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Abstract—We propose new grouping methods for group shuffled (GS) decoding of both regular and irregular low-density parity check cods. These methods are independent of the checkto-variable message formula used. Integer-valued metrics for measuring the reliability of each tentative variable node (VN) decision and the associated likelihood of being corrected are developed. The metrics are used to determine the VN updating priority so the grouping may vary in each iteration. We estimate the computation complexity needed to adaptively regroup VNs. Numerical results show that our GS algorithms improve the performance of some existing GS belief-propagation decoders.

Index Terms—LDPC codes, belief propagation, group shuffled decoding, adaptive decoding schedule.

I. INTRODUCTION

Conventional belief-propagation (BP) based algorithms for decoding low-density parity check (LDPC) codes update all variable nodes (VNs) and check nodes (CNs) in parallel [1]. The group shuffled (GS) BP algorithm [2] partitions the VNs into groups and performs group-wise parallel decoding serially which in effect divides a decoding iteration into several subiterations. A GSBP decoder can thus pass newly updated messages obtained in a sub-iteration to the neighboring groups in the ensuing sub-iterations and achieve faster convergence with reduced parallel decoding complexity. The GSBP algorithm generalizes the original shuffled BP algorithm [2] or the column-based layer BP decoder [3] by allowing a group to contain more than one VN. Some improved GSBP algorithms have been proposed to further improve the decoding performance [4]-[6]. A variation of the GSBP approach which greedily selects the 'best' edge(s) for updating is the class of informed dynamic scheduling based BP decoders [7], [8]. However, the greedy search requires high computing complexity and some VNs may never or seldom be updated.

One of the basic ideas of our GS decoding schedule is to prioritize the updates of the VNs which most probably have erroneous tentative decisions and are likely to be corrected. Early-updating such VNs enables us to invert an incorrect tentative local decision, avert potential error propagation, and strengthen the reliability of the passed messages. This concept is an extension of those inspire the algorithms presented in [7],[8]. We not only determine the set of VNs with the most unreliable tentative decision but also evaluate and compare the impact/benefit of updating these VNs. In this letter, we develop a set of simple binary/integer based rules that dynamically re-group VNs in each iteration for GS decoding. As will be seen, the improved performance comes at the expense of extra integer and binary operations (against the conventional GSBP decoders) and provides additional complexity-performance trade-offs in designing a BP decoding schedule.

The rest of this letter is organized as follows. In Section II, we define the basic system parameters and give a brief review of the standard GSBP decoding algorithm. In Section III, we introduce our adaptive GSBP (AGSBP) algorithm and two adaptive grouping approaches. Numerical results are presented in Section IV, and the complexity of the AGSBP algorithms are also analyzed in the same section. Finally, conclusion remarks are drawn in Section V.

II. GROUP SHUFFLED BP DECODING

A binary (N, K) LDPC code C is a linear block code of rate R = K/N described by an $M \times N$ parity check matrix Hwhich has $d^v(n)$ ones in the *n*th column and $d^c(m)$ ones in the *m*th row. H can be viewed as a bipartite graph with N VNs corresponding to the encoded bits and M CNs corresponding to the parity-check functions represented by the rows of H. The conventional GSBP algorithm [2] divides the VNs into Ggroups of equal size $N/G = N_G$ according to their natural order, i.e., if we define $\mathcal{G}_i = \{n|i \cdot N_G \leq n < (i+1) \cdot N_G - 1\}$ where $i = 0, 1, \ldots, N - 1$, then VN v_n belongs to the *i*th VN group if $n \in \mathcal{G}_i$. In each GSBP decoding iteration, groups are sequentially processed and the VNs belonging to the same group are updated in parallel.

