

On the Partial Decoding Delay of Sparse Network Coding

Amir Zarei, Peyman Pahlavani and Mansoor Davoodi

Abstract—Sparse Network Coding (SNC) is a promising technique for reducing the complexity of Random Linear Network Coding (RLNC), by selecting a sparse coefficient matrix to code the packets. However, the performance of SNC for the Average Decoding Delay (ADD) of the packets is still unknown. In this paper, we study the performance of ADD and propose a Markov Chain Model to analyze this SNC metric. This model provides a lower bound for decoding delay of a generation as well as a lower bound for decoding delay of a portion of a generation. Our results show that although RLNC provides a better decoding delay of an entire generation, SNC outperforms RLNC in terms of ADD per packet. Sparsity of the coefficient matrix is a key parameter for ADD per packet to transmit stream data. The proposed model enables us to select the appropriate degree of sparsity based on the required ADD. Numerical results validate that the proposed model would enable a precise evaluation of SNC technique behavior.

Index Terms—Random Linear Network Coding, Sparse Network Coding, Markov Chain Model

I. INTRODUCTION

RANDOM Linear Network Coding (RLNC) is decentralized scheme for Network Coding covered by many research studies that include rate characterization, error-protection coding, and construction of codes. Especially, determining the coding specifications has its importance in providing the premise for an efficient transmission [1] and [2].

High computational complexity and cost of large decoding delay of RLNC are two main practical barrier for extensive usage [3]. In order to reduce computational load, Sparse Network Coding (SNC) was introduced [4]- [7]. Unlike traditional RLNC, in SNC, the coefficients are chosen sparsely, i.e., mostly zero which enables a faster decoding speed at the receiver side. SNC organizes source packets in a group, called a generation, to be linearly combined using randomly selected coefficients from the elements of a finite field F_q . A coded packet is w -sparse if it contains exactly w non-zero coding coefficients.

Sparse coding techniques were originally conceived to reduce coding complexity. The first model is presented by [8], that analyzes the SNC technique behavior. The proposed model would enable the approximation assessment of the SNC behavior to compare to some of the more general bounds that have been recently used. However, they did not provide a closed form for the probability of receiving a linearly independent packet and conducted a Monte Carlo analysis to empirically obtain this probability.

The previous studies have emphasized on SNC as a new scheme to reduce the computational complexity of decoding. A profound understanding of the relationship between decoding delay (i.e. the expected number of transmissions to decode an entire generation) and Average value of Decoding Delay (ADD) per packet (i.e. the

expected number of transmissions required to decode a source packet from a generation) over all the packets of a generation of SNC technique is beneficial to its application, such as real-time video transmission systems. In this paper, we focus on SNC as a mechanism to reduce ADD per packet which it is a key parameter for transmitting streaming data. Our contribution can be separated into two main themes.

Mathematical Analysis using Absorbing Markov Chain: We propose an Absorbing Markov Chain based on the concept of covering source packets, i.e., the non-zero columns at the decoding matrix. This allows us to introduce states with the possibility to decode a generation entirely or partially. The model provides a lower bound for decoding delay of a generation and decoding delay of a portion of a generation (decoding at least x of the n source packets). To the best of our knowledge, there is no similar work in related literature. Finally, we use ADD metric for evaluation of partial decoding in SNC scheme and present a lower bound for ADD per packet of a generation.

Performance evaluation of the proposed model for erasure free and erasure channels: We validate our analytical models using simulation results and show that the difference between the Absorbing Markov Chain model and the simulation results are negligible for all analyzed generation sizes and packet loss rates. Deviation in terms of square average of the difference between our analytical model and the simulation results for the expected number of transmissions to decode a generation and a portion of a generation are 6 % and 7 %, respectively. Furthermore, we simulated ADD per packet to decode a generation for different generation size and value of w . The comparison between the simulation and analytical results show that, generation size and value of w are two key parameters for using SNC. So that, for small w and moderate generations size, SNC provides 14 % improvement compared to RLNC in terms of ADD per packet, which is significant. However, for large w ($w > 4$) the impact is marginal.

The remainder of this paper is organized as follows: Section II provides an overview of related work. In Section III, we describe the proposed model and exploit the characteristics of absorbing Markov chain to evaluate the performance of SNC technique. In section IV, we validate our model by means of a simulation. We finally conclude the paper in Section V, and provide an outlook of the characteristics that will be addressed in our future research.

II. BACKGROUND AND RELATED WORK

One of the main limitations of the RLNC is that if n source packets are considered, decoding at a destination node begins after receiving n linearly independent coded packets. The expected number of packet transmissions required by a receiver to decode a generation has been expressed in a function of a predefined deadline imposed on packet transmissions in [9]- [11]. If a receiver obtains an insufficient number of packets, it is extremely

A. Zarei, P. Pahlavani and M. Davoodi are with Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan, Iran. (e-mail: {amirzareei, pahlavani, mdmonfared}@iasbs.ac.ir).

unlikely that it can decode any of the source packets. However, the requirement for a large number of received coded packets to begin the decoding process introduces undesirable ADD at the receiving nodes. In a study to alleviate this problem, rank-deficient decoding was proposed in [12] for decoding of a portion of source packets when fewer than n coded packets have been received. Gadouleau et al. in [13] considered the probability of recovering some of the n source packets when k coded packets have been collected, where k can be smaller than, equal to or greater than n . The authors showed that partial decoding is highly unlikely. Claridge et al. [14] derived exact expressions for the probability of decoding a portion of a generation on the reception of an arbitrary number of coded packets.

