Alternative Mathematical Models for the Optimal Transmission Switching Problem

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***Abstract*—In this work, the Optimal Transmission Switching (OTS) problem is solved in order to optimize the operation cost of an electrical power system. This is accomplished by disconnecting some transmission lines, which enables a change to the profile of the power flow distribution in the system, allowing for increased generation at the buses with lower costs; thus, the hourly operation cost of the generation is minimized to meet the demand of the system. This paper presents contributions to topics related to the issue of the high number of transmission lines that are disconnected when the OTS problem is solved, the problem of system islanding, and the causes of Braess's paradox, in the context of the OTS problem. Finally, tests are conducted using the 41-bus southern Brazilian system and the 92-bus Colombian system. The results demonstrate the effectiveness of the proposals for reducing the number of lines disconnected from the system and for avoiding islanding.**

***Index Terms*—Braess's paradox, mixed-integer linear programming, optimal transmission switching, system islanding, transmission systems operation planning.**

I. Introduction

DECADES ago, a phenomenon was observed, one that appears to be in conflict with the elemental logic of an electrical engineer. Intuitively, the addition of a transmission line should improve the performance of an electrical power system, since one additional line should lead to a better distribution of the power flow, and therefore, the system should have a greater transmission capacity, which should improve some performance indicators. Thus, for example, electrical losses should be reduced, and the voltage regulation should be improved by the principle of minimum effort. Conversely, disconnecting a transmission line should worsen the performance of an electrical power system. However, it has been experimentally observed that, when some transmission lines are disconnected from the system, some performance indicators may improve [1]–[6]. In fact, in some cases, a transmission system that violates the capacity limits of some lines may operate without violating any limits after the disconnection of one or more transmission lines [4]. This atypical behavior of an electrical power system is known as Braess's paradox [7], as mentioned in [8]. Reference [9] shows that Wheatstone bridges can be associated with this paradox.

In this context, the optimal transmission switching (OTS) problem can be used to optimize the operation cost of an electrical power system. In this type of problem, the hourly operation cost of generation is minimized to meet a specified demand profile when some transmission lines are disconnected from the network with the objective of achieving a more flexible operation [10].

The OTS problem, with the objective of minimizing the generation cost, is, in fact, part of a family of problems that use the strategy of disconnecting transmission lines from an operating system to improve some aspect of the network's operation. Indeed, in the literature, several works propose to disconnect transmission lines in order to improve the performance of some aspects of the electrical power systems' operation by, for example, improving the voltage profile, reducing losses in the system [3], eliminating congestion at certain transmission lines [4], [5], [11], and improving the economic operation of the system by dispatching generators with cheaper generation costs [10]. It is worth noting, however, that almost all of the studies found in the literature are only concerned with solving the OTS problem, *i.e.*, finding an optimization proposal that identifies the transmission lines that can be disconnected from the system to optimize a specific goal. There are few attempts to understand, from a conceptual point of view, Braess's paradox in the context of the OTS problem.

With respect to the OTS problem, to improve the economic operation of the system for a specified demand profile, the parameters are the demand at each load bus, the minimum and maximum generation limits at each generation bus, the generation cost of each generation bus, and the existing lines that are in operation in the base topology, with their respective electrical parameters (resistance, reactance, line charging capacitance, and transmission capacity). In this context, the optimization of the OTS problem identifies the transmission lines that must be disconnected, maintaining the proper operation of the rest of the system, as well as providing a reduction in the value of the hourly operation cost of the system.

Fisher *et al.* [10] are considered pioneers for having described and solved the OTS problem for the economical operation of an electrical power system in 2008. From this fundamental work, many works related to this subject were developed. Reference [10] clearly shows the idea of switching transmission lines with the objective of reducing the generation cost of an electrical power system. In this proposal, the linear disjunctive model was chosen to represent the DC operation of the network,

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and the generation costs were linear. However, the first difficulties appeared when the proposal was used to solve the IEEE 118-bus system in the tests. For example, the CPLEX solver needed a high computational time and was unable to find a solution with a gap equal to zero. The same research group presented relevant contributions in [12], in which they analyzed related topics, such as the effect of the OTS on nodal prices, load payments, and generation revenues. In [13], they considered the N–1 security criterion in the OTS problem.

For the OTS problem considering the DC network model and a fixed demand profile, which is the research topic addressed in this paper, the following subjects are still relevant: (i) the network model traditionally used is the DC model, but it may be more appropriate to use the AC model; (ii) the high number of transmission lines disconnected from the system, as suggested by the traditional models, may not be acceptable to electrical companies, since it may degrade the system's security without any benefit; (iii) the processing times are still high when solving more complex models and highly complex systems; (iv) the problem of system islanding; (v) proposals for the reduction of the number of candidate lines that can be disconnected from the system, *i.e.*, to consider disconnecting only those lines that produce a significant reduction in the operating costs; and (vi) a scientific explanation for the Braess's paradox.

As noted, a high number of works in the literature solve the OTS problem. These works can be categorized as being published before or after [10]. The first publications related to the switching of transmission lines to improve the operation of a power system were conducted in the 1980s as a control mechanism of the operation, together with other strategies, such as capacitor banks operation and the adjustment of transformers' taps. Thus, [1] reveals that the switching of a transmission line can change the magnitudes and/or directions of the power flows on the other lines, while losses, bus voltage magnitudes, and the short-circuit currents at some points of the system may increase or decrease. References [2] and [3], from the same research group that published [1], present new proposals whereby lines are switched as a control mechanism, while linear programming techniques and distribution factors are used to switch lines or separate a bus to reduce the power losses in the system (a bus is, in fact, formed by several nodes of the electrical system). References [11] and [4] present new heuristic algorithms and proposals for the reduction of the search space of the problem, with the objective of minimizing losses and controlling overloads on the transmission lines. Other interesting proposals related to the OTS problem as a control mechanism for loss reduction and elimination of overloads can be found in [5], [6], and [14].

Reference [15] presents a proposal to disconnect a previously specified maximum number of transmission lines to reduce the computational time and find good-quality solutions for the OTS problem. In [16], the same research group proposes an improved and generalized version for the OTS problem, using the AC model for the power system operation. Reference [17] addresses the problem of the symmetry of power systems, which, in the case of the OTS problem, can lead to very high computational times when the existing models are considered. The symmetry problem appears when there is more than one line of the same type connecting two buses. Reference [18] presents a proposal to reduce the number of candidate lines for switching and, therefore, reduce the search space of the problem, but without overly degrading the quality of the final solution. The tests show a significant reduction in the computational time and a solution in which the operation is only 0.17% more expensive than the cost of the operation of the solution obtained using the traditional OTS model. Reference [19] presents a proposal for the analysis of the OTS problem for a period of operation of, for example, 24 hours, differing from the traditional proposal, which considers only one hour of operation. A new model of the OTS problem is presented in [20], which prevents partial or optimal solutions from presenting islanded topologies. This key topic is addressed in this paper.