A binary codeword $\boldsymbol{u} = (u_0, u_1, \cdots, u_{N-1})$ is BPSKmodulated and transmitted over an zero-mean AWGN channel with noise variance σ^2 . The corresponding received and binary decoded sequences are denoted by $\boldsymbol{r} = (r_0, r_1, \cdots, r_{N-1})$ and $\hat{\boldsymbol{u}} = (\hat{u}_0, \hat{u}_1, \cdots, \hat{u}_{N-1})$. We define $m_{n \to m}^v$ as the variableto-check (V2C) message from the *n*th VN v_n to the *m*th CN c_m and $m_{m \to n}^c$ as the check-to-variable (C2V) message from c_m to v_n . Let $\mathcal{N}(m)$ be the index set of VNs which are connected to c_m and $\mathcal{M}(n)$ be that of CNs connected to v_n in the code graph. $\mathcal{N}(m) \setminus n$ is the set $\mathcal{N}(m)$ with nexcluded; $\mathcal{M}(n) \setminus m$ is similarly defined. We further define the sign function sgn(x) = 1 if x > 0, sgn(x) = -1 if x < 0, and takes the value 1 or -1 equally likely if x = 0. We assume that the log-domain BP decoding is used. When processing the *i*th VN group, the C2V messages $m_{m \to n}^c$, $\forall m \in \mathcal{M}(n), n \in \mathcal{G}_i$ are computed by

$$m_{m \to n}^{c} = \prod_{n' \in \mathcal{N}(m) \setminus n} \alpha_{n' \to m} \cdot \phi\left(\sum_{n' \in \mathcal{N}(m) \setminus n} \phi(\beta_{n' \to m})\right) \quad (1)$$

where $\phi(x) = -\log[\tanh(x/2)], \ \alpha_{n' \to m} = \operatorname{sgn}(m_{n' \to m}^v),$ and $\beta_{n' \to m} = |m_{n' \to m}^v|$. The V2C messages $m_{n \to m}^v, \forall m \in \mathcal{M}(n), n \in \mathcal{G}_i$, are updated via

$$m_{n \to m}^{v} = \frac{2r_n}{\sigma^2} + \sum_{m' \in \mathcal{M}(n) \setminus m} m_{m' \to n}^{c}, \qquad (2)$$

the total log-likelihood ratio (LLR) of v_n is

$$L_n = \frac{2r_n}{\sigma^2} + \sum_{m \in \mathcal{M}(n)} m_{m \to n}^c.$$
 (3)

III. ADAPTIVE GROUPING AND AGSBP DECODING

A. General Adaptive GSBP Decoder

Let the index set of the updated VNs (in an iteration) be denoted by \mathcal{V} and its complement by $\mathcal{V}^c = \mathcal{Z}_N \setminus \mathcal{V}$, where $\mathcal{Z}_N \triangleq \{0, 1, \dots, N-1\}$. Suppose *l* is the iteration counter and l_{\max} is the maximum iteration number, then a generic AGSBP decoding algorithm can be described by **Algorithm 1**.

| Algorithm 1 Adaptive Group Shuffled BP Algorithm |
|---|
| Initialization Set the iteration counter $l = 0$. |
| Step 1 Set $\mathcal{V}^c = \mathcal{Z}_N$ and $\mathcal{V} = \mathcal{G} = \emptyset$. Let $l \leftarrow l + 1$. |
| Step 2 Perform an adaptive grouping method (e.g., |
| Algorithms 2 or 3) to form \mathcal{G} . |
| Step 3 Propagate $m_{m \to n}^c$ and then update $m_{n \to m}^v \forall m \in$ |
| $\mathcal{M}(n), \ n \in \mathcal{G}.$ |
| Step 4 Let $\mathcal{V} \leftarrow \mathcal{V} \cup \mathcal{G}$ and $\mathcal{V}^c \leftarrow \mathcal{V}^c \setminus \mathcal{G}$. If $\mathcal{V}^c = \emptyset$, go to |
| Step 5; otherwise, go to Step 2. |
| Step 5 If a valid codeword is obtained or $l = l_{max}$, stop |
| decoding; otherwise, go to Step 1. |

Note that the VNs are grouped sequentially and the grouping is likely to vary in each iteration. Once a new VN group is determined by **Step 2**, we update the C2V and V2C messages associated with the VNs belong to this group by **Step 3** before searching for the next VN group for updating.

