# Generalized Analysis of Convergence of Absolute Trust in Peer to Peer Networks

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Abstract—Open and anonymous nature of peer to peer networks provides an opportunity to malicious peers to behave unpredictably in the network. This leads the lack of trust among the peers. To control the behavior of peers in the network, reputation system can be used. In a reputation system, aggregation of trust is a primary issue. Algorithm for aggregation of trust should be designed such that, it can converge to a certain finite value. Absolute Trust is one of the algorithm, which is used for the aggregation of trust in peer to peer networks. In this letter, we present the generalized analysis of convergence of the Absolute Trust algorithm.

Index Terms-Non-negative matrix, Eigenvector, Matrix Norm.

### I. INTRODUCTION

**R** EPUTATION systems have been proposed by many authors in the recent past [1], [5], [6], [7], [8], to prevent the attacks by rogue peers. Absolute Trust [1] is one such model. This model can characterize the past behavior of peers in the network. It can be implemented as a truly distributed system. In this model, peers evaluate each other locally, and the local trust for each other is aggregated in the whole network. The aggregated trust is called global trust. The global trust is evaluated recursively.

In recursive solution of any equation, error in each iteration must reduce and for large number of iteration it should tend to zero. This will guarantee the uniqueness of the solution. It was shown in [1] that if error in global trust is less compare to the actual solution, then it will converge to zero. But analysis for large error was not presented. In this letter, we will show that in any step, if error is very large compared to the actual solution then it will converge much faster in that step.

#### II. PEER TO PEER MODEL AND ABSOLUTE TRUST

Let there be N peers in a peer to peer network. In this network, peer i can be evaluated by peer j based on service provided by peer i in the past. Evaluated value  $T_{ji}$  can be represented by a number from one to ten. One is for worst service and ten is for best service. If there is no interaction between peers,  $T_{ji}$  will be zero.  $T_{ji}$  is called local trust of peer i evaluated by peer j. All local trust values evaluated by various nodes can be aggregated in the whole network. Aggregated global trust of peer i can be given by [1]

$$t_i = \left[ \left( \frac{\sum_{j \in S_i} T_{ji} t_j}{\sum_{j \in S_i} t_j} \right)^p \cdot \left( \frac{\sum_{j \in S_i} t_j^2}{\sum_{j \in S_i} t_j} \right)^q \right]^{\frac{1}{(p+q)}}.$$
 (1)

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Yatindra Nath Singh is with the Department of Electrical Engineering, Indian Institute of Technology, Kanpur, India, e-mail: (ynsingh@iitk.ac.in) Here,  $S_i$  is the set of peers evaluating the service of peer *i*,  $t_j$  is global trust of peer *j*, and *p*, *q* are suitably chosen constants. Equation 1 can be rearranged as follows (see [1]).

$$\begin{split} t_{i} &= \left[ \left( \frac{\mathbf{e_{i}T^{t}t}}{\mathbf{e_{i}Ct}} \right)^{p} \left( \frac{\mathbf{e_{i}C.diag(t).t}}{\mathbf{e_{i}Ct}} \right)^{q} \right]^{\frac{1}{p+q}} \\ &= \left[ \left( \frac{\mathbf{e_{i}T^{t}t}}{\mathbf{e_{i}Ct}} \right)^{\frac{1}{1+\alpha}} \left( \frac{\mathbf{e_{i}C.diag(t).t}}{\mathbf{e_{i}Ct}} \right)^{\frac{\alpha}{1+\alpha}} \right] \end{split}$$

Here, **t** is global trust vector. Its  $i^{th}$  entry is a global trust of peer *i*. **diag**(**t**) is a diagonal matrix with its *ii* entry as  $t_i$ and other entry as zero. **T** is trust matrix with its element  $T_{ij}$ as a local trust of peer *j* evaluated by peer *i*. **T**<sup>t</sup> is transpose of matrix **T**. **C** is incidence matrix corresponding to matrix **T**<sup>t</sup> i.e.  $C_{ij} = 1$  if  $T_{ji} > 0$  otherwise  $C_{ij} = 0$ . **e**<sub>i</sub> is the row vector with its  $i^{th}$  entry as '1' and all the other entry as zero. Here,  $\alpha = q/p$ .

