Phase and TV Based Convex Sets for Blind Deconvolution of Microscopic Images

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Abstract—In this article, two closed and convex sets for blind deconvolution problem are proposed. Most blurring functions in microscopy are symmetric with respect to the origin. Therefore, they do not modify the phase of the Fourier transform (FT) of the original image. As a result blurred image and the original image have the same FT phase. Therefore, the set of images with a prescribed FT phase can be used as a constraint set in blind deconvolution problems. Another convex set that can be used during the image reconstruction process is the epigraph set of Total Variation (TV) function. This set does not need a prescribed upper bound on the total variation of the image. The upper bound is automatically adjusted according to the current image of the restoration process. Both of these two closed and convex sets can be used as a part of any blind deconvolution algorithm. Simulation examples are presented.

Index Terms—Projection onto Convex Sets, Blind Deconvolution, Inverse Problems, Epigraph Sets

I. INTRODUCTION

A wide range of deconvolution algorithms has been developed to remove blur in microscopic images in recent years [1]–[15]. In this article, two new convex sets are introduced for blind deconvolution algorithms. Both sets can be incorporated to any iterative deconvolution and/or blind deconvolution method.

One of the sets is based on the phase of the Fourier transform (FT) of the observed image. Most point spread functions blurring microscopic images are symmetric with respect to origin. Therefore, Fourier transform of such functions do not have any phase. As a result, FT phase of the original image and the blurred image have the same phase. The set of images with a prescribed phase is a closed and convex set and projection onto this convex set is easy to perform in Fourier domain.

The second set in the Epigraph Set of Total Variation (ESTV) function. Total variation (TV) value of an image can be limited by an upper-bound to stabilize the restoration process. In fact, such sets were used by many researchers in inverse problems [13], [16]–[20]. In this paper, the epigraph of the TV function will be used to automatically estimate an upper-bound on the TV value of a given image. This set is also a closed and convex set. Projection onto ESTV function can be also implemented effectively. ESTV can be incorporated into any iterative blind deconvolution algorithm.

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Image reconstruction from Fourier transform phase information was first considered in 1980's [21]–[24] and total variation based image denoising was introduced in 1990's [25]. However, FT phase information and ESTV have not been used in blind deconvolution problem to the best of our knowledge.

The paper is organized as follow. In Section II, we review image reconstruction problem from th FT phase and describe the convex set based on phase. In Section III, we describe the epigraph set of TV function.

II. CONVEX SET BASED ON THE PHASE OF FOURIER TRANSFORM

In this section, we introduce our notation and describe how the phase of Fourier transform can be used in deconvolution problems.

Let $x_o[n_1, n_2]$ be the original image and $h[n_1, n_2]$ be the point spread function. The observed image y is obtained by the convolution of h with x:

$$y[n_1, n_2] = h[n_1, n_2] * x_o[n_1, n_2], \tag{1}$$

where * represents the two-dimensional convolution operation. Discrete-time Fourier transform Y of y is, therefore, given by

$$Y(w_1, w_2) = H(w_1, w_2) X_o(w_1, w_2).$$
(2)

When $h[n_1, n_2]$ is symmetric with respect to origin (0,0)) phase of $H(w_1,w_2)$ is zero, $(h[n_1, n_2])$ = i.e., our assumption is $H(w_1, w_2)$ $|H(w_1, w_2)|.$ = Point spread functions satisfying this assumption includes uniform Gaussian blurs. Therefore, phase of $Y(w_1, w_2)$ $= |Y(w_1, w_2)| exp(j \measuredangle Y(w_1, w_2))$ and $X_o(w_1, w_2) = |X_0(w_1, w_2)| exp(j \measuredangle X_o(w_1, w_2))$ are the same:

$$\measuredangle Y(w_1, w_2) = \measuredangle X_o(w_1, w_2), \tag{3}$$

for all (w_1, w_2) values. Based on the above observation the following set can be defined:

$$C_{\phi} = \{x[n_1, n_2] \mid \measuredangle X(w_1, w_2) = \measuredangle X_o(w_1, w_2)\}, \quad (4)$$

which is the set of images whose FT phase is equal to a given prescribed phase $\angle X_o(w_1, w_2)$.