B. Adaptive VN Grouping Methods

We now present integer-valued metrics for selecting VNs from \mathcal{V}^c . The selected VNs have the least reliable tentative decisions and the highest probability of being corrected if updated.

Using the syndrome (checksum) vector $\boldsymbol{s} = (s_0, s_1, \cdots, s_{M-1}) = \hat{\boldsymbol{u}} \cdot \boldsymbol{H}^T \pmod{2}$, we define an integer-valued unreliability index

$$E_n = \Omega_n \left(\sum_{m \in \mathcal{M}(n)} s_m \right),\tag{4}$$

where

$$\Omega_n(x) = \left\lfloor \frac{x}{d^v(n)} \cdot d^v_{\max} \right\rfloor$$
(5)

and $d_{\max}^v = \max_n d^v(n)$. When C is a regular code, (4) becomes the flipping function of Gallager's BF decoding algorithm [1] and is equal to the number of unsatisfied check nodes (UCNs) associated with VN v_n . We further define

$$F_n = \sum_{m \in \mathcal{M}(n)} q_{mn} s_m,\tag{6}$$

where

$$q_{mn} = \begin{cases} 1, & \text{if } \max_{n' \in \mathcal{N}(m)} E_{n'} = E_n \text{ and } E_n \ge \eta \\ 0, & \text{otherwise} \end{cases}$$
(7)

and η is a numerically-optimized integer. (6) counts the number of UCNs connected to VN v_n for which it is a local E_n -maximizing VN. This function is similar to the reliability metric used in the parallel weighted BF decoder [9]. As a bit

decision associated with a large F_n is likely to be incorrect, we consider the VNs in \mathcal{V}^c with the largest F_n as the ones which have the least reliable decision.

Let \oplus denote the exclusive or (XOR) operation and $I_{m \to n}^c$ be the pre-computed sign bit of the C2V message to be sent from c_m to v_n , i.e., $I_{m \to n}^c \triangleq \operatorname{bsgn}(m_{m \to n}^c) = \bigoplus_{n' \in \mathcal{N}(m) \setminus n} \operatorname{bsgn}(m_{n' \to m}^v)$, where $\operatorname{bsgn}(x) = [1 - \operatorname{sgn}(x)]/2$. We need another integervalued indicator

$$A_n = \Omega_n \left(\sum_{m \in \mathcal{M}(n)} I_{m \to n}^c \oplus \operatorname{bsgn}(L_n) \right), \tag{8}$$

where the argument of $\Omega_n(\cdot)$ counts (predicts) the number of *future* incoming messages whose signs are different from that of the total LLR of v_n . The normalized count A_n is then used to quantify the force of driving a bit decision to change after updating, as a larger A_n implies that the decision of v_n may have a higher probability of being flipped. It is thus reasonable to conjecture that, among these local F_n -maximizing VNs, the VNs which are most likely to be corrected after receiving related CN messages are the ones which have the largest A_n . The VN selection procedure is summarized below.

Algorithm 2 Adaptive Grouping Method I

Initialization Set $\mathcal{G} = \emptyset$.

Step 1 $\forall n \in \mathcal{V}^c$, compute F_n , and find $F^* = \max_{n \in \mathcal{V}^c} F_n$. If $F^* = 0$, stop and output $\mathcal{G} = \mathcal{V}^c$.

- Step 2 Let $S = \{n | F_n = F^*, n \in \mathcal{V}^c\}$ and compute A_n for all $n \in S$.
- Step 3 Find $A^* = \max_{n \in S} A_n$ and form the candidate set $\tilde{\mathcal{G}} = \{n | A_n = A^*, n \in S\}$.
- Step 4 Select n^* arbitrarily from $\tilde{\mathcal{G}}$ and add n^* to \mathcal{G} . Then, remove all $n \in \mathcal{N}(m), m \in \mathcal{M}(n^*)$ from $\tilde{\mathcal{G}}$.