In [8], the authors present an alternative mathematical model for the OTS problem, one capable of providing equivalent solutions to the proposal presented in [10], but with lower computational times. In [21], the authors extend the proposal of [8] to include the N–1 security criterion in the OTS problem. In [22], the OTS problem is analyzed considering seasonal demand variations. In each period, the demand changes, but the topology found must be unique. Reference [23] proposes a formulation to strengthen the convex relaxation of the DC OTS problem when a connected spanning subnetwork with fixed lines exists in the system. Reference [24] presents a stochastic model for unit commitment considering transmission switching for power systems with large-scale renewable integration. Reference [25] proposes a method for capacity expansion and switch installation in transmission systems, considering short-term switching operations as a response to attacks. Other proposals related to the OTS problem can be found in [26], which considers wind power generation in the system; [27], which takes into account energy and spinning reserve markets; [28], which considers a multi-objective formulation for the OTS problem for the minimization of generation costs and the maximization of a probabilistic reliability; [29] which presents a contingency-constrained unit commitment model considering transmission switching; and [30] which presents a robust model for the OTS problem accounting for the uncertainty of the demand.

Regarding the OTS problem considering the AC operation of the network, the authors of [31] propose an AC model for the system operation within the context of the OTS problem. The resulting formulation is a mixed-integer nonlinear programming problem that is transformed into a mixed-integer second-order cone programming problem. Reference [32] presents a linearized AC formulation that identifies candidate transmission lines to be switched off in order to prevent load shedding caused by contingencies. Reference [33] presents a multi-objective artificial immune algorithm for congestion management using OTS with the objective of minimizing total operating costs and maximize probabilistic reliability. Reference [34] presents relaxations for the AC OTS problem with the objective of improving the precision of convex formulations. Finally, [35] presents an optimization model that considers a convex formulation to represent the AC of the network with the objective of protecting power systems from geomagnetically induced currents, that can saturate transformers, induce hot spot heating, and increase reactive power losses, through line switching, generator redispatch, and load shedding.

The goal of this work on the OTS problem is to identify the transmission lines that must be disconnected from the network in order to obtain a more flexible system that allows a lower operation cost. As previously mentioned, by disconnecting some transmission lines, it is possible to change the profile of the distribution of power flow on the transmission lines, which allows increasing the generation at the buses with lower operating costs. The DC model is used to represent the system's operation, while an equivalent linear disjunctive formulation is used to formulate the OTS problem for a fixed demand profile. A discussion is conducted on the extension of the formulations to consider the AC operation of the system.

The contributions of this work are as follows: (i) new formulations for the OTS problem that avoid the switching of an excessive number of lines, without degrading the quality of the solution and ensuring that the operation of the resulting system is connected; and (ii) an intuitive explanation of why switching off a line can improve the operation of an electrical system, leading to a reduction in its operating costs. Item (i) is particularly important as switching off an excessive number of lines may compromise the reliability of the system and, in particular, due to the simplicity with which this problem is solved when compared to other proposals presented in the literature. Item (ii) is also very important because it is well known in the literature that transmission switching can improve the operation of an electrical system, but there is no explanation as to why this apparently contradictory behavior occurs. This work shows that transmission switching eliminates constraints related to Kirchhoff's voltage law from the model for loops that disappear with the switching operation. From the optimization point of view, the problem becomes less constrained, but Kirchhoff's current law, on the other hand, becomes more restrictive. In this context, transmission switching can lead to an optimization gain if the benefit provided by eliminating some of the constraints associated with Kirchhoff's voltage law is greater than the loss of capacity caused in Kirchhoff's current law.

Finally, it should be emphasized that all the strategies, as well as the explanation for Braess's paradox presented in this paper for the DC model are also valid when the AC operation of the network is considered. Section II introduces the formulations using the DC model to facilitate the understanding of the approaches. Section III extends the discussion and explanations to the AC operation model. Section IV presents the results for the tests conducted using the 41-bus southern Brazilian system and the 92-bus Colombian system. Section V presents the conclusions of the work.

II. DC Models for the OTS Problem

This section presents several mathematical models for the OTS problem with the objective of solving it and analyzing its characteristics considering the DC formulation for the operation of the system. Therefore, this section includes (i) a mathematical model that allows for control of the number of lines to be disconnected from the system according to the profit obtained from the switching operations, (ii) two mathematical models that avoid islanding of the system's topology, both during the optimization procedure and in the final solution, and (iii) a mathematical model that allows us to explain Braess's paradox within the OTS problem context.

Initially, we present the existing model for DC optimal power flow and its extension to the OTS problem.

A. Model 1 – Traditional Model for DC OPF

In the traditional optimal power flow (OPF) problem that optimizes the operation of transmission systems with the objective of minimizing the generation costs, all of the existing lines are operating, *i.e.*, connected to the system.

The formulation for obtaining the economic operation of a power system considering the DC model of the network (DC OPF model) is shown in (1)–(6).

$$\text{minimize}\, \upsilon = \sum_{i \in \Omega_g} c_i \, g_i \tag{1}$$

subject to:

$$\sum_{ji \in \Omega_l} f_{ji} - \sum_{ij \in \Omega_l} f_{ij} + g_i = d_i \qquad \forall i \in \Omega_b \tag{2}$$

$$f_{ij} = \overline{n}_{ij} \frac{\theta_i - \theta_j}{x_{ij}} \qquad \forall ij \in \Omega_l \tag{3}$$

$$|f_{ij}| \leq \overline{n}_{ij} \overline{f}_{ij} \qquad \forall ij \in \Omega_l \tag{4}$$

$$\underline{g}_i \leq g_i \leq \overline{g}_i \qquad \forall i \in \Omega_b \tag{5}$$

$$\theta_{ref} = 0 \tag{6}$$

The previous mathematical model is a linear programming (LP) problem that is easy to solve. In this model, υ, in (1), is the objective function related to the hourly operation cost of the system. The sets are as follows: Ω_g is the set of generation buses, Ω_b is the set of buses, and Ω_l is the set of branches. The parameters of the model are as follows: c_i is the generation cost at bus i; d_i is the demand at bus i; $\underline{g}_i$ and $\overline{g}_i$ are the lower and upper limits of the generation at bus i, respectively; x_{ij} is the reactance of a transmission line on branch ij; $\overline{f}_{ij}$ is the maximum power flow capacity of a line in branch ij; and $\overline{n}_{ij}$ is the number of existing lines on branch ij. The decision variables related to the operation of the system are as follows: g_i is the generation at bus i, θ_i is the voltage phase angle at bus i, and f_{ij} is the power flow on branch ij.

Constraint (2) is the active power balance at each bus of the system (Kirchhoff's current law), (3) represents the application of Kirchhoff's voltage law at each fundamental loop of the system, formed by one line and the loads and/or the generators connected to the buses at its ends, (4) is the transmission capacity of a branch of the system, (5) is the limit of generation capacity of a bus, and (6) imposes the angular reference to the system.

To illustrate a solution for the OPF model, consider the optimal operation for the 3-bus system presented in Fig. 1, in which the power base is 100 MVA and bus 1 is the reference.