#### III. ANALYSIS OF CONVERGENCE OF ABSOLUTE TRUST

## A. Center Point of the Matrix

**Definition 1.** Center point of non-negative matrix  $\mathbf{T}^{\mathbf{t}}$  can be defined as the column vector  $\mathbf{t}$ . Where, its  $i^{th}$  element  $t_i$  will be  $(\mathbf{e_i}\mathbf{T}^{\mathbf{t}}\mathbf{t}/\mathbf{e_i}\mathbf{Ct})$ .

**Definition 2.** A non-negative matrix **M** is said to be mutually exclusive with a non-negative matrix **N**, if  $M_{ij} > 0$  implies  $N_{ij} = 0$ . It also implies that when  $N_{ij} > 0$  then  $M_{ij} = 0$ 

**Lemma 1.** Center point of any non-negative, irreducible matrix  $\mathbf{T}^{t}$  is unique and can be calculated by an iterative function

$$\mathbf{t}^{\mathbf{k}} = \phi_1(\mathbf{t}^{\mathbf{k}-1}) = [\mathbf{diag}(d_1, d_2 \dots d_N)]^{-1} \cdot \mathbf{T}^{\mathbf{t}} \cdot \mathbf{t}^{\mathbf{k}-1}$$

where  $d_i = \mathbf{e_iCt}$ 

*Proof.*  $i^{th}$  element of iterative function  $\phi_1(\mathbf{t^{k-1}})$  is

$$t_i^k = \frac{(\mathbf{e_i T^t}.\mathbf{t^{k-1}})}{(\mathbf{e_i C}.\mathbf{t^{k-1}})}$$
(2)

Let  $t_i^k$  and  $t_i^{k-1}$  are, far from actual solution  $t_i$  by  $\delta t_i^k$  and  $\delta t_i^{k-1}$  respectively, then

$$t_i + \delta t_i^k = \left[ \frac{\left( \mathbf{e_i T^t} \cdot (\mathbf{t} + \delta \mathbf{t^{k-1}}) \right)}{\left( \mathbf{e_i C} \cdot (\mathbf{t} + \delta \mathbf{t^{k-1}}) \right)} \right]$$
$$= \left[ \frac{\left( \mathbf{e_i T^t} \cdot \mathbf{t} \right)}{\left( \mathbf{e_i C} \cdot \mathbf{t} \right)} \right] \left[ \frac{\left( 1 + \frac{\mathbf{e_i} \cdot \mathbf{T^t} \cdot \delta \mathbf{t^{k-1}}}{\mathbf{e_i} \cdot \mathbf{T^t} \cdot \mathbf{t}} \right)}{\left( 1 + \frac{\mathbf{e_i} \cdot \mathbf{C} \cdot \delta \mathbf{t^{k-1}}}{\mathbf{e_i} \cdot \mathbf{C} \cdot \mathbf{t}} \right)} \right]$$

From definition 1,  $(\mathbf{e_i T^t} t / \mathbf{e_i Ct})$  is  $i^{th}$  element of center point *Proof.* Iterative function can be written as of matrix T<sup>t</sup>. So we can write,

