Multi-antenna Wireless Powered Communication with Co-channel Energy and Information Transfer

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Abstract

This letter studies a multi-antenna wireless powered communication (WPC) system with co-channel energy and information transfer, where a wireless device (WD), powered up by wireless energy transfer (WET) from an energy transmitter (ET), communicates to an information receiver (IR) over the same frequency band. We maximize the achievable data rate from the WD to the IR by jointly optimizing the energy beamforming at the ET and the information beamforming at the WD, subject to their individual transmit power constraints. We obtain the optimal solution to this problem in closed-form, where the optimal energy beamforming at the ET achieves a best energy/interference tradeoff between maximizing the energy transfer efficiency to the WD and minimizing the co-channel interference to the IR. Numerical results show that our proposed optimal co-channel design is superior to other reference schemes.

Index Terms

Wireless powered communication (WPC), co-channel energy and information transfer, multi-antenna, co-channel interference.

I. INTRODUCTION

Radio-frequency (RF) signals enabled wireless energy transfer (WET) has been recognized as a promising technology to provide perpetual and convenient power supply to energy-constrained wireless networks. This motivates an appealing wireless powered communication (WPC) system, in which wireless information transfer (WIT) from wireless devices (WDs) to information receivers (IRs), e.g., sensor nodes delivering information to fusion centers, is powered up by the means of WET from dedicatedly deployed energy transmitters (ETs). In practice, the IR and the ET can be either separately located as two nodes (e.g., an information access point and a power beacon) [1]-[4], or co-located as a single node (e.g., a hybrid access point) [5]-[8]. In both cases, WET and WIT are usually assumed to be implemented over orthogonal time/frequency resources [1]-[8], so as to avoid co-channel interference from WET to WIT links at a cost of reduced spectrum utilization efficiency.

To improve the spectrum utilization efficiency, in this letter, we propose a new co-channel energy and information transfer scheme in a WPC system with separately located IR and ET, as shown in Fig. 1, where the WET and WIT links concurrently utilize the same frequency band, and the WD operates in a full-duplex mode with concurrent

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Fig. 1. WPC system model with co-channel energy and information transfer.



Fig. 2. A full-duplex wireless powered WD with concurrent energy harvesting and information transmission.

energy harvesting and information transmission. In this case, the co-channel interference from WET to WIT links is a key issue that limits the system performance. Therefore, it is essential for the ET to design its transmitted energy signal to balance the tradeoff between maximizing the energy transfer efficiency to the WD and minimizing the co-channel interference to the IR. On the other hand, the WD also needs to efficiently utilize its harvested energy from the ET and the recycled self-energy due to the full-duplex operation, so as to assure its information transmission quality to the IR. To this end, we apply multi-antenna techniques by assuming the ET and WD are both deployed with multiple transmit antennas.

Under this setup, in this letter, we maximize the achievable data rate from the WD to the IR, by jointly optimizing the transmit energy signal at the ET and the transmit information signal at the WD subject to their individual transmit power constraints. We obtain the optimal solution to this problem in closed-form, where the obtained energy beamforming at the ET optimally balances the aforementioned energy/interference tradeoff. It is also revealed that the optimal energy and information beamforming can be implemented at the ET and the WD in a distributed manner by only using their respective local channel state information (CSI). Numerical results show that the proposed optimal co-channel design achieves large throughput gains over three reference schemes.

In the literature, there are only limited studies considering multi-antenna co-channel WET and WIT [9] and [10]. However, [9] studied the coexistence of two separate WET and WIT systems, instead of a single WPC system as in this paper, while [10] ignored the co-channel interference from WET to WIT links. To our best knowledge, our results on the multi-antenna WPC system with co-channel energy and information transfer are new and have not been reported in the literature.