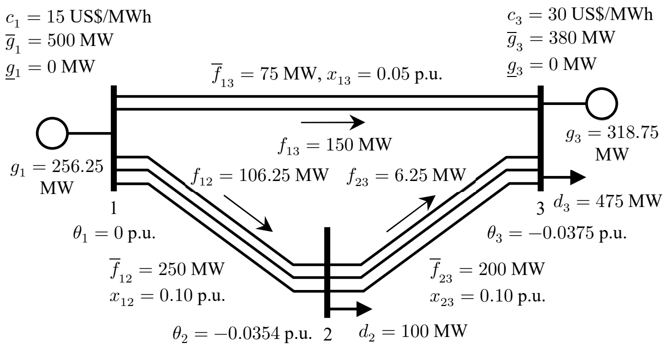

Fig. 1. The solution of the DC OPF model for the 3-bus system.

In Fig. 1, it can be verified that the lines of branch 1–3 are operating at their limits, with $\theta_{13} = \theta_1 - \theta_3 = x_{13} f_{13}/\overline{n}_{13} = 0.05 \cdot 1.5/2 = 0.0375$ p.u. Kirchhoff's voltage law will then require that $\theta_{12} + \theta_{23} = \theta_{13}$, limiting the power transfer on the lines of branches 1–2 and 2–3, and requiring that the generator at bus 3, which has a higher operating cost than the one at bus 1, generate more power. The cost of the operation for the solution presented in Fig. 1 is 13,406.25 US$/h.

If constraint (3) is not considered in this formulation, then we have the transportation model for the problem of the generation's cost minimization. Fig. 2 presents the optimal operation of the 3-bus system when constraint (3) is not considered in the model. In this case, the operation cost is 9,750.00 US$/h.

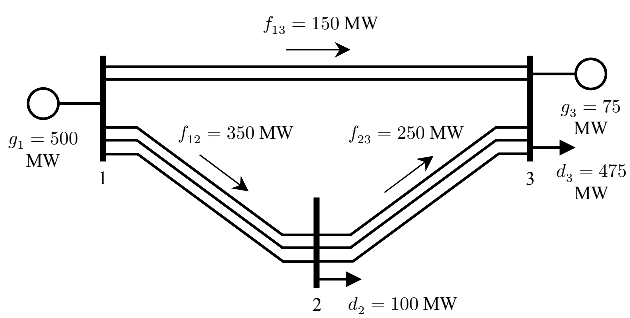

Fig. 2. Operation of the transportation model for the 3-bus system.

The solution presented in Fig. 2 is infeasible for the DC model and can only be used as a lower bound for the operation cost in the OTS problem. The model (1)–(6) will be used later to help to explain Braess's paradox in the context of the OTS problem.

Finally, it should be noted that the simplest example of a situation in which one line may limit the power transfer capacity on other lines is when two or more lines are in parallel on the same branch ij and they have different values of $x_{ij}\overline{f}_{ij}$. In this case, according to Kirchhoff's voltage law, the value of the angular opening between buses i and j, $\theta_i - \theta_j$, will be limited by the line with the smallest value of $x_{ij}\overline{f}_{ij}$.

B. *Model 2 – Traditional Model for the OTS Problem*

The model presented in [10] for the OTS problem is a mixed-integer linear programming (MILP) problem, and it is shown in (7)–(13).

$$\text{minimize}\, v = \sum_{i \in \Omega_g} c_i\, g_i \tag{7}$$

subject to: (5), (6) (8)

$$\sum_{ji \in \Omega_l} \sum_{y=1}^{\overline{n}_{ji}} f_{ji,y} - \sum_{ij \in \Omega_l} \sum_{y=1}^{\overline{n}_{ij}} f_{ij,y} + g_i = d_i \qquad \forall i \in \Omega_b \tag{9}$$

$$\left| x_{ij} f_{ij,y} - (\theta_i - \theta_j) \right| \le M(1 - w_{ij,y}) \tag{10}$$

$$\left| f_{ij,y} \right| \le w_{ij,y} \overline{f}_{ij} \tag{11}$$

$$\forall ij \in \Omega_l, y = 1, \ldots, \overline{n}_{ij}$$

$$w_{ij,y} \le w_{ij,y-1} \qquad \forall ij \in \Omega_l, y = 2, \ldots, \overline{n}_{ij} \tag{12}$$

$$w_{ij,y} \in \{0{,}1\} \qquad \forall ij \in \Omega_l, y = 1, \ldots, \overline{n}_{ij} \tag{13}$$

In this model, in addition to the previously defined sets, parameters, and variables, M is a large number that shifts from the transformation of the DC optimal power flow (OPF) model into the linear disjunctive model for the OTS problem and must be chosen adequately. The variable $f_{ij,y}$ represents the power flow on line y of branch ij, and the binary decision variable $w_{ij,y}$ indicates whether line y of branch ij is connected to the system. Thus, $w_{ij,y} = 1$ if the transmission line y of branch ij is connected, and $w_{ij,y} = 0$ otherwise. The total power flow on branch ij can be calculated as $f_{ij} = \sum_{y=1}^{\overline{n}_{ij}} f_{ij,y}$.

Constraint (9) represents the active power balance at each bus of the system (Kirchhoff's current law), and constraint (10) represents the application of Kirchhoff's voltage law at each fundamental loop of the system. Regarding this constraint, if $w_{ij,y} = 1$ (line connected to the system), then Kirchhoff's voltage law must be fulfilled in the fundamental loop formed by line ij and the generators or loads or by both generators and loads connected between bus i and the ground and bus j and the ground. If $w_{ij,y} = 0$, then this constraint is not active, and the angle difference, $\theta_i - \theta_j$, is limited by the parameter M. Note that a fundamental loop that does not exist does not need to comply with Kirchhoff's voltage law. Constraint (11) limits the power flow of a transmission line to the maximum amount allowed. The surrogate constraint (12) requires that the transmission lines existing in a certain branch ij must be disconnected from the system sequentially. It should be noted that this constraint is not in the original formulation proposed by [10]. It has since been incorporated, taking into account the similarity between the OTS problem and the transmission network expansion planning problem [36], for which this constraint has already been used. Finally, constraint (13) requires that the decision variable $w_{ij,y}$ be binary. It should be noted that, for the DC OPF model to transform into the OTS disjunctive linear model, each transmission line of a branch must be treated separately.

As a solution for the OTS model (7)–(13), Fig. 3 shows the operation of the 3-bus system after the lines of branch 1–3 are disconnected from the system. In this case, the transmission capacity of the system is reduced, but the nonfundamental loop (*i.e.,* any loop in the system that contains two or more lines) formed with the lines of branch 1–3 disappears, allowing a more economical operation of the system, with a cost of 9,750.00 US$/h.

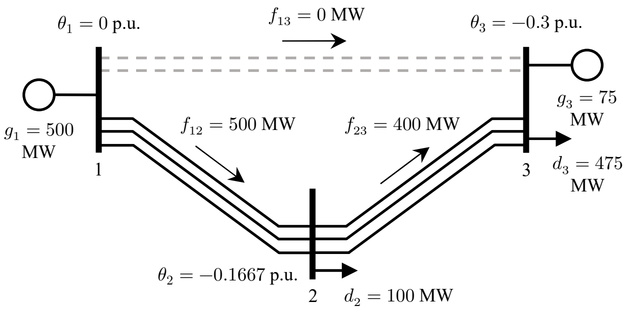

Fig. 3. OTS solution for reducing the operation cost of the 3-bus system considering the DC formulation.