$$\begin{split} t_i + \delta t_i^k &= t_i. \left[ \frac{\left(1 + \frac{\mathbf{e}_i.\mathbf{T}^t.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{T}^t.\mathbf{t}}\right)}{\left(1 + \frac{\mathbf{e}_i.\mathbf{C}.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{C}.\mathbf{t}}\right)} \right] \\ \delta t_i^k &= t_i. \left[ \frac{\left(1 + \frac{\mathbf{e}_i.\mathbf{T}^t.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{T}^t.\mathbf{t}}\right)}{\left(1 + \frac{\mathbf{e}_i.\mathbf{C}.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{C}.\mathbf{t}}\right)} - 1 \right] \\ \delta t_i^k &= \frac{t_i}{\left(1 + \frac{\mathbf{e}_i.\mathbf{C}.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{C}.\mathbf{t}}\right)} \cdot \left[ \frac{\mathbf{e}_i.\mathbf{T}^t.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{T}^t.\mathbf{t}} - \frac{\mathbf{e}_i.\mathbf{C}.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{C}.\mathbf{t}} \right] \\ &= \frac{1}{\left(1 + \frac{\mathbf{e}_i.\mathbf{C}.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{C}.\mathbf{t}}\right)} \cdot \left[ \frac{t_i \mathbf{e}_i.\mathbf{T}^t}{\mathbf{e}_i.\mathbf{T}^t.\mathbf{t}} - \frac{t_i \mathbf{e}_i.\mathbf{C}.}{\mathbf{e}_i.\mathbf{C}.\mathbf{t}} \right] \cdot \delta \mathbf{t}^{k-1} \\ &= \frac{1}{\left(1 + \frac{\mathbf{e}_i.\mathbf{C}.\delta \mathbf{t}^{k-1}}{\mathbf{e}_i.\mathbf{C}.\mathbf{t}}\right)} \cdot \left[ \mathbf{A}_i - \mathbf{B}_i \right] \cdot \delta \mathbf{t}^{k-1} \\ &= f_i(\delta \mathbf{t}^{k-1}) \cdot \left[ \mathbf{A}_i - \mathbf{B}_i \right] \cdot \delta \mathbf{t}^{k-1} \end{split}$$

where  $A_i$  and  $B_i$  are  $i^{th}$  row of NXN matrices A and B respectively. It can be observed easily that

and

$$\mathbf{B} \mathbf{t} = \mathbf{t}$$

 $\mathbf{A} \cdot \mathbf{t} = \mathbf{t}$ 

Matrices A, B have non zero elements at same position as matrix T<sup>t</sup>, hence A, B are also irreducible. Therefore spectral radius of  $\mathbf{A}, \mathbf{B}$  will be '1'(see[2]).

Now

$$f_i(\delta \mathbf{t^{k-1}}) = \frac{1}{(1 + \frac{\mathbf{e_i.C.}\delta \mathbf{t^{k-1}}}{\mathbf{e_i.C.t}})}$$

If  $\delta \mathbf{t^{k-1}} \ll \mathbf{t}$  then  $f_i(\delta \mathbf{t^{k-1}}) \approx 1$ 

Hence,

$$\delta \mathbf{t}^{\mathbf{k}} = [\mathbf{A} - \mathbf{B}] . \delta \mathbf{t}^{\mathbf{k}-1}$$
$$lim_{k \to \infty} \delta \mathbf{t}^{\mathbf{k}} = lim_{k \to \infty} [\mathbf{A} - \mathbf{B}]^k \delta \mathbf{t}^{\mathbf{0}} =$$

(see Theorem 1 in [1]) here  $\delta t^0$  is initial error in t

Now, for the case when  $\delta \mathbf{t^{k-1}} > \mathbf{t}$  then  $f_i(\delta \mathbf{t^{k-1}}) < 1$ , in each step,  $\delta t_i^k$  will decrease more rapidly. If  $\delta \mathbf{t^{k-1}} \approx \mathbf{t}$  then  $f_i(\delta \mathbf{t^{k-1}}) \approx 1/2$ , in this case each  $\delta t_i^k$  will reduced to half in  $k^{th}$  step.

Hence we can conclude that center point of any non negative, irreducible matrix can be calculated by above iterative function. Error in each step will depend upon the error in past step. Error will reduce very fast, if it is far from actual solution in an step and after large iterations it will go to zero. 

