individual UCRs. Different user preferences and physical states affect subjective scores. For example, users who prefer high-quality videos may give stricter subjective scores than those who do not require high video quality. Generally, many users found sitting to provide more comfort than standing, as indicated by a higher UCR in most seated scenarios. However, it's important to note that individual differences were observed, and this trend did not hold true for all participants.

To summarize, extensive simulation results above confirm the effectiveness of our proposed algorithm.

X. CONCLUSION

In this paper, we optimize the system utility-cost ratio (UCR) for the Metaverse over wireless networks. The optimization variables include the allocation of both communication and computation resources as well as the resolutions of virtual reality (VR) videos. Our human-centric utility measure represents users' subjective assessment of the VR video quality, and is supported by real datasets. We tackle the non-convex system UCR optimization by proposing a novel technique for fractional programming. Our computationally efficient algorithm for the system UCR optimization is validated by extensive simulations. Three future directions are as follows. Firstly, since the current paper solves the optimization problem via alternating optimization (AO) of video frame resolution and other variables, a future task is to see whether we can optimize all variables simultaneously to obtain the globally optimal solution. Secondly, we may incorporate the priorities of different users into computing the system utility

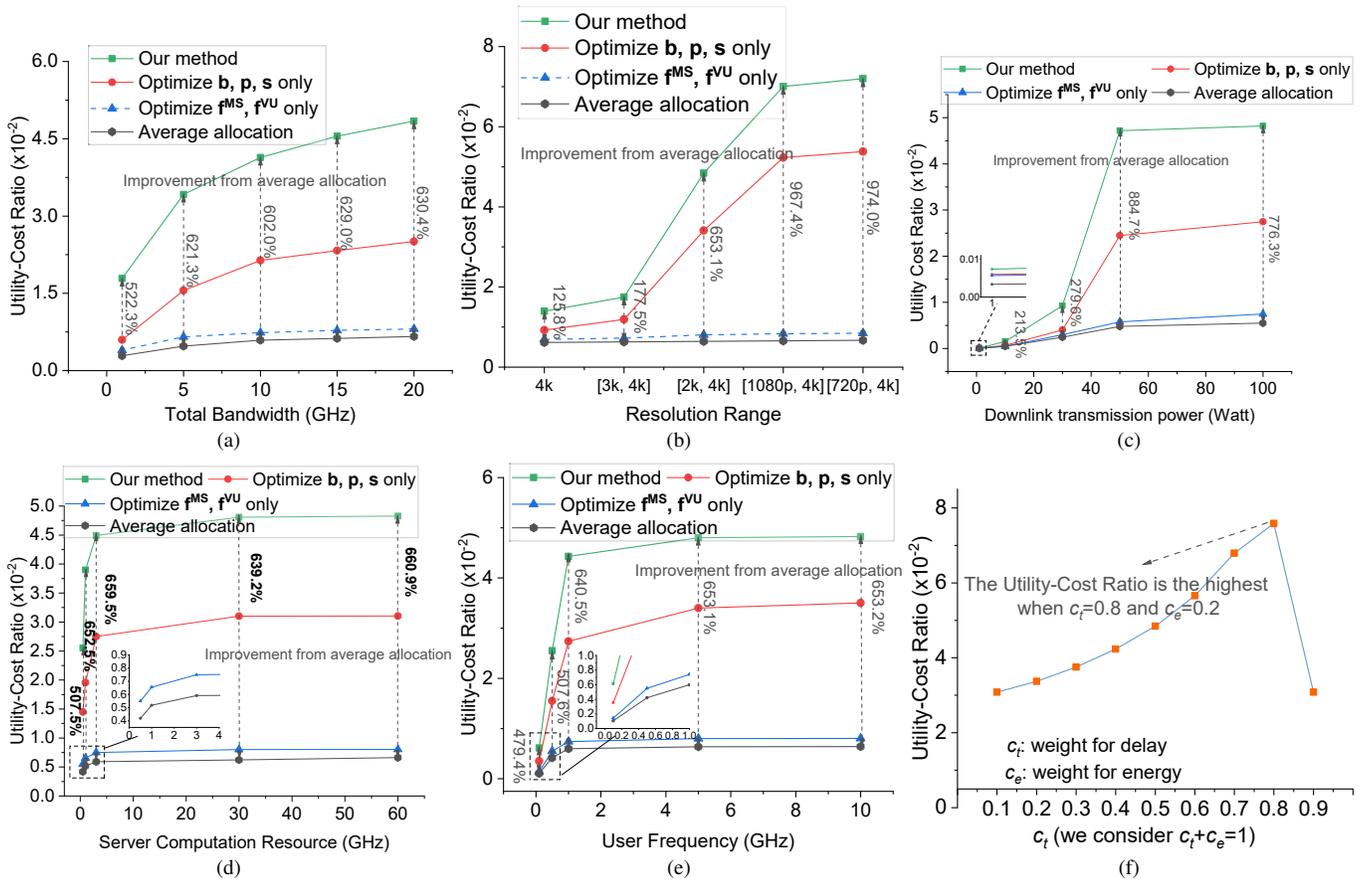


Fig. 5: The system utility-cost ratio (UCR) versus various parameters.

(e.g., using a weighted sum with weights representing users' priorities), and investigate the impact of such formulation on the optimization. Thirdly, while the current paper contains extensive simulation results to support the analysis, we can implement real-world systems to evaluate the performance of our proposed algorithm in practice.

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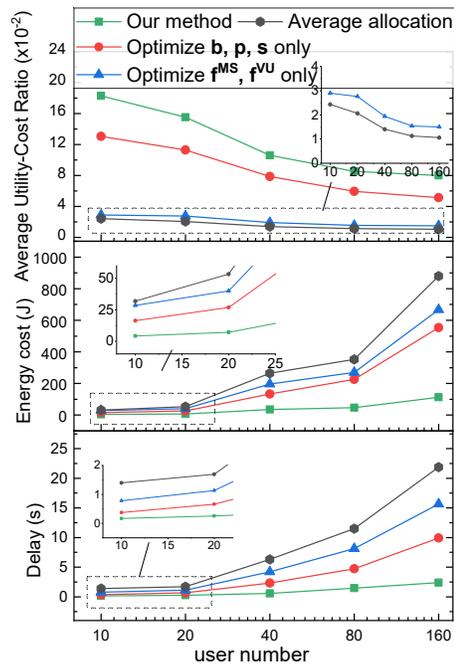
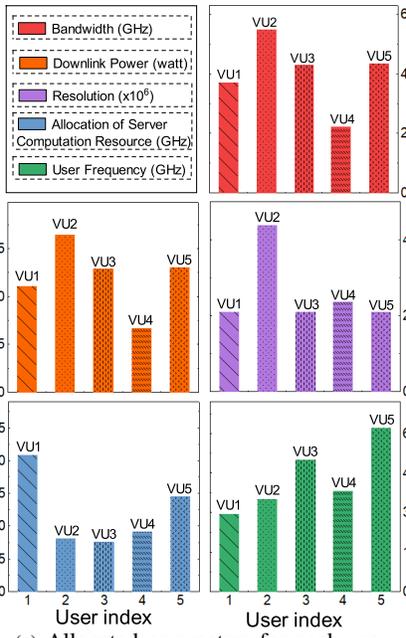
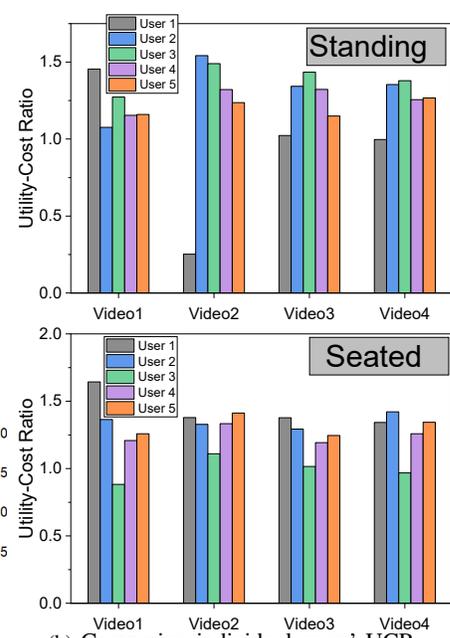


Fig. 6: Metrics with respect to the number of users.