II. SYSTEM MODEL

We consider a WPC system consisting of one ET, one WD, and one IR, as shown in Fig. 1, where the ET delivers wireless energy to the WD, and at the same time the WD communicates to the IR over the same channel, by using the harvested energy from the ET and the recycled self-energy from its own transmission. We assume that the ET is equipped with $N_E > 1$ antennas for energy transmission, and the IR is deployed with a single antenna for information reception. Moreover, the WD is deployed with $1 + N_W$ antennas, $N_W > 1$, with one for

energy harvesting and the other N_W for information transmission.¹ In practice, to simultaneously harvest energy and transmit information, the WD can use a full-duplex rechargeable battery that can be charged and discharged at the same time [1], [10], an example of which is shown in Fig. 2. Denote the baseband equivalent channels from the ET to the WD and the IR by vectors $h \in \mathbb{C}^{N_E \times 1}$ and $f \in \mathbb{C}^{N_E \times 1}$, respectively, and that from the WD to the IR by $g \in \mathbb{C}^{N_W \times 1}$. Also denote the loop channel from the N_W transmit antennas of the WD to its receive antenna by $\varphi \in \mathbb{C}^{N_W \times 1}$. It is assumed that all channels are quasi-static flat-fading, where h, f, g, and φ remain constant during each transmission block with a length of T > 0, but can vary from one block to another. It is further assumed that the ET perfectly knows the CSI of h and f, while the WD accurately knows g and φ .² We denote the energy signal transmitted by the ET as $x_E \in \mathbb{C}^{N_E \times 1}$ and the information signal transmitted by the WD as $x_I \in \mathbb{C}^{N_W \times 1}$.

Consider first the WET from the ET to the WD. We consider that the transmitted energy signal x_E by the ET is a circularly symmetric complex Gaussian (CSCG) random sequence to meet certain power spectral density (PSD) requirements set by spectrum regulators. Accordingly, we denote the energy covariance matrix at the ET as $Q = \mathbb{E} (x_E x_E^H) \succeq 0$, where $\mathbb{E}(\cdot)$ denotes the statistical expectation, the superscript H denotes the conjugate transpose, and $A \succeq 0$ means that A is positive semi-definite. Here, we consider that Q is of a general rank without loss of generality [11]. Denote the maximum transmit power at the ET by P. The transmit power constraint at the ET is given by

$$\mathbb{E}\left(\|\boldsymbol{x}_E\|^2\right) = \operatorname{tr}(\boldsymbol{Q}) \le P,\tag{1}$$

where tr(A) denotes the trace of a square matrix A.

Next, consider the co-channel WIT from the WD to the IR. We consider transmit information beamforming at the WD. Thus, the information signal x_I transmitted by the WD can be expressed as $x_I = ws$, where $w \in \mathbb{C}^{N_W \times 1}$ denotes the information beamforming vector at the WD and *s* denotes the desired signal for the IR. We assume that *s* is a CSCG random variable with zero mean and unit variance, and is independent of the energy signal x_E sent by the ET. Due to the co-channel energy and information transfer, the received signal at the IR is $y = g^H ws + f^H x_E + n$, where $g^H ws$ is the desirable information signal, $f^H x_E$ is the interference due to the energy signal sent from the ET, and *n* is the additive white Gaussian noise (AWGN) with zero mean and variance σ^2 . Hence, the received signal-to-interference-plus-noise ratio (SINR) at the IR is given by $\gamma = |g^H w|^2 / (f^H Q f + \sigma^2)$, where $f^H Q f$ is the received interference power at the IR due to the ET's energy transfer. Therefore, the achievable data rate from the WD to the IR (in bps/Hz) is expressed as

$$R = \log_2(1+\gamma) = \log_2\left(1 + \frac{|\boldsymbol{g}^H \boldsymbol{w}|^2}{\boldsymbol{f}^H \boldsymbol{Q} \boldsymbol{f} + \sigma^2}\right).$$
(2)

Finally, consider the energy harvesting at the WD. The WD can harvest the wireless energy from the ET and also recycle its self-energy via the loop channel. The totally harvested energy at the WD is thus expressed as

$$E = \eta \mathbb{E} \left(|\boldsymbol{h}^{H} \boldsymbol{x}_{E} + \boldsymbol{\varphi}^{H} \boldsymbol{w} \boldsymbol{s}|^{2} \right) = \eta \boldsymbol{h}^{H} \boldsymbol{Q} \boldsymbol{h} + \eta \left| \boldsymbol{\varphi}^{H} \boldsymbol{w} \right|^{2},$$
(3)

¹Note that using multiple receive antennas for improving the energy harvesting efficiency is equivalent to using a single receive antenna with a larger aperture area. In practice, it is cost-effective for the WD to install a single receive antenna or rectenna for harvesting the RF energy.