Step 5 If $\tilde{\mathcal{G}} = \emptyset$, stop and output \mathcal{G} ; otherwise, go to **Step 4**.

In **Step 1**, we compute the F_n 's of those un-updated VNs. If all the resulting F_n 's are zero, i.e., $F^* = 0$, we conclude that they are reliable and put these VNs in the same group. When $F^* > 0$, we suspect that some incorrect decisions may still exist in \mathcal{V}^c . Thus in **Step 2** and **Step 3**, we collect the VNs with largest F_n value and select the ones having maximum A_n to form a tentative set $\tilde{\mathcal{G}}$. In **Step 4**, we randomly select one index from $\tilde{\mathcal{G}}$, say n^* , to join \mathcal{G} and remove it along with the indices of the VNs which are connected with those CNs linked to v_{n^*} (i.e., $\mathcal{M}(n^*)$) from $\tilde{\mathcal{G}}$. The purpose of removing these VNs is to prevent potential mutual erroneous message exchanges. **Step 4** is repeated until $\tilde{\mathcal{G}}$ is emptied.

Since finding $\max_{n' \in \mathcal{N}(m)} E_{n'}$ for each UCN ($s_m = 1$) in (7) requires extra computational effort, a simple alternative is to use (4) as the reliability metric directly. Moreover, as the sign of the a V2C message from a VN is likely to be the same as that of its LLR value in later iterations, we have

$$I_{m \to n}^{c} = \bigoplus_{n' \in \mathcal{N}(m) \setminus n} \operatorname{bsgn}(m_{n' \to m}^{v})$$
$$\approx \bigoplus_{n' \in \mathcal{N}(m) \setminus n} \operatorname{bsgn}(L_{n'})$$
(9)

and therefore

$$A_n \approx \Omega_n \left(\sum_{m \in \mathcal{M}(n)} \bigoplus_{n' \in \mathcal{N}(m)} \operatorname{bsgn}(L_{n'}) \right) = E_n, \qquad (10)$$

which indicates that the VNs with the largest E_n may have the highest probability of being corrected as well. By adopting (4) and (10), we obtain a simplified version of **Algorithm 2**.

| Algorithm 3 Adaptive Grouping Method II |
|--|
| Initialization Set $\mathcal{G} = \emptyset$. |
| Step 1 Compute E_n for all $n \in \mathcal{V}^c$. |
| Step 2 Let $E^* = \max_{n \in \mathcal{V}^c} E_n$. If $E^* < \delta$, stop and output |
| $\mathcal{G} = \mathcal{V}^c$; otherwise, set $\tilde{\mathcal{G}} = \{n E_n = E^*, n \in \mathcal{V}^c\}.$ |
| Step 3 Select n^* arbitrarily from $\tilde{\mathcal{G}}$ and add n to \mathcal{G} . Then, |
| remove all n where $n \in \mathcal{N}(m), m \in \mathcal{M}(n^*)$ from $\tilde{\mathcal{G}}$. |
| Step 4 If $\tilde{C} = \emptyset$ stop and output C : otherwise go to Step 3 |

The integer reliability threshold δ in **Step 2** is numerically optimized through simulation. No matter whether **Algorithms 2** or **3** is used as the grouping method in **Step 2** of the AGSBP algorithm (**Algorithm 1**), the set of parameters, $\{A_n, F_n, E_n\}$ is immediately updated once any related message is renewed. The selection of next \mathcal{G} is based on the updated information and therefore the group partition and the corresponding group sizes for different iterations may not be the same.

