

An Iterative Algorithm for Optimal Carrier Sensing Threshold in Random CSMA/CA Wireless Networks

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Abstract—We investigate the optimal carrier sensing threshold in random CSMA/CA networks considering the effect of binary exponential backoff. We propose an iterative algorithm for optimizing the carrier sensing threshold and hence maximizing the area spectral efficiency. We verify that simulations are consistent with our analytical results.

Index Terms—CSMA/CA, carrier sensing threshold, iterative algorithm, stochastic geometry.

I. INTRODUCTION

To enhance wireless connectivity and capacity, efficient multiple access schemes for spatially randomly distributed nodes are necessary. The most widely used multiple access scheme is Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA). In this letter, we propose an iterative algorithm for finding the optimal carrier sensing threshold of spatially randomly distributed CSMA/CA wireless networks.

The fundamental processes in CSMA/CA are carrier sensing and random backoff. Carrier sensing provides the spatial resolution to concurrent transmitters and random backoff gives the temporal resolution to concurrent transmitters at the nearby place. The IEEE 802.11 Distributed Coordination Function (DCF) [1] utilizes physical carrier sensing (optionally virtual carrier sensing) and binary exponential backoff (BEB). In physical carrier sensing, such as the energy detection method, the node senses the medium to measure the aggregate interference, and transmission can begin only if the measured interference is below the carrier sensing threshold. In virtual carrier sensing, the node that intends to transmit performs proactive actions to prevent nodes in the vicinity from transmitting simultaneously with it. When the medium becomes idle, multiple transmitters would access simultaneously, causing collisions. By utilizing BEB, contention conflict is avoidable.

The carrier sensing threshold is a significant parameter to balance the tradeoff between the spatial reuse and the packet collision by controlling the aggregate interference. In [2], it is noted that optimizing carrier sensing is important to increase the throughput performance. The authors of [2] investigate the optimal carrier sensing range under the regular hexagonal topology, whereas we consider the spatially randomly distributed interferers to realistically capture the effect of interference using stochastic geometry [3].

Some researches conducted to determine the spatial distribution of transmitting nodes in the CSMA/CA network. In one such work [4], the authors applied the Matérn hard-core process (MHP) [3] to model the spatial transmitter pattern.

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MHP is a dependent thinning process of the Poisson point process (PPP) used to create separation of the marked points by at least a certain minimum distance. In [5], the authors proposed a simple sequential inhibition (SSI) point process to model for the same purpose, which is not mathematically tractable. In [4] and [5], backoff scheme was not considered and the optimal carrier sensing threshold is not provided.

Later, the authors of [6] investigated the throughput performance of dense CSMA networks from the stochastic geometry point of view. However, the authors of [6] did not find the optimal carrier sensing threshold and did not consider the effect of random backoff either. In [7], the authors investigated the optimal carrier sensing threshold based on a lower bound for the outage probability. They considered the effect of one strong interference and simplified the backoff scheme, whereas we consider the effect of aggregate interference and the effect of BEB, such as, collisions in the contention period, increasing of the backoff interval and backoff freezing behavior.

II. PROBLEM DEFINITION

The *area spectral efficiency* η (ASE), which is defined as the product of successfully transmitting node density and the data rate, provides a framework to quantify the capacity of the wireless network [8]. Our problem is to find the optimal carrier sensing threshold I_s^* that maximizes ASE η as follows:

$$I_s^* = \arg \max_{I_s} \lambda_t \log_2 (1 + \beta) p_s, \quad (1)$$

where λ_t denotes the active transmitter density in the contention free period of CSMA/CA and β means the target SIR. The transmission success probability is denoted by p_s .

We propose an iterative algorithm for finding I_s^* as described in Algorithm 1. We will explain how the proposed algorithm is obtained and show the performance of the algorithm.