 2 CSI acquisition at the ET and the WD may practically require the corresponding (energy/information) receivers to feed back certain information (see, e.g., [11]), and there may exist CSI imperfections due to, e.g., feedback errors. However, the detailed CSI acquisition methods and the effects of CSI imperfections are beyond the scope of this letter.

where $\eta \in (0, 1]$ is a constant denoting the energy harvesting efficiency at the WD. Since the transmit energy at the WD cannot exceed its totally harvested energy E in (3), we obtain the energy harvesting constraint at the WD

$$\mathbb{E}(\|\boldsymbol{x}_{I}\|^{2}) = \|\boldsymbol{w}\|^{2} \leq \eta \boldsymbol{h}^{H} \boldsymbol{Q} \boldsymbol{h} + \eta \left|\boldsymbol{\varphi}^{H} \boldsymbol{w}\right|^{2},$$
(4)

or equivalently,

as

$$\boldsymbol{w}^{H}(\boldsymbol{I}-\eta\boldsymbol{\varphi}\boldsymbol{\varphi}^{H})\boldsymbol{w}\leq\eta\boldsymbol{h}^{H}\boldsymbol{Q}\boldsymbol{h}.$$
(5)

III. OPTIMAL CO-CHANNEL DESIGN

This section studies the optimal co-channel transmission design. In particular, we aim to maximize the achievable data rate R from the WD to the IR in (2), by jointly optimizing the ET's energy covariance matrix Q and the WD's information beamforming vector w, subject to the transmit power constraint at the ET in (1) and the energy harvesting constraint at the WD in (5). Therefore, the optimization problem is formulated as

(P1):
$$\max_{\boldsymbol{Q} \succeq \boldsymbol{0}, \boldsymbol{w}} \log_2 \left(1 + \frac{|\boldsymbol{g}^H \boldsymbol{w}|^2}{\boldsymbol{f}^H \boldsymbol{Q} \boldsymbol{f} + \sigma^2} \right)$$
s.t. (1) and (5).

Note that since the objective of (P1) is non-convex, problem (P1) is non-convex in general. Despite this fact, we can still obtain the optimal solution to (P1) by first deriving the optimal w under any given Q, and then obtaining the optimal Q.

First, we optimize w under any given Q. It is observed that the objective function of (P1) is monotonically increasing over $|g^Hw|^2$. Optimizing w under a given Q is thus equivalent to solving the following problem.

(P2):
$$\max_{\boldsymbol{w}} |\boldsymbol{g}^{H}\boldsymbol{w}|^{2},$$
s.t. (5).

To solve (P2), we define $\tilde{\boldsymbol{w}} \triangleq (\boldsymbol{I} - \eta \varphi \varphi^H)^{1/2} \boldsymbol{w}$, and thus $\boldsymbol{w} = (\boldsymbol{I} - \eta \varphi \varphi^H)^{-1/2} \tilde{\boldsymbol{w}}$, where $\boldsymbol{I} - \eta \varphi \varphi^H \succeq \boldsymbol{0}$ holds practically since φ is the loop channel vector with high attenuation. By replacing \boldsymbol{w} as $(\boldsymbol{I} - \eta \varphi \varphi^H)^{-1/2} \tilde{\boldsymbol{w}}$, (P2) can be rewritten as