(a) Allocated parameters for each user.



(b) Comparing individual users' UCR.

Fig. 7: Reviewing the allocated results of different users and the impact of user scenarios on UCR.

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APPENDIX

A. Proving the relationship between FP-minimization in (10) and minimizing $W(\mathbf{x}, \mathbf{y}) := G(\mathbf{x}) + \sum_{n=1}^N K_n(\mathbf{x}, y_n)$ subject to $\mathbf{x} \in \mathcal{S}$ and $y_n \in \mathbb{R}^+$, stated in Section IV on Pages 4 and 5

Recall that FP-minimization in (10) means minimizing $H(\mathbf{x}) := G(\mathbf{x}) + \sum_{n=1}^N \frac{A_n(\mathbf{x})}{B_n(\mathbf{x})}$ subject to \mathbf{x} in a convex set \mathcal{S} , for convex $A_n(\mathbf{x})$ and concave $B_n(\mathbf{x})$.

We now consider alternating optimization (AO) of \mathbf{x} and \mathbf{y} to minimize $W(\mathbf{x}, \mathbf{y}) := G(\mathbf{x}) + \sum_{n=1}^N K_n(\mathbf{x}, y_n)$ subject to $\mathbf{x} \in \mathcal{S}$ and $\mathbf{y} := [y_1, \dots, y_N] \in (\mathbb{R}^+)^N$, where $K_n(\mathbf{x}, y_n) := [A_n(\mathbf{x})]^2 y_n + \frac{1}{4[B_n(\mathbf{x})]^2 y_n}$, and \mathbb{R}^+ denotes the set of positive numbers. We will show that

if the above AO process of minimizing $W(\mathbf{x}, \mathbf{y})$ converges to $(\mathbf{x}^*, \mathbf{y}^*)$, \mathbf{x}^* is a stationary point for FP-minimization in (10).

(80)

The AO process is as follows: We start with a randomly initialized $\mathbf{x}^{(0)} \in \mathcal{S}$. Then we optimize \mathbf{y} with \mathbf{x} being $\mathbf{x}^{(0)}$ to minimize $W(\mathbf{x}, \mathbf{y})$, and denote the obtained $\mathbf{y} \in (\mathbb{R}^+)^N$ as $\mathbf{y}^{(1)}$. Given \mathbf{y} as $\mathbf{y}^{(1)}$, we optimize \mathbf{x} to minimize $W(\mathbf{x}, \mathbf{y})$, and denote the obtained $\mathbf{x} \in \mathcal{S}$ as $\mathbf{x}^{(1)}$. Given \mathbf{x} as $\mathbf{x}^{(1)}$, we optimize \mathbf{y} to minimize $W(\mathbf{x}, \mathbf{y})$, and denote the obtained $\mathbf{y} \in (\mathbb{R}^+)^N$ as $\mathbf{y}^{(2)}$. The above process continues iteratively. The i -th iteration includes the following two steps:

- Given \mathbf{x} as $\mathbf{x}^{(i-1)}$, we optimize \mathbf{y} to minimize $W(\mathbf{x}, \mathbf{y})$, and denote the obtained $\mathbf{y} \in (\mathbb{R}^+)^N$ as $\mathbf{y}^{(i)}$.
- Given \mathbf{y} as $\mathbf{y}^{(i)}$, we optimize \mathbf{x} to minimize $W(\mathbf{x}, \mathbf{y})$, and denote the obtained $\mathbf{x} \in \mathcal{S}$ as $\mathbf{x}^{(i)}$.

The above AO process converges when the relative difference between $W(\mathbf{x}^{(i-1)}, \mathbf{y}^{(i-1)})$ and $W(\mathbf{x}^{(i)}, \mathbf{y}^{(i)})$ is smaller than a predefined small error tolerance.

To examine the AO process of minimizing $W(\mathbf{x}, \mathbf{y})$, we will analyze 1) optimizing \mathbf{y} given \mathbf{x} , and 2) optimizing \mathbf{x} given \mathbf{y} . Optimizing \mathbf{y} given \mathbf{x} means minimizing $K_n(\mathbf{x}, y_n) := [A_n(\mathbf{x})]^2 y_n + \frac{1}{4[B_n(\mathbf{x})]^2 y_n}$ with respect to y_n for each $n = 1, 2, \dots, N$; i.e., letting y_n be $y_n^\#(\mathbf{x}) := \frac{1}{2A_n(\mathbf{x})B_n(\mathbf{x})}$. If such \mathbf{y} is substituted back to $K_n(\mathbf{x}, y_n)$, then $K_n(\mathbf{x}, y_n)$ will become the desired $\frac{A_n(\mathbf{x})}{B_n(\mathbf{x})}$. Moreover, we now show that the partial derivative of $W(\mathbf{x}, \mathbf{y})$ with respect to \mathbf{x} at \mathbf{y} being $\mathbf{y}^\#(\mathbf{x})$ is the same as the derivative of $H(\mathbf{x})$ with respect to \mathbf{x} . In fact, we have

$$\frac{\partial K_n(\mathbf{x}, y_n)}{\partial \mathbf{x}} = 2A_n(\mathbf{x})y_n \frac{\partial A_n(\mathbf{x})}{\partial \mathbf{x}} - \frac{1}{2[B_n(\mathbf{x})]^3 y_n} \cdot \frac{\partial B_n(\mathbf{x})}{\partial \mathbf{x}}, \quad (81)$$

and

$$\frac{\partial \left(\frac{A_n(\mathbf{x})}{B_n(\mathbf{x})} \right)}{\partial \mathbf{x}} = \frac{\frac{\partial A_n(\mathbf{x})}{\partial \mathbf{x}} \cdot B_n(\mathbf{x}) - \frac{\partial B_n(\mathbf{x})}{\partial \mathbf{x}} \cdot A_n(\mathbf{x})}{(B_n(\mathbf{x}))^2}. \quad (82)$$

From (81) and (82), it holds that

$$\left(\frac{\partial K_n(\mathbf{x}, y_n)}{\partial \mathbf{x}} \right) \Big|_{y_n = \frac{1}{2A_n(\mathbf{x})B_n(\mathbf{x})}} = \frac{\partial \left(\frac{A_n(\mathbf{x})}{B_n(\mathbf{x})} \right)}{\partial \mathbf{x}}, \quad (83)$$

which further implies

$$\left(\frac{\partial W(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}} \right) \Big|_{\mathbf{y} = \mathbf{y}^*(\mathbf{x})} = \frac{\partial H(\mathbf{x})}{\partial \mathbf{x}}. \quad (84)$$

Moreover, as explained, we have

$$W(\mathbf{x}, \mathbf{y}) \Big|_{\mathbf{y} = \mathbf{y}^*(\mathbf{x})} = H(\mathbf{x}). \quad (85)$$

Using (84) and (85), we now show (80). The AO process of minimizing $W(\mathbf{x}, \mathbf{y})$ is non-decreasing. Specifically, we have $W(\mathbf{x}^{(i)}, \mathbf{y}^{(i)}) \leq W(\mathbf{x}^{(i-1)}, \mathbf{y}^{(i)}) \leq W(\mathbf{x}^{(i-1)}, \mathbf{y}^{(i-1)})$. For lower-bounded $W(\mathbf{x}, \mathbf{y})$, we know $W(\mathbf{x}^{(i)}, \mathbf{y}^{(i)})$ converges as the iteration number $i \rightarrow \infty$. Supposing the variable solution of the AO process converges to $(\mathbf{x}^*, \mathbf{y}^*)$, we know that

- ① \mathbf{y}^* is the optimal \mathbf{y} for minimizing $W(\mathbf{x}, \mathbf{y})$ given \mathbf{x} as \mathbf{x}^* (i.e., $y_n^* := \frac{1}{2A_n(\mathbf{x}^*)B_n(\mathbf{x}^*)}$), and
- ② \mathbf{x}^* is the optimal \mathbf{x} for minimizing $W(\mathbf{x}, \mathbf{y})$ given \mathbf{y} as \mathbf{y}^* .

