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Large System Decentralized Detection Performance Under Communication Constraints

Sudharman K. Jayaweera, *Member, IEEE*

***Abstract*— The problem of decentralized detection in a sensor network subjected to a total average power constraint and all nodes sharing a common bandwidth is investigated. The bandwidth constraint is taken into account by assuming non-orthogonal communication between sensors and the data fusion center via direct-sequence code-division multiple-access (DS-CDMA). In the case of large sensor systems and random spreading, the asymptotic decentralized detection performance is derived assuming independent and identically distributed (iid) sensor observations via random matrix theory. The results show that, even under both power and bandwidth constraints, it is better to combine many not-so-good local decisions rather than relying on one (or a few) very-good local decisions.**

***Index Terms*— Data fusion, distributed detection, large-system analysis, sensor networks.**

I. Introduction

This paper considers decentralized detection in energy-constrained, large wireless sensor networks in noisy, band-limited channels. Although there is a considerable amount of previous work on the subject of distributed detection, most of it used to ignore the effect of noisy channels between the local sensors and data fusion center. Even less is the attention received by bandlimited noisy channels in the context of decentralized detection. For example, while distributed detection performance of an energy-constrained wireless sensor network over a noisy channel has been considered recently [1], it assumes orthogonal sensor-to-fusion center communication leading to an infinite bandwidth assumption. However, in applications involving dense, low-power, distributed wireless sensor networks it is more likely that all nodes will share a common available bandwidth. In this case, the assumption of large sensor systems implies non-orthogonal communication between the sensor nodes and the fusion sensor.

An important design objective in low-power wireless sensor systems is to extend the whole network lifetime. Thus, a sensible constraint on the sensor system is a finite total power [1]. In this paper, the bandwidth constraint is taken into account by assuming non-orthogonal direct-sequence code-division multiple-access (DS-CDMA) communication between sensors and the data fusion center. The main contribution of this paper is the derivation of the decentralized detection performance, in closed-form, under a total power constraint when the communication channel between the local sensors and the fusion center is both bandlimited and noisy. As we will see, the performance is a function of the exact signalling codes used by the distributed sensors for any finite-size sensor network. However, in the case of random spreading we are able to derive an elegant and simple closed-form expression that is independent of the exact spreading codes once we consider asymptotically large sensor systems. This is our main result and, as we will see, it allows us to draw general conclusions regarding the design of wireless sensor systems under such total power constraints in noisy and bandlimited channels.

The remainder of the paper is organized as follows: In Section II we present our system model. Next, in Section III we use random matrix theory to derive a closed-form expression for the decentralized detection performance in a large sensor system followed by a discussion of our analysis. Finally, in Section IV we conclude by summarizing our results.

II. System Model Description

We consider a binary hypothesis testing problem in an N_s-node wireless sensor network connected to a data fusion center via distributed parallel architecture. Let us denote by H_0 and H_1 the null and alternative hypotheses, respectively, having corresponding prior probabilities $P(H_0) = p_0$ and $P(H_1) = p_1$. We will consider that the observed stochastic process under each hypothesis consists of one of two possible Gaussian signals, denoted by $X_{0,n}$ and $X_{1,n}$, corrupted by additive white Gaussian noise. Under the two hypotheses the n-th local sensor observation z_n, for $n = 1, \cdots N_s$, can be written as

$$\begin{aligned} H_0: \quad z_n &= X_{0,n} + v_n \\ H_1: \quad z_n &= X_{1,n} + v_n \end{aligned} \tag{1}$$

where the observation noise v_n is assumed to be zero-mean Gaussian with the collection of noise samples having a covariance matrix Σ_v. Each local sensor processes its observation z_n independently to generate a local decision $u_n(z_n)$ which are sent to the fusion center. Let us denote by $\mathbf{r}(u_1(z_1), u_2(z_2), \cdots, u_{N_s}(z_{N_s}))$ the received signal at the fusion center. The fusion center makes a final decision based on the decision rule $u_0(\mathbf{r})$. The problem at hand is to choose $u_0(\mathbf{r}), u_1(z_1), u_2(z_2), \cdots, u_{N_s}(z_{N_s})$ so that a chosen performance metric is optimized.