(P2-E):
$$\max_{\tilde{\boldsymbol{w}}} \left| \boldsymbol{g}^{H} \left(\boldsymbol{I} - \eta \boldsymbol{\varphi} \boldsymbol{\varphi}^{H} \right)^{-1/2} \tilde{\boldsymbol{w}} \right|^{2}$$
s.t. $\| \tilde{\boldsymbol{w}} \|^{2} \leq \eta \boldsymbol{h}^{H} \boldsymbol{Q} \boldsymbol{h}.$

It is easy to verify that for any arbitrary Q, the optimal \tilde{w} for problem (P2-E), denoted by $\tilde{w}(Q)$, is obtained as $\tilde{w}(Q) = \sqrt{\eta h^H Q h} \frac{\tilde{g}}{\|\tilde{g}\|}$, where $\tilde{g} = (I - \eta \varphi \varphi^H)^{-1/2} g$. As a result, for any arbitrary Q, the optimal w for problem (P2), denoted by w(Q), is obtained as

$$\boldsymbol{w}(\boldsymbol{Q}) = \left(\boldsymbol{I} - \eta \boldsymbol{\varphi} \boldsymbol{\varphi}^{H}\right)^{-1/2} \tilde{\boldsymbol{w}}(\boldsymbol{Q})$$

= $\sqrt{\eta \boldsymbol{h}^{H} \boldsymbol{Q} \boldsymbol{h}} \frac{\left(\boldsymbol{I} - \eta \boldsymbol{\varphi} \boldsymbol{\varphi}^{H}\right)^{-1} \boldsymbol{g}}{\left\| \left(\boldsymbol{I} - \eta \boldsymbol{\varphi} \boldsymbol{\varphi}^{H}\right)^{-1/2} \boldsymbol{g} \right\|}.$ (6)

Next, with the optimal w(Q) for any given Q at hand, we derive the optimal Q for problem (P1). By substituting w(Q) in (6) into (P1), the optimization of Q becomes solving the following problem.

(P3):
$$\max_{\boldsymbol{Q} \succeq \boldsymbol{0}} \log_2 \left(1 + \eta \| \tilde{\boldsymbol{g}} \|^2 \frac{\boldsymbol{h}^H \boldsymbol{Q} \boldsymbol{h}}{\boldsymbol{f}^H \boldsymbol{Q} \boldsymbol{f} + \sigma^2} \right),$$

s.t. (1).

The optimal solution to problem (P3), denoted by Q^* , is given in the following proposition.

<u>Proposition</u> 3.1: Let $v = \left(ff^H + \frac{\sigma^2}{P}I\right)^{-1}h$. The optimal Q^* for problem (P3) and thus (P1) is

$$\boldsymbol{Q}^* = \frac{P\boldsymbol{v}\boldsymbol{v}^H}{\|\boldsymbol{v}\|^2} \tag{7}$$

Proof: Please refer to Appendix A.

It follows from Proposition 3.1 that $rank(Q^*) = 1$, which indicates that a single energy beam is sufficient for the ET to achieve the optimality. It is also observed from (7) that the optimal energy beamforming at the ET balances the energy/interference tradeoff, between maximizing the energy transfer efficiency over the WET link h and minimizing the resulted co-channel interference over the interference link f.

Finally, by substituting Q^* into w(Q) given in (6), we obtain the optimal w^* for problem (P1) as

$$\boldsymbol{w}^{*} = \boldsymbol{w}(\boldsymbol{Q}^{*}) = \sqrt{\eta \boldsymbol{h}^{H} \boldsymbol{Q}^{*} \boldsymbol{h}} \frac{\left(\boldsymbol{I} - \eta \boldsymbol{\varphi} \boldsymbol{\varphi}^{H}\right)^{-1} \boldsymbol{g}}{\left\| \left(\boldsymbol{I} - \eta \boldsymbol{\varphi} \boldsymbol{\varphi}^{H}\right)^{-1/2} \boldsymbol{g} \right\|}.$$
(8)

By combining Q^* in (7) and w^* in (8), the optimal solution to problem (P1) is obtained.