Result “②” means that \mathbf{x}^* satisfies the KKT conditions for optimizing \mathbf{x} to minimize $W(\mathbf{x}, \mathbf{y}^*)$ subject to $\mathbf{x} \in \mathcal{S}$. Suppose \mathbf{x} is M -dimensional, and $\mathbf{x} \in \mathcal{S}$ means

$$\begin{cases} \mathcal{Q}_q(\mathbf{x}) \leq 0, & q = 1, 2, \dots, Q, \\ \mathcal{R}_r(\mathbf{x}) = 0, & r = 1, 2, \dots, R, \\ \mathbf{x} \in \mathbb{R}^M. \end{cases}$$

Then with α and β denoting the multipliers, the KKT conditions mean

$$\begin{cases} \text{Stationarity:} & \frac{\partial}{\partial x_m} (W(\mathbf{x}^*, \mathbf{y}^*) + \sum_{q=1}^Q \alpha_q \mathcal{Q}_q(\mathbf{x}^*) + \sum_{r=1}^R \beta_r \mathcal{R}_r(\mathbf{x}^*)) = 0, \\ & \text{for } m = 1, 2, \dots, M, \\ \text{Primal feasibility:} & \mathcal{Q}_q(\mathbf{x}^*) \leq 0, \\ & \text{for } q = 1, 2, \dots, Q, \\ & \mathcal{R}_r(\mathbf{x}^*) = 0, \\ & \text{for } r = 1, 2, \dots, R, \\ \text{Dual feasibility:} & \alpha_q \geq 0, \\ & \text{for } q = 1, 2, \dots, Q, \\ \text{Complementary slackness:} & \alpha_q \mathcal{Q}_q(\mathbf{x}^*) = 0 \\ & \text{for } q = 1, 2, \dots, Q. \end{cases} \quad (86a)-(86e)$$

Using (84) in (86a), we know that (86a)–(86e) are equivalent

to

$$\begin{cases} \text{Stationarity:} & \frac{\partial}{\partial x_m} (H(\mathbf{x}^*) + \sum_{q=1}^Q \alpha_q \mathcal{Q}_q(\mathbf{x}^*) + \sum_{r=1}^R \beta_r \mathcal{R}_r(\mathbf{x}^*)) = 0, \\ & \text{for } m = 1, 2, \dots, M, \\ \text{Primal feasibility:} & \mathcal{Q}_q(\mathbf{x}^*) \leq 0, \\ & \text{for } q = 1, 2, \dots, Q, \\ & \mathcal{R}_r(\mathbf{x}^*) = 0, \\ & \text{for } r = 1, 2, \dots, R, \\ \text{Dual feasibility:} & \alpha_q \geq 0 \\ & \text{for } q = 1, 2, \dots, Q, \\ \text{Complementary slackness:} & \alpha_q \mathcal{Q}_q(\mathbf{x}^*) = 0 \\ & \text{for } q = 1, 2, \dots, Q. \end{cases} \quad (87a)-(87e)$$

The above (87a)–(87e) mean that \mathbf{x}^* is a stationary point for optimizing \mathbf{x} to minimize $H(\mathbf{x})$ subject to $\mathbf{x} \in \mathcal{S}$; i.e., \mathbf{x}^* is a stationary point of FP-minimization in (10). Hence, we have proved (80). \square

B. Proving that the left-hand side of (57) is non-increasing with respect to γ

With $v(x)$ defined by $\zeta_n \frac{A_n(s_n, \Lambda_n)}{x^2} - y_{ce} \cdot 2\kappa^{\text{MS}} \mathcal{F}_n(s_n, \Lambda_n)x$, clearly $v(x)$ is non-increasing with respect to x . Then $\text{PositiveRoot}(v(x) = \gamma)$, denoting the positive root satisfying $v(x) = \gamma$, is non-increasing with respect to γ . Thus, the left-hand side of (57) is non-increasing with respect to γ . \square

C. Proving that the left-hand side of (64) is non-increasing with respect to p_n

From (62), we know (i) $\psi_n(p_n, \alpha, \beta \mid \star)$ is positive and decreasing in p_n , where “ \star ” denotes “ z, y, s ”. In addition, (ii) the function $\log_2(1+x) - \frac{x}{(1+x)\ln 2}$ is increasing and positive for $x > 0$ since the derivative $\frac{x}{(1+x)^2 \ln 2}$ is positive for $x > 0$, and $\log_2(1+x) - \frac{x}{(1+x)\ln 2}$ at $x = 0$ equals 0. From the above Results “(i)” and “(ii)”, we obtain (iii) $\left[\log_2(1 + \psi_n(p_n, \alpha, \beta \mid \star)) - \frac{\psi_n(p_n, \alpha, \beta \mid \star)}{(1 + \psi_n(p_n, \alpha, \beta \mid \star)) \ln 2} \right]$ is positive and decreasing in p_n .

We also have (iv) the function $\frac{\log_2(1+x)}{x}$ is decreasing for $x > 0$ since the derivative $\frac{1}{x^2} \left[\frac{x}{(1+x)\ln 2} - \log_2(1+x) \right]$ is negative due to Result “(ii)” above. From the above Results “(i)” and “(iv)”, we obtain (v) $\bar{r}_n(p_n, \alpha, \beta \mid \star)$ defined in (63) is increasing in p_n . Since the utility function $U_n(r_n, s_n)$ is concave and non-decreasing in r_n , $\frac{\partial U_n(r_n, s_n)}{\partial r_n}$ is non-negative and non-increasing in r_n . Then (vi) $\frac{\partial U_n(r_n, s_n)}{\partial r_n} + \frac{y_{ce}}{2z_n r_n^3 \nu_n^2} + \frac{\zeta_n s_n \mu_n \Lambda_n}{r_n^2 \nu_n}$ is positive and non-increasing in r_n . From the above Results “(v)” and “(vi)”, we obtain (vii) $\left(\frac{\partial U_n(r_n, s_n)}{\partial r_n} \right) \Big|_{r_n = \bar{r}_n(p_n, \alpha, \beta \mid \star)} + \frac{y_{ce}}{2z_n \cdot [\bar{r}_n(p_n, \alpha, \beta \mid \star)]^3 \nu_n^2} + \frac{\zeta_n s_n \mu_n \Lambda_n}{[\bar{r}_n(p_n, \alpha, \beta \mid \star)]^2 \nu_n}$ is positive and non-increasing in p_n .

From the above Results “(iii)” and “(vii)”, the left hand side of (64) is decreasing as p_n increases. \square

D. Further explanations of the step-by-step analysis in (45) on Page 8 for Problem $\mathbb{P}_5(z, y, s)$, which will be useful for Appendices E, F, and H

In this part, we provide further explanations of the step-by-step analysis in (45) on Page 8 for Problem $\mathbb{P}_5(z, y, s)$, which will be useful to prove the results in Appendices E, F, and H later.