**Lemma 2.** If vector  $\mathbf{t}$  is the center point of matrix  $\mathbf{C}.\mathbf{diag}(\mathbf{t})$ , then center point will lie on the vector e. It can be calculated by iterative function

$$\mathbf{t}^{\mathbf{k}} = \phi_2(\mathbf{t}^{\mathbf{k}-1}) = [\mathbf{diag}(d_1, d_2, \dots, d_N)]^{-1} \cdot \mathbf{C} \cdot \mathbf{diag}(\mathbf{t}^{\mathbf{k}-1}) \cdot \mathbf{t}^{\mathbf{k}-1}$$

where **e** is a vector with each element as '1' and  $d_i = \mathbf{e_iCt}$ .

$$\begin{aligned} \mathbf{t}^{\mathbf{k}} &= \mathbf{M}(\mathbf{k}-1).\mathbf{t}^{\mathbf{k}-1} \\ &= \mathbf{M}(\mathbf{k}-1).\mathbf{M}(\mathbf{k}-2)....\mathbf{M}(0).\mathbf{t}^{0} \end{aligned}$$

where  $i^{th}$  row of matrix  $\mathbf{M}(\mathbf{k})$  is  $\left(\frac{\mathbf{e_i C.diag}(\mathbf{t}^k)}{\mathbf{e_i C.t^k}}\right)$ . We can easily prove that for matrix  $\mathbf{M}(\mathbf{k})$ , sum of its each row is one. Hence it has '1' as an eigen value. The corresponding eigen vector will be e. For positive initial guess of  $t^0$ , all matrices M(k)will be non-negative and irreducible. So we can conclude that spectral radius of all M(k) is '1'. Hence (see Lemma 2 in [1])

$$lim_{k\to\infty}\mathbf{t}^{\mathbf{k}} = lim_{k\to\infty}\mathbf{M}(\mathbf{k}-\mathbf{1}).\mathbf{M}(\mathbf{k}-\mathbf{2})....\mathbf{M}(\mathbf{0}).\mathbf{t}^{\mathbf{0}} = \mathbf{e}$$
$$lim_{k\to\infty}\mathbf{t}^{\mathbf{k}} = lim_{k\to\infty}\phi_2(\mathbf{t}^{\mathbf{k}-\mathbf{1}}) = \mathbf{e}$$

## B. Properties of Center Point

**Property 1.** If center point of matrix M is t then center point of matrix  $k\mathbf{M}$  will be  $k\mathbf{t}$ . Where k is any arbitrary scalar.

*Proof.* Let  $t_i$  is  $i^{th}$  element of t then

$$t_i = \frac{\mathbf{e_i} \cdot \mathbf{M} \cdot \mathbf{t}}{\mathbf{e_i} \cdot \mathbf{C} \cdot \mathbf{t}}$$
$$kt_i = \frac{\mathbf{e_i} \cdot k \cdot \mathbf{M} \cdot \mathbf{t}}{\mathbf{e_i} \cdot \mathbf{C} \cdot \mathbf{t}}$$

$$(kt_i) = \frac{\mathbf{e_i}.(k\mathbf{M}).(k\mathbf{t})}{\mathbf{e_i}.\mathbf{C}.(k\mathbf{t})}$$

hence kt is center point of matrix kM

**Property 2.** If all entries of the NXN matrix M are positive( $M_{ij} > 0 \ \forall i, j$ ) then center point will lie on the principle eigen vector of matrix.

*Proof.* If all the entry of matrix M are positive then

$$\mathbf{e_i.C.t} = \sum_{j=1}^{N} t_j = \lambda \quad \forall i$$

Here  $\lambda$  is a constant. Further,

$$t_i = \frac{\mathbf{e_i}.\mathbf{M}.\mathbf{t}}{\mathbf{e_i}.\mathbf{C}.\mathbf{t}} = \frac{\mathbf{e_i}.\mathbf{M}.\mathbf{t}}{\lambda}.$$

Hence  $Mt = \lambda t$ . Hence  $\lambda$  will be spectral radius and t will be principle eigen vector of matrix  $\mathbf{M}$  (see[2]) 

**Property 3.** If center point of *m* non-negative, irreducible and mutually exclusive matrices  $M_1, M_2, ..., M_m$  are same, then the center point of their sum will also be the same.