It can easily be shown that this set is closed and convex in $\mathbb{R}^{N_1} \times \mathbb{R}^{N_2}$, for images of size $N_1 \times N_2$.

Projection of an arbitrary image x onto C_{ϕ} is implemented in Fourier domain. Let the FT of x be $X(w_1, w_2) = |X(w_1, w_2)|e^{j\phi(w_1, w_2)}$. The FT X_p of its projection x_p is obtained as follows:

$$X_p(w_1, w_2) = |X(w_1, w_2)| e^{j \angle X_o(w_1, w_2)},$$
(5)

where the magnitude of $X_p(w_1, w_2)$ is the same as the magnitude of $X(w_1, w_2)$ but its phase is replaced by the prescribed phase function $\angle X_o(w_1, w_2)$. After this step, $x_p[n_1, n_2]$ is obtained using the inverse FT. The above operation is implemented using the FFT and implementation details are described in Section IV.

Obviously, projection of y onto the set C_{ϕ} is the same as itself. Therefore, the iterative blind deconvolution algorithm should not start with the observed image. Image reconstruction from phase (IRP) has been extensively studied by Oppenheim and his coworkers [21]–[24]. IRP problem is a robust inverse problem. In Figure 1, phase only version of the well-known Lena image is shown. The phase only image is obtained as follows:

$$v = \mathcal{F}^{-1}[Ce^{\phi(w_1, w_2)}]$$
(6)

where \mathcal{F}^{-1} . represents the inverse Fourier transform, C is a constant and $\phi(w_1, w_2)$ is the phase of Lena image. Edges of the original image are clearly observable in the phase only image. Therefore, the set C_{ϕ} contains the crucial edge information of the original image x_o .

When the support of x_o is known it is possible to reconstruct the original image from its phase within a scale factor. Oppenheim and coworkers developed Papoulis-Gerchberg type iterative algorithms from a given phase information. In [23] support and phase information are imposed on iterates in space and Fourier domains in a successive manner to reconstruct an image from its phase.

In blind deconvolution problem the support regions of x_o and y are different from each other. Exact support of the original image is not precisely known; therefore, C_{ϕ} is not sufficient by itself to solve the blind deconvolution problem. However, it can be used as a part of any iterative blind deconvolution method.

When there is observation noise, Eq. (1) becomes:

$$\mathbf{y}_o = \mathbf{y} + \boldsymbol{\nu},\tag{7}$$

where ν represents the additive noise. In this case, phase of the observed image is obviously different from the phase of the original image. Luckily, phase information is robust to noise as shown in Fig. 1c which is obtained from a noisy version of Lena image. In spite of noise, edges of Lena are clearly visible in the phase only image. Gaussian noise with variance $\sigma = 30$ is added to Lena image in Fig. 1a.

FTs of some symmetric point spread function may take negative values for some (w_1, w_2) values. In such (w_1, w_2) values, phase of the observed image $Y(w_1, w_2)$ differs from $X(w_1, w_2)$ by π . Therefore, phase of $Y(w_1, w_2)$ should be corrected as in phase unwrapping algorithms. Or some of the (w_1, w_2) values around $(w_1, w_2) = (0, 0)$ can be used during the image reconstruction process. It is possible to estimate the main lobe of the FT of the point spread function from the observed image. Phase of FT coefficients within the main lobe are not effected by a shift of π .

In this article, the set C_{ϕ} will be used as a part of the iterative blind deconvolution scheme developed by Dainty *et al* and together with the epigraph set of total variation function which will be introduced in the next section.



(a)



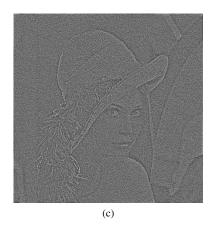


Fig. 1: (a) noisy "Lena" image, (b) Phase only version of the "Lena" image, and (c) phase only version of the noisy "Lena" image.

