# Out-of-Band Power Reduction in NC-OFDM with **Optimized Cancellation Carriers Selection**

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Abstract-In this letter, we propose a computationally efficient method for joint selection of cancellation carriers (CCs) and calculation of their values minimizing the out-of-band (OOB) power in non-contiguous (NC-) OFDM transmission. The proposed new CCs selection method achieves higher OOB power attenuation than algorithms known from literature as well as noticable reception performance improvement.

Index Terms-NC-OFDM, spectrum shaping, cancellation carriers, out-of-band power reduction

#### I. INTRODUCTION

**P**RACTICAL spectrum shaping algorithms are of great importance for the development of a single state portance for the development of wireless communications systems as they allow for systems coexistence in the adjacent frequency bands and higher spectrum utilization. In case of non-contiguous orthogonal frequency division multiplexing (NC-OFDM) high out-of-band (OOB) radiation is observed at the output of the power amplifier (PA) caused by: high Peakto-Average Power Ratio (PAPR) and related PA nonlinear distorions, as well as by the subcarrier spectrum sidelobes having the shape of the sinc-like function. Although digital filtering can reduce the OOB radiation it involves high computational complexity especially when high filter-selectivity is required [1]. Moreover, in the dynamically changing radio frequency opportunities, on-line filter design is usually unacceptable. Cancellation carriers method presented in [1], [2] is a flexible way of reducing OOB power by minimizing the spectrum sidelobes. It does not have limitations of filtering, and provides significant OOB power attenuation, especially when steep and narrow spectrum notches are required. Unfortunately, in some cases CCs can use significant fraction of the OFDM symbol power. In [2], [3] optimization mechanisms for each OFDM symbol are suggested to overcome this problem. However, online optimization are computationally complex and impractical in hardware implementation. Extension of CCs utilizing subcarriers non-orthogonal to the OFDM subcarriers and frequency bins in the OOB region has been presented in [4]. The method results in high OOB radiation suppression at the cost of increased computational complexity and introduced self interference. An overview of the existing sidelobes suppression

methods can be found in [5]. There, the combination of the CCs and the time-domain windowing is discussed, and its performance results are presented showing higher OOB power suppression than the CCs method alone. The drawback of the CC method is that it has to be adjusted for each particular subcarriers pattern matching fragmented frequency bands.

In this letter, we show a computationally efficient method of the CCs calculation based on stochastic approach, which dynamically adjusts to the subcarriers-pattern. The mean CCs power is also constrained. The computational complexity of CCs calculation is decreased for each OFDM symbol in comparison with [2] as it boils down to matrix-vector multiplication. Additionally, determination of the CCs-calculation matrix is made less computationally expensive. The CCs-calculation matrix is constant for a given set of system parameters and can be used at the receiver for improving reception quality.

Moreover, in this letter, the location of CCs is also revised. Typically equal number of CCs is placed on each side of data-occupied subcarriers blocks [1], [2]. Here, we propose optimized CCs selection (OCCS), i.e. the heuristic approach to choose CCs locations iteratively. Our method outperforms significantly traditional approachs in terms of the OOB power reduction for a given number of CCs, what is obtained at the only cost of the iterative off-line low-computationally complex design of the CCs-calculation matrix.

In Section II we describe the system model. In Section III we state the optimization problem, and present its computationally-efficient solution and the OCCS heuristic. Simulation results are shown in Section IV followed by the conclusions in Section V.

#### **II. SYSTEM MODEL**

We consider a wireless digital communication link based on NC-OFDM technique. In every OFDM symbol interval,  $\alpha$ complex data symbols e.g. QAM, constituting a vector  $d_{DC}$ modulate the data carriers (DC) which are selected input bins of the N-th order Inverse Fast Fourier Transform (IFFT). Another set of  $\beta$  subcarriers is used as CCs, and constite vector  $d_{CC}$ . Merging of both vectors  $d_{DC}$  and  $d_{CC}$  results in vector d of length  $\alpha + \beta \leq N$ . Vectors of data and cancellation carriers indices are denoted as  $I_{DC} = \{I_{DC_i}\}$  and  $I_{CC} =$  $\{I_{CCl}\}$  respectively (where  $j = 1, ..., \alpha$  and  $l = 1, ..., \beta$ ). Merging of both vectors gives a vector  $I_C = [I_{CC}, I_{DC}]$ containing subcarriers indices from the set of N IFFT inputs indexed as:  $\{-N/2, ..., N/2 - 1\}$ . After IFFT,  $N_{\rm CP}$  samples of cyclic prefix (CP) are inserted and symbol samples  $y_n$  $(n = -N_{\rm CP}, ..., N - 1)$  are subject to the D/A conversion.