Therefore, it can be concluded that the transmission capacity lost by disconnecting the lines of branch 1–3 was compensated for by the removal of the loop, in which Kirchhoff's voltage law needed to be satisfied.

Besides that, contrary to common sense and as a result of Braess's paradox, it can be verified that the disconnection of the lines of branch 1–3 also improves the security of the network in relation to the N–1 criterion. For the network shown in Fig. 3, it can be easily verified that after the disconnection of a line of branches 1–2 or 2–3, the system maintains the operation within its limits. However, it is not possible to find a feasible operation point for the network topology shown in Fig. 1 after a fault occurs on one line of branch 1–3.

This behavior was observed by the authors in the real southern Brazilian and Colombian systems, that are tested in this work. However, the OTS problem considering the N–1 security criterion and the analysis of the security of these systems is outside the scope of this work.

Three alternative optimal solution to the one presented in Fig. 3 for the traditional OTS model (7)–(13) are: (i) switch off two lines of branch 1–3 and one line of branch 1–2; (ii) switch off two lines of branch 1–3 and one line of branch 2–3; and (iii) switch off two lines of branch 1–3, one line of branch 1–2, and one line of branch 2–3. However, due to reliability aspects, these solutions should be avoided. Next section will introduce a formulation that avoids the disconnection of unnecessary lines from the system in the OTS problem.

C. Model 3 – Formulation for the OTS Problem that Avoids Switching Unnecessary Lines

The first mathematical model proposed in this work for the OTS problem is shown in (14) and (15).

$$\text{minimize}\, v = \sum_{i \in \Omega_g} c_i\, g_i + c^s \sum_{ij \in \Omega_l} \sum_{y=1}^{\overline{n}_{ij}} (1 - w_{ij,y}) \qquad (14)$$

subject to: (5), (6), (9)–(13) (15)

It should be noted that the only difference between the above formulation and the proposal presented in [10] is the objective function, in which a new term appears. In (14), c^s is a parameter that represents the minimum profit for a line to be disconnected from the system. Therefore, the optimization process should only disconnect lines if doing so generates a profit greater than or equal to c^s. The system operator can define c^s according to the savings he is willing to obtain in the operation cost of the system by disconnecting one line. This strategy produces two significant changes in the optimization process: (i) lines that produce a very small operating cost reduction should not be disconnected, and (ii) the processing time decreases significantly, which may be very important in systems of high complexity.

The second part of the objective function was also introduced in the model to eliminate the possibility of system islanding in the OTS problem, and therefore, it represents an alternative to proposals like the one presented in [20], [37]. Note that, since a single line that connects two parts of a system does not form any loops with other lines, then by using the arguments presented previously (and that will be proved in the following sections), no improvement in the operation cost of the system can be obtained by switching it off, and, therefore, the modified objective function presented in (14) will ensure that this line is not disconnected, maintaining the system connected.

It should be noted that optimization solvers, such as CPLEX, use a sophisticated branch-and-cut algorithm in which a node of the tree represents a partial solution to the problem. In this context, in the solution process of the OTS problem, unconnected partial solutions can appear, and the optimal solution itself may be unconnected. Therefore, it is fundamental that the mathematical model avoid islanded partial solutions. Also, it has been experimentally observed that a significant number of transmission lines are indifferent in relation to the OTS optimization problem, *i.e.*, they produce the same objective function in both connected or disconnected statuses. In this context, in addition to accelerating the optimization process, the proposed strategy avoids the improper disconnection of lines, and it avoids islanding.

D. Model 4 –Model that Avoids Islanding of the System

The second mathematical model proposed in this work avoids the formation of unconnected partial and optimal solutions, *i.e.*, it avoids islanding in the optimization process. It should be noted that a partial solution (a node of the branch-and-cut tree) of the OTS problem may be unconnected, which can cause convergence issues in the solver for several reasons. For instance, an unconnected partial solution may have issues with the angle θ_i at bus i if the initially connected system is divided into several islands. In this context, only the angular values of the portion of the system in which the reference bus is connected will not present issues. In the other islanded systems, in which the value θ_i was defined as being the angular difference in relation to the reference bus, the reference is lost. Thus, the values of θ_i at the buses of the islanded parts of the system separated from the reference bus usually present numerical stability problems.

The following alternative mathematical model for the OTS problem avoids the generation of partial or final unconnected solutions, *i.e.*, every valid solution proposal is always connected. According to graph theory, a graph is connected if there is a path between a bus and each of the other buses of the graph.

The mathematical model, taking this requirement into account, is shown in (16)–(21).

$$\text{minimize}\, v = \sum_{i\in\Omega_g} c_i\, g_i + c^s \sum_{ij\in\Omega_l} \sum_{y=1}^{\overline{n}_{ij}} (1 - w_{ij,y}) \tag{16}$$

subject to: (5), (6), (9)–(13) (17)

$$\sum_{ji\in\Omega_l} h_{ji} - \sum_{ij\in\Omega_l} h_{ij} + H_i = 1 \qquad \forall i \in \Omega_b \tag{18}$$

$$|h_{ij}| \le (|\Omega_b| - 1) w_{ij,1} \qquad \forall ij \in \Omega_l \tag{19}$$

$$H_i = 0 \qquad \forall i \in \Omega_b | i \neq ref \tag{20}$$

$$H_{ref} = |\Omega_b| \tag{21}$$

In this model, the new constraints (18)–(21) were added in order to represent artificial flows in the electrical network, with an artificial generation bus and every other bus of the system being a demand bus with a unity artificial demand consumption. The need to provide a unity artificial demand to all of the system's buses from a reference bus allows for a path from the reference bus to each system's bus (other than the reference); therefore, the system must always be connected. The variable H_i is the artificial power generated at bus i, h_{ij} is the artificial flow on branch ij, and the parameter $|\Omega_b|$ is the cardinality of the set Ω_b, *i.e.*, the number of buses in the system. Thus, constraint (18) allows the unity artificial demand at bus i to be met, and constraint (19) ensures that if at least one line of branch ij is connected, considering the order imposed by (12), then the maximum artificial flow through this branch is limited to $|\Omega_b| - 1$. Constraints (20) and (21) ensure that only the reference bus can have artificial generation, with a capacity of $|\Omega_b|$ units, enough to meet each of the $|\Omega_b|$ buses that have unity artificial demands.

It should be noted that, model (16)–(21) can provide connected solutions to the OTS problem with the same operation cost obtained by the traditional model (7)–(13), since, as discussed previously, switching off lines that do not form loops with other lines cannot lead to a reduction in the operation cost of the system.