III. EPIGRAPH SET OF TOTAL VARIATION FUNCTION

Bounded total variation is widely used in various image denoising and related applications [16], [17], [26]–[29]. The set C_{TV} of images whose TV values is bounded by a prescribed number ϵ is defined as follows:

$$C_{\rm TV} = \{ \mathbf{x} : {\rm TV}(\mathbf{x}) \le \epsilon \},\tag{8}$$

where TV of an image is defined, in this paper, as follows:

$$TV(\mathbf{x}) = \sum_{i,j=1}^{M} |x^{i+1,j} - x^{i,j}| + \sum_{i,j=1}^{M} |x^{i,j+1} - x^{i,j}|.$$
 (9)

This set is closed and convex set in $\mathbb{R}^{N_1 \times N_2}$. Set C_{TV} can be used in blind deconvolution problems. But the upper bound ϵ has to be determined somehow a priori.

In this article we increase the dimension of the space by 1 and consider the problem in $\mathbb{R}^{N_1 \times N_2 + 1}$. We define the epigraph set of the TV function:

$$\mathbf{C}_{ESTV} = \{ \underline{\mathbf{x}} = [x^T \ z]^T \mid \mathsf{TV}(\mathbf{x}) \le z \}, \tag{10}$$

where T is the transpose operation and we use bold face letters for N dimensional vectors and underlined bold face letters for N + 1 dimensional vectors, respectively.

The concept of the epigraph set is graphically illustrated in Fig. 2. Since $TV(\mathbf{x})$ is a convex function in $\mathbb{R}^{N_1 \times N_2}$ set the C_{ESTV} is closed and convex in $\mathbb{R}^{N_1 \times N_2+1}$. In Eq. (10) one does not need to specify a prescribed upper bound on TV of an image. An orthogonal projection onto the set C_{ESTV} reduces the total variation value of the image as graphically illustrated in Fig. 2 because of the convex nature of the TV function. Let \mathbf{v} be an $N = N_1 \times N_2$ dimensional image to be projected onto the set C_{ESTV} . In orthogonal projection operation, we select the nearest vector \mathbf{x}^* on the set C_{ESTV} to \mathbf{w} . The projection vector \mathbf{x}^* of an image \mathbf{v} is defined as:

$$\underline{\mathbf{w}}^{\star} = \arg \min_{\underline{\mathbf{w}} \in C_{\text{ESTV}}} \|\underline{\mathbf{v}} - \underline{\mathbf{w}}\|^2, \tag{11}$$

where $\underline{\mathbf{v}} = [\mathbf{v}^T \ 0]$. The projection operation described in (11) is equivalent to:

$$\underline{\mathbf{w}}^{\star} = \begin{bmatrix} \mathbf{w}_p \\ \mathrm{TV}(\mathbf{w}_p) \end{bmatrix} = \arg\min_{\underline{\mathbf{w}}\in\mathrm{C}_{\mathrm{f}}} \| \begin{bmatrix} \mathbf{v} \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbf{w} \\ \mathrm{TV}(\mathbf{w}) \end{bmatrix} \|, \quad (12)$$

where $\underline{\mathbf{w}}^{\star} = [\mathbf{w}_p^T, \text{TV}(\mathbf{w}_p)]$ is the projection of $(\mathbf{v}, 0)$ onto the epigraph set. The projection $\underline{\mathbf{w}}^{\star}$ must be on the boundary of the epigraph set. Therefore, the projection must be on the form $[\mathbf{w}_p^T, \text{TV}(\mathbf{w}_p)]$. Equation (12) becomes:

$$\underline{\mathbf{w}}^{\star} = \begin{bmatrix} \mathbf{w}_p \\ \mathrm{TV}(\mathbf{w}_p) \end{bmatrix} = \arg\min_{\underline{\mathbf{w}}\in\mathrm{C}_\mathrm{f}} \|\mathbf{v} - \mathbf{w}\|_2^2 + \mathrm{TV}(\mathbf{w})^2.$$
(13)