We recall Problem $\mathbb{P}_5(z, y, s)$ (i.e., $\mathbb{P}_5(\star)$) from (16):

$$\begin{aligned} \mathbb{P}_5(\star) : \max_{\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T} & F(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T \mid \mathbf{s}, y) \\ & - y c_e \cdot \sum_{n \in \mathcal{N}} \{ [(p_n + p_n^{\text{cir}}) s_n \mu_n \Lambda_n]^2 z_n \\ & + \frac{1}{4(r_n(b_n, p_n) \nu_n)^2 z_n} \} \\ \text{s.t. (7a): } & \sum_{n \in \mathcal{N}} b_n \leq b_{\max}, \\ \text{(7b): } & \sum_{n \in \mathcal{N}} p_n \leq p_{\max}, \\ \text{(7d): } & \sum_{n \in \mathcal{N}} f_n^{\text{MS}} \leq f_{\max}^{\text{MS}}, \\ \text{(7e): } & f_n^{\text{VU}} \leq f_{n, \max}^{\text{VU}}, \forall n \in \mathcal{N}, \\ \text{(11a): } & t_n(b_n, p_n, s_n, f_n^{\text{MS}}, f_n^{\text{VU}}) \leq T, \forall n \in \mathcal{N}. \end{aligned}$$

As shown at the beginning of Section VI on Page 7, $\mathbb{P}_5(\star)$ belongs to convex optimization, and $\alpha, \beta, \gamma, \delta, \zeta$ denote the Lagrange multipliers for (7a), (7b), (7d), (7e), and (11a), respectively.

Suppose we already know ζ . We move ζ and (11a) to the objective function, and construct the following problem (recall that for an optimization problem \mathbb{P}_i , we use $H_{\mathbb{P}_i}$ to denote its objective function):

$$\begin{aligned} \mathbb{P}_7(\star, \zeta) : \min_{\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T} & -H_{\mathbb{P}_5}(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T \mid \star) \\ & + \sum_{n \in \mathcal{N}} [\zeta_n \cdot (t_n(b_n, p_n, s_n, f_n^{\text{MS}}, f_n^{\text{VU}}) - T)] \\ \text{s.t. (7a): } & \sum_{n \in \mathcal{N}} b_n \leq b_{\max}, \\ \text{(7b): } & \sum_{n \in \mathcal{N}} p_n \leq p_{\max}, \\ \text{(7d): } & \sum_{n \in \mathcal{N}} f_n^{\text{MS}} \leq f_{\max}^{\text{MS}}, \\ \text{(7e): } & f_n^{\text{VU}} \leq f_{n, \max}^{\text{VU}}, \forall n \in \mathcal{N}. \end{aligned}$$

We define

- Statement $\mathcal{V}_{\mathbb{P}_5}$: $\left\{ \begin{array}{l} [\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), (\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), \\ T^\#(\star), \alpha^\#(\star), \beta^\#(\star), \gamma^\#(\star), \delta^\#(\star), \zeta^\#(\star)] \\ \text{is a solution to the KKT conditions } \mathcal{S}_{\text{KKT}} \text{ in (34)} \\ \text{for Problem } \mathbb{P}_5(\star). \end{array} \right\}$,
and
- Statement $\mathcal{V}_{\mathbb{P}_7}$: $\left\{ \begin{array}{l} [\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), (\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), \\ T^\#(\star), \alpha^\#(\star), \beta^\#(\star), \gamma^\#(\star), \delta^\#(\star)] \\ \text{is a solution to the KKT conditions of} \\ \text{Problem } \mathbb{P}_7(\star, \zeta^\#(\star)). \end{array} \right\}$.

By checking the KKT conditions of $\mathbb{P}_5(\star)$ and $\mathbb{P}_7(\star, \zeta^\#(\star))$, we build the following relationship between

$\mathbb{P}_5(\star)$ and $\mathbb{P}_7(\star, \zeta)$:

$$\text{Statement } \mathcal{V}_{\mathbb{P}_5} \Leftrightarrow \left\{ \begin{array}{l} \text{Statement } \mathcal{V}_{\mathbb{P}_7} \text{ holds,} \\ \text{and } [\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), (\mathbf{f}^{\text{MS}})^\#(\star), \\ (\mathbf{f}^{\text{VU}})^\#(\star), T^\#(\star), \zeta^\#(\star)] \text{ satisfies} \\ \mathcal{S}_{2.1} \cup \mathcal{S}_{2.2} := \{(24), (29), (11a), (31e)\}. \end{array} \right\}. \quad (88)$$

In $\mathbb{P}_7(\star, \zeta)$, the optimizations of $[\mathbf{b}, \mathbf{p}]$, $[\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}]$ and T are independent and thus separable. This independence holds because ζ is already given for $\mathbb{P}_7(\star, \zeta)$. We do not have such independence in optimizing $\mathbb{P}_5(\star)$ where ζ is not decided yet. Hence, $\mathbb{P}_7(\star, \zeta)$ is equivalent to the combination of $\mathbb{P}_8(\star, \zeta)$, $\mathbb{P}_9(\star, \zeta)$, and $\mathbb{P}_{10}(\star, \zeta)$ defined below:

$$\begin{aligned} \text{Problem } \mathbb{P}_8(\star, \zeta) : \min_T & y c_e T - T \sum_{n \in \mathcal{N}} \zeta_n \\ \text{Problem } \mathbb{P}_9(\star, \zeta) : \min_{\mathbf{b}, \mathbf{p}} & \left\{ \begin{array}{l} -\mathcal{U}(\mathbf{b}, \mathbf{p}, \mathbf{s}) \\ + y c_e \cdot \sum_{n \in \mathcal{N}} \{ [(p_n + p_n^{\text{cir}}) s_n \mu_n \Lambda_n]^2 z_n \\ + \frac{1}{4(r_n(b_n, p_n) \nu_n)^2 z_n} \} \\ + \sum_{n \in \mathcal{N}} (\zeta_n \cdot t_n^{\text{Tx}}(b_n, p_n, s_n)) \end{array} \right\} \end{aligned} \quad (89)$$

s.t. (7a), (7b),

$$\text{Problem } \mathbb{P}_{10}(\star, \zeta) : \min_{\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}} \left\{ \begin{array}{l} y \cdot [c_e \cdot (\sum_{n \in \mathcal{N}} E_n^{\text{MS:Pro}}(s_n, f_n^{\text{MS}})) \\ + \sum_{n \in \mathcal{N}} E_n^{\text{VU:Pro}}(s_n, f_n^{\text{VU}})] \\ + \sum_{n \in \mathcal{N}} (\zeta_n \cdot [t_n^{\text{MS:Pro}}(s_n, f_n^{\text{MS}}) \\ + t_n^{\text{VU:Pro}}(s_n, f_n^{\text{VU}})]) \end{array} \right\} \quad (90)$$

s.t. (7d), (7e),

Then after defining

- Statement $\mathcal{V}_{\mathbb{P}_8}$: $\{ T^\#(\star) \text{ is a solution to Problem } \mathbb{P}_8(\star, \zeta^\#(\star)). \}$,
- Statement $\mathcal{V}_{\mathbb{P}_9}$: $\left\{ \begin{array}{l} [\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), \alpha^\#(\star), \beta^\#(\star)] \\ \text{is a solution to the KKT conditions of} \\ \text{Problem } \mathbb{P}_9(\star, \zeta^\#(\star)). \end{array} \right\}$, and
- Statement $\mathcal{V}_{\mathbb{P}_{10}}$: $\left\{ \begin{array}{l} (\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), \gamma^\#(\star), \delta^\#(\star) \\ \text{is a solution to the KKT conditions of} \\ \text{Problem } \mathbb{P}_{10}(\star, \zeta^\#(\star)). \end{array} \right\}$