The solution to this problem is known to be too complicated under the most general conditions [2]. While optimal local processing schemes have been derived under certain special assumptions, a class of especially important local processers

The author is with the Department of Electrical and Computer Engineering, Wichita State University, Wichita, KS, 67260, USA. E-mail: sudharman.jayaweera@wichita.edu.

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are those that simply amplify the observations before retransmission to the fusion center [1], [3]. Thus, the local sensor decisions sent to the fusion center are given by, $u_n = gz_n$ for $n = 1, \cdots N_s$ where $g > 0$ is the analog relay amplifier gain at each node. In our model all sensor nodes share a common bandwidth and a total available energy. For analytical reasons, as well as due to their practical relation to DS-CDMA communications, we consider bandwidth sharing non-orthogonal communication based on spreading in which each sensor node is assigned a signature code of length N. If the n-th sensor node is assigned the code $\mathbf{s}_n$, the received chip-matched filtered and sampled discrete-time signal at the fusion center can be written as $\mathbf{r} = g\sum_{n=1}^{N_s} \mathbf{s_n} z_n + \mathbf{w} = g\mathbf{S}\mathbf{z} + \mathbf{w}$ where $\mathbf{r}$ and $\mathbf{w}$ are N-dimensional received signal and receiver noise vectors, respectively and the n-th column of the $N \times N_s$ matrix $\mathbf{S}$ is equal to the vector $\mathbf{s}_n$. We assume that the receiver noise is a white Gaussian noise process so that the filtered noise vector $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma_w^2 \mathbf{I}_N)$.Then we have that

$$\begin{aligned} H_0 : \quad & \mathbf{r} \sim \mathcal{N}(\mathbf{m}_0, \Sigma_0) \\ H_1 : \quad & \mathbf{r} \sim \mathcal{N}(\mathbf{m}_1, \Sigma_1) \end{aligned} \tag{2}$$

where, for $j = 0, 1$, $\mathbf{m}_j = g\mathbf{S}\mathbb{E}\{\mathbf{X}_j\}$ and $\Sigma_j = g^2\mathbf{S}\left(Cov(\mathbf{X}_j) + \Sigma_v\right)\mathbf{S}^T + \sigma_w^2\mathbf{I}_N$.

To be specific, consider the detection of a deterministic signal so that $\mathbf{X}_1 = -\mathbf{X}_0 = m\mathbf{1}$ is known ($m > 0$) and $\Sigma_0 = \Sigma_1 = \Sigma$ where ($\mathbf{1}$ is the vector of all ones) $\Sigma = g^2\mathbf{S}\Sigma_v\mathbf{S}^T + \sigma_w^2\mathbf{I}_N$. With these assumptions we also have that $\mathbf{m}_1 = -\mathbf{m}_0 = gm\mathbf{S}\mathbf{1}$ and the radiated power of node n is then given by $\mathbb{E}\{|u_n|^2\} = g^2\mathbb{E}\{|z_n|^2\} = g^2(m^2 + \sigma_v^2)$ where σ_v^2 is the observation noise variance. Let us define the total power constraint the whole sensor system is subjected to as P, so that the amplifier gain g is given by

$$g = \sqrt{\frac{P}{N_s(m^2 + \sigma_v^2)}}. \tag{3}$$

Then, it can be shown that the optimal threshold rule at the fusion center is of the form

$$u_0(\mathbf{r}) = \begin{cases} 1 & \\ & \text{if} \quad T(\mathbf{r}) \begin{matrix} \geq \\ < \end{matrix} \tau' , \\ 0 & \end{cases} \tag{4}$$