Proof. Let the incidence matrix corresponding to matrices  $M_1, M_2, ..., M_m$  are  $C_1, C_2, ..., C_m$  respectively. Now if  $\mathbf{M}=\mathbf{M_1}+\mathbf{M_2}+....\mathbf{M_m}$  then incidence matrix corresponding to matrix  $\mathbf{M}$  will be  $\mathbf{C} = \mathbf{C_1} + \mathbf{C_2} + \dots \mathbf{C_m}$  because matrices  $M_1, M_2....M_m$  are mutually exclusive. Let  $i^{th}$  element of center point of matrices  $M_1, M_2, \dots, M_m$  be  $t_i$  then

$$t_i = \frac{\mathbf{e_i}.\mathbf{M_j}.\mathbf{t}}{\mathbf{e_i}.\mathbf{C_j}.\mathbf{t}} \quad \forall j$$

or

0

or

or

$$t_i \mathbf{e_i} \cdot \mathbf{C_j} \cdot \mathbf{t} = \mathbf{e_i} \cdot \mathbf{M_j} \cdot \mathbf{t}$$

taking summation on both side w.r.t. j

$$\sum_{j=1}^{m} t_i \mathbf{e_i.C_j.t} = \sum_{j=1}^{m} \mathbf{e_i.M_j.t}$$
$$t_i \mathbf{e_i.} \left(\sum_{j=1}^{m} \mathbf{C_j}\right) \cdot \mathbf{t} = \mathbf{e_i.} \left(\sum_{j=1}^{m} \mathbf{M_j}\right) \cdot \mathbf{t}$$
$$t_i \mathbf{e_i.C.t} = \mathbf{e_i.M.t}$$

or

$$t_i = \frac{\mathbf{e_i}.\mathbf{M}.\mathbf{t}}{\mathbf{e_i}.\mathbf{C}.\mathbf{t}}$$

hence t is also the center point of matrix M.

C. Convergence of Absolute Trust for Large Error in Initial Guess

It was stated in [1] that global trust in equation 1 can be calculated by iterative function

$$\mathbf{t}^{\mathbf{k}} = \phi(\mathbf{t}^{\mathbf{k}-1}) = [\mathbf{diag}(d_1, d_2 \dots d_N) \mathbf{T}^{\mathbf{t}} \cdot \mathbf{t}^{\mathbf{k}-1}]^{\frac{1}{1+\alpha}}$$
  
where  $d_{\mathbf{k}} = \left[ \left( \mathbf{e}_{\mathbf{i}} \mathbf{C} \cdot \mathbf{diag}(\mathbf{t}^{\mathbf{k}-1}) \cdot \mathbf{t}^{\mathbf{k}-1} \right)^{\alpha} \right]$  Proof was derived of

where  $d_i$  $(\mathbf{e_iCt^{k-1}})^{(1+\alpha)}$ . Proof was derived only for the small error that is, if initial guess of t is very close to

the actual solution. However global trust can be calculated for any positive initial guess. In this subsection we will show that it will converge faster in any step if error is large compared to t.

Let  $t_i^k$  and  $t_i^{k-1}$  are, far from actual solution  $t_i$  by  $\delta t_i^k$  and  $\delta t_i^{k-1}$  respectively, then

$$\begin{split} t_i + \delta t_i^k &= \left[ \frac{\left( \mathbf{e_i T^t}.(\mathbf{t} + \delta \mathbf{t^{k-1}}) \right)}{\left( \mathbf{e_i C}(\mathbf{t} + \delta \mathbf{t^{k-1}}) \right)} \right]^{\frac{1}{1+\alpha}} \\ & \left[ \frac{\left( \mathbf{e_i C}.\mathrm{diag}(\mathbf{t} + \delta \mathbf{t^{k-1}}).(\mathbf{t} + \delta \mathbf{t^{k-1}}) \right)}{\left( \mathbf{e_i C}(\mathbf{t} + \delta \mathbf{t^{k-1}}) \right)} \right]^{\frac{\alpha}{1+\alpha}} \end{split}$$