IV. NUMERICAL RESULTS

We present the frame error rate (FER) performance of the conventional GSBP algorithm and AGSBP algorithms with the proposed grouping methods in decoding MacKay's (1008, 504) regular LDPC code (504.504.3.504 [10], $d^{v}(n) = 3$), (806, 272) regular code $(816.1A4.845 [10], d^{v}(n) = 4)$, and WiFi (1944, 972) quasi-cyclic (QC) LDPC code [11]. The frame size is assumed to be equal to the codeword length, hence FER is the same as the codeword error probability. AGSBP-I and AGSBP-II in the following figures denote the AGSBP algorithms that use Adaptive Grouping Method I and II (Algorithm 2 and 3), respectively. For further decoding complexity reduction, we also consider the min-sum (MS) approximation [12] of the C2V updating formula (1). We denote the MS-based GS algorithms with the conventional grouping method by GSMS. Similarly, the MS-based adaptive group shuffled decoders using proposed grouping methods I and II are denoted by AGSMS-I and AGSMS-II, respectively. Table I lists the optimized parameters used for AGSBP and AGSMS algorithms in decoding different codes.

A. Numerical Examples

Figs. 1 and 2 respectively plot the FER performance of the (1008, 504) and (806, 272) regular MacKay codes using various GS algorithms with $l_{max} = 25$. For the (1008, 504) code, the AGSBP-I and AGSBP-II algorithms achieve about 0.3 dB and 0.25 dB gains against the conventional GSBP decoder at FER $\approx 10^{-5}$. The AGSMS algorithms outperform the GSMS decoder as well. When decoding the (806, 272) code, the AGSBP-I algorithm gives a 0.25 dB gain against the

TABLE I PARAMETER VALUES USED FOR AGSBP/AGSMS ALGORITHMS

| Code | AGSBP-I | AGSBP-II | AGSMS-I | AGSMS-II | |
|-------------------|---------|----------|---------|----------|--|
| Code | η | δ | η | δ | |
| MacKay (1008,504) | 1 | 1 | 1 | 2 | |
| MacKay (816,272) | 1 | 2 | 2 | 2 | |
| WiFi (1944,972) | 1 | 4 | 6 | 6 | |



Fig. 1. FER performance of Mackay's (1008, 504) regular LDPC code using various GS decoding algorithms.

conventional GSBP decoder at FER $\approx 10^{-5}$. The AGSBP-II algorithm outperforms the GSBP one by 0.2 dB at the same FER. Furthermore, the decoding gains of both AGSMS-I/-II over the GSMS decoder are about 0.3 dB at FER $\approx 10^{-5}$.

Fig. 3 presents the FER performance of various algorithms in decoding the length-1944 WiFi code where $l_{\rm max} = 50$. The performance of the local girth based GSBP (LGSBP) [5], informed fixed scheduling based GSBP (IFSGSBP) [6] algorithms, and their MS-based variants (denoted by LGSMS and IFSGSMS) is also shown in Fig. 3 for reference purpose. To limit the implementation parallelism, we set the constraint for the WiFi code that the group size determined by our grouping methods be less than 1944/3=648 VNs. Simulation results show that the AGSBP algorithms yield about 0.25 dB performance gain over the GSBP, IFSGSBP, and LGSBP algorithms at the FER $\approx 2 \times 10^{-6}$. Moreover, by applying our grouping methods, AGSMS algorithms also offer improved performance against the GSMS, IFSGSMS, and LGSMS algorithms.

B. Complexity Analysis

All GSBP (GSMS) algorithms discussed, including our AGSBP (AGSMS) decoder, need the same basic computing complexity. For our AGSBP and AGSMS algorithms, extra computation is needed whenever **Steps 2** of **Algorithm 1** is activated. $\Omega_n(x)$ can be obtained by using a look-up table since $d^v(n), d_{max}^v$ are known and x is an integer. The computation of E_n can be done by having each UCN sending a triggering signal to the counter associated with its connected VNs. The UCN number of an ungrouped VN can then be accumulated (added), so the number of required additions is (at most) $\sum_{m:s_m=1} d^c(m)$. The AGSBP-II and



Fig. 2. FER performance of Mackay's (816, 272) LDPC code using various GS decoding algorithms.