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The OFDM symbol spectrum S(v) at normalized frequency v can be obtained by Fourier transformation of a time domain signal  $y_n$  as in [1], [5]:

$$S(v) = \frac{1}{\sqrt{N}} \sum_{k \in \mathbf{I_C}} d_k S(v, k) , \qquad (1)$$

where S(v, k) is the k-th subcarrier spectrum:

$$S(v,k) = \sum_{n=-N_{CP}}^{N-1} \exp\left(j2\pi \frac{n(k-v)}{N}\right).$$
 (2)

# III. NEW IMPROVED CANCELLATION-CARRIERS METHOD

The idea of CCs is to modulate selected carriers with specific non-information symbols which minimize the OOB power measured at certain spectrum sampling points constituting vector  $\mathbf{V} = \{V_i\}$  of  $\gamma$  elements  $(i = 1, ..., \gamma)$ . Given vectors  $\mathbf{V}$  and  $\mathbf{I_{DC}}$  we can define matrix  $\mathbf{P_{DC}} = \{P_{\text{DC}i,j}\}$ of size  $\gamma \times \alpha$  with elements  $P_{\text{DC}i,j} = S(V_i, I_{\text{DC}j})$ . Similarly we can define matrix  $\mathbf{P_{CC}} = \{P_{\text{CC}i,l}\}$  of size  $\gamma \times \beta$ , where  $P_{\text{CC}i,l} = S(V_i, I_{\text{CC}l})$ . Then, the optimization problem to find vector  $\mathbf{d_{CC}}$  resulting in the minimum of the OOB power:

$$\min_{\mathbf{d}_{\mathbf{CC}}} \|\mathbf{P}_{\mathbf{CC}} \mathbf{d}_{\mathbf{CC}} + \mathbf{P}_{\mathbf{DC}} \mathbf{d}_{\mathbf{DC}} \|^2$$
(3)  
s.t.  $\|\mathbf{d}_{\mathbf{CC}}\|^2 \le \beta$ ,

where  $\|\cdot\|$  is the Euclidean vector norm. Here above, it is assumed that the power of CCs should be equal or lower than  $\beta$ making mean CC power equal to or lower than the normalized data symbols power, which is assumed to be 1.

# A. New efficient way of problem solution

Let us study the Lagrange function for problem (3):

$$f(\mathbf{d}_{\mathbf{CC}}, \theta) = \|\mathbf{P}_{\mathbf{CC}}\mathbf{d}_{\mathbf{CC}} + \mathbf{P}_{\mathbf{DC}}\mathbf{d}_{\mathbf{DC}}\|^2 + \theta \left(\|\mathbf{d}_{\mathbf{CC}}\|^2 - \beta\right)$$
(4)

where  $\theta$  is the Lagrange multiplier active, i.e. taking the value higher than zero, when CCs power is to be limited and 0 otherwise, according to Karush-Kuhn-Tucker conditions. The solution of  $\frac{\partial f(\mathrm{dcc},\theta)}{\partial \mathrm{dcc}} = 0$  gives

$$\mathbf{d_{CC}} = -\left(\mathbf{P_{CC}}^{\mathcal{H}}\mathbf{P_{CC}} + \theta \mathbf{I}\right)^{-1}\mathbf{P_{CC}}^{\mathcal{H}}\mathbf{P_{DC}}\mathbf{d_{DC}} = \mathbf{W}\mathbf{d_{DC}}$$
(5)

where  $()^{\mathcal{H}}$  and **I** denote Hermitian transpose and identity matrix respectively, and **W** is our CCs-calculation matrix. Typically, as in [2],  $\theta$  has to be found to satisfy the constraint from (3) for each OFDM symbol. Here, we focus on satisfying the condition of the mean CCs power assuming that random symbols in  $\mathbf{d}_{\mathbf{DC}}$  are independent with zero mean and unit variance. This mean CCs power equals:

$$\mathbb{E}[\|\mathbf{d}_{\mathbf{CC}}\|^{2}] = tr\left(\mathbb{E}\left[\mathbf{d}_{\mathbf{DC}}^{\mathcal{H}}\mathbf{W}^{\mathcal{H}}\mathbf{W}\mathbf{d}_{\mathbf{DC}}\right]\right) = (6)$$

$$\mathbb{E}\left[tr\left(\mathbf{d}_{\mathbf{DC}}^{\mathcal{H}}\mathbf{W}^{\mathcal{H}}\mathbf{W}\mathbf{d}_{\mathbf{DC}}\right)\right] = tr\left(\mathbb{E}\left[\mathbf{d}_{\mathbf{DC}}\mathbf{d}_{\mathbf{DC}}^{\mathcal{H}}\right]\mathbf{W}^{\mathcal{H}}\mathbf{W}\right) = tr\left(\mathbf{W}^{\mathcal{H}}\mathbf{W}\right) = tr\left(\mathbf{W}^{\mathcal{H}}\mathbf{P}_{\mathbf{CC}}\left(\mathbf{P}_{\mathbf{CC}}^{\mathcal{H}}\mathbf{P}_{\mathbf{CC}} + \theta\mathbf{I}\right)^{-2}\mathbf{P}_{\mathbf{CC}}^{\mathcal{H}}\mathbf{P}_{\mathbf{DC}}\right),$$

where  $\mathbb{E}[\]$  and  $tr(\)$  denote the expectation and the matrix trace, respectively. Expression (6) is obtained due to the linearity of trace and expectation operators and cyclic property of trace. As this equation is nonlinear, finding the value of  $\theta$ , for which  $\mathbb{E}\left[\mathbf{d}_{CC}^{\mathcal{H}}\mathbf{d}_{CC}\right] \leq \beta$  requires the use of the Newton method, where calculation of matrix inverse and a number of matrix-by-matrix multiplications is done in each iteration.

Decreased computational complexity without results accuracy deterioration can be obtained by replacing  $\mathbf{P}_{CC}$  by its singular value decomposition (SVD), i.e.  $\mathbf{P}_{CC} = \mathbf{USV}^{\mathcal{H}}$ , where U and V are unitary matrices, and S is  $\gamma \times \beta$  diagonal matrix with  $\delta$  singular values on its diagonal. Assuming full rank  $\mathbf{P}_{CC}$  (what is usually the case), i.e.  $\delta$  equal to the minimum of  $\beta$  and  $\gamma$  we obtain:

$$\mathbb{E}[\|\mathbf{d}_{\mathbf{CC}}\|^{2}] =$$

$$tr\left(\mathbf{P}_{\mathbf{DC}}\mathbf{P}_{\mathbf{DC}}^{\mathcal{H}}\mathbf{U}\mathbf{S}\mathbf{V}^{\mathcal{H}}\left(\mathbf{V}\left(\mathbf{S}^{\mathcal{H}}\mathbf{S}+\theta\mathbf{I}\right)\mathbf{V}^{\mathcal{H}}\right)^{-2}\mathbf{V}\mathbf{S}^{\mathcal{H}}\mathbf{U}^{\mathcal{H}}\right) =$$

$$tr\left(\mathbf{P}_{\mathbf{DC}}\mathbf{P}_{\mathbf{DC}}^{\mathcal{H}}\mathbf{U}\mathbf{S}\left(\mathbf{S}^{\mathcal{H}}\mathbf{S}+\theta\mathbf{I}\right)^{-2}\mathbf{S}^{\mathcal{H}}\mathbf{U}^{\mathcal{H}}\right),$$

where the properties of trace, unitary matrix and matrix inversion have been used to obtain this result. With a modicum of algebra applied to (7) we get

$$tr\left(\mathbf{A} \ diag\left(\frac{|S_{1,1}|^2}{(\theta+|S_{1,1}|^2)^2}, ..., \frac{|S_{\delta,\delta}|^2}{(\theta+|S_{\delta,\delta}|^2)^2}, 0, ..., 0\right)\right) \leq \beta$$
(8)

where diag() denotes diagonal matrix with diagonal entries given in brackets and **A** is the positive semidefinite Hermitian matrix defined as:  $\mathbf{A} = \mathbf{U}^{\mathcal{H}} \mathbf{P}_{\mathbf{DC}} \mathbf{P}_{\mathbf{DC}}^{\mathcal{H}} \mathbf{U}$ . By analyzing the matrix and trace properties we finally obtain:

$$\mathbb{E}\left[\|\mathbf{d}_{\mathbf{CC}}\|^{2}\right] = \sum_{i=1}^{\delta} \frac{A_{i,i}|S_{i,i}|^{2}}{(\theta + |S_{i,i}|^{2})^{2}} \le \beta .$$
(9)