It is also possible to use $\lambda TV(.)$ as a the convex cost function and Eq. 13 becomes:

$$\underline{\mathbf{w}}^{\star} = \begin{bmatrix} \mathbf{w}_p \\ \mathrm{TV}(\mathbf{w}_p) \end{bmatrix} = \arg\min_{\underline{\mathbf{w}}\in\mathrm{C}_{\mathrm{f}}} \|\mathbf{v} - \mathbf{w}\|_2^2 + \lambda^2 \mathrm{TV}(\mathbf{w})^2. \quad (14)$$

The solution of (11) can be obtained using the method that we discussed in [28], [30]. The solution is obtained in an iterative manner and the key step in each iteration is an orthogonal projection onto a supporting hyperplane of the set C_{ESTV} .

In current TV based denoising methods [17], [27] the following cost function is used:

$$\min \|\mathbf{v} - \mathbf{w}\|_2^2 + \lambda \mathrm{TV}(\mathbf{w}). \tag{15}$$

However, we were not able to prove that 15 corresponds to a non-expensive map or not. On the other hand, minimization problem in Eq. (13) and (14) are the results of projection onto convex sets, as a result they correspond to non-expensive maps [5], [16], [26], [31], [31]–[37]. Therefore, they can be incorporated into any iterative deblurring algorithm without effecting the convergence of the algorithm.

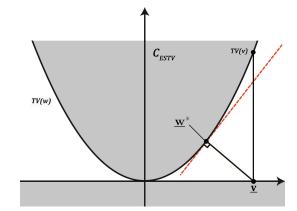


Fig. 2: Graphical representation of the orthogonal projection onto the set C_{ESTV} defined in (11). The observation vector $\mathbf{y} = [v^T \ 0]^T$ is projected onto the set C_{ESTV} , which is the epigraph set of TV function

IV. How to incorporate C_{ESTV} and C_{ϕ} into a deblurring method

One of the earliest blind deconvolution methods is the iterative space-Fourier domain method developed by Ayers and Dainty [38]. In this approach, iterations start with a $x_o[n] = x_o[n_1, n_2]$, where we introduce a new notation to specify equations $[n] = [n_1, n_2]$. For example, we rewrite Eq. (1) as follows:

$$y[n] = h[n] * x_o[n],$$
 (16)

The method successively updates h[n] and x[n] in a Wiener filter-like equation. Here is the i^{th} step of the algorithm:

- 1) Compute $\hat{X}_i(w) = \mathcal{F}\{x_i[n]\}\)$, where \mathcal{F} represents the FT operation and $w = (w_1, w_2)$, with some abuse of notation.
- 2) Estimate the point-spread filter response using the following equation

$$\tilde{H}_i(w) = \frac{Y(w) \tilde{X}_i^*(w)}{|\hat{X}_i(w)|^2 + \alpha/|\hat{H}_i(w)|^2},$$
(17)

where α is a small real number.

- 3) Compute $\tilde{h}_i[n] = \mathcal{F}^{-1}{\{\tilde{H}_i(w)\}}$
- 4) Impose the positivity constraint and finite support constraints on $\tilde{h}_i[n]$. Let the output of this step be $\hat{h}_i[n]$.
- 5) Compute $\hat{H}_i(w) = \mathcal{F}\{\hat{h}_i[n]\}$
- 6) Update the image

$$\tilde{X}_{i}(w) = \frac{Y(w)\tilde{H}_{i}^{*}(w)}{|\hat{H}_{i}(w)|^{2} + \alpha/|\hat{X}_{i}(w)|^{2}},$$
(18)

- 7) Compute $\hat{x}_i[n] = \mathcal{F}^{-1}\{\hat{X}_i(w)\}$
- 8) Impose spatial domain positivity and finite support constraint on $\hat{x}_i[n]$ to produce the next iterate $\hat{x}_{i+1}[n]$.