Fig. 3. FER performance of IEEE 802.11 (1944, 972) rate-1/2 LDPC code using various GS decoding algorithms.

AGSMS-II decoders need to find E^* which requires $|\mathcal{V}^c| - 1$ comparisons, where $|\cdot|$ denotes set cardinality. For the AGSBP-I and AGSMS-I algorithms, besides E_n , they also have to compute F_n, A_n and find F^* and A^* . Given E_n , we need $\sum_{m:s_m=1} [d^c(m) - 1]$ comparisons to find $\max_{n' \in \mathcal{N}(m)} E_{n'}$ for all UCNs; see (7). As for F_n , it can be computed in a way similar to that of computing E_n , and finding F^* requires another $|\mathcal{V}^c| - 1$ comparisons. To compute A_n and find A^* , we need $\sum_{n \in S} [d^v(n) - 1]$ additions and $|\mathcal{S}| - 1$ comparisons. Computing checksums and the XOR operations in (8) are omitted for they involve binary operations only.

Shown in Table II is the basic computational complexity per iteration in decoding MacKay's (1008, 504) code. The corresponding extra average per-iteration computational complexity at some selected iterations for the proposed algorithms to decode the same code are listed in Table III. The complexity is evaluated at specific SNRs and iterations. Note that the operations listed in Table II are real-number operations while those shown in Table III are integer based; in fact, each integer addition is only a simple 'add one' (accumulation) operation.

TABLE II BASIC COMPLEXITY $(\times 10^3)$ of GSBP and GSMS Algorithms

| | Addition/Subtraction | Comparison | $\phi(\cdot)$ -Operation |
|------|----------------------|------------|--------------------------|
| GSBP | 18.144 | 0 | 18.144 |
| GSMS | 6.048 | 12.096 | 0 |

TABLE III SIMULATED AVERAGE EXTRA CONDITIONAL COMPLEXITY ($\times 10^3$) for AGSBP and AGSMS Decoders (AD: Addition; CP: Comparison)

| SNR I | Itor | AGSBP-I | | AGSBP-II | | AGSMS-I | | AGSMS-II | |
|------------|------|---------|------|----------|------|---------|------|----------|------|
| | nei. | AD | CP | AD | CP | AD | CP | AD | CP |
| 2.75 dB | 5 | 0.31 | 5.16 | 0.13 | 4.97 | 0.65 | 6.89 | 0.19 | 3.85 |
| | 10 | 1.85 | 13.8 | 0.95 | 14.6 | 3.97 | 21.2 | 0.52 | 4.87 |
| | 15 | 4.75 | 26.0 | 1.91 | 22.7 | 7.04 | 33.2 | 1.13 | 6.46 |
| | 20 | 4.71 | 27.3 | 1.90 | 24.1 | 8.33 | 37.3 | 1.18 | 6.82 |
| 3.0 dB | 5 | 0.18 | 4.13 | 0.09 | 3.97 | 0.35 | 6.04 | 0.12 | 3.38 |
| | 10 | 1.83 | 13.3 | 1.02 | 14.5 | 3.34 | 18.5 | 0.42 | 4.46 |
| | 15 | 3.71 | 21.1 | 1.92 | 22.4 | 6.67 | 31.0 | 1.10 | 6.34 |
| | 20 | 3.04 | 19.6 | 1.56 | 21.2 | 7.16 | 33.5 | 1.29 | 6.84 |

V. CONCLUSION

We have developed new VN grouping methods for use in GS decoding of LDPC codes. The proposed methods employ integer based metrics to sequentially select the VNs for updating and can be applied to both BP and MS based decoding algorithms. The extra binary/integer computational efforts needed for the adaptive VN grouping methods are evaluated. We present some numerical behaviors of the proposed AGSBP and AGSMS decoding algorithms and demonstrate that both of them are able to provide improved performance in comparison with some known grouping methods in decoding either regular or irregular LDPC codes.

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