

# Stochastic-Geometry Based Characterization of Aggregate Interference in TVWS Cognitive Radio Networks

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**Abstract**—In this paper, we characterize the worst-case interference for a finite-area TV white space heterogeneous network using the tools of stochastic geometry. We derive closed-form expressions on the probability distribution function (PDF) and an average value of the aggregate interference for various values of path loss exponent. The proposed characterization of the interference is simple and can be used in improving the spectrum access techniques. Using the derived PDF, we demonstrate the performance gain in the spectrum detection of an eigenvalue-based detector for cognitive radio networks.

**Keywords:** Aggregate interference, TVWS, stochastic geometry, cognitive radio.

## I. INTRODUCTION

Dynamic spectrum access by cognitive radios (CRs) is central to next-generation communication networks to bridge the scarce utilization of the spectrum by licensed services [1]. Efficient spectrum sensing is required to limit the interference caused by the secondary users on the primary users. The knowledge of the interference statistics at the primary and secondary receivers can limit the interference to the primary system by designing robust spectrum sensing mechanism. Aggregate interference in a heterogeneous secondary network is characterized by different types of services, network topology, operational behavior, and channel fading [2].

There has been a lot of research interest in how to characterize the aggregate interference for a CR network (see [3], [4], [5], [6], [7], [8], [9]). In [3] and [4], a Gaussian approximation of the aggregate interference

was presented. Wen et al. [3] approximated the aggregate interference to a Gaussian random variable when the average number of nodes in the forbidden range is greater than one. Babaei et al. [4] used the central limit theorem to show the Gaussianity of the aggregate interference considering power control at the secondary nodes. Authors in [5] [6] characterized the aggregate interference in an infinite-area network by considering a Poisson's point process for the interfering nodes. These papers presented complicated approximations and bounds on the distribution function using moment generating function (MGF).

Recently, aggregate interference from a finite annular region around the primary receiver has been studied using the MGF and closed form expressions were presented for different values of the path loss exponent,  $\alpha$  [7] [8], [9]. Specifically, authors in [7] presented the probability distribution function (PDF) of the aggregate interference in a closed-form for  $\alpha = 4$ . The tools of stochastic geometry are becoming useful for the analysis of wireless networks, especially the interference characterization in large wireless networks and to study the fundamental limits of networks (see [10], [11], [12] and the references therein).

In this paper, we study to characterize the worst-case interference for a finite-area TV white space heterogeneous network using the tools of stochastic geometry. We derive closed-form expressions on the PDF of the aggregate interference for various values of path loss exponent,  $\alpha = \{2, 3, 4, 6\}$ . The derived PDF is useful in improving the spectrum access techniques for CR networks. We also derive the expected value of the aggregate interference in terms of finite-area of the annular region.

The paper is organized as follow: First, we present the system model in Section II. In Section III, we derive closed form expressions of the PDF and expected value of the aggregate interference. Simulation results are depicted in Section IV. Finally, Section V concludes the paper.

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## II. SYSTEM MODEL

We consider a simplex primary system, in which the primary receivers are passive (e.g. Terrestrial TV network) and the cognitive users form a distributed and heterogeneous ad-hoc network. The primary receivers are assumed within the coverage area described by  $\mathcal{A}(r_{dec}, r_{dec})$ , where  $r_{dec}$  is the detectability radius of the primary receivers and  $\mathcal{A}$  is a general area measure on  $\mathbb{R}^2$ . The secondary transmitters are assumed to be distributed over the Euclidean space  $\mathbb{R}^2$  according to the PPP  $\Phi$ . We define a forbidden range from the primary network as  $\mathcal{A}_{f_i} = \mathcal{A}(0, r_{f_i})$  for the  $i$ -th primary receiver. Here,  $r_{f_i}$  denotes the radius of the contention region within which no secondary transmission is allowed; and we define  $A_f = \bigcup_i \mathcal{A}_{f_i}$  as a protection region of the radius  $r_p$  from the primary transmitter.