<u>Remark</u> 3.1 (distributed implementation): From (7), the optimal Q^* only depends on the ET's local CSI of h and f, and as a result, the optimal energy beamforming can be implemented at the ET locally. Nevertheless, it is observed from (8) that the optimal information beamforming vector w^* generally relies on h, f, g, and φ . Despite this fact, w^* can still be obtained by the WD with its local CSI of g and φ , by noticing that $\eta h^H Q^* h$ is the WD's harvested energy from the ET and thus can be measured by the WD locally. Therefore, the optimal joint design of Q^* and w^* can be implemented at the ET and the WD in a distributed manner.

IV. REFERENCE SCHEMES

This section considers three reference schemes for performance comparison. First, under the co-channel energy and information transfer, we develop two heuristic schemes for the ET to maximize its transferred energy to the WD or cancel its resultant interference to the IR, respectively. Next, we consider a time-division based orthogonal transfer scheme.

1) Co-channel Scheme with Harvested Energy Maximization: In this scheme with co-channel energy and information transfer, we first optimize the ET's energy covariance matrix Q to maximize its transferred energy E to the WD in (3) subject to the transmit power constraint in (1), for which we have the optimal energy covariance matrix at the ET as $Q_E^* = \frac{Phh^H}{\|h\|^2}$. Next, under the optimal Q_E^* , we optimize the WD's information beamforming vector w to maximize its data rate R in (2) subject to the energy harvesting constraint in (5), for which we obtain the optimal information beamforming vector at the WD as $w_E^* = w(Q_E^*)$ with w(Q) given in (6).

<u>Remark</u> 4.1: It can be shown that the optimal Q_E^* and w_E^* in this reference scheme is the optimal solution Q^* and w^* to problem (P1) in the special case when $P \to 0$.



Fig. 3. Time-division based orthogonal energy and information transfer.

2) Co-channel Scheme with Interference Nulling: We also consider co-channel energy and information transfer in this scheme. First, we optimize the ET's energy covariance matrix Q to maximize its transferred energy E to the WD in (3), under the ET's transmit power constraint in (1) and the zero-interference constraint $f^{H}Qf = 0$, i.e.,

(P4):
$$\max_{\boldsymbol{Q} \succeq \boldsymbol{0}} \boldsymbol{h}^{H} \boldsymbol{Q} \boldsymbol{h},$$

s.t. (1) and $\boldsymbol{f}^{H} \boldsymbol{Q} \boldsymbol{f} = 0$

Let $\boldsymbol{u} = \left(\boldsymbol{I} - \boldsymbol{f}\boldsymbol{f}^{H}/\|\boldsymbol{f}\|^{2}\right)\left(\boldsymbol{I} - \boldsymbol{f}\boldsymbol{f}^{H}/\|\boldsymbol{f}\|^{2}\right)^{H}\boldsymbol{h}$. It can be shown that the optimal \boldsymbol{Q} to problem (P4) is given by $\boldsymbol{Q}_{IC}^{*} = \frac{P\boldsymbol{u}\boldsymbol{u}^{H}}{\|\boldsymbol{u}\|^{2}}$. Next, under the optimal \boldsymbol{Q}_{IC}^{*} , we derive the WD's optimal information beamforming vector of this scheme, denoted by \boldsymbol{w}_{IC}^{*} , to maximize the WD's data rate R in (2) subject to the energy harvesting constraint in (5), for which we obtain $\boldsymbol{w}_{IC}^{*} = \boldsymbol{w}(\boldsymbol{Q}_{IC}^{*})$ with $\boldsymbol{w}(\boldsymbol{Q})$ given in (6).

<u>Remark</u> 4.2: The optimal Q_{IC}^* and w_{IC}^* to problem (P4) can be shown to be the optimal solution Q^* and w^* to problem (P1) in the special case when $P \to \infty$.