we obtain

$$\begin{aligned} \text{Statement } \mathcal{V}_{\mathbb{P}_5} & \Leftrightarrow \left\{ \begin{array}{l} \text{Statements } \mathcal{V}_{\mathbb{P}_8}, \mathcal{V}_{\mathbb{P}_9}, \text{ and } \mathcal{V}_{\mathbb{P}_{10}} \text{ hold,} \\ \text{and } [\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), (\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), \\ T^\#(\star), \zeta^\#(\star)] \text{ satisfies} \\ \mathcal{S}_{2.1} \cup \mathcal{S}_{2.2} := \{(24), (29), (11a), (31e)\}. \end{array} \right\}, \\ & \Leftrightarrow \left\{ \begin{array}{l} \text{Statements } \mathcal{V}_{\mathbb{P}_9} \text{ and } \mathcal{V}_{\mathbb{P}_{10}} \text{ hold,} \\ \text{and } [\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), (\mathbf{f}^{\text{MS}})^\#(\star), (\mathbf{f}^{\text{VU}})^\#(\star), \\ T^\#(\star), \zeta^\#(\star)] \text{ satisfies} \\ \mathcal{S}_{2.1} \cup \mathcal{S}_{2.2} := \{(24), (29), (11a), (31e)\}. \end{array} \right\} \end{aligned} \quad (91)$$

where the last step means $\mathbb{P}_8(\star, \zeta)$ can be neglected since (24) induces the objective function of $\mathbb{P}_8(\star, \zeta)$ to be always 0.

Now we analyze Problem $\mathbb{P}_9(\star, \zeta)$. Note that β is the Lagrange multiplier for (7b). Suppose we already know β .

We move β and (7b) to the objective function, and construct the following problem:

Problem $\mathbb{P}_{11}(\star, \zeta, \beta)$:

$$\min_{\mathbf{b}, \mathbf{p}} \{H_{\mathbb{P}_9}(\mathbf{b}, \mathbf{p} \mid \star, \zeta) + \beta \cdot (\sum_{n \in \mathcal{N}} p_n - p_{\max})\} \quad (92)$$

$$\text{s.t. (7a),} \quad (93)$$

where $H_{\mathbb{P}_9}(\mathbf{b}, \mathbf{p} \mid \star, \zeta)$ denotes the objective function of Problem \mathbb{P}_9 . Then after defining

- Statement $\mathcal{V}_{\mathbb{P}_{11}}$:

$$\left\{ \begin{array}{l} [\mathbf{b}^\#(\star), \mathbf{p}^\#(\star), \alpha^\#(\star)] \\ \text{is a solution to the KKT conditions of} \\ \text{Problem } \mathbb{P}_{11}(\star, \zeta^\#(\star), \beta^\#(\star)). \end{array} \right\}$$

and checking the KKT conditions of $\mathbb{P}_9(\star, \zeta^\#(\star))$ and $\mathbb{P}_{11}(\star, \zeta^\#(\star), \beta^\#(\star))$, we have

$$\text{Statement } \mathcal{V}_{\mathbb{P}_9} \Leftrightarrow \left\{ \begin{array}{l} \text{Statement } \mathcal{V}_{\mathbb{P}_{11}} \text{ holds,} \\ \text{and } [\beta^\#(\star), \mathbf{p}^\#(\star)] \text{ satisfies} \\ \mathcal{S}_{1.2.2.2} := \{(26), (7b), (31b)\}. \end{array} \right\} \quad (94)$$

Now we analyze Problem $\mathbb{P}_{11}(\star, \zeta, \beta)$. Note that α is the Lagrange multiplier for (7a). Suppose we already know α . We move α and (7a) to the objective function, and construct the following problem:

Problem $\mathbb{P}_{12}(\star, \zeta, \beta, \alpha)$:

$$\min_{\mathbf{b}, \mathbf{p}} \{H_{\mathbb{P}_{11}}(\mathbf{b}, \mathbf{p} \mid \star, \zeta, \beta) + \alpha \cdot (\sum_{n \in \mathcal{N}} b_n - b_{\max})\}, \quad (95)$$

where $H_{\mathbb{P}_{11}}(\mathbf{b}, \mathbf{p} \mid \star, \zeta, \beta)$ denotes the objective function of Problem \mathbb{P}_{11} . Then after defining

- Statement $\mathcal{V}_{\mathbb{P}_{12}}$:
 $[\mathbf{b}^\#(\star), \mathbf{p}^\#(\star)]$ is a globally optimal solution to Problem $\mathbb{P}_{12}(\star, \zeta^\#(\star), \beta^\#(\star), \alpha^\#(\star))$,

and checking the KKT conditions of $\mathbb{P}_{11}(\star, \zeta^\#(\star), \beta^\#(\star))$ and $\mathbb{P}_{12}(\star, \zeta^\#(\star), \beta^\#(\star), \alpha^\#(\star))$, we get

$$\text{Statement } \mathcal{V}_{\mathbb{P}_{11}} \Leftrightarrow \left\{ \begin{array}{l} \text{Statement } \mathcal{V}_{\mathbb{P}_{12}} \text{ holds,} \\ \text{and } [\alpha^\#(\star), \mathbf{b}^\#(\star)] \text{ satisfies} \\ \mathcal{S}_{1.2.2.1} := \{(32), (33)\}. \end{array} \right\} \quad (96)$$

E. Proving that the left-hand side of (68) is non-increasing with respect to β

From the conditions of Proposition 1.2.1 and Proposition 1.2.2.1, setting $[\mathbf{b}, \mathbf{p}, \alpha]$ as $[\tilde{\mathbf{b}}(\tilde{\alpha}(\beta, \zeta \mid \star), \beta, \zeta \mid \star), \tilde{\mathbf{p}}(\tilde{\alpha}(\beta, \zeta \mid \star), \beta, \zeta \mid \star), \tilde{\alpha}(\beta, \zeta \mid \star)]$ satisfies $\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} = \{(20), (21), (32), (33)\}$; i.e., the KKT conditions of convex optimization $\mathbb{P}_{11}(\star, \zeta, \beta)$. Hence,

$$[\tilde{\mathbf{b}}(\tilde{\alpha}(\beta, \zeta \mid \star), \beta, \zeta \mid \star), \tilde{\mathbf{p}}(\tilde{\alpha}(\beta, \zeta \mid \star), \beta, \zeta \mid \star)] \quad (97)$$

is a globally optimal solution to $\mathbb{P}_{11}(\star, \zeta, \beta)$.

To prove the desired result, we consider the case where β equals β_1 , and the case where β equals β_2 , respectively, for arbitrarily chosen β_1 and β_2 . Due to Result (97) above, for $H_{\mathbb{P}_{11}}(\mathbf{b}, \mathbf{p} \mid \star, \zeta, \beta)$ denoting the objective function of Problem \mathbb{P}_{11} , we obtain

$$\begin{aligned} & H_{\mathbb{P}_{11}}(\tilde{\mathbf{b}}(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star), \tilde{\mathbf{p}}(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star \mid \star, \zeta, \beta_1)) \\ & \leq \\ & H_{\mathbb{P}_{11}}(\tilde{\mathbf{b}}(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star), \tilde{\mathbf{p}}(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star \mid \star, \zeta, \beta_1)), \end{aligned} \quad (98)$$

and

$$\begin{aligned} & H_{\mathbb{P}_{11}}(\tilde{\mathbf{b}}(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star), \tilde{\mathbf{p}}(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star \mid \star, \zeta, \beta_2)) \\ & \leq \\ & H_{\mathbb{P}_{11}}(\tilde{\mathbf{b}}(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star), \tilde{\mathbf{p}}(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star \mid \star, \zeta, \beta_2)). \end{aligned} \quad (99)$$