E. Model 5 – Model That Explains the Occurrence of Braess's Paradox

The third mathematical model proposed in this paper explains Braess's paradox in the context of the OTS problem. In the literature, practically all works related to the OTS problem attempt to solve the problem using mathematical models; however, they fail to explain why the system's operation can be improved by disconnecting some lines from the network. In this work, we intend to explain this paradox by experimentally proving the following hypothesis: (i) the need to comply with Kirchhoff's current law at each bus of the system is not responsible for the occurrence of Braess's paradox; (ii) the need to comply with Kirchhoff's voltage law on every nonfundamental loop (formed by two or more transmission lines) of the electrical system is responsible for the occurrence of Braess's paradox in the OTS problem; and (iii) the other constraints are not responsible for the occurrence of Braess's paradox.

In order to experimentally prove the previous hypothesis, the LP problem (22)–(25) model must be solved.

$$\text{minimize}\, v = \sum_{i\in\Omega_g} c_i\, g_i + c^d \sum_{ij\in\Omega_l} \sum_{y=1}^{\overline{n}_{ij}} |\varphi_{ij,y}| \tag{22}$$

subject to: (5), (6), (9), (11) (23)

$$f_{ij,y} = \frac{\theta_i - \theta_j + \varphi_{ij,y}}{x_{ij}} \qquad \forall ij \in \Omega_l, y = 1, \ldots, \overline{n}_{ij} \tag{24}$$

$$\underline{\varphi}_{ij} \le \varphi_{ij,y} \le \overline{\varphi}_{ij} \qquad \forall ij \in \Omega_l, y = 1, \ldots, \overline{n}_{ij} \tag{25}$$

The model optimizes the generation cost by adding artificial phase shifters at each branch of the electrical system, with a negligible cost c^d. The value of c^d must be small enough, so that it do not compete with the generation cost reduction obtained with line switching, e.g., $c^d = 0.0001$ US\$. The other new parameters in the model are $\underline{\varphi}_{ij}$ and $\overline{\varphi}_{ij}$, that represent the minimum and maximum phase-shift for the transformer at branch ij. The variable $\varphi_{ij,y}$ is the phase-shift of the transformer at branch ij, line y. It should be noted that the proposal of allocating artificial phase shifters is only a mathematical artifice. Experimental tests show that the solution of (22)–(25) is exactly the same as the solution for the transportation model obtained from (1)–(6) after eliminating constraint (3) from the formulation. This proves that constraint (3) is responsible for the occurrence of Braess's paradox in the OTS problem.

Fig. 4 illustrates the 3-bus system with two phase shifters allocated at the lines of branch 1–3, according to the solution of model (22)–(25).

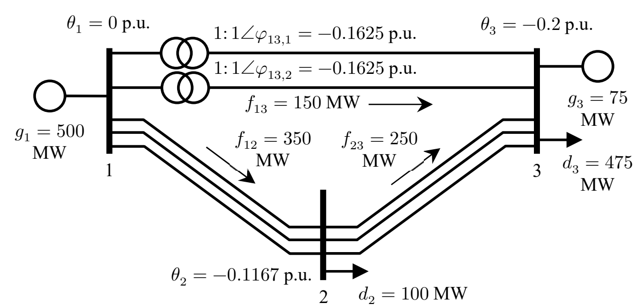

Fig. 4. Operation of the DC OPF model for the 3-bus system with artificial phase shifters allocated at the lines of branch 1–3.

In Fig. 4, it can be verified that $\varphi_{13,1} = \varphi_{13,2} = \varphi_{13} = -0.1625$ p.u. It can also be verified that the lines of branch 1–3 are operating at their limits, with $\theta_{13} + \varphi_{13} = \theta_1 - \theta_3 + \varphi_{13} = x_{13} f_{13} / \overline{n}_{13} = 0.05 \cdot 1.5/2 = 0.0375$ p.u. Kirchhoff's voltage law will then require that $\theta_{12} + \theta_{23} = \theta_{13}$, with $\theta_{12} = \theta_1 - \theta_2 = x_{12} f_{12}/\overline{n}_{12} = 0.10 \cdot 3.5/3 = 0.1167$ p.u. and $\theta_{23} = \theta_2 - \theta_3 = x_{23} f_{23} / \overline{n}_{23} = 0.10 \cdot 2.5/3 = 0.0833$ p.u. It should be noted that the phase shifter changes Kirchhoff's voltage law relation in the nonfundamental loop, making it easier to be satisfied. The cost of the operation of the solution presented in Fig. 4 is 9,750.00 US\$/h, the same value obtained by the transportation model for the solution presented in Fig. 2.

The allocation of artificial phase shifters helps the model to comply with Kirchhoff's voltage law on some nonfundamental loops of the original system. Braess's paradox can be explained as follows in the context of electrical power systems: (i) the disconnection of a transmission line reduces the transmission capacity of an electrical system and, therefore, increases the "difficulty" of the system in complying with both Kirchhoff's current law and the line capacity limits simultaneously; (ii) the disconnection of a transmission line can improve the power transfer capability in the electrical system if such disconnection eliminates problematic nonfundamental loops; (iii) therefore, the disconnection of a transmission line that eliminates problematic nonfundamental loops and does not compromise the power transfer capability (the reduction in the power transfer capability, *i.e.*, the increase in the "difficulty" of the system in complying with both Kirchhoff's current law and the line capacity limits simultaneously, is surpassed by the flexibility provided by not attending Kirchhoff's voltage law on some nonfundamental loops) can improve the performance of the electrical system.

Finally, it should be noted that this model can be used to identify problematic lines in the network that constrains the power flow on other lines they form loops with.

F. Model 6 – Model that Justifies the Occurrence of Braess's Paradox in the OTS Problem

Finally, a mathematical model that more consistently explains Braess's paradox in the context of the OTS problem is presented in (26)–(29).

$$\text{minimize}\, \upsilon = \sum_{i\in\Omega_g} c_i\, g_i + c^s \sum_{ij\in\Omega_l} \sum_{y=1}^{\overline{n}_{ij}} (1 - w_{ij,y}) + c^d \sum_{ij\in\Omega_l} \sum_{y=1}^{\overline{n}_{ij}} |\varphi_{ij,y}| \tag{26}$$

$$\text{subject to: (5), (6), (9), (11)–(13)} \tag{27}$$

$$\left| x_{ij} f_{ij,y} - (\theta_i - \theta_j + \varphi_{ij,y}) \right| \le M(1 - w_{ij,y}) \tag{28}$$

$$w_{ij,y}\underline{\varphi}_{ij} \le \varphi_{ij,y} \le w_{ij,y}\overline{\varphi}_{ij} \qquad \forall ij \in \Omega_l, y = 1, \dots, \overline{n}_{ij} \tag{29}$$

All sets, parameters, and variables of this model have been defined previously. This model minimizes the cost of operation whereby transmission lines can be disconnected from the system, but phase shifters, with a negligible cost $c^d \ll c^s$, e.g., $c^d = 0.0001$ US\$, can also be installed. Exhaustive experimental tests show that, when using this model, no line is disconnected and some phase shifters are allocated in the problematic nonfundamental loops of the system. The solution for this model for the 3-bus system is the same as one presented in Fig. 4.

III. Extension of the Formulations for AC Operation

In this section, the discussion presented for the DC OTS problem is extended to consider the AC operation of the network.