3) Time-Division based Orthogonal Scheme: For the time-division based orthogonal transfer, as shown in Fig. 3, each block of duration T is divided into two phases: the first phase of duration τT , $\tau \in (0, 1)$, is used for energy transfer from the ET to the WD, and the second phase of duration $(1-\tau)T$ is for information transfer from the WD to the IR. For any information beamforming vector w, the WD's consumed energy during the information transfer phase is given by $(1-\tau)T \|w\|^2$, which cannot exceed its harvested energy during the energy transfer phase, given by $\tau T \eta h^H Q h$. Hence, we have $\|w\|^2 \leq \eta \frac{\tau}{1-\tau} h^H Q h$ as the WD's energy harvesting constraint for the orthogonal case. Moreover, since the WET does not generate interference to the IR for the orthogonal case, the signal-to-noise-ratio at the IR is $|g^H w|^2 / \sigma^2$, and thus the WD's achievable data rate is $R_o = (1-\tau) \log_2 \left(1 + |g^H w|^2 / \sigma^2\right)$. We thereby formulate the data rate maximization problem in this scheme as [8]

(P5):
$$\max_{\tau \in (0,1), \boldsymbol{Q} \succeq \boldsymbol{0}, \boldsymbol{w}} (1-\tau) \log_2 \left(1 + \left| \boldsymbol{g}^H \boldsymbol{w} \right|^2 / \sigma^2 \right)$$
s.t. (1) and $\|\boldsymbol{w}\|^2 \leq \eta \frac{\tau}{1-\tau} \boldsymbol{h}^H \boldsymbol{Q} \boldsymbol{h}.$

The following proposition gives the optimal Q, w, and τ to problem (P5), which are denoted by Q_o^* , w_o^* , and τ^* , respectively.

<u>Proposition</u> 4.1: The optimal solutions to problem (P5) are given by $\mathbf{Q}_{o}^{*} = \frac{P \mathbf{h} \mathbf{h}^{H}}{\|\mathbf{h}\|^{2}}, \ \mathbf{w}_{o}^{*} = \sqrt{\eta \frac{\tau^{*}}{1-\tau^{*}} \mathbf{h}^{H} \mathbf{Q}_{o}^{*} \mathbf{h}} \frac{\mathbf{g}}{\|\mathbf{g}\|},$ and τ^{*} being the unique solution to $\frac{df(\tau)}{d\tau} = 0$, where $f(\tau) \triangleq (1-\tau) \log_{2}(1+\kappa \tau/(1-\tau))$ with $\kappa = \frac{\|\mathbf{g}\|^{2} \eta}{\sigma^{2}} \mathbf{h}^{H} \mathbf{Q}_{o}^{*} \mathbf{h}.$

Proposition 4.1 can be easily verified by first deriving the optimal Q and w under any given τ based on a similar method used to solve (P1) in Section III, and then finding the optimal τ by solving a simple single-variable convex optimization problem. The proof is thus omitted here for brevity.



Fig. 4. WD's throughput versus ET's transmit power.

Note that among the three reference schemes, the two co-channel schemes can be implemented at the ET and the WD in a distributed manner, while the time-division based orthogonal scheme requires the ET and the WD to coordinate in deciding the optimal time division ratio τ^* .

V. NUMERICAL RESULTS

This section provides numerical results to validate our studies. We use a similar channel model as in [3] and [9], by considering Rican fading for the WET link, and Rayleigh fading for the WIT and interference links. We set $N_E = N_W = 2$, $\eta = 0.4$, and $\sigma^2 = -90$ dBm. As in [10], we model the loop channel as $\varphi = \sqrt{\beta} [1 \ 1]^T$ with $\beta = -15$ dB.

Fig. 4 compares the WD's throughput achieved by different transmission schemes versus the transmit power P at the ET. It is observed that the throughput achieved by the optimal co-channel scheme is always higher than that achieved by the two reference co-channel schemes. In particular, when P is sufficient large (or small), the throughput under the reference co-channel scheme with interference nulling (or harvested energy maximization) is observed to be identical to the optimal co-channel scheme. This is consistent with Remark 4.1 and Remark 4.2. Moreover, it is observed that as compared to the orthogonal scheme, a large throughput gain is achieved by our proposed co-channel design, since both energy and information transfer are efficiently operated across the entire time block with carefully controlled co-channel interference to achieve higher spectral efficiency.