From (98) and (99), it follows that

$$\begin{aligned} & \left[\begin{array}{l} H_{\mathbb{P}_{11}}(\tilde{\mathbf{b}}(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star), \\ \tilde{\mathbf{p}}(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star \mid \star, \zeta, \beta_1)) \\ - H_{\mathbb{P}_{11}}(\tilde{\mathbf{b}}(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star), \\ \tilde{\mathbf{p}}(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star \mid \star, \zeta, \beta_2)) \end{array} \right] \\ & + \left[\begin{array}{l} H_{\mathbb{P}_{11}}(\tilde{\mathbf{b}}(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star), \\ \tilde{\mathbf{p}}(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star \mid \star, \zeta, \beta_2)) \\ - H_{\mathbb{P}_{11}}(\tilde{\mathbf{b}}(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star), \\ \tilde{\mathbf{p}}(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star \mid \star, \zeta, \beta_1)) \end{array} \right] \leq 0. \end{aligned} \quad (100)$$

Since $H_{\mathbb{P}_{11}}(\mathbf{b}, \mathbf{p}, \mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, T \mid \beta, \zeta, \mathbf{z}, \mathbf{y}, \mathbf{s})$ equals $H_{\mathbb{P}_9}(\mathbf{b}, \mathbf{p} \mid \star, \zeta) + \beta \cdot (\sum_{n \in \mathcal{N}} p_n - p_{\max})$ from (93), the term inside the first “[.]” of (100) equals $(\beta_1 - \beta_2) \cdot (\sum_{n \in \mathcal{N}} \tilde{p}_n(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star) - p_{\max})$, and the term inside the second “[.]” of (100) equals $(\beta_2 - \beta_1) \cdot (\sum_{n \in \mathcal{N}} \tilde{p}_n(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star) - p_{\max})$. Then we obtain

$$\begin{aligned} & \left\{ (\beta_1 - \beta_2) \cdot (\sum_{n \in \mathcal{N}} \tilde{p}_n(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star) - p_{\max}) \right\} \\ & \left\{ + (\beta_2 - \beta_1) \cdot (\sum_{n \in \mathcal{N}} \tilde{p}_n(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star) - p_{\max}) \right\} \\ & \leq 0; \end{aligned} \quad (101)$$

i.e., $(\beta_1 - \beta_2) \cdot (\sum_{n \in \mathcal{N}} \tilde{p}_n(\tilde{\alpha}(\beta_1, \zeta \mid \star), \beta_1, \zeta \mid \star) - \sum_{n \in \mathcal{N}} \tilde{p}_n(\tilde{\alpha}(\beta_2, \zeta \mid \star), \beta_2, \zeta \mid \star)) \leq 0$. Hence, $\sum_{n \in \mathcal{N}} \tilde{p}_n(\tilde{\alpha}(\beta, \zeta \mid \star), \beta, \zeta \mid \star)$, i.e., the left-hand side of (68), is non-increasing as β increases. \square

F. Proving that the left-hand side of (66) is non-increasing with respect to α

From Proposition 1.2.1’s condition, setting $[\mathbf{b}, \mathbf{p}]$ as $[\tilde{\mathbf{b}}(\alpha, \beta, \zeta \mid \star), \tilde{\mathbf{p}}(\alpha, \beta, \zeta \mid \star)]$ satisfies $\mathcal{S}_{1.2.1} = \{(20), (21)\}$; i.e., the KKT conditions of convex optimization $\mathbb{P}_{12}(\star, \zeta, \beta, \alpha)$. Hence,

$$[\tilde{\mathbf{b}}(\alpha, \beta, \zeta \mid \star), \tilde{\mathbf{p}}(\alpha, \beta, \zeta \mid \star)] \text{ is a globally optimal solution to } \mathbb{P}_{12}(\star, \zeta, \beta, \alpha). \quad (102)$$

To prove the desired result, we consider the case where α equals α_1 , and the case where α equals α_2 , respectively, for arbitrarily chosen α_1 and α_2 . Due to Result (102) above, for $H_{\mathbb{P}_{12}}(\mathbf{b}, \mathbf{p} \mid \star, \zeta, \beta, \alpha)$ denoting the objective function of Problem \mathbb{P}_{12} , we obtain

$$\begin{aligned} & H_{\mathbb{P}_{12}}(\tilde{\mathbf{b}}(\alpha_1, \beta, \zeta \mid \star), \tilde{\mathbf{p}}(\alpha_1, \beta, \zeta \mid \star) \mid \star, \zeta, \beta, \alpha_1) \leq \\ & H_{\mathbb{P}_{12}}(\tilde{\mathbf{b}}(\alpha_2, \beta, \zeta \mid \star), \tilde{\mathbf{p}}(\alpha_2, \beta, \zeta \mid \star) \mid \star, \zeta, \beta, \alpha_1), \end{aligned} \quad (103)$$

and

$$\begin{aligned} & H_{\mathbb{P}_{12}}(\tilde{\mathbf{b}}(\alpha_2, \beta, \zeta \mid \star), \tilde{\mathbf{p}}(\alpha_2, \beta, \zeta \mid \star) \mid \star, \zeta, \beta, \alpha_2) \leq \\ & H_{\mathbb{P}_{12}}(\tilde{\mathbf{b}}(\alpha_1, \beta, \zeta \mid \star), \tilde{\mathbf{p}}(\alpha_1, \beta, \zeta \mid \star) \mid \star, \zeta, \beta, \alpha_2). \end{aligned} \quad (104)$$

From (103) and (104), it follows that

$$\begin{aligned} & [H_{\mathbb{P}_{12}}(\tilde{\mathbf{b}}(\alpha_1, \beta, \zeta | \star), \tilde{\mathbf{p}}(\alpha_1, \beta, \zeta | \star) | \star, \zeta, \beta, \alpha_1) \\ & - H_{\mathbb{P}_{12}}(\tilde{\mathbf{b}}(\alpha_1, \beta, \zeta | \star), \tilde{\mathbf{p}}(\alpha_1, \beta, \zeta | \star) | \star, \zeta, \beta, \alpha_2)] \\ & + [H_{\mathbb{P}_{12}}(\tilde{\mathbf{b}}(\alpha_2, \beta, \zeta | \star), \tilde{\mathbf{p}}(\alpha_2, \beta, \zeta | \star) | \star, \zeta, \beta, \alpha_2) \\ & - H_{\mathbb{P}_{12}}(\tilde{\mathbf{b}}(\alpha_2, \beta, \zeta | \star), \tilde{\mathbf{p}}(\alpha_2, \beta, \zeta | \star) | \star, \zeta, \beta, \alpha_1)] \\ & \leq 0. \end{aligned} \quad (105)$$

Since $H_{\mathbb{P}_{12}}(\mathbf{b}, \mathbf{p} | \star, \zeta, \beta, \alpha)$ equals $H_{\mathbb{P}_{11}}(\mathbf{b}, \mathbf{p} | \star, \zeta, \beta) + \alpha \cdot (\sum_{n \in \mathcal{N}} b_n - b_{\max})$ from (95), the term inside the first “[.]” of (105) equals $(\alpha_1 - \alpha_2) \cdot (\sum_{n \in \mathcal{N}} \tilde{b}_n(\alpha_1, \beta, \zeta | \star) - b_{\max})$, and the term inside the second “[.]” of (105) equals $(\alpha_2 - \alpha_1) \cdot (\sum_{n \in \mathcal{N}} \tilde{b}_n(\alpha_2, \beta, \zeta | \star) - b_{\max})$. Then we obtain

$$\begin{aligned} & (\alpha_1 - \alpha_2) \cdot \left(\sum_{n \in \mathcal{N}} \tilde{b}_n(\alpha_1, \beta, \zeta | \star) - b_{\max} \right) \\ & + (\alpha_2 - \alpha_1) \cdot \left(\sum_{n \in \mathcal{N}} \tilde{b}_n(\alpha_2, \beta, \zeta | \star) - b_{\max} \right) \leq 0; \end{aligned} \quad (106)$$

i.e., $(\alpha_1 - \alpha_2) \cdot (\sum_{n \in \mathcal{N}} \tilde{b}_n(\alpha_1, \beta, \zeta | \star) - \sum_{n \in \mathcal{N}} \tilde{b}_n(\alpha_2, \beta, \zeta | \star)) \leq 0$. Hence, $\sum_{n \in \mathcal{N}} \tilde{b}_n(\alpha, \beta, \zeta | \star)$, i.e., the left-hand side of (66), is non-increasing as α increases. \square