III. SYSTEM MODEL

1) Topology and Channel Modeling: Consider a wireless network, in which all transmitters communicate with their receivers over a common wireless channel. Transmitters are located according to a homogeneous PPP with intensity λ . This kind of network topology is called the *Poisson bipolar network* [9]. Each transmitter i has infinite backlogged data to transmit. The transmitter/receiver pairs vary over time, but we focus on a snapshot of the overall communication process. The channel gain from transmitter i to receiver j is modeled by $g_{i,j} d_{i,j}^{-\alpha}$, where $g_{i,j}$ is an independently and identically distributed exponential random variable with unit mean, which reflects the effect of Rayleigh fading. The distance between nodes i and j is denoted by $d_{i,j}$ with the path loss exponent α . Using a common channel, different communication pairs can interfere with one another. Let P be the transmit power and

Algorithm 1 Proposed Algorithm.

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1: Initialize  $I_s^{\text{next}}$  with a small value less than  $r_t^{-\alpha}P$ 
2:  $I_s^{\text{current}} \leftarrow I_s^{\text{next}} + 1$ 
3: while  $I_s^{\text{next}} \neq I_s^{\text{current}}$  do
4:    $I_s^{\text{current}} \leftarrow I_s^{\text{next}}$ 
5:    $\tau_{\text{current}} \leftarrow 1, \tau_{\text{next}} \leftarrow 0$ 
6:   while  $\tau_{\text{next}} \neq \tau_{\text{current}}$  do
7:      $\tau_{\text{current}} \leftarrow \tau_{\text{next}}$ 
8:      $\tau_{\text{next}} \leftarrow \tau_{\text{current}} - \frac{(\tau_{\text{current}} - h(\tau_{\text{current}}))}{(1 - h'(\tau_{\text{current}}))} \triangleright h$  is RHS of (5)
9:   end while
10:   $\tau \leftarrow \tau_{\text{next}}$ 
11:  update  $\eta$  with  $\tau$   $\triangleright \eta$  is (10)
12:   $I_s^{\text{next}} \leftarrow I_s^{\text{current}} - \frac{\eta'(I_s^{\text{current}})}{\eta''(I_s^{\text{current}})}$ 
13: end while
14:  $I_s^* \leftarrow I_s^{\text{next}}$   $\triangleright$  get the optimal carrier sensing threshold

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an associated receiver j is at a distance of r_t from the typical transmitter i . Assuming the network is interference-limited and the receiver noise is ignored, then the signal-to-interference-ratio (SIR) γ_j is given by:

$$\gamma_j = \frac{g_{i,j} r_t^{-\alpha} P}{\sum_{u \in \mathcal{T}_i, j \notin \mathcal{T}_i} g_{u,j} d_{u,j}^{-\alpha} P} = \frac{g_{i,j} r_t^{-\alpha} P}{I}, \quad (2)$$

where I denotes the aggregate interference and \mathcal{T}_i denotes the set of concurrently transmitting (interfering) nodes when node i transmits. For a given target SIR β , a transmission succeeds if γ_j is greater than or equal to β . The data rate of the typical transmitter i is a function of β . We use Shannon's formula $\log_2(1+\beta)$ in which we assume a unit bandwidth.

2) *CSMA/CA Modeling*: Let us assume that the network employs the CSMA/CA random access scheme, especially, RTS/CTS mode [1]. In CSMA/CA with BEB, if the channel is idle during the predetermined time (DIFS in IEEE 802.11 DCF), the transmitters enter the contention period. Each transmitter should defer its transmission during a randomly selected slotted contention window. The backoff counter is decremented in each slot time if the channel is still sensed idle. When the backoff counter is expired, every contending communication pair exchanges control packets (RTS/CTS) to reserve a wireless channel. The transmitters who conducted this process successfully enter the contention-free period, and proceed data transmissions. If the transmission is failed, the contention window size increases exponentially.