Fig. 5 presents complete data for the 3-bus system, used in the previous section, for an AC analysis, in which $\overline{S}_i^G$ is the apparent power generation capacity of bus i, $\overline{S}_{ij}$ is the apparent power transmission limit of a line in branch ij, r_{ij} is the resistance of a line of branch ij, b_{ij}^{SH} is the total line charging capacitance of a line of branch ij, P_i^D is the active power demand at bus i, and Q_i^D is the reactive power demand at bus i.

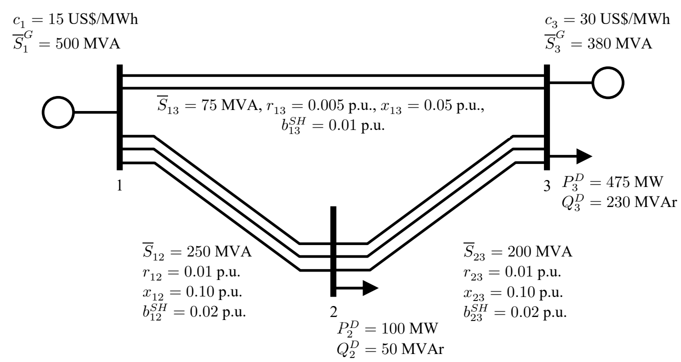

Fig. 5. Data for the 3-bus system.

The results of the AC OPF, without considering line switching, for the 3-bus system is presented in Fig. 6, in which V_i is the voltage magnitude at bus i, P_{ij} and Q_{ij} are, respectively, the active and reactive power flows on branch ij, from bus i to bus j, and P_i^G and Q_i^G are, respectively, the active and reactive power generations at bus i. In this case, the operation cost is 13,586.25 US\$/h, which is greater than the cost of the DC solution presented in Fig. 1 due to the losses.

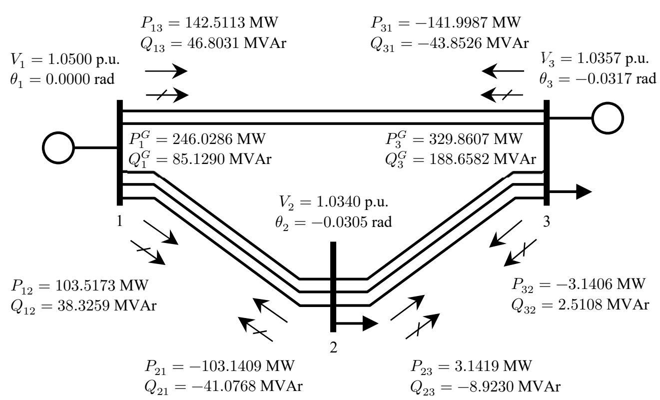

Fig. 6. The solution of the AC OPF model for the 3-bus system.

In the solution presented in Fig. 6, as in the solution of Fig. 1, the lines of branch 1–3 are operating at their limits, with $\dot{V}_{13} = \dot{V}_1 - \dot{V}_3 = 0.0360\angle 1.1467$ p.u., in which $\dot{V}_i$ is the phasor voltage at bus i. Kirchhoff's voltage law then requires that $\dot{V}_{12} + \dot{V}_{23} = \dot{V}_{13}$, again limiting the power transfer on the lines of branches 1–2 and 2–3, and requiring that the generator at bus 3, which has a higher operating cost than the one at bus 1, generate more power. In this case, the losses on the lines of branches 1–3, 1–2, and 2–3 are 0.5126 MW, 0.3764 MW, and 0.0013 MW, respectively, and the total losses in the system is 0.8903 MW.

Fig. 7 illustrates the optimal AC operation of the 3-bus system after the lines of branch 1–3 are switched off. As for the solution presented in Fig. 3, the transmission capacity of the system is reduced. However, in the resulting system, Kirchhoff's voltage law does not need to be satisfied in the nonfundamental loop formed with the lines of branches 1–2, 1–3, and 2–3, allowing a more economical operation, with a cost of 10,146.59 US$/h. The total losses in the system in this case increases to 12.8731 MW, since the losses on branches 1–2 and 2–3 increase to 7.7413 MW and 5.1318 MW, respectively.

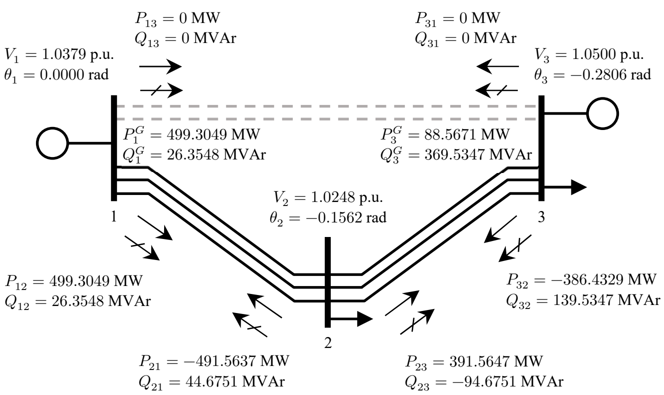

Fig. 7. OTS solution for reducing the operation cost of the 3-bus system considering the AC formulation.

The test considering the allocation of phase shifters at the lines of branch 1–3 of the 3-bus system is illustrated in Fig. 8. In this case, the cost of operation is 9,937.65 US$/h, slightly lower than the cost for the solution presented in Fig. 7, due to the reduction of the total losses to 6.2445 MW. The losses on branches 1–3, 1–2, and 2–3 are 0.5102 MW, 3.7426 MW, and 1.9917 MW, respectively.

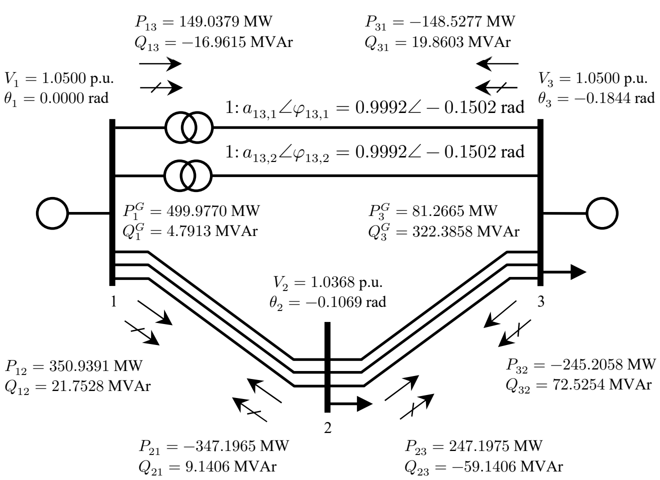

Fig. 8. Operation of the AC OPF for the 3-bus system with artificial phase shifters allocated at the lines of branch 1–3.

In Fig. 8, for the phase shifters, the taps $a_{13,1} = a_{13,2} = a_{13} = 0.9992$ and $\varphi_{13,1} = \varphi_{13,2} = \varphi_{13} = -0.1502$ rad. The lines of branch 1–3 are operating at their limits, with $\dot{V}_{12} = \dot{V}_1 a_{13} \angle \varphi_{13} - \dot{V}_2$. Kirchhoff's voltage law requires that $\dot{V}_{12} + \dot{V}_{23} = \dot{V}_{13}$. Note that the only difference between the systems presented in Fig. 6 and Fig. 8 is related to Kirchhoff's voltage law in the nonfundamental loop.