G. Proving Page 13’s Lemma 2

From Lemma 3 to be presented in Appendix I, $t_n(\hat{b}_n(\zeta | \star), \hat{p}_n(\zeta | \star), s_n, \hat{f}_n^{\text{MS}}(\zeta | \star), \hat{f}_n^{\text{VU}}(\zeta | \star))$ is non-increasing as ζ_n increases. For $h_n(\zeta | T)$ defined in (71), given $[\zeta_1, \dots, \zeta_{n-1}, \zeta_{n+1}, \dots, \zeta_N]$ and T , we either have (71a) or (71b). In either case, “ $t_n(\hat{b}_n(\zeta | \star), \hat{p}_n(\zeta | \star), s_n, \hat{f}_n^{\text{MS}}(\zeta | \star), \hat{f}_n^{\text{VU}}(\zeta | \star)) - T$ ” or “ $-\zeta_n$ ” defined for $h_n(\zeta | T)$ is non-increasing as ζ_n increases. Hence, $h_n(\zeta | T)$ is non-increasing as ζ_n increases, given $[\zeta_1, \dots, \zeta_{n-1}, \zeta_{n+1}, \dots, \zeta_N]$ and T . \square

H. Proving that the left-hand side of (79) is non-increasing with respect to T

We recall from (72) and (73) that

setting ζ as $\hat{\zeta}(T | \star)$ ensures $h_n(\zeta | T) = 0$ for any $n \in \mathcal{N}$, (107)

where $h_n(\zeta | T) = 0$ is defined in (71).

From Lemma 3 to be presented in Appendix I below, we can prove that $\sum_{n \in \mathcal{N}} \hat{\zeta}_n(T | \star)$; i.e., the left-hand side of (79) is non-increasing with respect to T . The proof is similar to those in Appendices E and F. \square

I. Lemma 3 and its proof

Lemma 3. Given “ \star ” (i.e., $[z, y, s]$) and $[\zeta_1, \dots, \zeta_{n-1}, \zeta_{n+1}, \dots, \zeta_N]$, we have: given $[\zeta_1, \dots, \zeta_{n-1}, \zeta_{n+1}, \dots, \zeta_N]$, and “ \star ” (i.e., “ z, y, s ”), then as ζ_n increases,

- i) $t_n^{\text{Tx}}(\hat{b}_n(\zeta | \star), \hat{p}_n(\zeta | \star), s_n)$ is non-increasing;
- ii) $t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta | \star)) + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta | \star))$ is non-increasing; and
- iii) $t_n(\hat{b}_n(\zeta | \star), \hat{p}_n(\zeta | \star), s_n, \hat{f}_n^{\text{MS}}(\zeta | \star), \hat{f}_n^{\text{VU}}(\zeta | \star))$ is non-increasing.

Proof of Lemma 3:

Below we prove Results “i)”, “ii)”, and “iii)”, respectively.

Proving Lemma 3’s Result “i)”:

From Proposition 1.2’s condition, setting $[\mathbf{b}, \mathbf{p}, \alpha, \beta]$ as $[\hat{\mathbf{b}}(\zeta | \star), \hat{\mathbf{p}}(\zeta | \star), \hat{\alpha}(\zeta | \star), \hat{\beta}(\zeta | \star)]$ satisfies $\mathcal{S}_{1.2.1} \cup \mathcal{S}_{1.2.2.1} \cup \mathcal{S}_{1.2.2.2} = \{(20), (21), (26), (7b), (31b), (32), (33)\}$; i.e., the KKT conditions of convex optimization $\mathbb{P}_9(\star, \zeta)$. Hence,

$$[\hat{\mathbf{b}}(\zeta | \star), \hat{\mathbf{p}}(\zeta | \star)] \text{ is a globally optimal solution to } \mathbb{P}_9(\star, \zeta). \quad (108)$$

To prove the desired result, we consider the case where ζ_n equals $\zeta_n^{(1)}$, and the case where ζ_n equals $\zeta_n^{(2)}$, respectively, for arbitrarily chosen $\zeta_n^{(1)}$ and $\zeta_n^{(2)}$. Due to Result (108) above, after defining

$$\begin{aligned} \zeta^{(n,1)} & := [\zeta_1, \dots, \zeta_{n-1}, \zeta_n^{(1)}, \zeta_{n+1}, \dots, \zeta_N], \\ \zeta^{(n,2)} & := [\zeta_1, \dots, \zeta_{n-1}, \zeta_n^{(2)}, \zeta_{n+1}, \dots, \zeta_N], \end{aligned}$$

then with $H_{\mathbb{P}_9}(\mathbf{b}, \mathbf{p} | \star, \zeta)$ denoting the objective function of Problem \mathbb{P}_9 , we obtain

$$\begin{aligned} & H_{\mathbb{P}_9}(\hat{\mathbf{b}}(\zeta^{(n,1)} | \star), \hat{\mathbf{p}}(\zeta^{(n,1)} | \star) | \star, \zeta^{(n,1)}) \\ & \leq H_{\mathbb{P}_9}(\hat{\mathbf{b}}(\zeta^{(n,1)} | \star), \hat{\mathbf{p}}(\zeta^{(n,1)} | \star) | \star, \zeta^{(n,2)}), \end{aligned} \quad (109)$$

and

$$\begin{aligned} & H_{\mathbb{P}_9}(\hat{\mathbf{b}}(\zeta^{(n,2)} | \star), \hat{\mathbf{p}}(\zeta^{(n,2)} | \star) | \star, \zeta^{(n,2)}) \\ & \leq H_{\mathbb{P}_9}(\hat{\mathbf{b}}(\zeta^{(n,2)} | \star), \hat{\mathbf{p}}(\zeta^{(n,2)} | \star) | \star, \zeta^{(n,1)}). \end{aligned} \quad (110)$$

From (109) and (110), it follows that

$$\begin{aligned} & [H_{\mathbb{P}_9}(\hat{\mathbf{b}}(\zeta^{(n,1)} | \star), \hat{\mathbf{p}}(\zeta^{(n,1)} | \star) | \star, \zeta^{(n,1)}) \\ & - H_{\mathbb{P}_9}(\hat{\mathbf{b}}(\zeta^{(n,1)} | \star), \hat{\mathbf{p}}(\zeta^{(n,1)} | \star) | \star, \zeta^{(n,2)})] \\ & + [H_{\mathbb{P}_9}(\hat{\mathbf{b}}(\zeta^{(n,2)} | \star), \hat{\mathbf{p}}(\zeta^{(n,2)} | \star) | \star, \zeta^{(n,2)}) \\ & - H_{\mathbb{P}_9}(\hat{\mathbf{b}}(\zeta^{(n,2)} | \star), \hat{\mathbf{p}}(\zeta^{(n,2)} | \star) | \star, \zeta^{(n,1)})] \\ & \leq 0. \end{aligned} \quad (111)$$