The seminal works of [10] and [11] show that the effect of BEB can be appropriately modeled by p -persistence medium access analysis. We denote τ as a steady state medium access probability. Each transmitter accesses the medium by Bernoulli trial with probability τ . Therefore, after the medium becomes idle, contending node density is $\lambda\tau$. Let p_c be the collision probability of control messages in the contention period, and using the result of the [9], we obtain p_c as follows:

$$p_c = 1 - \exp\left(-\lambda\tau r_t^2 \beta_c^{\frac{2}{\alpha}} \frac{2\pi^2}{\alpha \sin(2\pi/\alpha)}\right), \quad (3)$$

where β_c denotes the target SIR for the control messages.

Let p_b be the channel busy probability, which is the probability that the aggregate interference is greater than or equal to

TABLE I: The medium access probability τ .

λ	0.0001				0.001				0.01			
I_s (dBm)	-40	-10	-40	-10	-40	-10	-40	-10	-40	-10	-40	-10
β_c (dB)	3	10	3	10	3	10	3	10	3	10	3	10
τ (simul.)	.053	.043	.055	.051	.023	.016	.026	.015	.005	.003	.007	.004
τ (anlys.)	.053	.047	.055	.048	.025	.017	.028	.018	.006	.004	.007	.004

a given carrier sensing threshold I_s . In [12], the authors derived the cumulative distribution function of the interference in PPP networks with Rayleigh fading. We modified their result with a transmit power term P , then p_b is

$$p_b = \Pr[I \geq I_s] = \text{erf}\left(\frac{\pi^2 \lambda \tau}{4} \sqrt{\frac{P}{I_s}}\right). \quad (4)$$

In [13], the authors derived τ considering the effect of BEB, such as, collision from overlapping of backoff counter, increase of backoff window size and freezing of backoff counter. However, they assumed a node can access the medium without backoff after successful transmission. This assumption is not compatible with the IEEE 802.11 [1]. We newly derived τ by correcting the erroneous assumption in [13] as follows:¹

$$\tau = \frac{2(1-p_b)(1-2p_c)}{(1-2p_c)(1-2p_b+W_0(2p_c)^m) + W_0(1-p_c)(1-(2p_c)^m)}, \quad (5)$$

where m is the maximum backoff stage and W_0 is the initial backoff window size. The value τ is a function of collision probability p_c and channel busy probability p_b . By substituting (3) and (4) into (5), let the right-hand side of (5) be $h(\tau)$. We obtain the fixed point formulation $\tau = h(\tau)$, which can be numerically solved by Newton's method as follows:

$$\tau_{n+1} = \tau_n - \frac{\tau_n - h(\tau_n)}{1 - h'(\tau_n)}, \quad (6)$$

where τ_n denotes the value of τ at n -th iteration and h' denotes a derivative of h with respect to τ . The iterative method (6) works well with an initial value $\tau_0=0$. The value τ is validated by simulations performed in NS-3 with various λ , I_s and β_c . In simulations, W_0 is 32 and increases up to 1024. The transmitters continuously generate 1KB packets to model the saturated traffic. The transmit power is 30dBm. The simulation area is 10km \times 10km. To model the PPP network, the number of transmitters N in the network is generated according to the Poisson distribution with $\lambda \times 10\text{km} \times 10\text{km}$. For example, if λ is 0.0001nodes/m², the average number of nodes in the area is 10⁴, where the transmitters are uniformly distributed.² The receivers are located at a distance r_t with random directions from transmitters. The Rayleigh fading channel is modeled by the Nakagami propagation loss component in NS-3 with proper parameter settings. The simulation results are averaged over hundreds of simulation runs. As shown in Table I, the simulation results are consistent with the analytical results.

After the RTS/CTS handshaking, more nodes are silenced, refraining the nearby nodes from transmitting simultaneously. The realistic backoff scheme along with the carrier sensing should be considered to model \mathcal{T}_i in (2) properly. By using

¹Due to space limitation, we omit the derivation of (5). Please see <http://hertz.yonsei.ac.kr/tau.pdf>.

