Local Stability of a New Adaptive Queue Management (AQM) Scheme

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***Abstract*—In this letter, we propose a new Adaptive Queue Management scheme based on the virtual queue size and the aggregate flow rate. Our proposal is tailored for the widely deployed TCP Reno and does not require modifications to TCP end users. We prove that our scheme can guarantee local stability for a network with an arbitrary topology and heterogeneous round-trip times for TCP users.**

***Index Terms*—Adaptive Queue Management (AQM), stability, TCP, virtual queue.**

I. Introduction

ACTIVE Queue Management (AQM) has been a very active research area in recent years. Many AQM mechanisms have been proposed, e.g., random early detection (RED) [1], random exponential marking (REM) [2], PI controller [3], adaptive virtual queue (AVQ) [4], and state feedback control (SFC) [5]. AQM schemes control the traffic rate and buffer occupancy by schematically dropping packets. If Explicit Congestion Control (ECN) is enabled, packets are marked instead of being dropped. Without loss of generality, we assume ECN is enabled in this letter. End TCP users adapt their transmission rates based on the marking feedback from AQM.

RED is a queue length based AQM that marks packets with probability proportional to the current average queue length. AVQ is a typical rate based scheme. REM, PI, and SFC use the queue length and the aggregate flow rate to compute the marking probability.

The TCP/AQM system has been modeled as a close loop control system. One of the major concerns about such a system is its stability property. [3]–[5] studied the local stability conditions for the network with PI, AVQ, and SFC, respectively. These works only considered a network of a single bottleneck link with homogeneous round-trip times, and neglected the backward propagation delays. [6] took the heterogeneous round-trip time and backward propagation delay into consideration, and provided the local stability condition for RED in a network with a single bottleneck link.

In this letter, we propose a virtual queue and rate based AQM scheme (referred to as VQR), which uses the virtual queue size and the aggregate flow rate to compute the marking probability. We provide and prove the local stability condition of our scheme for a network with an arbitrary topology rather than a single bottleneck link, with heterogeneous rather than homogeneous round-trip times, and with the consideration of rather than negligence of the backward propagation delays.

The remainder of this letter is organized as follows. Section II presents the framework of the TCP/AQM dynamic model. Section III presents our proposed VQR scheme and the corresponding local stability condition. Section IV presents the conclusions.

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II. TCP/AQM Dynamics

In this section, we adopt the work in [6] to present the TCP/AQM dynamic model used in this letter. Consider a network with L links, each with capacity $C_l (l = 1, 2, \ldots L)$. Assume there exist S TCP sources, which share the L links. The routing policy is expressed by an $L \times S$ matrix $\mathbf{R}$ with elements R_{li} defined as

$$R_{li} = \begin{cases} 1, & \text{if source } i \text{ uses link } l \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Each link l marks packets with probability $p_l(t)$ at time t. Each TCP source i is associated with a round-trip time $\tau_i(t)$

$$\tau_i(t) = d_i + \sum_l R_{li} \frac{b_l(t)}{C_l} \tag{2}$$

where d_i is the round-trip propagation delay for source i and $b_l(t)$ is the queue length of link l at time t. The round-trip time delay $\tau_i(t)$ can also be expressed as

$$\tau_i(t) = \tau_{li}^f(t) + \tau_{li}^b(t) \tag{3}$$

where $\tau_{li}^f(t)$ is the forward delay from source i to link l and $\tau_{li}^b(t)$ is the backward delay from link l back to source i.

The transmitting rate of source i, $x_i(t)$, is modeled as

$$x_i(t) = \frac{w_i(t)}{\tau_i(t)} \tag{4}$$

where $w_i(t)$ is the window size of i. The aggregate transmission rate at link l is

$$y_l(t) = \sum_i R_{li} x_i \left(t - \tau_{li}^f(t)\right). \tag{5}$$

With the assumption that $p_l(t)$ is small, the end-to-end marking probability $q_i(t)$ for source i can be obtained by summing up the marking probability at each link traversed by i

$$q_i(t) = \sum_l R_{li} p_l \left(t - \tau_{li}^b(t)\right). \tag{6}$$

We adopt the fluid TCP model studied in [3] and [6], in which the TCP Reno window size evolves according to

$$\dot{w}_i(t) = \frac{x_i(t-\tau_i(t))(1-q_i(t))}{w_i(t)} - x_i(t-\tau_i(t))q_i(t)\frac{w_i(t)}{2}. \quad (7)$$