For a given set of CCs and DCs only three matrix operations, i.e. one singular value decomposition and two matrix-bymatrix multiplications, and a few scalar-based Newton algorithm iterations have to be performed to find  $\theta$ .

Additionally we can use matrices A, S and V for calculation of the mean OOB radiation and the final CCs-calculation matrix. The OOB radiation power, being minimized in problem (3) can be reformulated by using (5) as follows:

$$\min_{\mathbf{d}_{\mathbf{C}\mathbf{C}}} \|\mathbf{G}\mathbf{d}_{\mathbf{D}\mathbf{C}}\|^2 , \qquad (10)$$

where

$$\mathbf{G} = \mathbf{P}_{\mathbf{C}\mathbf{C}}\mathbf{W} + \mathbf{P}_{\mathbf{D}\mathbf{C}}.$$
 (11)

The mean OOB radiation power

$$P_{\text{OOB}} = \frac{1}{\gamma} \mathbb{E} \left[ \|\mathbf{G}\mathbf{d}_{\mathbf{DC}}\|^2 \right]$$
(12)

can be found by averaging the norm in (10) over all possible  $d_{DC}$  vectors (by repeating operations presented in (6)) for the number of spectrum sampling points  $\gamma$ . By following similar routine as in derivation from (6) to (9) we obtain:

$$P_{\text{OOB}} = \frac{1}{\gamma} \left( \sum_{i=\delta+1}^{\gamma} A_{i,i} + \sum_{i=1}^{\delta} A_{i,i} \left( \frac{\theta}{\theta + |S_{i,i}|^2} \right)^2 \right), \quad (13)$$

while for a basic system not using CCs, the OOB power equals:  $P_{\text{OOB}} = \frac{1}{\gamma} \|\mathbf{P}_{\mathbf{DC}}\|^2$ . Finally, CCs calculation matrix can be obtained as:

$$\mathbf{W} = -\mathbf{V}_{\delta} diag \left( \frac{S_{1,1}^{\mathcal{H}}}{\theta + |S_{i,i}|^2}, ..., \frac{S_{\delta,\delta}^{\mathcal{H}}}{\theta + |S_{\delta,\delta}|^2} \right) \mathbf{U}_{\delta}^{\mathcal{H}} \mathbf{P}_{\mathbf{DC}} ,$$
(14)

where  $V_{\delta}$  and  $U_{\delta}$  are submatrices containing only first  $\delta$  columns of matrices V and U, respectively. The optimum  $d_{CC}$  vector can be now easily calculated using this matrix W as in formula (5).

#### B. Computational complexity

The computational complexity of the original CCs algorithm is relatively high. According to [6] for each OFDM symbol about  $\alpha\gamma + \delta\beta + 2\delta M$  operations are needed for the calculation of the CCs symbols, where M is number of Newton method steps. In our case, (5) can be used directly or with precalculation of  $\mathbf{P_{DC}d_{DC}}$  ( $\mathbf{P_{DC}}$  decomposed from  $\mathbf{W}$  for  $\gamma \ll \beta$ ), what requires  $\alpha\beta$  or  $\alpha\gamma + \beta\gamma$  operations respectively. The computational complexity of our method in any case is significantly lower than the original one from [2].