VI. CONCLUSION

This letter proposed a new co-channel energy and information transfer scheme for the WPC system with separated ET and IR. By considering multiple antennas at the ET and the WD, we jointly designed the ET's energy beamforming and WD's information beamforming, so as to maximize the WD's throughput subject to the individual transmit power constraints at the ET and the WD. It was shown that the optimal transmitted energy

signal can properly balance ET's energy transfer efficiency to the WD and its resultant interference to the IR. It was also revealed that the optimal joint design can be implemented at the ET and WD in a distributed manner. It is our hope that this work can provide insights on the joint WET and WIT design in WPC systems with co-channel energy and information transfer, and it will be an interesting future direction to extend this work to multi-user WPC networks with co-channel energy and information transfer.

APPENDIX A

PROOF TO PROPOSITION 3.1

As the throughput R monotonically increases with the SINR γ , by omitting the constant $\eta \|\tilde{g}\|^2$ in γ , optimizing Q in problem (P3) is equivalent to solving the following problem.

(P3-1):
$$\max_{\boldsymbol{Q} \succeq \boldsymbol{0}} \frac{\boldsymbol{h}^{H} \boldsymbol{Q} \boldsymbol{h}}{\boldsymbol{f}^{H} \boldsymbol{Q} \boldsymbol{f} + \sigma^{2}},$$

s.t. (1). (9)

It is easy to verify that the optimal solution to problem (P3-1) is achieved with the constraint (1) being met with equality. Accordingly, by substituting tr(Q)/P = 1 into (P3-1) and omitting the constraint in (P3-1), we have

(P3-2):
$$\max_{\boldsymbol{Q}\succeq\boldsymbol{0}} \hat{\gamma} \triangleq \frac{\boldsymbol{h}^{H}\boldsymbol{Q}\boldsymbol{h}}{\boldsymbol{f}^{H}\boldsymbol{Q}\boldsymbol{f} + \frac{\sigma^{2}\operatorname{tr}(\boldsymbol{Q})}{P}} = \frac{\boldsymbol{h}^{H}\boldsymbol{Q}\boldsymbol{h}}{\operatorname{tr}\left(\left(\boldsymbol{f}^{H}\boldsymbol{f} + \frac{\sigma^{2}}{P}\boldsymbol{I}\right)\boldsymbol{Q}\right)}.$$
(10)

Since any feasible solution to (P3-1) is also feasible for (P3-2), optimal value achieved by (P3-2) is indeed an upper bound of that by (P3-1). Thus, if an optimal solution to (P3-2) is feasible to (P3-1), it is also the optimal solution to (P3-1). Based on such an observation, we can solve (P3-1) by finding an optimal solution to (P3-2) that is feasible to (P3-1).

To solve (P3-2), let $\hat{Q} \triangleq \left(\boldsymbol{f} \boldsymbol{f}^{H} + \frac{\sigma^{2}}{P} \boldsymbol{I} \right)^{1/2} \boldsymbol{Q} \left(\boldsymbol{f} \boldsymbol{f}^{H} + \frac{\sigma^{2}}{P} \boldsymbol{I} \right)^{1/2}$, and thus $\boldsymbol{Q} = \left(\boldsymbol{f} \boldsymbol{f}^{H} + \frac{\sigma^{2}}{P} \boldsymbol{I} \right)^{-1/2} \hat{\boldsymbol{Q}} \left(\boldsymbol{f} \boldsymbol{f}^{H} + \frac{\sigma^{2}}{P} \boldsymbol{I} \right)^{-1/2}$ and tr $\left(\left(\boldsymbol{f} \boldsymbol{f}^{H} + \frac{\sigma^{2}}{P} \boldsymbol{I} \right) \boldsymbol{Q} \right) = \operatorname{tr} \left(\hat{\boldsymbol{Q}} \right)$. By replacing \boldsymbol{Q} with $\hat{\boldsymbol{Q}}$, we rewrite $\hat{\gamma}$ in (10) as