 $\begin{array}{ll} If & error & \delta t^{k-1} > t \\ t + \delta t^{k-1} \approx \delta t^{k-1} \text{ hence} \end{array}$ then approximate we can

$$\delta t_i^k = \left[\frac{\left(\mathbf{e_i T^t}.\delta \mathbf{t^{k-1}}\right)}{\left(\mathbf{e_i C}.\delta \mathbf{t^{k-1}}\right)}\right]^{\frac{1}{1+\alpha}} \cdot \left[\frac{\left(\mathbf{e_i C.diag}(\delta \mathbf{t^{k-1}}).\delta \mathbf{t^{k-1}}\right)}{\left(\mathbf{e_i C}.\delta \mathbf{t^{k-1}}\right)}\right]^{\frac{\alpha}{1+\alpha}}$$

Using Young's Inequality [4], i.e.

$$c.d \le \frac{1}{1+\alpha}c^{1+\alpha} + \frac{\alpha}{1+\alpha}d^{(1+\alpha)/\alpha}$$

and taking

$$c = \left[\frac{\left(\mathbf{e_i T^t}.\delta \mathbf{t^{k-1}}\right)}{\left(\mathbf{e_i C}.\delta \mathbf{t^{k-1}}\right)}\right]^{\frac{1}{1+\alpha}}$$

and

$$d = \left[\frac{\left(\mathbf{e_i C. diag}(\delta \mathbf{t^{k-1}}).\delta \mathbf{t^{k-1}}\right)}{\left(\mathbf{e_i C}.\delta \mathbf{t^{k-1}}\right)}\right]^{\frac{\alpha}{1+\alpha}},$$

We can write

$$\delta t_i^k \leq \frac{1}{1+\alpha} \left[ \frac{\left( \mathbf{e_i T^t} . \delta \mathbf{t^{k-1}} \right)}{\left( \mathbf{e_i C} . \delta \mathbf{t^{k-1}} \right)} \right] + \frac{\alpha}{1+\alpha} \left[ \frac{\left( \mathbf{e_i C} . \mathbf{diag}(\delta \mathbf{t^{k-1}}) . \delta \mathbf{t^{k-1}} \right)}{\left( \mathbf{e_i C} . \delta \mathbf{t^{k-1}} \right)} \right].$$
$$\delta \mathbf{t^k} \leq \frac{1}{1+\alpha} \phi_1(\delta \mathbf{t^{k-1}}) + \frac{\alpha}{1+\alpha} \phi_2(\delta \mathbf{t^{k-1}}).$$

Here  $\phi_1(.)$  and  $\phi_2(.)$  are as defined in Lemma 1 and in Lemma 2 respectively.

$$\delta \mathbf{t}^{\mathbf{k}} \leq \frac{1}{1+\alpha} \mathbf{M}'_{\mathbf{1}} \cdot \delta \mathbf{t}^{\mathbf{k}-\mathbf{1}} + \frac{\alpha}{1+\alpha} \mathbf{M}'_{\mathbf{2}} \cdot \delta \mathbf{t}^{\mathbf{k}-\mathbf{1}}$$
(3)

It is convex combination of iterative function  $\phi_1$  and  $\phi_2$ .  $i^{th}$  row of matrix  $\mathbf{M}'_2$  is  $\left[\frac{\left(\mathbf{e_iC.diag}(\delta \mathbf{t}^{\mathbf{k}-1})\right)}{\left(\mathbf{e_iC.\delta t^{\mathbf{k}-1}}\right)}\right]$  and sum of each row is one. Hence  $\infty$ -norm of matrix  $\mathbf{M}'_{\mathbf{2}}$  is  $|\mathbf{M}'_{\mathbf{2}}|_{\infty} = 1$  using the property of norm [3]

$$|\mathbf{M}_{2}^{'}.\delta\mathbf{t}^{\mathbf{k}-1}|_{\infty} \leq |\mathbf{M}_{2}^{'}|_{\infty}.|\delta\mathbf{t}^{\mathbf{k}-1}|_{\infty} = |\delta\mathbf{t}^{\mathbf{k}-1}|_{\infty}$$
(4)

Function  $\phi_1$  is converging function toward center point. It is shown in Lemma 1 that in any step, if t<sup>k</sup> is very far from the center point then it will tend toward the center point very rapidly. Hence for  $\delta t^{k-1} > t$ 