²A homogeneous Poisson point process with λ in infinite area becomes a uniform distribution of k nodes on the finite area of size \mathcal{A} , where $k = \lambda\mathcal{A}$.

MHP [3], the active transmitter density λ_t in the contention-free period can be modeled as follows:

$$\lambda_t = \frac{1 - \exp(-\lambda\tau\pi R_s^2)}{\pi R_s^2}, \quad (7)$$

where R_s denotes the sensing range. The value τ is the probability that an arbitrary transmitter in the network completes the BEB process (i.e., backoff counter reaches 0) and accesses the channel. Therefore, $\lambda\tau$ accurately represents the node density of contending nodes. In this regard, we used a thinned node density $\lambda\tau$ instead of λ to model the effect of BEB in (7).

The sensing range R_s is a function of the physical carrier sensing threshold I_s . In [14], the authors used the mean value of the sensing range, which is given as follows:

$$R_s = D_5 \int_0^{D_0} f(r) dr + \sum_{i=1}^5 D_{5-i} \int_{D_{i-1}}^{D_i} f(r) dr + D_0 \int_{D_5}^{\infty} f(r) dr, \quad (8)$$

where $f(r) = 2\lambda\tau\pi r \exp(-\lambda\tau\pi r^2)$ and $D_i = \sqrt[3]{(i+1)P/I_s}$ for $i=0, \dots, 5$. The value D_i means the minimum distance from an arbitrary node to the interferers. It is clear that R_s is in inverse proportion to I_s . However, the dynamics between them is affected by the node density. In the sparse node density case, it is most probable that there is only one interferer nearby the sensing node. As node density grows, at most six strong interferers can exist at the same distance (refer to [14] for detail). In this regard, R_s can be approximated as $R_s \approx D_0$ and $R_s \approx D_5$ for sparse and dense cases, respectively. In next section, we will find the optimal carrier sensing threshold I_s^* .

IV. OPTIMAL CARRIER SENSING THRESHOLD

When I_s is high, most transmitters are simultaneously transmitting, making the success probability low. For lower I_s , more transmitters are silent, and the aggregate interference is less, leading to a higher success probability. Thus, there exists an optimal carrier sensing threshold that maximizes the ASE of (1). Fig. 1 shows the ASE of the CSMA/CA scheme as a function of I_s . The simulation and our analytical results are congruent. We observe that an optimal I_s exists, which is obtained by solving (1). To this end, the transmission success probability p_s in (1) is derived in the next subsection.

A. Transmission Success Probability of CSMA/CA

With the carrier sensing range R_s , the other transmitters within R_s should be silenced. The transmission of a typical transmitter is successful if $\gamma_j \geq \beta$ is satisfied. Assuming path loss exponent $\alpha=4$, which is validated for urban area, the transmission success probability p_s can be approximated in closed-form as follows:

$$p_s \approx \exp\left(-\pi\lambda_t \sqrt{\beta} r_t^2 \arctan\left(\frac{\sqrt{\beta} r_t^2}{R_s^2}\right)\right). \quad (9)$$

Details of the derivation are contained in Appendix.

B. Proposed Algorithm

We now explain our main result for the optimal carrier sensing threshold. Using (9), the ASE of (1) is as follows:

$$\eta = \lambda_t \log_2(1 + \beta) \exp\left(-\pi\lambda_t \sqrt{\beta} r_t^2 \arctan\left(\frac{\sqrt{\beta} r_t^2}{R_s^2}\right)\right). \quad (10)$$