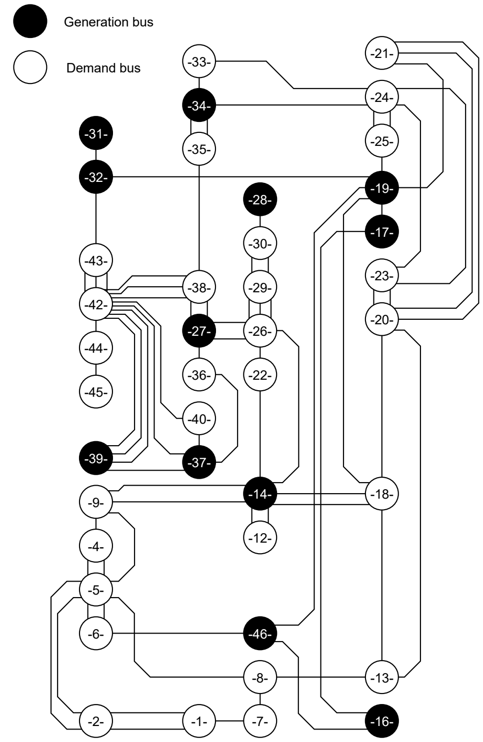

Fig. 9. Topology of the 41-bus southern Brazilian system.

The examples presented in this section show that the discussion regarding Braess's paradox introduced in Section II for the DC OTS model is also valid for the AC formulation. Thus, the formulations (14) and (15), and (16)–(21) can be adapted to the AC OTS problem, providing the same benefits presented previously.

IV. Tests and Results

This section presents the results of the tests performed with two systems typically used for tests with the transmission network expansion planning problem: the southern Brazilian system and the Colombian system [38]. In this paper, however, both systems are modified adding the lines of the optimal expansion plan as part of the network.

The proposed models were implemented in the AMPL [39] modeling language and solved with the CPLEX [40] solver version 12.9 on a computer with a 3.2 GHz Intel® Core™ i7-8700 processor with 16 GB of RAM.

A. *Modified Southern Brazilian System*

The southern Brazilian system, presented in Fig. 9, has 41 buses and 55 branches with 78 lines. The total demand is 6880 MW, and the total generation capacity is 10545 MW. The generation cost without disconnecting any lines, provided by Model 1, is 185,111.80 US$/h.

When Model 2 is used to solve the OTS problem in this system, the generation cost is 184,063.00 US$/h, and 13 lines are disconnected: 1–7, 7–8, 4–5, 14–22, 23–24, 33–34, 37–39, 40–

TABLE I
RESULTS USING MODEL 4 FOR THE MODIFIED SOUTHERN BRAZILIAN SYSTEM

c^s (US$/line)	# of lines disconnected	Operational cost (US$/h)	Disconnected lines	Time (s)
0	9	184,063.00	4–5, 20–23, 33–34, 37–39, 40–42, 38–42×(3), 42–43	0.23
1	5	184,063.00	33–34, 37–39, 38–42×(3)	0.22
50	4	184,084.47	35–38, 37–39, 38–42×(2)	0.12
100	3	184,168.71	35–38, 38–42×(2)	0.11
300	1	184,590.89	35–38	0.08
600	0	185,111.80	–	0.06

42, 38–42×(3), 46–19, and 42–43. In this case, bus 7, which does not have load, is disconnected from the system. The computational time to solve the problem is less than 1 second. In fact, Model 2 can obtain several alternative optimal solutions for this problem with the same value of the objective function, with five up to 17 lines disconnected from the system.

Table I shows the results obtained when Model 4 is used to solve the OTS problem. Model 3 provides the same results as Model 4, with similar computational times, with one exception: when $c^s = 0$ US$/line, Model 3 provides the same solution as Model 2.

The results show that the solution provided by the traditional model for OTS, Model 2, has up to 17 lines disconnected from the system, while Model 4 finds a solution with the same generation cost, but disconnecting only five lines, and ensuring that the resulting systems is connected.

Also, by disconnecting four lines instead of 17, Model 4 provides a solution that is only 0.01% more expensive than the solution provided by Model 2. Note that, the marginal profit obtained in the operational cost by disconnecting an additional line decreases as the number of disconnected lines increases.

The solution provided by Model 1 when constraint (3) is not considered in the formulation (transportation model for the optimal power flow problem, without disconnecting lines) provides an operation cost of 184,063.00 US$/h, which is the same cost obtained by Model 4 for OTS with $c^s = 0$ US$/line or $c^s = 1$ US$/line.

This same solution is obtained by Model 5, by allocating phase shifters at branches 9–14, 27–38, and 39–42, and by Model 6, which does not disconnect any lines and allocates phase shifters at the same branches as Model 5.

The results provided by Model 1 without constraint (3), Model 5, and Model 6 indicate that Kirchhoff's voltage law is therefore responsible for Braess's paradox in the OTS problem, and by eliminating some nonfundamental loops from the systems, it is possible to obtain a more efficient operation, as previously discussed.

B. *Modified Colombian System*

The Colombian system, presented in Fig. 10, has 92 buses and 146 branches with 212 lines. The total demand is 14559 MW, and the total generation capacity is 17473 MW. The generation cost without disconnecting any lines, provided by Model 1, is 438,755.37 US$/h.

In this case, when Model 2 is used to solve the OTS problem, the generation cost is 434,749.44 US$/h, and 45 lines are disconnected: 25–29, 14–60×(2), 2–4×(2), 15–17, 15–76, 35–44, 38–68, 10–78, 1–59, 59–67, 8–59×(2), 1–3, 3–6, 46–53, 9–69, 32–34, 16–23, 31–60, 47–49×(2), 18–20, 5–6, 1–71, 27–44, 73–74, 29–64, 33–34, 48–63, 23–24, 26–28, 12–76, 50–54, 54–56×(4), 60–62, 62–82, 11–92, 1–93, 92–93, and 91–92. In this case, Model 2 can obtain alternative optimal solutions for this problem with the same value of the objective function and up to 67 lines disconnected from the system. In addition, buses 92 and 93 are disconnected from the system (bus 89 is not connected in the solution of the expansion planning problem; therefore, it is not considered in the OTS problem).