Iterations are stopped when there is no significant change between successive iterates. We can easily modify this algorithm using the convex sets defined in Section II and III. We introduce step 6-a as follows:

6-a) Impose the phase information

$$\bar{X}_i(w) = |\tilde{X}_i(w)| e^{j \measuredangle Y(w)}, \tag{19}$$

where $\measuredangle Y(w)$ is the phase of Y(w). This step is the projection onto the set C_{ϕ} . As a result step 7 becomes $\tilde{x}_i[n] = \mathcal{F}^{-1}\{\bar{X}_i(w)\}$. We also introduce a new step to Ayers and Dainty's algorithm as follows: Project $\tilde{x}_i[n]$ onto the set C_{ESTV} to obtain $\hat{x}_{i+1}[n]$. The flowchart of the proposed algorithm is shown in Fig. 3.

Since the filter is a zero-phase filter in microscompic image analysis $h[n_1, n_2] = h[-n_1, -n_2] = h[-n_1, n_2] = h[n_1, -n_2]$ this condition is also imposed on the current iterate in Step 4.

Global convergence of Ayers-Dainty method has not been proved. In fact, we experimentally observed that it may diverge in some FL microscopy images. Projections onto convex sets are non-expansive maps [37], [39], [40], therefore, they do not cause any divergence problems in an iterative image debluring algorithm.

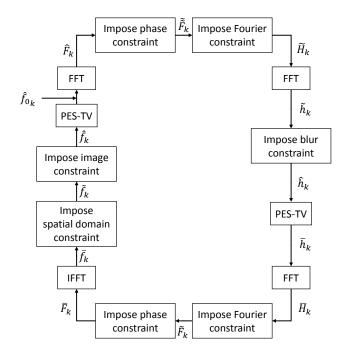


Fig. 3: Flow chart of the proposed algorithm. PES-TV stands for Projection onto the Epigraph Set of TV function.

V. EXPERIMENTAL RESULTS

The proposed algorithm is evaluated using different florescence (FL) microscopy images obtained at Bilkent University. Gaussian and uniform filters are used to blur the images. These images are blurred by Gaussian filter with disc sizes d = 5, 10, and 15 and $\sigma = 1, 2$, and 3. The blind deconvolution results are presented for $\sigma = 1, 2$, and 3 in Tables I, II, and III, respectively.

In Tables I, II, and III bold font is used for the highest PSNR. Clearly, the modified deblurring method using C_{ϕ} and C_{ESTV} produces better PSNR values in almost all cases.

Four sample of images used in this set of experiments are shown in Fig. 4. As an example, the Im-11 shown in Fig. 5a in is blurred using Gaussian filter with d = 5 and $\sigma = 3$. The blurred image is shown in Fig. 5b. The deblurred image obtained using the proposed algorithm and Ayers and Dainty's algorithm are shown in Fig. 5c and 5d, respectively. In another set of experiments, we used the FL image shown in Fig. 6a which is blurred by an unknown filter or captured with a focus blur [41]. This image is deblurred using the blind deconvolution by phase information and its output is compared with Ayers and Dainty's and Xu *et al*'s algorithm [7]. The deblurred image using the blind deconvolution by phase information and $C_{\rm ESTV}$, Ayers and Dainty's algorithm, and the Xu *et al*'s algorithm are shown in Fig. 6b, 6c, and 6d, respectively.

Ayers and Dainty's method does not converge as shown in Fig. 6c. Xu *et al*'s algorithm also diverges when we select "default" option. It does not diverge when we select "small" kernel option but the result is far from perfect as shown in Fig. 6. Sets C_{ϕ} and C_{ESTV} can be also incorporated into Xu *et al*'s method for symmetric kernels but we do not have an access to the source code. We get the best results when we use C_{ϕ} and C_{ESTV} in a successive manner as shown in Fig. 5c and 6b.

Iterations are stopped after 300 rounds in all cases. In the following web-page you may find the MATLAB code of projections onto C_{ϕ} and C_{ESTV} and the example FL images which four of them are shown in Fig. 4. Web-page: http://signal.ee.bilkent.edu.tr/BlindDeconvolution.html.