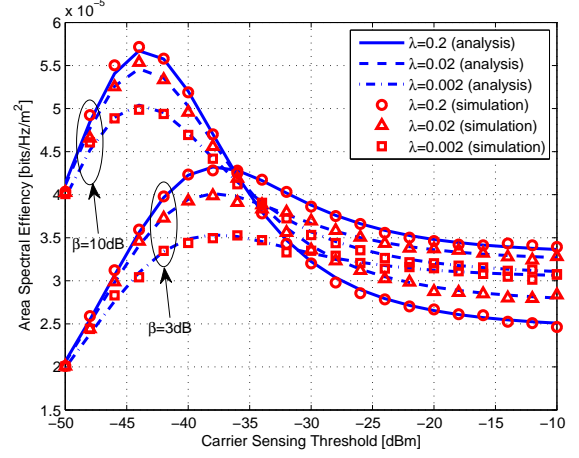


Fig. 1: Area spectral efficiency of CSMA/CA as a function of the carrier sensing threshold ($W_0 = 16$, $m = 32$, $r_t = 50$ m, $P = 30$ dBm).

Equation (10) is a function of R_s , and R_s is a function of I_s as shown in (8). The value λ_t is obtained using (7). Unfortunately, the closed-form solution of (1) is hard to find. One way to deal with the problem is making an algorithm where the transmitters update their sensing thresholds iteratively and distributively. Our proposed algorithm is given as follows:

- 1) First, initialize the carrier sensing threshold $I_s^{(0)}$ with a small value less than $r_t^{-\alpha} P$.
- 2) Next, find the value τ using Newton's method (6).
- 3) Update $I_s^{(n)}$ using the following Newton's method:

$$I_s^{(n+1)} = I_s^{(n)} - \frac{\eta'(I_s^{(n)})}{\eta''(I_s^{(n)})}, \quad (11)$$

where η' and η'' denote the first and second derivatives of η of (10) with respect to I_s .

- 4) Repeat procedures 2) and 3) until the solution is found.

The pseudo code of proposed algorithm is described in Algorithm 1. The proposed algorithm converges to an optimal value within a few iterations. The transmission distance r_t can be estimated by the received signal strength (RSS) and/or Global Positioning System (GPS) information. We conducted simulation based on the RSS method [15]. The RSS measurements are relatively inexpensive and simple to implement in hardware. To estimate the node density, a node collects the received power samples from its nearest neighbors and performs the maximum likelihood estimation. According to [16], the estimation results are highly accurate and the procedure is uncomplicated.

As shown in Fig. 2a, the results of the iterative solution and exhaustive search are coherent. The optimal carrier sensing threshold varies with the target SIR β . If β increases, I_s should be decreased to lower the active transmitter density. For the comparison, we also plot the optimal carrier sensing threshold obtained by ignoring the BEB (dashed line in Fig 2). In this case, the optimal carrier sensing range can be approximated as $1.1278\sqrt{\beta}r_t$ (derivation in Appendix). By ignoring the BEB, the active transmitter density is overestimated, where the corresponding optimal carrier sensing threshold is lower, causing performance degradation as shown in Fig. 2b. Fig. 2 shows

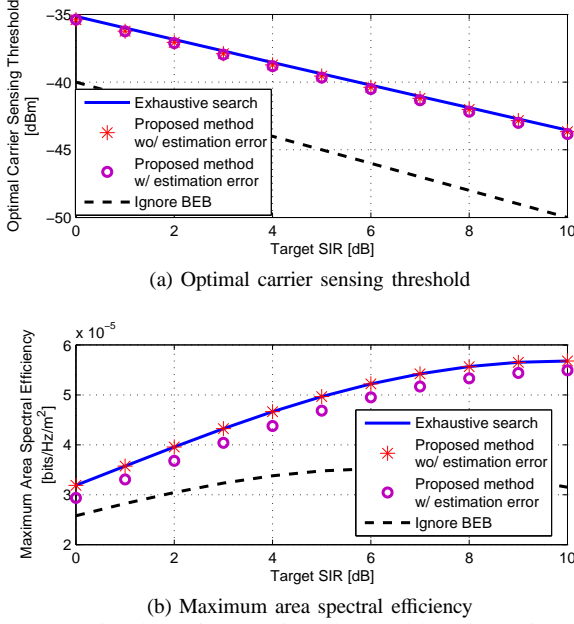


Fig. 2: Optimal carrier sensing threshold and maximum area spectral efficiency as a function of the target SIR ($\lambda=0.2$, $W_0=16$, $m=32$, $r_t=50$ m, $P=30$ dBm).

the impact of estimation errors on r_t and λ . The $r_t=50$ m is estimated by 54.8547m and the $\lambda=0.2$ is estimated by 0.1911. Even though we adopted rather primitive estimation methods, the proposed algorithm shows acceptable performance.