$$\hat{\gamma} = \frac{\boldsymbol{h}^{H} \left(\boldsymbol{f} \boldsymbol{f}^{H} + \frac{\sigma^{2}}{P} \boldsymbol{I}\right)^{-1/2} \hat{\boldsymbol{Q}} \left(\boldsymbol{f} \boldsymbol{f}^{H} + \frac{\sigma^{2}}{P} \boldsymbol{I}\right)^{-1/2} \boldsymbol{h}}{\operatorname{tr}\left(\hat{\boldsymbol{Q}}\right)}.$$
(11)

It is noted that the term in (11) is a Rayleigh quotient. Denote $\hat{Q}^* = \left(ff^H + \frac{\sigma^2}{P}I\right)^{-1/2}hh^H \left(ff^H + \frac{\sigma^2}{P}I\right)^{-1/2}$. Thus, any positively scaled matrix of \hat{Q}^* is optimal to maximize $\hat{\gamma}$ in (11). Accordingly, any positively scaled matrix of $Q^* = \left(ff^H + \frac{\sigma^2}{P}I\right)^{-1}hh^H \left(ff^H + \frac{\sigma^2}{P}I\right)^{-1}$ is optimal to (P3-2). By using this result together with $P/\operatorname{tr}(Q) = 1$ for (P3-1), it is then easy to obtain $Q^* = PQ^*/\operatorname{tr}(Q^*) = Pvv^H/||v||^2$. is optimal for (P3-2) and is feasible for (P3-1), and thus is optimal for (P3-1). Proposition 3.1 thus follows.

REFERENCES

- K. Huang and V. K. N. Lau, "Enabling wireless power transfer in cellular networks: architecture, modeling and deployment," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 902-912, Feb. 2014.
- [2] C. Zhong, G. Zheng, Z. Zhang, and G. K. Karagiannidis, "Optimum wirelessly powered relaying", *IEEE Signal Process. Lett.*, vol. 22. no. 10, pp. 1728-1732, Oct., 2015.
- [3] Q. Sun, G. Zhu, C. Shen, X. Li, and Z. Zhong, "Joint beamforming design and time allocation for wireless powered communication networks", *IEEE Commun. Lett*, vol. 18, no. 10, pp. 1783-1786, Oct. 2014.

- [4] W. Liu, X. Zhou, S. Durrani, and P. Popovski, "Secure Communication with a wireless-powered friendly jammer," *IEEE Trans. Wireless Commun.*, to appear.
- [5] X. Zhou, C. K. Ho, and R. Zhang, "Wireless power meets energy harvesting: a joint energy allocation approach in OFDM-based system." Available [online] at http://arxiv.org/abs/1410.1266.
- [6] Y. L. Che, L. Duan, and R. Zhang, "Spatial throughput maximization of wireless powered communication networks," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 8, pp. 1534-1548, Aug. 2015.
- [7] H. Ju and R. Zhang, "Throughput maximization in wireless powered communication networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 1, pp. 418-428, Jan. 2014.
- [8] L. Liu, R. Zhang, and K. C. Chua, "Multi-antenna wireless powered communication with energy beamforming," *IEEE Trans. Wireless Commun.*, vol. 62, no. 12, pp. 4349-4361, Dec., 2014.
- [9] J. Xu, S. Bi, and R. Zhang, "Multiuser MIMO wireless energy transfer with coexisting opportunistic communication," *IEEE Wireless Commun. Lett.*, vol. 4, no. 3, pp. 273-276, Jun. 2015.
- [10] Y. Zeng and R. Zhang, "Full-duplex wireless-powered relay with self-energy recycling," *IEEE Wireless Commun. Lett.*, vol. 4, no. 2, pp. 201-204, Apr. 2015.
- [11] J. Xu and R. Zhang, "Energy beamforming with one-bit feedback," *IEEE Trans. Signal Process.*, vol. 62, no. 20, pp. 5370-5381, Oct. 2014.