where we have defined the decision variable T as $T(\mathbf{r}) = (\mathbf{m}_1 - \mathbf{m}_0)^T \Sigma^{-1}\mathbf{r} = 2gm\mathbf{1}^T\mathbf{S}^T\left(g^2\mathbf{S}\Sigma_v\mathbf{S}^T + \sigma_w^2\mathbf{I}_N\right)^{-1}\mathbf{r}$ and τ' is the threshold that depends on the specific optimality criteria. It can be shown that the false-alarm P_f and miss P_m probabilities of the detector (4) are given by

$$P_f = Q\left(\frac{\tau' + 2g^2m^2\mathbf{1}^T\mathbf{S}^T\Sigma^{-1}\mathbf{S}\mathbf{1}}{2gm\sqrt{\mathbf{1}^T\mathbf{S}^T\Sigma^{-1}\mathbf{S}\mathbf{1}}}\right), \tag{5}$$

and

$$P_m = Q\left(\frac{2g^2m^2\mathbf{1}^T\mathbf{S}^T\Sigma^{-1}\mathbf{S}\mathbf{1} - \tau'}{2gm\sqrt{\mathbf{1}^T\mathbf{S}^T\Sigma^{-1}\mathbf{S}\mathbf{1}}}\right). \tag{6}$$

For example, in the case of Neyman-Pearson optimality at the fusion center, τ' is chosen to minimize P_m subject to an upper bound on P_f. On the other hand under Bayesian minimum probability of error optimality one would choose τ' to minimize $P_e = p_0P_f + p_1P_m$. As one would expect, the performance of course depends on the particular codes assigned to each sensor node as seen from (5) and (6). Thus, while it is possible to evaluate the performance for specific systems via (5) and (6), it is rather difficult to draw general conclusions regarding the design of decentralized detection systems. However, such conclusions can be reached for large systems through asymptotic analysis, as we show next.

III. Large Sensor System Performance Analysis

Let us assume that the spreading codes are chosen randomly so that each element of $\mathbf{s}_n$ takes either $\frac{1}{\sqrt{N}}$ or $-\frac{1}{\sqrt{N}}$ with equal probability. Moreover, we take independent sensor observations such that $\Sigma_v = \sigma_v^2\mathbf{I}$. Let us assume a large sensor system such that both N_s and N are large such that $\lim_{N\longrightarrow\infty} \frac{N_s}{N} = \alpha$. Now using a theorem on the convergence of the empirical distribution of eigenvalues of a large random matrix proven in [4], we may prove the following proposition, which is the main result of this paper:

Proposition 1: With $\mathbf{S}$ and Σ defined as above,

$$g^2\mathbf{1}^T\mathbf{S}^T\Sigma^{-1}\mathbf{S}\mathbf{1} \longrightarrow \left(\frac{\sigma_v^2}{N_s} + \frac{m^2 + \sigma_v^2}{P\beta_0}\right)^{-1}, \tag{7}$$

almost surely, as $N \longrightarrow \infty$, where

$$\beta_0 = \frac{\sqrt{\left(\gamma + \sigma_w^2\right)^2\alpha^2 + 2\gamma(\sigma_w^2 - \gamma)\alpha + \gamma^2} - \left(\gamma + \sigma_w^2\right)\alpha + \gamma}{2\gamma\sigma_w^2} \tag{8}$$

with $\gamma = \frac{P}{N}\left(1 + \frac{m^2}{\sigma_v^2}\right) - 1$ and $\Sigma_v = \sigma_v^2\mathbf{I}$.

Proof: See Appendix I.

The proposition 1 leads to the following corollary on the asymptotically large sensor system performance of decentralized detection in noisy bandlimited channels:

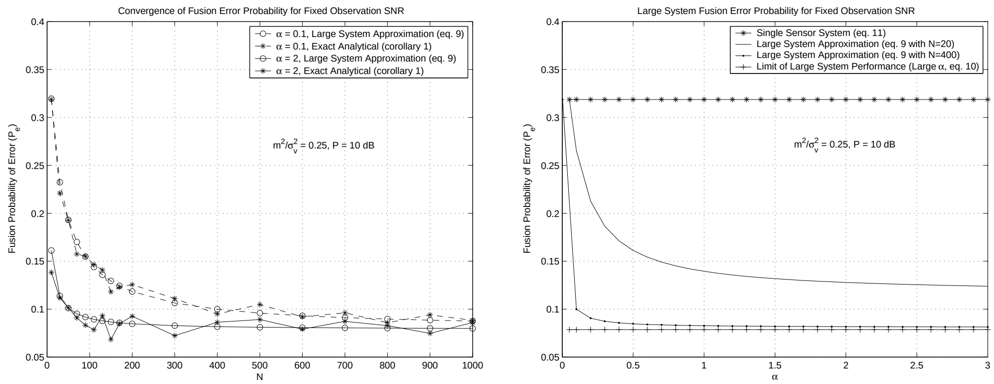
Corollary 1: With all notation as defined above, when $\lim_{N\longrightarrow\infty} \frac{N_s}{N} = \alpha$, the large sensor network performance of the decentralized detection is given by $P_f \longrightarrow Q\left(\sqrt{\mu}(\tau' + \frac{2m^2}{\mu})/2m\right)$ and $P_m \longrightarrow Q\left(\sqrt{\mu}(\frac{2m^2}{\mu} - \tau')/2m\right)$ where $\mu = \frac{\sigma_v^2}{N_s} + \frac{m^2+\sigma_v^2}{P\beta_0}$.

The above corollary leads to insights on large sensor system performance of decentralized detection in noisy, bandlimited channels. For instance, in the special case of minimum probability of error optimality at the fusion center, according to corollary 1, the large system probability of error is asymptotically given by

$$P_e(\alpha) \longrightarrow Q\left(m/\sqrt{\mu}\right), \tag{9}$$

where convergence is almost surely and μ is as defined above.

Figure 1a shows the convergence of the random-spreading based decentralized detection performance as predicted by (9). Note that the exact analysis in Fig. 1a was obtained for a random choice of the code matrix $\mathbf{S}$. As can be seen from Fig. 1a, (9) provides a good approximation to the detection performance for large spreading lengths N, and thus for large-sensor systems (since $N_s = N\alpha$). More importantly, we can observe from Fig. 1a that for each fixed N, increasing α improves the decentralized detection performance. Since this is equivalent to increasing the number of sensors N_s allowed



(a) (b)

Fig. 1. Decentralized Detection Performance in a Noisy, Bandlimited Channel Subjected to a Total Power Constraint (a) Large Sensor System Approximation (b) Limit of Large Sensor System Approximation when $\alpha \longrightarrow \infty$.

in the system for a fixed bandwidth we conclude that it is better to allow as many sensors to send their local decisions to the fusion center.

In fact, for large alpha, one can show that $\beta_0 \longrightarrow \frac{1}{\sigma_w^2}$, and as a result, in this case the error probability in (9) goes to

$$P_e(\alpha) \quad \longrightarrow \quad Q\left(\sqrt{\frac{P}{\sigma_w^2}\left(1+\frac{\sigma_v^2}{m^2}\right)^{-1}}\right). \qquad (10)$$

On the other hand, if one were to allocate all available power P and the total bandwidth to just one sensor node the fusion center performance will be given by

$$P_{e,1} \quad = \quad Q\left(\sqrt{\frac{P}{\sigma_w^2}\left(\frac{P/\sigma_w^2}{m^2/\sigma_v^2}+\left(1+\frac{\sigma_v^2}{m^2}\right)\right)^{-1}}\right). \qquad (11)$$

Comparison of (10) and (11) shows that allowing more sensor nodes in the network is better even if the channel is both noisy and bandlimited. This comparison is shown in Fig. 1b. First, we can observe from Fig. 1b that as N increases the fusion center performance improves. Secondly we see that as $N \longrightarrow \infty$, the performance for large α indeed goes to (10). Third, Fig. 1b confirms that combining more local decisions is better than allocating all available power and bandwidth to one sensor. Moreover, the performance improves monotonically with increasing α (for a fixed N) showing that it is better to combine as many local decisions as possible at the fusion center. We should divide the available power among all nodes and allow all of them to share the available bandwidth even if they are to interfere with each other due to non-orthogonality.