 $\phi_1(\delta \mathbf{t^{k-1}}) < \delta \mathbf{t^{k-1}}$ 

or

$$|\mathbf{M}_{1}^{'}.\delta\mathbf{t}^{\mathbf{k}-1}|_{\infty} < |\delta\mathbf{t}^{\mathbf{k}-1}|_{\infty} \tag{5}$$

Now taking  $\infty$ -norm on both side of equation 3

$$\begin{split} |\delta \mathbf{t}^{\mathbf{k}}|_{\infty} &\leq |\frac{1}{1+\alpha} \mathbf{M}_{\mathbf{1}}^{'}.\delta \mathbf{t}^{\mathbf{k}-\mathbf{1}} + \frac{\alpha}{1+\alpha} \mathbf{M}_{\mathbf{2}}^{'}.\delta \mathbf{t}^{\mathbf{k}-\mathbf{1}}|_{\infty} \\ &\leq \frac{1}{1+\alpha} |\mathbf{M}_{\mathbf{1}}^{'}.\delta \mathbf{t}^{\mathbf{k}-\mathbf{1}}|_{\infty} + \frac{\alpha}{1+\alpha} |\mathbf{M}_{\mathbf{2}}^{'}.\delta \mathbf{t}^{\mathbf{k}-\mathbf{1}}|_{\infty} \end{split}$$

Using equation 4 and 5,

 $|\delta \mathbf{t}^{\mathbf{k}}|_{\infty} < \frac{1}{1+\alpha} |\delta \mathbf{t}^{\mathbf{k}-\mathbf{1}}|_{\infty} + \frac{\alpha}{1+\alpha} |\delta \mathbf{t}^{\mathbf{k}-\mathbf{1}}|_{\infty} = |\delta \mathbf{t}^{\mathbf{k}-\mathbf{1}}|_{\infty}$ Hence  $|\delta \mathbf{t^k}|_{\infty} < |\delta \mathbf{t^{k-1}}|_{\infty}$ 

Therefore error in every step will decrease. In any step, it will decrease faster if 
$$\mathbf{t}^{\mathbf{k}}$$
 is very far from actual solution. Speed of convergence depends upon the  $\alpha$ . For lower  $\alpha$ , impact of  $\phi_2$  will be lower and  $\phi_1$  will dominate the speed of convergence. Hence for smaller  $\alpha$  speed of convergence will be high.

## **IV. NUMERICAL RESULTS**

In order to verify what has been discussed in earlier sections, we have taken the values of the trust matrix T as

[0	5	6	6
8	0	5	$\begin{bmatrix} 6 \\ 5 \end{bmatrix}$
5	6	0	$\begin{bmatrix} 2\\0 \end{bmatrix}$
0	4	0	0

The value of center point and the global trust in each iteration is calculated and shown in Table I, II, III, IV and V.

Table I: Center point in each iteration, when initial guess is close to center point

i	1	2	3	4
$t^0$	1.0000	2.0000	3.0000	4.0000
t <sup>1</sup>	6.2000	4.8750	5.3333	3.6667
$t^2$	6.4327	5.1096	5.5598	4.4027
$t^3$	6.4367	5.0706	5.5573	4.4008
$t^4$	6.4313	5.0705	5.5594	4.4002
$t^5$	6.4310	5.0707	5.5592	4.3994
$t^6$	6.4311	5.0708	5.5591	4.3994
t <sup>7</sup>	6.4311	5.0708	5.5591	4.3994

Table II: Center point in each iteration, when initial guess is very far from center point

i	1	2	3	4
t <sup>0</sup>	100.0000	300.0000	200.0000	100.0000
t <sup>1</sup>	6.8000	5.2500	5.2500	4.1667
$t^2$	6.5000	5.0668	5.5643	4.4827
$t^3$	6.4298	5.0654	5.5620	4.4050
$t^4$	6.4299	5.0706	5.5593	4.3987
$t^5$	6.4310	5.0708	5.5591	4.3993
$t^6$	6.4311	5.0708	5.5591	4.3994
t <sup>7</sup>	6.4311	5.0708	5.5591	4.3994

## A. Convergence of the Center Point

The convergence of center point of matrix  $\mathbf{T}^{\mathbf{t}}$  is shown in Table I and II. In Table I initial guess  $\mathbf{t}^{\mathbf{0}} = [1 \ 2 \ 3 \ 4]^t$ , it is close to the center point and in Table II initial guess  $\mathbf{t}^{\mathbf{0}} = [100 \ 300 \ 200 \ 100]^t$ , which is significantly far from center point. But we can see in both the cases that it will converge in seven iterations. In the latter case, error is very large in  $0^{th}$  step and it becomes less then  $\mathbf{t}$  with in one step.