In this paper, we consider the worst case interference scenario for the primary as well the secondary receivers. Hence, we assume that a victim receiver can be located at the origin  $\mathcal{O}(0, 0)$  of the  $\Phi$ , such that the primary receiver is at a very low SINR (which can be affected severely by a nearby secondary transmitter) and the secondary receiver experience the maximum interference from the secondary transmitters and the primary transmitter located at  $C_1$ . Further, we consider that there is interference from a finite-area network of radius  $r_{max}$ , which contributes to the maximal aggregate interference to a victim receiver. The  $r_{max}$  is chosen such that the mean interference generated by the finite secondary network at least matches up to a factor to that from an infinite network i.e.  $r \rightarrow \infty$  s.t.  $1 - r_{max}^{2-\alpha} > 1 - \epsilon \Rightarrow r > \epsilon^{-1/\alpha-2}$ , where  $\alpha$  is the path loss exponent.

The interference at a victim receiver located at an arbitrary location  $z$  from a number of CR transmitters is:

$$I(z) = \sum_{x \in \Phi} P_x h_x l(z - x), \quad (1)$$

where  $P_x$  is the transmit power of the CR located at  $x$ ,  $h_x$  is the channel gain, and  $l(z - x) = \|z - x\|^{-\alpha}$  is the isotropic path loss function with  $\alpha$  as the path loss exponent.

## III. STATISTICS OF AGGREGATE INTERFERENCE

In this section, we derive closed form expression of the PDF and expected values of aggregate interference.

### A. PDF of Aggregate Interference

We use Laplace transform (LT) to characterize the interference [2] [6]. Considering the transmit power of the CRs to unity and using  $\|x\| = \|z - x\|$ ,

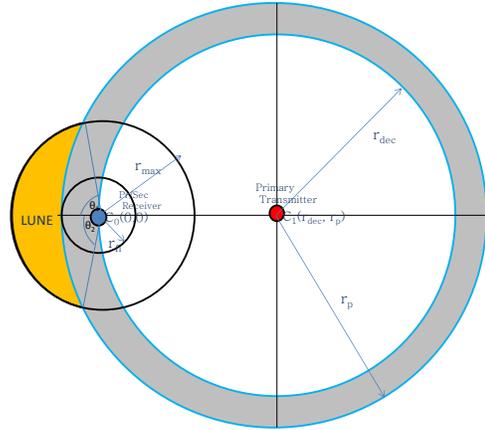


Fig. 1: System model for the worst case interference.

the conditional Laplace transform of the interference,  $\mathcal{L}_{I(z)}^{!o}(s) \triangleq \mathbb{E}(e^{-sI(z)})$  in (1) excluding the origin  $\mathcal{O}(0, 0)$  from the PPP, which is the location of the victim receiver:

$$\begin{aligned} \mathcal{L}_{I(o)}^{!o}(s) &= \mathbb{E}_h \left[ \exp(-s \sum_{x \in \Phi \setminus \{o\}} h_x \|x\|^{-\alpha}) \right] \\ &= \tilde{G} \left[ \exp(-s \sum_{x \in \Phi \setminus \{o\}} f(x)) \right] \\ &= \exp \left[ - \int_{\mathbb{R}^2 \setminus \{o\}} (1 - \exp(-sf(x))) \Lambda(dx) \right] \end{aligned} \quad (2)$$

where  $\mathbb{E}_h(\cdot)$  denotes the expectation over channel realizations,  $f(x) = h_x \|x\|^{-\alpha}$ ,  $\Lambda(\cdot)$  is the intensity function of the PPP, and  $\tilde{G}(\cdot)$  denotes the conditional probability generating functional (CPGFL) [11].

Since CPGFL of the PPP satisfies  $\tilde{G}(\cdot) = G(\cdot)$ , where  $G(\cdot)$  denotes the CPGFL including the origin, we have

$$\begin{aligned} \mathcal{L}_{I_o}(s) &= \mathcal{L}_{I(o)}^{!o}(s) \\ &= \mathbb{E}_h \left[ \exp(-\pi \lambda \int_{\mathbb{R}^2} (1 - \exp(-sh_x \|x\|^{-\alpha})) dx) \right] \end{aligned} \quad (3)$$

where  $\mathcal{L}_{I_o}(s)$  is the LT of the interference including the origin,  $\lambda$  denotes the intensity of the PPP,  $\mathbb{E}_h$  denotes the expectation over fading channel, and integration depicts the average over  $\Phi$ .