The first term on the right side of (7) describes the additive window increment, and the second term stands for the multiplicative window decrement. The sending rate of TCP i evolves according to

$$\dot{x}_i(t) \triangleq \frac{d}{dt}\left(\frac{w_i(t)}{\tau_i(t)}\right) = \frac{\dot{w}_i(t)}{\tau_i(t)} - \frac{w_i(t)\dot{\tau}_i(t)}{\tau_i^2(t)}. \quad (8)$$

The dynamics of the queue length $b_l(t)$ at link l can be expressed as

$$\dot{b}_l(t) = y_l(t) - C_l. \quad (9)$$

Denote w_i^*, p_l^*, q_i^* as the equilibrium values of $w_i(t)$, $p_l(t)$, $q_i(t)$, respectively. At equilibrium, we have $x_i^* = w_i^*\tau_i$, and $q_i^* = \sum_l R_{li}p_l^*$. Here, $\tau_i = d_i + \sum_l R_{li}(b_l^*/C_l)$ is the equilibrium round-trip time for source i. It can be shown from (7) that

$$w_i^* = \sqrt{\frac{2(1-q_i^*)}{q_i^*}}. \quad (10)$$

Let $\delta w_i(t) = w_i(t) - w_i^*$, $\delta p_l(t) = p_l(t) - p_l^*$, $\delta x_i(t) = x_i(t) - x_i^*$. Linearizing (7), (8), and (5) around the equilibrium state with the first-order approximation, we have

$$\delta\dot{w}_i(t) = -\frac{1}{\tau_i q_i^*}\sum_l R_{li}\delta p_l\left(t-\tau_{li}^b\right) - \frac{q_i^* w_i^*}{\tau_i}\delta w_i(t) \quad (11)$$

$$\delta\dot{x}_i(t) = \frac{\delta\dot{w}_i(t)}{\tau_i} - \frac{w_i^*\delta\dot{\tau}_i}{\tau_i^2} \quad (12)$$

$$\delta\dot{y}_l(t) = \sum_i R_{li}\delta\dot{x}_i\left(t-\tau_{li}^f\right). \quad (13)$$

III. The VQR Algorithm

VQR maintains a virtual queue $B_l(t)$, which is updated at each packet arrival according to

$$\dot{B}_l(t) = y_l(t) - \gamma C_l \quad (14)$$

where γ is a positive number less than but close to 1 (e.g., 0.95). The marking probability is updated by

$$p_l(t) = \frac{g}{\gamma C_l}\{a(B_l(t) - B_{l,ref}) + (y_l(t) - \gamma C_l)\} \quad (15)$$

where $B_{l,ref}$ is a reference virtual queue length, and g and a are positive numbers. If $p_l(t) < 0$, it is set to zero; if $p_l(t) > 1$, it is set to one.

At the equilibrium state, we have $\dot{B}_l(t) = 0$. Thus, the aggregate rate y_l^* is γC_l, which is slightly smaller than the link capacity C_l. Although this could lead to slight link under-utilization, there are some benefits of doing so. First, the queuing delay is zero at the equilibrium state. Second, since we only consider small perturbation around the equilibrium state, it can reasonably be assumed that $y_l(t)$ is always less than the link capacity. As a result, the queue length $b_l(t)$ is always zero, the round-trip time $\tau_i(t)$ is reduced to d_i, and $\delta\tau_i(t)$ is always zero under such an assumption. Thus, the second term on the right side of (12) is negligible. Together with (11), we can rewrite (12) as

$$\begin{aligned}\delta\dot{x}_i(t) &= \frac{\delta\dot{w}_i(t)}{\tau_i}\\ &= -\frac{1}{\tau_i^2 q_i^*}\sum_l R_{li}\delta p_l\left(t-\tau_{li}^b\right) - \frac{q_i^* w_i^*}{\tau_i}\delta x_i(t). \quad (16)\end{aligned}$$

Linearizing (14) and (15) around the equilibrium state, we have

$$\delta\dot{B}_l(t) = \delta y_l(t), \quad (17)$$

$$\delta p_l(t) = \frac{g}{\gamma C_l}(a\delta B_l(t) + \delta y_l(t)). \quad (18)$$