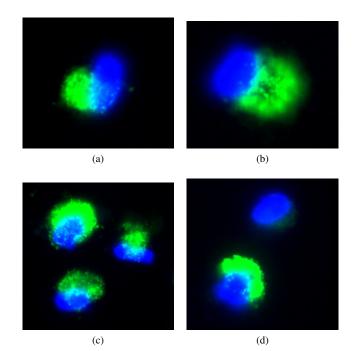


Fig. 4: Sample florescence microscopic images used in experiments (a) Im-5, (b) Im-7 (c) Im-10, and (d) Im-11.

VI. CONCLUSION

FT phase and the epigraph of the TV function are closed and convex sets. They can be used as a part of iterative microscopic image deblurring algorithms. Both sets not only provide additional information about the desired solution but they also stabilize the deconvolution algorithms. We experimentally observed that they significantly improved the deblurring results of Ayers and Dainty's method.

TABLE I: Deconvolution results for florescence microscopic images blurred by Gaussian filter with disc size d and $\sigma = 1$. PSNR (dB) values are presented for the proposed algorithm and Ayers and Dainty's algorithm.

Method	Filter radius	Im-1	Im-2	Im-3	Im-4	Im-5	Im-6	Im-7	Im-8	Im-9	Im-10	Im-11	Im-12	Im-13	Im-14
Ayers	d = 5	5.39	17.98	6.13	9.20	8.22	5.52	12.31	7.52	6.79	5.67	6.53	13.34	14.49	8.13
Modified	d = 5	9.16	11.00	10.36	11.80	9.85	10.37	16.49	8.90	10.86	8.47	10.34	16.72	17.48	10.68
Ayers	d = 10	4.57	5.25	5.12	8.70	7.61	10.93	13.51	6.52	7.70	5.96	6.44	10.17	16.80	7.53
Modified	d = 10	11.14	9.41	9.81	11.61	9.96	12.80	16.04	9.45	11.45	16.43	11.71	15.42	18.14	11.37
Ayers	d = 15	4.94	5.30	5.90	7.81	6.33	7.76	11.49	8.47	5.56	4.01	6.38	10.23	18.67	7.22
Modified	d = 15	9.00	10.60	11.51	12.04	10.31	10.49	14.16	10.72	10.99	7.83	9.49	15.65	17.69	10.71

TABLE II: Deconvolution results for florescence microscopic images blurred by Gaussian filter with disc size d and $\sigma = 2$. PSNR (dB) values are presented for the proposed algorithm and Ayers and Dainty's algorithm.

Method	Filter radius	Im-1	Im-2	Im-3	Im-4	Im-5	Im-6	Im-7	Im-8	Im-9	Im-10	Im-11	Im-12	Im-13	Im-14
Ayers	d = 5	6.37	6.92	7.03	8.10	9.08	8.69	15.64	8.44	10.17	14.19	10.98	16.28	22.95	11.41
Modified	d = 5	17.24	12.61	11.79	12.08	12.82	10.36	18.81	12.36	9.38	13.20	8.40	16.66	17.07	13.27
Ayers	d = 10	16.36	7.04	8.04	9.63	24.20	10.58	19.26	9.58	16.78	8.07	6.57	18.63	22.35	15.21
Modified	d = 10	15.30	13.01	12.58	11.86	16.86	16.33	20.44	11.61	10.47	14.18	11.74	22.17	21.66	18.39
Ayers	d = 15	10.52	13.85	12.71	14.85	13.49	15.64	18.38	9.01	7.20	7.10	6.35	22.07	19.60	13.48
Modified	d = 15	20.96	11.70	16.14	12.99	20.76	19.80	21.23	15.08	11.42	13.84	12.32	21.91	23.05	18.12

TABLE III: Deconvolution results for florescence microscopic images blurred by Gaussian filter with disc size d and $\sigma = 3$. PSNR (dB) values are presented for the proposed algorithm and Ayers and Dainty's algorithm.