## B. Convergence of the Global Trust

Impact of initial guess and parameter  $\alpha$  on the convergence of global trust is shown in Table III, IV and V. In Table III and IV  $\alpha$  is taken as 1/3 but initial guess is different. Again we can see that for large initial guess of global trust it takes only one more iteration to converge to final value. It converge very fast when error( $\delta t$ ) is very large compare to global trust(t)

In Table III and V initial guess is taken same but  $\alpha$  is different and we can see that for  $\alpha = 1/6$  it converges only in eight iterations.

# V. CONCLUSION

In this letter, we analyzed the convergence of global trust given by equation 1. We have shown that in recursive calculation of global trust, error will decrease even if initial guess is very far from the actual solution. In any step, convergence is faster if error is larger. We have shown that speed of convergence depends upon the value of  $\alpha$ . For smaller  $\alpha$  it will converge more faster.

#### REFERENCES

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Table III: Global trust in each iteration, when initial guess is close to global trust ( $\alpha = 1/3$ )

i	1	2	3	4
t <sup>0</sup>	1.0000	2.0000	3.0000	4.0000
t <sup>1</sup>	4.9893	4.4051	3.9876	3.2749
$t^2$	5.8801	4.8299	5.3148	4.4838
$t^3$	6.0621	5.1110	5.5131	4.6039
$t^4$	6.1418	5.1543	5.5636	4.6490
$t^5$	6.1550	5.1683	5.5802	4.6631
$t^6$	6.1593	5.1717	5.5834	4.6657
$t^7$	6.1603	5.1725	5.5844	4.6665
$t^8$	6.1605	5.1727	5.5846	4.6667
$t^9$	6.1606	5.1728	5.5847	4.6667
$t^{10}$	6.1606	5.1728	5.5847	4.6667

Table IV: Global trust in each iteration, when initial guess is very far from global trust ( $\alpha = 1/3$ )

i	1	2	3	4
t <sup>0</sup>	100.0000	300.0000	200.0000	100.0000
$t^1$	16.9093	12.1379	13.7913	11.3982
$t^2$	7.6469	6.5685	7.1369	5.9630
$t^3$	6.5419	5.4824	5.9029	4.9291
$t^4$	6.2499	5.2477	5.6685	4.7377
$t^5$	6.1829	5.1918	5.6048	4.6834
$t^6$	6.1662	5.1775	5.5897	4.6710
$t^7$	6.1620	5.1740	5.5859	4.6678
$t^8$	6.1610	5.1731	5.5850	4.6670
$t^9$	6.1607	5.1729	5.5847	4.6668
t <sup>10</sup>	6.1606	5.1728	5.5847	4.6667
t <sup>11</sup>	6.1606	5.1728	5.5847	4.6667

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Table V: Global trust in each iteration for lesser value of  $\alpha$ ( $\alpha = 1/6$ )

i	1	2	3	4
t <sup>0</sup>	1.0000	2.0000	3.0000	4.0000
$t^1$	5.4761	4.6006	4.5168	3.4374
$t^2$	6.1901	5.0137	5.4742	4.5092
$t^3$	6.2506	5.1176	5.5682	4.5424
$t^4$	6.2693	5.1288	5.5767	4.5486
$t^5$	6.2714	5.1304	5.5790	4.5510
$t^6$	6.2716	5.1307	5.5793	4.5511
$t^7$	6.2717	5.1307	5.5793	4.5512
$t^8$	6.2717	5.1307	5.5793	4.5512