V. CONCLUDING REMARKS

We proposed a tractable approach for the optimal carrier sensing threshold of the random CSMA/CA networks. Most previous works using stochastic geometry overlooked the effect of the random backoff. We considered the effect of the practical backoff scheme and verified accuracy of our analysis by NS-3 simulations. Our analytical results could be employed in the design and optimization of high performing CSMA/CA networks. The spectrum sensing based cognitive radio network (CRN) is one of the viable applications of our work. If the spectrum is sensed as available, the multiple secondary transmitters would access concurrently, causing collisions. To avoid this situation, the CSMA/CA-based MAC protocol for the CRN is desirable. Our results can be used to find an optimal spectrum sensing level.

APPENDIX

1) *Derivation of (9)*: We denote I_{R_s} as the aggregate interference from the outside of region with the radius R_s . Using the fact that the channel gain $g_{i,j}$ is an exponential random variable and taking expectation of I_{R_s} , then p_s is:

$$p_s \approx \Pr \left[\frac{g_{i,j} r_t^{-\alpha} P}{I_{R_s}} \geq \beta \right] = \mathbb{E}_{I_{R_s}} \left[\exp \left(-\frac{\beta r_t^\alpha}{P} I_{R_s} \right) \right]. \quad (12)$$

By substituting $s = \beta r_t^\alpha / P$, (12) becomes the Laplace transform of shot-noise process I_{R_s} . Using the result of [9], (12) is

$$p_s \approx \exp \left(-2\pi\lambda_t \int_{R_s}^{\infty} \left(1 - \mathbb{E}_G \left[e^{-sGPv^{-\alpha}} \right] \right) v dv \right), \quad (13)$$

where v is a dummy variable representing the distance to a random interferer. Using the moment generating function of the exponential random variable, the probability p_s is

$$p_s \approx \exp \left(-2\pi\lambda_t \int_{R_s}^{\infty} \left(\frac{\beta}{\beta + v^\alpha / r_t^\alpha} \right) v dv \right). \quad (14)$$

Assuming $\alpha=4$, closed-form is obtained as follows:

$$p_s \approx \exp \left(-\pi\lambda_t \sqrt{\beta} r_t^2 \arctan \left(\frac{\sqrt{\beta} r_t^2}{R_s^2} \right) \right). \quad (15)$$

2) *Derivation of Optimal Sensing Threshold ignoring BEB*: Assuming high node density and $\alpha=4$, the value λ_t approximates $\lambda_t \approx 1/\pi R_s^2$. The objective function of (1) becomes:

$$\eta = \frac{\log_2(1+\beta)}{\pi R_s^2} \exp \left(-\frac{\sqrt{\beta} r_t^2}{R_s^2} \arctan \left(\frac{\sqrt{\beta} r_t^2}{R_s^2} \right) \right). \quad (16)$$

Differentiating (16) with R_s and simplifying exponential term:

$$\frac{\partial \eta}{\partial R_s} \approx \frac{2 \log_2(1+\beta)}{\pi R_s^3} \exp \left(-\frac{\beta r_t^4}{R_s^4} \right) \left(\frac{\beta r_t^4}{R_s^4 + \beta r_t^4} + \frac{\beta r_t^4}{R_s^4} - 1 \right) = 0. \quad (17)$$

By solving (17), R_s^* is obtained:

$$R_s^* = \left(0.5 (1 + \sqrt{5}) \beta r_t^4 \right)^{1/4} \approx 1.1278 \beta^{1/4} r_t. \quad (18)$$

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