Considering the protection and forbidden region as defined in the system model, the maximal aggregate interference comes from the transmitters in the shaded region  $A_1$ , as shown in Fig. 1. First, we consider the

region  $A_{\max}$  of radius  $r_{\max}$ , and use Campbell theorem [2] to get

$$\mathcal{L}_{I_o}(s) = \mathbb{E}_h \left[ \exp(-\pi\lambda \int_0^{r_{\max}} ((1 - \exp(-sh_t t^{-\frac{1}{\eta}})) dt) \right], \quad (4)$$

where  $\eta = \frac{2}{\alpha}$ . Substituting  $y \leftarrow t^{\frac{1}{\eta}}$  in (4), we get

$$\mathcal{L}_{I_o}(s) = \exp \left( -\pi\lambda s^\eta \mathbb{E}(h)^\eta \int_{y=0}^{y=r_{\max}} y^{-\eta} e^{-y} dy \right). \quad (5)$$

For the infinite-area network  $r_{\max} \rightarrow \infty$ , (5) can be calculated as [2]:

$$\mathcal{L}_{I_o}(s) = \exp \left( -\pi\lambda s^\eta \frac{\pi\eta}{\sin(\pi\eta)} \right) \quad (6)$$

To calculate the Laplace transform of the aggregate interference in the shaded lune  $A_1$ , we convert the integration in Eq. (5) into polar form by computing the ranges of  $r$  and  $\theta$ . It can be easily seen that  $r_p \leq r \leq r_{\max}$ . The angles of intersection of circle  $C_0 : u^2 + v^2 = r_{\max}^2$  and  $C_1 : (u - r_{dec})^2 + v^2 = r_p^2$  are denoted by  $\theta_1$  and  $\theta_2$ . Using  $u = r \cos \theta_1$  and  $v = r \sin \theta_1$  for  $C_1$ , we get  $\theta_1 = \cos^{-1}(x/r_{\max})$ . From Fig. 1, it can be seen that  $\theta_2 = 2x - \theta_1$ . Thus, the integral in (5) can be represented as

$$T = 2 \int_{\theta=0}^{\theta_1} \left[ \int_{r=0}^{r_{\max}} r^{-\eta} e^{-r} dr - \int_{r=0}^{r_p} r^{-\eta} e^{-r} dr \right] d\theta \quad (7)$$

Solving the above integral in terms of standard Gamma function, and substituting in (5), we get

$$\mathcal{L}_{I_o(A_1)}(s) = \exp(-Ks^\eta), \quad (8)$$

where  $K = \pi\lambda\Gamma(1+\eta)2\theta_1[\Gamma(1-\eta, r_{\max}) - \Gamma(1-\eta, r_p)]$ . The expression in (8) denotes the complimentary cumulative distribution function (CCDF) of the interference which provides the worst case outage probability for both primary or secondary receivers.

Finally, we list the PDF of the interference for various  $\alpha$  by taking inverse Laplace transform of (8) [13]:

$$f_I(r) = \begin{cases} \delta(K), & \alpha = 2 \\ \frac{2^{4/3}}{3^{3/2}\sqrt{\pi}K^2r^{7/3}} \exp\left(-\frac{4}{27(K^3r^2)}\right) \\ \times U\left[1/6, 4/3, 4/27(K^{3/4}r^{-2})\right], & \alpha = 3 \\ \frac{1}{2\sqrt{\pi}K^3r^{3/2}} \exp\left(-\frac{1}{4K^2r}\right), & \alpha = 4 \\ \frac{3}{3Kr^{4/3}} A_i(3Kr^{1/3}), & \alpha = 6 \end{cases} \quad (9)$$

where  $U(\cdot)$  is confluent hyper-geometric function and  $A_i(\cdot)$  is the Airy function as defined in [13]. It is important to note that we get the same expression for PDF when  $\alpha = 4$  as in [7].

## B. Expected Value of Aggregate Interference

The expected value of the aggregate interference  $\bar{I} = \int_0^{r_{\max}} r f_I(r) dr$  over the annular region  $r_{\max}$  for  $\alpha = 2$  and  $\alpha = 4$  is easily integrable and can be expressed as:

$$\bar{I} = \begin{cases} \delta(K) \frac{r_{\max}^2}{2}, & \alpha = 2 \\ \text{erfc}\left(\frac{1}{2K\sqrt{r_{\max}}}\right), & \alpha = 4 \end{cases} \quad (10)$$

where  $\text{erfc}$  is complementary error function.