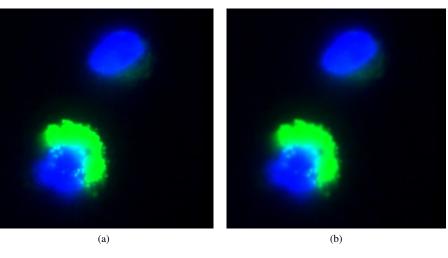
Method	Filter radius	Im-1	Im-2	Im-3	Im-4	Im-5	Im-6	Im-7	Im-8	Im-9	Im-10	Im-11	Im-12	Im-13	Im-14
Ayers	d = 5	7.40	6.33	6.39	7.43	6.51	9.39	15.92	7.20	6.35	6.71	6.62	17.47	22.19	8.88
Modified	d = 5	8.08	23.06	17.18	22.16	11.66	23.18	21.75	23.83	20.26	23.20	30.91	17.77	23.59	8.91
Ayers	d = 10	8.80	20.03	14.95	16.08	22.06	22.57	21.18	22.97	17.39	22.03	32.26	21.55	24.74	20.89
Modified	d = 10	8.02	25.66	24.74	26.44	24.15	29.61	23.99	24.44	20.67	26.04	39.92	24.03	27.05	27.30
Ayers	d = 15	18.29	28.88	14.36	18.77	27.50	27.54	24.15	24.91	21.64	26.09	34.00	22.60	23.69	26.76
Modified	d = 15	23.86	28.62	32.23	28.55	36.93	27.80	24.31	24.33	21.34	29.63	40.84	23.16	27.44	35.31

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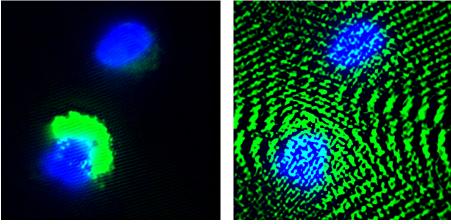


Fig. 5: The deconvolution results for Im-11 (a) Original Im-11, (b) blurred Im-11 (d = 5, $\sigma = 3$) (c) Deblurred by the proposed algorithm (PSNR = 30.91 dB), and (d) Deblurred by Ayers and Dainty's algorithm (PSNR = 6.62 dB).

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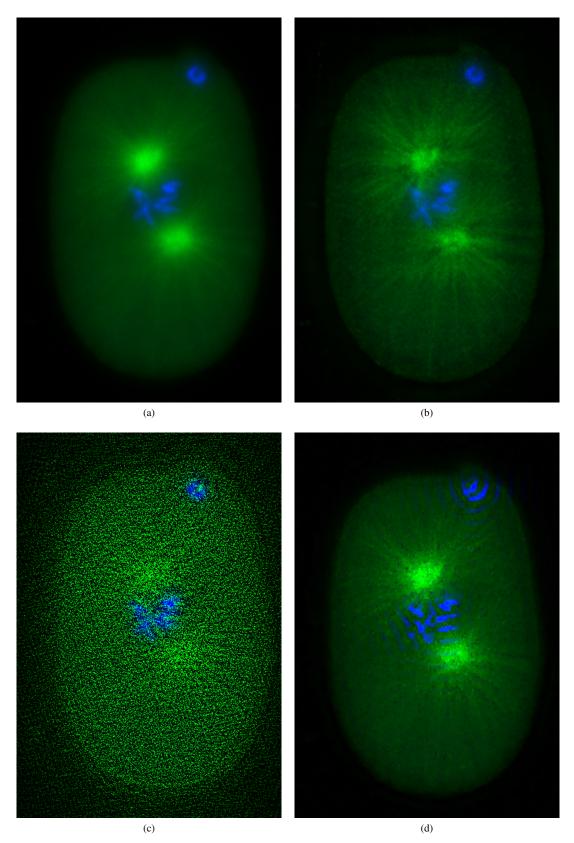


Fig. 6: The deconvolution results for FL image downloaded from [http://bigwww.epfl.ch/algorithms/mltldeconvolution/] (a) blurred image, (b) deblurred by the blind deconvolution using phase information, (c) deblurred by Ayers and Dainty's algorithm (PSNR = 34.00 dB), and (d) Deblurred by Xu et al's algorithm [7].