Following [6], we can express (13), and (16)–(18) in the Laplace domain in the matrix form:

$$\boldsymbol{\delta}\mathbf{y}(s) = \mathbf{R_f}(s)\boldsymbol{\delta}\mathbf{x}(s), \quad (19)$$

$$\boldsymbol{\delta}\mathbf{x}(s) = -(s\mathbf{I} + \mathbf{H_1})^{-1}\mathbf{H_2}\mathbf{R_b^T}(s)\boldsymbol{\delta}\mathbf{p}(s), \quad (20)$$

$$\boldsymbol{\delta}\mathbf{p}(s) = \mathbf{A}(s)\mathbf{G}\boldsymbol{\delta}\mathbf{y}(s) \quad (21)$$

where $\mathbf{G} = \mathrm{diag}(g/\gamma C_l)$, $\mathbf{A}(s) = \mathrm{diag}(s + a/s)$, $\mathbf{H_1} = \mathrm{diag}(q_i^* w_i^*/\tau_i)$, $\mathbf{H_2} = \mathrm{diag}\left(1/\tau_i^2 q_i^*\right)$, and

$$[R_f]_{li} = \begin{cases} e^{-\tau_{li}^f s}, & \text{if source } i \text{ uses link } l\\ 0, & \text{otherwise}\end{cases} \quad (22)$$

$$[R_b]_{li} = \begin{cases} e^{-\tau_{li}^b s}, & \text{if source } i \text{ uses link } l\\ 0, & \text{otherwise.}\end{cases} \quad (23)$$

Equations (19)–(21) form a closed-loop control system with the return ratio

$$\mathbf{L}(s) = \mathbf{R_f}(s)(s\mathbf{I} + \mathbf{H_1})^{-1}\mathbf{H_2}\mathbf{R_b^T}(s)\mathbf{A}(s)\mathbf{G}. \quad (24)$$

According to [6] and [7], the above control system is stable if the eigenvalues of $L(j\omega)(\omega \geq 0)$ do not encircle -1 in the complex plane. Define

$$\tilde{\mathbf{R}}(j\omega) = \mathrm{diag}\left(\sqrt{\frac{g}{\gamma C_l}}\right)\mathbf{R_f}(j\omega)\mathrm{diag}\left(\sqrt{x_i^*}\right). \quad (25)$$

Using the relationship

$$\mathbf{R_b}(s) = \mathbf{R_f}(-s)\mathrm{diag}(e^{-\tau_i s}) \quad (26)$$

and following the similar argument in [7], the eigenvalues of $L(j\omega)$ are the same as those of:

$$\mathbf{Z}(j\omega) = \mathrm{diag}\left(\frac{e^{-j\omega\tau_i}(j\omega + a)}{q_i^* x_i^*(j\omega\tau_i + q_i^* w_i^*)j\omega\tau_i}\right)\tilde{\mathbf{R}}^{\mathbf{T}}(-j\omega)\tilde{\mathbf{R}}(j\omega). \quad (27)$$

Denote E as the set of eigenvalues of $Z(j\omega)$. According to [7], we have

$$\begin{aligned}E \subset \rho&\left(\tilde{\mathbf{R}}^{\mathbf{T}}(-j\omega)\tilde{\mathbf{R}}(j\omega)\right)\\ &\times co\left(0 \cup \left(\frac{e^{-j\omega\tau_i}(j\omega + a)}{q_i^* x_i^*(j\omega\tau_i + q_i^* w_i^*)j\omega\tau_i}\right),\right.\\ &\qquad\left. i = 1, 2, \ldots, S\right) \quad (28)\end{aligned}$$

where $co(m_i, i = 1, 2, \ldots, S)$ denotes the convex hull of the set points $\{m_1, m_2, \ldots, m_S\}$. The spectral radius of $\tilde{R}(j\omega)$ satisfies [7]:

$$\begin{aligned}\rho\left(\tilde{\mathbf{R}}^{\mathbf{T}}(-j\omega)\tilde{\mathbf{R}}(j\omega)\right) &\\ &\leq \left\|\mathbf{R}_{\mathbf{f}}^{\mathbf{T}}(-j\omega)\mathrm{diag}\left(\frac{g}{\gamma C_l}\right)\mathbf{R}_{\mathbf{f}}(j\omega)\mathrm{diag}\left(x_i^*\right)\right\|_\infty \\ &\leq \left\|\mathbf{R}_{\mathbf{f}}^{\mathbf{T}}(-j\omega)\right\|_\infty \cdot \left\|\mathrm{diag}\left(\frac{g}{\gamma C_l}\right)\mathbf{R}_{\mathbf{f}}(j\omega)\mathrm{diag}\left(x_i^*\right)\right\|_\infty \\ &= Mg \end{aligned} \tag{29}$$

where M is the maximum number of links a TCP source traverses in the network.