### C. Heuristic approach to CCs allocation scheme: OCCS

Let us now review matrix **G** defined in (11). It projects data symbols  $d_{DC}$  onto spectrum samples in normalized frequencies **V** after CCs insertion. As data symbols are independent random variables with zero mean and unit mean power, coefficient  $|G_{i,j}|^2$  is the mean OOB radiation power caused at frequency sampling point  $V_i$  by data subcarrier indexed by  $I_{DCj}$ . If we partition **G** into vertical vectors  $\mathbf{g}_j$ , i.e.  $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, ..., \mathbf{g}_{\alpha}]$ , the mean OOB power over all spectrum sampling points caused by  $I_{DCj}$ -th data subcarrier equals:

$$\|\mathbf{g}_{\mathbf{j}}\|^{2} = \sum_{i=1}^{\gamma} |G_{i,j}|^{2}.$$
 (15)

It is obvious that the data subcarrier causing the highest OOB power has the biggest influence on the OOB region. As such it has the highest potential to be used as CC to be added in the counterphase to strong OOB components caused by other subcarriers, i.e. to cancel them. If it is used as CC it should decrease some  $|G_{i,j}|^2$  components. The OCCS criteria for finding index m of the DC to be used as CC is the following:

$$m = \underset{j}{\operatorname{argmin}} \|\mathbf{g}_{\mathbf{j}}\|^2.$$
(16)

The CCs selection has to be done iteratively as above. Single CC selection changes the matrix W changing also correlation properties between subcarriers. Typically, selection of a single CC causes that the other DCs in its neighborhood are not chosen to be the CC in the next step, because their influence on the OOB radiation is similar (highly correlated). The algorithm stops when the OOB radiation power achieves the required level  $P_{\rm OOBreq}$  (see the flow diagram of the OCCS in Fig.1).

Finally note, that the OCCS can be combined with the timedomain windowing similarly as the standard CCs method [5].

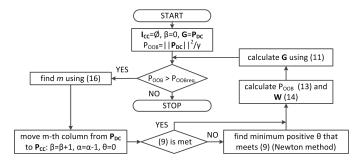


Fig. 1. Flow diagram of the OCCS algorithm.

# IV. SIMULATION RESULTS

For the proposed algorithm evaluation, we have considered an example NC-OFDM system with N = 256 subcarriers spaced by 15kHz (as in the LTE system [7]) and a set of occupied subcarriers indexed as:  $I_{C} = \{-80, ..., 16\} \cup \{49, ..., 80\}$ .

Notch spanning subcarriers from 17 to 48 (480kHz) can be occupied by a narrowband licensed system, e.g. a wireless microphone. The subcarrier indexed 0 is unoccupied. The algorithm uses  $\gamma = 485$  spectrum sampling points V distributed equally in the normalized frequency region  $\langle -125.75; -81 \rangle \vee$  $\langle 17; 48 \rangle \lor \langle 81; 125.75 \rangle$ . Matrix **W** is calculated for a number of CCs varying from 0 to 40, for various CP durations, for both standard CCs selection method and the OCCS heuristic. For the OCCS the heuristic algorithm in Fig. 1 has been used. For the standard selection of CCs, the index m indicates a subcarrier lying at the NC-OFDM band edges (not (16)). The resulting mean OOB power vs. CP duration is presented in Fig. 2. Note, that while standard CCs selection needs different number of CCs to obtain given mean OOB power, the OCCS method is nearly independent from  $N_{\rm CP}$ . Moreover, for each number of CCs, the OCCS method is not worse (in terms of the mean OOB power) than the standard method while outperforming it significantly as the required OOB power level decreases. Additionally, the shorter the CP, the higher saving in the number of required CCs. For example, if we require the mean OOB power of at least -40dB, for  $N_{\rm CP} = N/32$  we save about 44% of CCs (decrease from 34 to 19). These saved subcarriers can be used as DCs what increases the bit rate.

For the remaining results presentation we have chosen the system with the fixed number of CCs  $\beta = 19$ ,  $N_{\rm CP} = N/16$ and Gray mapped QPSK symbols. The comparison is done among the NC-OFDM reference system I, i.e. without any spectrum shaping mechanism where all  $\alpha + \beta$  subcarriers are DCs, and the systems applying standard CCs, our heuristic OCCS and OCCS combined with time-domain windowing (W) with  $\beta = 15$ , and window cyclic suffix  $N_{\rm CS} = 10$ , i.e. the parameters resulting in the same bit rate as systems using CCs only. The Rapp-model of the PA has been used with nonlinearity hardness parameter p = 10, and two values of the input back-offs (IBOs): 6 and 8 dB. In Fig. 3, we can see the power spectral densities (PSDs) calculated over 10000 OFDM symbols. The locations of selected CCs are also marked. While the standard selection of CCs decreases OOB power by 12 dB in comparison to reference system I, our OCCS method provides additional 6 dB of the OOB power

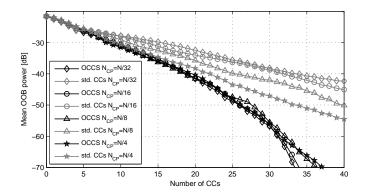


Fig. 2. The comparison of mean OOB power for standard (std.) CCs selection and OCCS scheme for different cyclic prefix durations at the PA input.