Note that $H_{\mathbb{P}_9}(\mathbf{b}, \mathbf{p} | \star, \zeta)$ is given by (89). Then the term inside the first “[.]” of (111) equals $(\zeta_n^{(1)} - \zeta_n^{(2)}) \cdot t_n^{\text{Tx}}(\hat{b}_n(\zeta^{(n,1)} | \star), \hat{p}_n(\zeta^{(n,1)} | \star), s_n)$, and the term inside the second “[.]” of (111) equals $(\zeta_n^{(2)} - \zeta_n^{(1)}) \cdot t_n^{\text{Tx}}(\hat{b}_n(\zeta^{(n,2)} | \star), \hat{p}_n(\zeta^{(n,2)} | \star), s_n)$. Then we obtain

$$\begin{aligned} & (\zeta_n^{(1)} - \zeta_n^{(2)}) \cdot t_n^{\text{Tx}}(\hat{b}_n(\zeta^{(n,1)} | \star), \hat{p}_n(\zeta^{(n,1)} | \star), s_n) \\ & + (\zeta_n^{(2)} - \zeta_n^{(1)}) \cdot t_n^{\text{Tx}}(\hat{b}_n(\zeta^{(n,2)} | \star), \hat{p}_n(\zeta^{(n,2)} | \star), s_n) \leq 0; \end{aligned} \quad (112)$$

i.e., $(\zeta_n^{(1)} - \zeta_n^{(2)}) \cdot (t_n^{\text{Tx}}(\hat{b}_n(\zeta^{(n,1)} | \star), \hat{p}_n(\zeta^{(n,1)} | \star), s_n) - t_n^{\text{Tx}}(\hat{b}_n(\zeta^{(n,2)} | \star), \hat{p}_n(\zeta^{(n,2)} | \star), s_n)) \leq 0$. Hence, $t_n^{\text{Tx}}(\hat{b}_n(\zeta | \star), \hat{p}_n(\zeta | \star), s_n)$ is non-increasing as ζ_n increases.

Proving Lemma 3’s Result “ii)”:

From Proposition 1.1’s condition, setting $[\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}}, \gamma, \delta]$ as $[\hat{\mathbf{f}}^{\text{MS}}(\zeta | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta | \star), \hat{\gamma}(\zeta | \star), \hat{\delta}(\zeta | \star)]$ satisfies $\mathcal{S}_{1.1} := \{(22), (23), (27), (28), (7d), (7e), (31c), (31d)\}$; i.e., the KKT conditions of convex optimization $\mathbb{P}_{10}(\star, \zeta)$. Hence,

$$[\hat{\mathbf{f}}^{\text{MS}}(\zeta | \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta | \star)] \text{ is a globally optimal solution to } \mathbb{P}_{10}(\star, \zeta). \quad (113)$$

To prove the desired result, we consider the case where ζ_n equals $\zeta_n^{(1)}$, and the case where ζ_n equals $\zeta_n^{(2)}$, respectively, for arbitrarily chosen $\zeta_n^{(1)}$ and $\zeta_n^{(2)}$. Due to Result (113) above,

after defining

$$\begin{aligned}\zeta^{(n,1)} &:= [\zeta_1, \dots, \zeta_{n-1}, \zeta_n^{(1)}, \zeta_{n+1}, \dots, \zeta_N], \\ \zeta^{(n,2)} &:= [\zeta_1, \dots, \zeta_{n-1}, \zeta_n^{(2)}, \zeta_{n+1}, \dots, \zeta_N],\end{aligned}$$

then with $H_{\mathbb{P}_{10}}(\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}} \mid \star, \zeta)$ denoting the objective function of Problem \mathbb{P}_{10} , we obtain

$$\begin{aligned}H_{\mathbb{P}_{10}}(\hat{\mathbf{f}}^{\text{MS}}(\zeta^{(n,1)} \mid \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{(n,1)} \mid \star) \mid \star, \zeta^{(n,1)}) &\leq \\ H_{\mathbb{P}_{10}}(\hat{\mathbf{f}}^{\text{MS}}(\zeta^{(n,1)} \mid \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{(n,1)} \mid \star) \mid \star, \zeta^{(n,2)}),\end{aligned}\quad (114)$$

and

$$\begin{aligned}H_{\mathbb{P}_{10}}(\hat{\mathbf{f}}^{\text{MS}}(\zeta^{(n,2)} \mid \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{(n,2)} \mid \star) \mid \star, \zeta^{(n,2)}) &\leq \\ H_{\mathbb{P}_{10}}(\hat{\mathbf{f}}^{\text{MS}}(\zeta^{(n,2)} \mid \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{(n,2)} \mid \star) \mid \star, \zeta^{(n,1)}).\end{aligned}\quad (115)$$

From (114) and (115), it follows that

$$\begin{aligned}&[H_{\mathbb{P}_{10}}(\hat{\mathbf{f}}^{\text{MS}}(\zeta^{(n,1)} \mid \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{(n,1)} \mid \star) \mid \star, \zeta^{(n,1)}) \\ &- H_{\mathbb{P}_{10}}(\hat{\mathbf{f}}^{\text{MS}}(\zeta^{(n,1)} \mid \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{(n,1)} \mid \star) \mid \star, \zeta^{(n,2)})] \\ &+ [H_{\mathbb{P}_{10}}(\hat{\mathbf{f}}^{\text{MS}}(\zeta^{(n,2)} \mid \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{(n,2)} \mid \star) \mid \star, \zeta^{(n,2)}) \\ &- H_{\mathbb{P}_{10}}(\hat{\mathbf{f}}^{\text{MS}}(\zeta^{(n,2)} \mid \star), \hat{\mathbf{f}}^{\text{VU}}(\zeta^{(n,2)} \mid \star) \mid \star, \zeta^{(n,1)})] \\ &\leq 0.\end{aligned}\quad (116)$$

Note that $H_{\mathbb{P}_{10}}(\mathbf{f}^{\text{MS}}, \mathbf{f}^{\text{VU}} \mid \star, \zeta)$ is given by (90). Then the term inside the first “[.]” of (116) equals $(\zeta_n^{(1)} - \zeta_n^{(2)}) \cdot [t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta^{(n,1)} \mid \star)) + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta^{(n,1)} \mid \star))]$, and the term inside the second “[.]” of (116) equals $(\zeta_n^{(2)} - \zeta_n^{(1)}) \cdot [t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta^{(n,2)} \mid \star)) + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta^{(n,2)} \mid \star))]$. Then we obtain

$$\left\{ \begin{array}{l} (\zeta_n^{(1)} - \zeta_n^{(2)}) \cdot [t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta^{(n,1)} \mid \star)) \\ + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta^{(n,1)} \mid \star))] \\ + (\zeta_n^{(2)} - \zeta_n^{(1)}) \cdot [t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta^{(n,2)} \mid \star)) \\ + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta^{(n,2)} \mid \star))] \end{array} \right\} \leq 0;\quad (117)$$

$$\text{namely, } (\zeta_n^{(1)} - \zeta_n^{(2)}) \cdot \left\{ \begin{array}{l} [t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta^{(n,1)} \mid \star)) \\ + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta^{(n,1)} \mid \star))] \\ - [t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta^{(n,2)} \mid \star)) \\ + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta^{(n,2)} \mid \star))] \end{array} \right\} \leq 0.$$

Therefore, $t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta \mid \star)) + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta \mid \star))$ is non-increasing as ζ_n increases.

Proving Lemma 3’s Result “iii”: From (3), $t_n(\hat{b}_n(\zeta \mid \star), \hat{p}_n(\zeta \mid \star), s_n, \hat{f}_n^{\text{MS}}(\zeta \mid \star), \hat{f}_n^{\text{VU}}(\zeta \mid \star))$ is the sum of $t_n^{\text{Tx}}(\hat{b}_n(\zeta \mid \star), \hat{p}_n(\zeta \mid \star), s_n)$ and $t_n^{\text{MS:Pro}}(s_n, \hat{f}_n^{\text{MS}}(\zeta \mid \star)) + t_n^{\text{VU:Pro}}(s_n, \hat{f}_n^{\text{VU}}(\zeta \mid \star))$. Then the desired result clearly follows from Lemma 3’s Results “i)” and “ii)”. \square