However, for  $\alpha = 6$ , we use the asymptotic approximation of the Airy function [14]:

$$A_i(x) \approx \frac{1}{2\sqrt{\pi}x^{1/4}} \exp(-2x^{3/2}/3) \quad (11)$$

Using the approximation on Airy function in (11), the average of aggregate interference for  $\alpha = 6$  is given by

$$\bar{I} \approx \frac{1.6633r_{\max}^{1/9}}{K} \left( 1.3541 - \Gamma(2/3, 5.1962K^{3/2}\sqrt{r_{\max}}) \right) - \frac{0.5618r_{\max}^{1/4}}{K^{9/4}} \Gamma(2/3, 1.5197K^{3/4}\sqrt{r_{\max}}) \quad (12)$$

For  $\alpha = 3$  involving the confluent hyper-geometric function  $U(\cdot)$ , we fix the  $r_{\max}$  to a constant value, say 3.4 such that  $K = 0.5598$  and hyper-geometric function reduces to a standard gamma function. Hence, for  $r_{\max} = 3.4$ , and  $\alpha = 3$ , the average of aggregate interference is given by

$$\bar{I} = 0.56\Gamma^{-1}(1/6, 0.8536r_{\max}^2), \quad (13)$$

where  $\Gamma^{-1}$  denotes the inverse Gamma function.

The expected values derived in (10), (12) and (13) are simple to evaluate, and can help design robust spectrum sensing techniques for CR networks.

## IV. SIMULATION RESULTS

Using typical values  $r_{\max} = 3.4$ ,  $r_p = 70$ ,  $\alpha = 4$  in (9), we plot the PDF of the aggregate interference in Fig. 2, which shows the non-Gaussian distribution of the interference. Next, we use the derived PDF to improve the detection probability of maximum-minimum eigenvalue (MME) [15]) spectrum detector by computing the distributional uncertainty of the aggregate interference in the form of differential entropy [16]:

$$\delta = \int_0^\infty f_I(r) \ln(f_I(r)) dr. \quad (14)$$

Substituting the derived PDF of (9) in (14), the uncertainty in the threshold computation is improved. This

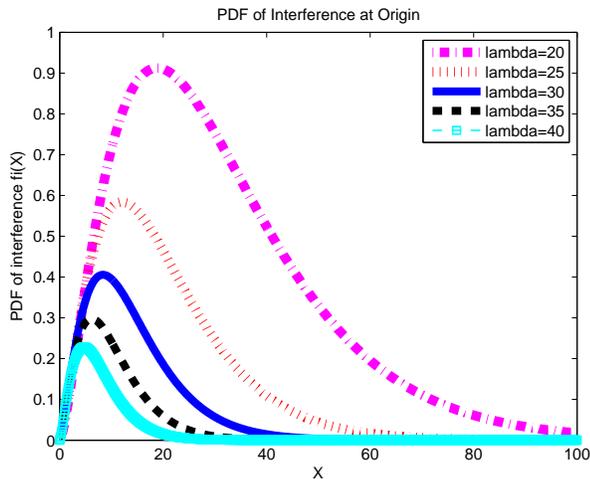


Fig. 2: Probability density function of the aggregate interference for  $\alpha = 4$ .

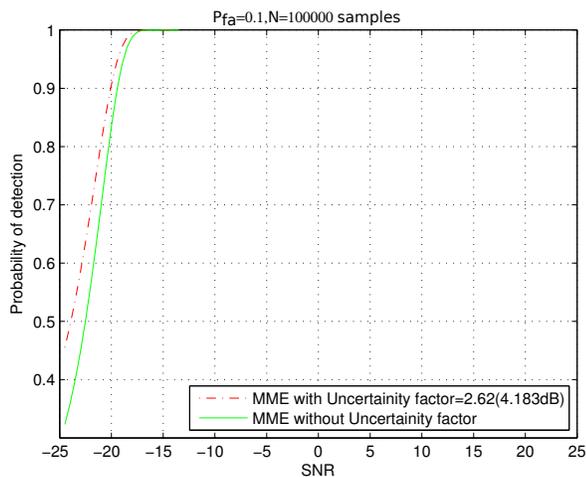


Fig. 3: Impact of uncertainty on the detection probability at  $\delta = 2.62$ , probability of false alarm  $P_{fa} = 0.1$ , number of samples  $N = 100000$ , and  $\alpha = 4$

improves the detection probability of the MME spectrum detector at low signal to noise ratio (SNR), as depicted in Fig. 3.