Denote

$$\Lambda_i(j\omega) = \frac{Mg \times e^{-j\omega\tau_i}(j\omega + a)}{q_i^* x_i^* \left(j\omega\tau_i + q_i^* w_i^*\right) j\omega\tau_i}. \tag{30}$$

If $\Lambda_i(j\omega)(i = 1, 2, \ldots, M)$ do not encircle -1 in the complex plane, E does not either [7]. Therefore, the system is stable. Next, we present a sufficient condition to guarantee the stability of such a system.

Theorem 1: If

$$a < \min_i \left\{(1/\tau_i)\sqrt{2q_i^*\left(1 - q_i^*\right)}; i = 1, \ldots, S\right\}$$

and $g < (\pi/2M)\min_i\left(\sqrt{2q_i^*\left(1 - q_i^*\right)}; i = 1, \ldots, S\right)$, the system is stable.

Proof: From (30), we can rewrite $\Lambda_i(j\omega)$ as

$$\Lambda_i(j\omega) = \frac{e^{-j\omega\tau_i}(j\omega\tau_i + a\tau_i)}{j\omega\tau_i\left(j\omega\tau_i + q_i^* w_i^*\right)}\frac{Mg}{q_i^* w_i^*}. \tag{31}$$

With (10), it can be shown that the two conditions in Theorem 1 are equivalent to

$$a < \min_i \left\{\frac{1}{\tau_i}q_i^* w_i^*, i = 1, \ldots, S\right\} \tag{32}$$

$$g < \frac{\pi}{2M}\min_i\left(q_i^* w_i^*, i = 1, \ldots, S\right). \tag{33}$$

Let

$$\frac{e^{-j\omega\tau_i}}{j\omega\tau_i}\frac{(j\omega\tau_i + a\tau_i)}{\left(j\omega\tau_i + q_i^* w_i^*\right)} = \rho e^{j\theta}\frac{e^{-j\omega\tau_i}}{j\omega\tau_i} \tag{34}$$

where

$$\begin{aligned}\theta &= \mathrm{phase}\left((j\omega\tau_i + a\tau_i)/(j\omega\tau_i + q_i^* w_i^*)\right)\\ \rho &= \left|\left((j\omega\tau_i + a\tau_i)/(j\omega\tau_i + q_i^* w_i^*)\right)\right|.\end{aligned}$$

If $a < \min_i\left\{\left(q_i^* w_i^*/\tau_i\right); i = 1, \ldots, S\right\}$, we have $0 < \theta < \pi/2$ and $0 < \rho < 1$. Let ω' be any frequencies at which $\rho e^{j\theta}\left(e^{-j\omega\tau_i}/j\omega\tau_i\right)$ cross the negative real axis, and we have

$$-\omega'\tau_i - \frac{\pi}{2} + \theta = -(2k+1)\pi, \qquad k = 0, 1, 2, \ldots. \tag{35}$$

Thus, from (34) and (35) we have

$$\frac{e^{-j\omega'\tau_i}}{j\omega'\tau_i}\frac{(j\omega'\tau_i + a\tau_i)}{\left(j\omega'\tau_i + q_i^* w_i^*\right)} = -\frac{\rho}{\left(2k + \frac{1}{2}\right)\pi + \theta}. \tag{36}$$

From (35) and with the properties of ρ and θ, we have

$$\frac{e^{-j\omega'\tau_i}}{j\omega'\tau_i}\frac{(j\omega'\tau_i + a\tau_i)}{\left(j\omega'\tau_i + q_i^* w_i^*\right)} > -\frac{2}{\pi}. \tag{37}$$

If $g < (\pi/2M)\min_i\left(q_i^* w_i^*; i = 1, \ldots, S\right)$, from (36) we have

$$\frac{e^{-j\omega'\tau_i}}{j\omega'\tau_i}\frac{(j\omega'\tau_i + a\tau_i)}{\left(j\omega'\tau_i + q_i^* w_i^*\right)}\frac{Mg}{q_i^* w_i^*} > -1. \tag{38}$$

In other words, $\Lambda_i(j\omega)$ only cross the real axis on the right side of point -1, and thus never encircle -1 in the complex plane. Therefore, the system is stable. ∎

IV. Conclusions

In this letter, we have proposed VQR, a new AQM scheme based on the virtual queue length and the aggregate flow rate. VQR can achieve high link utilization and near zero queuing delay. We have also presented and proved the local stability condition for a TCP/VQR based network with an arbitrary topology, heterogeneous round-trip times, and backward propagation delays.

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