IV. Conclusions

We analyzed the decentralized detection performance of a total average power constrained wireless sensor network in a noisy and bandlimited channel. Assuming that the sensors-to-fusion center communication is based on DS-CDMA, a closed form expression for the fusion performance, and its large system asymptotic under random spreading were derived. It was shown that in a noisy, bandlimited channel it is beneficial to combine as many sensor local decisions as possible even if this leads to non-orthogonal sensor-to-fusion center communication.

Appendix I
The Proof of Proposition 1

Proof: Using the definitions of $\mathbf{S}$ and $\mathbf{1}$, we can write

$$g^2\mathbf{1}^T\mathbf{S}^T\Sigma^{-1}\mathbf{S}\mathbf{1} = g^2\left(\sum_{n=1}^{N_s}\mathbf{s}_n^T\Sigma^{-1}\mathbf{s}_n + \sum_{n=1}^{N_s}\sum_{\substack{n'=1\\ n'\neq n}}^{N_s}\mathbf{s}_n^T\Sigma^{-1}\mathbf{s}_{n'}\right) \qquad (12)$$

Let $\mathcal{I}$ denote a set of sensor indices (i.e. $\mathcal{I} \subset \{1,2,\cdots,N_s\}$), $\mathbf{S}_{\mathcal{A}}$ denote the matrix $\mathbf{S}$ with column indices specified by set $\mathcal{A}$ deleted, $\mathbf{\Lambda}_n = g^2\sigma_v^2\mathbf{I}_n$ and $\mathbf{Q}_{\mathcal{A}} = \left(\mathbf{S}_{\mathcal{A}}\mathbf{\Lambda}_{N_s-|\mathcal{A}|}\mathbf{S}_{\mathcal{A}} + \sigma_w^2\mathbf{I}_N\right)$ where $\mathbf{I}_n$ and $|\mathcal{A}|$ are the $n\times n$ identity matrix and the cardinality of set $\mathcal{A}$, respectively. Then, for $n = 1,\cdots,N_s$, using the matrix inversion lemma we can show that $\mathbf{s}_n^T\Sigma^{-1}\mathbf{s}_n = \mathbf{s}_n^T\mathbf{Q}_{\{n\}}^{-1}\mathbf{s}_n/(1+g^2\sigma_v^2\mathbf{s}_n^T\mathbf{Q}_{\{n\}}^{-1}\mathbf{s}_n)$. But, applying Theorem 7 of [4] and using (3), we can show that $\mathbf{s}_n^T\mathbf{Q}_{\{n\}}^{-1}\mathbf{s}_n \longrightarrow \beta_0$ almost surely, where β_0 is as given by (8) and $\gamma = \frac{P}{N}\left(1+\frac{m^2}{\sigma_v^2}\right)^{-1}$. Combining these we have almost surely

$$\mathbf{s}_n^T\Sigma^{-1}\mathbf{s}_n \quad \longrightarrow \quad \left(\beta_0{}^{-1} + g^2\sigma_v^2\right)^{-1}. \qquad (13)$$

Similarly, repeated application of matrix inversion lemma twice show that,

$$\mathbf{s}_n^T\Sigma^{-1}\mathbf{s}_{n'} = \frac{\mathbf{s}_n^T\mathbf{Q}_{\{n,n'\}}^{-1}\mathbf{s}_{n'}}{\left(1+g^2\sigma_v^2\mathbf{s}_n^T\mathbf{Q}_{\{n\}}^{-1}\mathbf{s}_n\right)\left(1+g^2\sigma_v^2\mathbf{s}_{n'}^T\mathbf{Q}_{\{n,n'\}}^{-1}\mathbf{s}_{n'}\right)}. \qquad (14)$$

Now the use of Theorem 7 of [4] shows that RHS goes to zero almost surely, for $n \neq n'$. Substituting (13) and (14) in (12) gives (7), completing the proof.

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