## V. CONCLUSION AND FUTURE WORK

We have derived closed-form expressions on the distribution of the aggregate interference under Rayleigh fading channels for various values of path loss exponent,  $\alpha = \{2, 3, 4, 6\}$ . The derived distribution is shown to be non-Gaussian for the worst-case aggregate interference. The proposed characterization of the interference is simple. This can be used in improving the spectrum access techniques and for capacity analysis in CR networks. We

also derived the expected values of the aggregate interference which can help in designing robust spectrum sensing techniques for CR networks.

## REFERENCES

- [1] J. Mitola and G. Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug 1999.
- [2] M. Haenggi and R. K. Ganti, *Interference in Large Wireless Networks*. Now Foundations and Trends, 2009.
- [3] Y. Wen, S. Loyka, and A. Yongacoglu, "On distribution of aggregate interference in cognitive radio networks," in *2010 25th Biennial Symposium on Communications*, May 2010, pp. 265–268.
- [4] A. Babaei and B. Jabbari, "Internodal distance distribution and power control for coexisting radio networks," in *IEEE GLOBECOM 2008 - 2008 IEEE Global Telecommunications Conference*, Nov 2008, pp. 1–5.
- [5] A. Ghasemi and E. S. Souza, "Interference aggregation in spectrum-sensing cognitive wireless networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 41–56, Feb 2008.
- [6] C. han Lee and M. Haenggi, "Interference and outage in poisson cognitive networks," *IEEE Transactions on Wireless Communications*, vol. 11, no. 4, pp. 1392–1401, April 2012.
- [7] L. Vijayandran, P. Dharmawansa, T. Ekman, and C. Tellambura, "Analysis of aggregate interference and primary system performance in finite area cognitive radio networks," *IEEE Transactions on Communications*, vol. 60, no. 7, pp. 1811–1822, July 2012.
- [8] S. Kusaladharma and C. Tellambura, "Aggregate interference analysis for underlay cognitive radio networks," *IEEE Wireless Communications Letters*, vol. 1, no. 6, pp. 641–644, December 2012.
- [9] Z. Chen, C.-X. Wang, X. Hong, J. Thompson, V. S.A., X. Ge, X. Hailin, and F. Zhao, "Aggregate interference modeling in cognitive radio networks with power and contention control," *IEEE Transactions on Communications*, vol. 60, no. 2, pp. 456–468, February 2012.
- [10] F. Baccelli and B. Błaszczyszyn, "Stochastic geometry and wireless networks," *Applications, INRIA and Ecole normale superieure*, vol. 2, no. 45, Dec 2009.
- [11] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1029–1046, September 2009.
- [12] H. ElSawy, E. Hossain, and M. Haenggi, "Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey," *IEEE Communications Surveys Tutorials*, vol. 15, no. 3, pp. 996–1019, Third 2013.
- [13] E. Helfand, "On inversion of the Williams-Watts function for large relaxation times," *The Journal of Chemical Physics*, vol. 78, no. 4, pp. 1931–1934, Third 1983.
- [14] E. Meissen, "Probability distribution and entropy as a measure of uncertainty," Available at: [math.arizona.edu/meissen/docs/asymptotics.pdf](http://math.arizona.edu/meissen/docs/asymptotics.pdf) [online][Accessed 23 June 2018].
- [15] Y. Zeng and Y. C. Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio," *IEEE Transactions on Communications*, vol. 57, no. 6, pp. 1784–1793, June 2009.
- [16] N. Horiya and A. Sahai, "Probability distribution and entropy as a measure of uncertainty," *Journal of Physics A: Mathematical and Theoretical*, vol. 41, pp. 13–16, 2008.