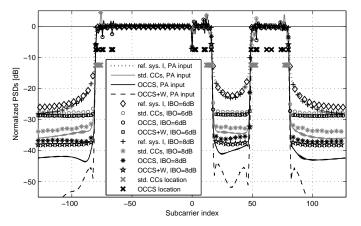


Fig. 3. PSDs of signals in ref. system I, the system with standard CCs, OCCS ( $\beta = 19$ ,  $N_{\rm CP} = N/16$ ), and OCCS with windowing (W) ( $\beta = 15$ ,  $N_{\rm CS} = 10$ ) at the PA input and output (with given IBO).

reduction. Moreover, our OCCS method lowers peaks rising in the in-band region, what helps the issue of satisfying the spectrum emission mask. Note that, as the IBO decreases, the intermodulations start to play dominant role in the OOB region and the effect of sidelobes minimization is less prominent.

Interestingly, our OCCS method results also in lower values of the complementary cumulative distribution function (CCDF) of PAPR than the standard CCs selection scheme, e.g. by 0.4 dB for  $CCDF(PAPR) = 10^{-4}$ , as shown in Fig.4.

In Fig. 5, the bit error rates (BERs) are shown for both CCs selection schemes considered with the option of matrix W available and nonavailable at the receiver. Both schemes are compared with the reference system II, in which subcarriers that could become standard CCs are instead modulated by zeros. This is for the sake of fair comparison of systems with equal bit rate. Matrix W can be used at the receiver to improve the performance making use of the redundancy symbols modulating CCs as shown in [5]. Here, the 9-paths Rayleigh channel model recommended for LTE [7] has been considered. There have been 50000 channel instances simulated with 1000 random OFDM symbols in each of them. The signalto-noise ratio (SNR) loss with respect to the reference system II is the same for both CC selection methods when the W matrix is not known in the receiver. This is due to sacrificing some transmission power to the CCs. This SNR-loss equals

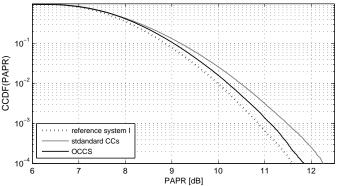


Fig. 4. CCDF of PAPR for NC-OFDM in the reference system I, and systems with standard and proposed CCs selection;  $\beta = 19$ ;  $N_{CP} = N/16$ 

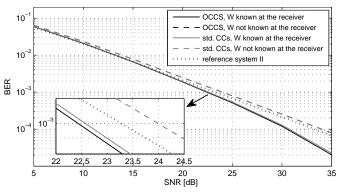


Fig. 5. BER vs. SNR for reference system II, the system with standard CCs alocation and OCCS;  $\beta = 19$ ,  $N_{\rm CP} = N/16$ 

 $10 \log_{10} (1 + \beta/\alpha)$ , i.e. about 0.7 dB in our scenario. The knowledge of the W matrix allows the receiver to decrease BER and obtain SNR gain even over reference system II (of 0.8 dB). The OCCS scheme outperforms the standard approach to CCs selection, although improvement of BER performance is relatively small (0.15 dB for BER= $10^{-3}$ ). As the CCs improve mostly the reception quality of data subcarriers in their frequency neighborhood, the OCCS providing sparse CCs pattern causes more DCs to be positively influenced.

## V. CONCLUSION

The CCs calculation and OCCS algorithms presented in this letter allow for lower computational complexity, lower OOB power and lower PAPR in comparison with the standard CCs algorithm. Moreover, the OCCS approach increases the SNR gain at the receiver if the decoder makes use of the known CCs calculation (coding) matrix. These properties make the proposed OCCS algorithm superior over the existing ones.

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