TABLE II
RESULTS USING MODEL 4 FOR THE MODIFIED COLOMBIAN SYSTEM

c^s (US$/line)	# of lines disconnected	Operational cost (US$/h)	Disconnected lines	Time (h)
0	46	434,749.44	14–31, 14–60×(2), 15–20, 24–75, 35–44×(2), 7–78, 30–72, 56–57×(2), 9–77, 59–67, 8–59×(2), 3–6, 46–53, 60–69, 32–34, 16–18, 31–33×(2), 31–60, 47–54, 47–49×(2), 5–6, 12–17, 1–71, 1–11, 64–65, 8–71, 48–63, 8–9, 39–43, 39–40, 50–54, 54–56×(4), 72–73, 55–82, 62–82, 7–90, 1–93	0.42
0.0001	23	434,749.44	25–29, 14–31, 14–60×(2), 2–4×(2), 7–78, 30–72, 1–59×(2), 59–67, 3–6, 8–71, 48–63, 8–9, 33–72, 50–54, 64–74, 54–56×(4), 1–93	3.74
50	10	434,932.02	14–60×(2), 59–67, 1–3, 1–71×(2), 1–8, 48–63, 50–54, 91–92	2.75
200	4	435,735.35	14–60×(2), 59–67, 50–54	0.19
2000	0	438,755.37	–	0.00

Table II shows the results obtained when Model 4 is used to solve the OTS problem for this system. Again, Model 3 provides the same results as Model 4, with similar computational times; the one exception occurs when $c^s = 0$ US$/line, in which case, Model 3 provides the same solution as Model 2.

By using Model 4 with $c^s = 0.0001$ US$/line, it was possible to obtain a solution for the problem with the same operational cost of 434,749.44 US$/h found by Model 2, but by disconnecting only 23 lines, instead of 67. Although the computational time to solve the problem is 3.74 h, the optimal solution was found in 30.64 s, and the rest of the time was used by CPLEX only to analyze the remaining nodes of the branch and cut tree.

The results for Model 4 also indicate that, by disconnecting only ten lines, instead of the 67 lines of the traditional OTS Model 2, it is possible to obtain a solution that is only 0.04% more expensive.

The solution provided by Model 1 when constraint (3) is not considered in the formulation provides an operation cost of 433,194.04 US$/h, which is lower than the cost obtained by Model 4 for OTS with $c^s = 0$ US$/line. Obviously, this solution is infeasible for Model 4, which considers Kirchhoff's voltage law.

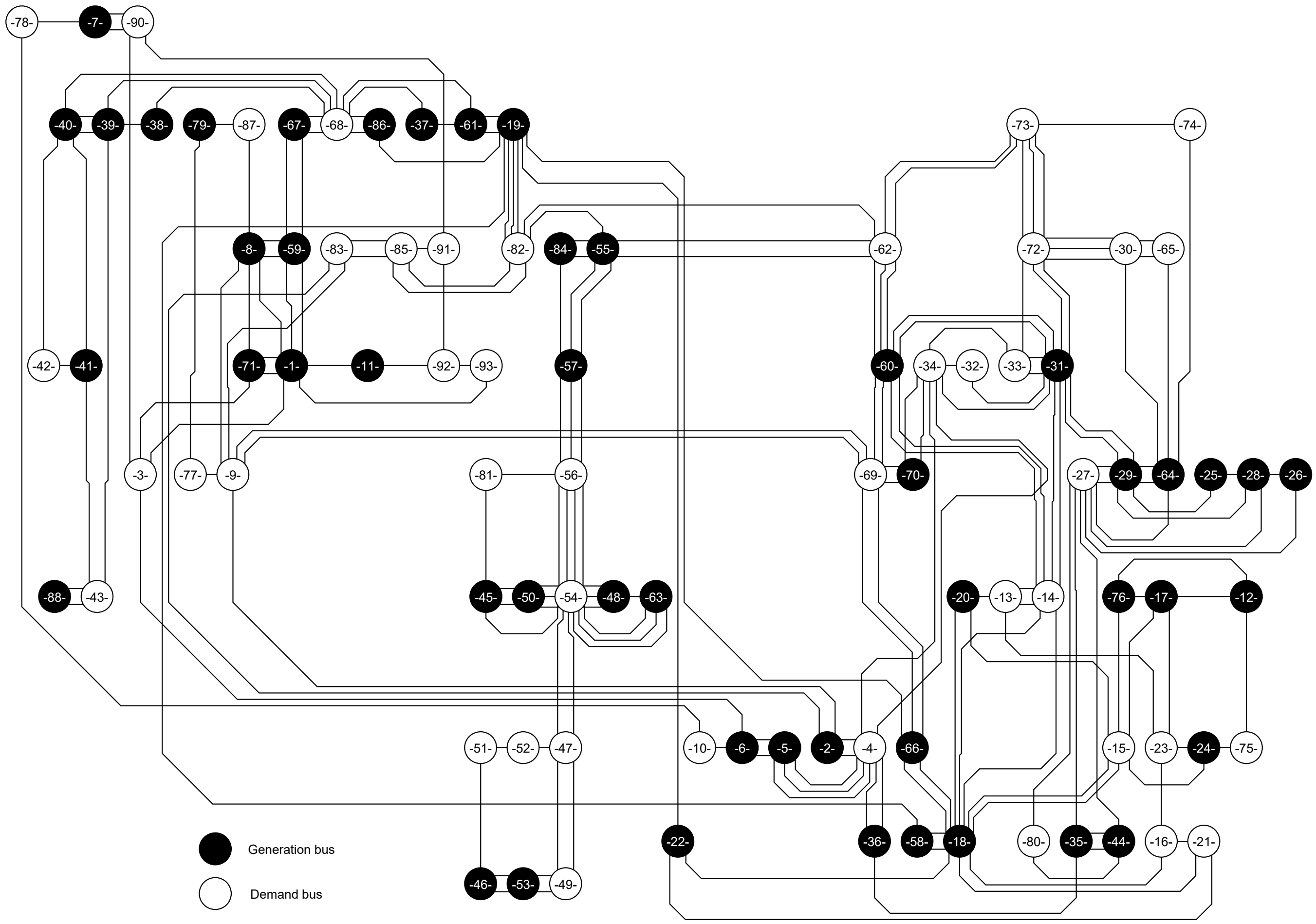

Fig. 10. Topology of the 92-bus Colombian system.

This same solution is obtained by Model 5 by allocating phase shifters at branches 19–61, 45–54, 45–50, 59–67, 18–58, 19–22, 48–54, and 19–82, and Model 6, which does not disconnect any lines and allocates phase shifters at branches 19–61, 45–54, 19–22, 19–58, 67–68, 48–54, 50–54, and 19–82.

The results again confirm the hypothesis presented in this paper for Braess's paradox in the context of the OTS problem.

V. Conclusion

In this work, we have presented new models for the optimal transmission switching (OTS) problem that provide a more stable performance than the models available in the literature. Indeed, these models provide solutions with a reduced number of disconnected lines, eliminate the possibility of generating islanded partial solutions, and ensure that the final solution provided by each model is also connected.

Braess's paradox in the context of the OTS problem has also been consistently explained through mathematical models and experimental tests. Furthermore, it should be noted that, if we accept the hypothesis about Braess's paradox presented in this work, then the new models will not provide solutions with islanded operation, and the system will be connected. Indeed, a line connecting two parts of the electrical system cannot be disconnected, since this line does not generate a nonfundamental loop and, therefore, does not interfere with Kirchhoff's voltage law in the system.

The proposals presented in this paper can be extended to more complex OTS problem formulations, such as the OTS problem considering the AC network operation model and the OTS problem considering reliability with the N–1 criterion.

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