ADD imposed to a system in a broadcast Scenario is the average number of time steps required by all receivers to decode a generation. Eryilmaz et al. [15] obtained an upper bound on ADD for one receiver, derived expressions for ADD of the system, and provided various scheduling strategies when receivers send feedback to acknowledge the recovery of the original packets. Lucani et al. [9] considered the case where each receiver regularly reports to the transmitter the number of linearly independent coded packets that are still missing. The ADD at a receiver was computed using a Markov chain model and was incorporated in an optimization strategy, which minimizes the number of coded packets that are transmitted before receivers are encouraged to send acknowledgments. More recently, Brown et al. [16] proposed a model which exploits the Stein-Chen method to derive a tight approximation to the probability of a user recovering a generation when SNC is used.

Tunable Sparse Network Coding (TSNC) was proposed by Feizi et al. [17], and was later broadened in [18] and [19]. TSNC can be seen as an improvement of SNC. It helps to reduce additional transmission overhead, which has been seen as one of the SNC's main constraints. In this approach, the system changes sparsity levels using a distribution or density parameter. Tuning process can reduce overhead transmissions (linearly dependent combinations) by using denser codes towards the end of transmissions process.

III. MARKOV CHAIN MODEL

In this section, the goal is to provide a Markov chain model for evaluating decoding delay of SNC. We consider a SNC system with a fixed w for all transmissions and assume that q is sufficiently large, that each randomly generated coded packet is linearly independent with high probability until all w source packets its constituent are not decoded.

The process can be modeled through a Markov chain model with states (c, t) , where t is the number of coded packets received by the destination for covering c packets. Based on these states, we can define a discrete Markov chain, $\delta_q(w, n)$. By reception at receiver the state changes from (c, t) to $(c + i, t + 1)$. For $w = 2$ and generation size n the transition probability between states (c, t) and $(c + i, t + 1)$, denoted by $p_{c,t}^{w,n}(i)$ is defined as follows:

$$p_{c,t}^{w,n}(i) = \begin{cases} 0 & \text{if } (c, t) \in \mathbb{A} \\ \frac{\binom{c}{2}}{\binom{n}{2}} & \text{if } (c, t) \notin \mathbb{A} \text{ and } i = 0, \\ \frac{\binom{c}{1} \times \binom{n-c}{1}}{\binom{n}{2}} & \text{if } (c, t) \notin \mathbb{A} \text{ and } i = 1, \\ \frac{\binom{n-c}{2}}{\binom{n}{2}} & \text{if } (c, t) \notin \mathbb{A} \text{ and } i = 2, \end{cases} \quad (1)$$

where \mathbb{A} is the set of absorbing states, as the states with the possibility of decoding x out of n source packets. In Eq. (1), since $w = 2$, two source packets are combined to construct a coded packet. By receiving a new coded packet, for $i = 0$, the pair of source packets is already covered by t received coded packets. For the case of $i = 1$ and $i = 2$, one and two new source packets are covered respectively by receiving the new coded packet.

By using Eq. (2), the transition probabilities can be easily extended to other w values. We use combinatorial mathematics to derive the transition probabilities of the absorbing Markov chain for arbitrary w as follows:

$$p_{c,t}^{w,n}(i) = \begin{cases} 0 & \text{if } (c, t) \in \mathbb{A}, \\ \frac{\binom{c}{w-t} \times \binom{n-c}{t}}{\binom{n}{w}} & \text{otherwise,} \end{cases} \quad (2)$$

where for $w > c, i \in [w - c, n - c]$ and, for $w \leq c, i \in [0, n - c]$. In our model, we assume that each coded packet includes w source packets with non-zero coefficients exactly. Thus, the states with $(c > \min(t \times w, n), t)$ are not possible since each transmission covers at most w new source packets. Moreover, the states with $(c < w, t)$ are also not possible since the first transmission covering w source packets. Therefore, we consider the probability of occurrence and leaving of these states equal to zero.

In addition, if the probability of occurrence of the state (c, t) , denoted by $pr_{c,t}$, then it can be calculated as follows:

$$pr_{c,t} = \begin{cases} 1 & \text{if } t = 0 \text{ or } 1, \\ \sum_{i=0}^w p_{c-i,t-1}^{w,n}(i) \times pr_{c-i,t-1} & \text{otherwise.} \end{cases} \quad (3)$$

A. Lower bound on decoding delay a generation entirely or partially

One of the features of SNC scheme is partial decoding, i.e., the ability to decode a portion of a generation after receiving a few coded packets. In this section, we provide a lower bound on the expected number of transmission to decode at least x out of n source packets. In fact, we restrict our attention to some condition on the parameters of the Markov chain, which enable us to determine a good lower bound in the following lemma.

Lemma 1. For decoding at least x out of n source packets, only states with $(c \geq x, t \geq (x + \lceil \frac{c-x}{w} \rceil))$ are absorbing.

proof : One key observation is that, for covering n source packets, at least $\lceil \frac{n}{w} \rceil$ received coded packets is required. For decoding x source packets, the destination requires to receive x coded packets. Thus, x received coded packets covers x source packets. According to above-mentioned observation, for covering $(c - x)$ remaining source packets, the destination needs at least $\lceil \frac{c-x}{w} \rceil$ received coded packets. As a result, the state (c, t) is absorbing, if $c \geq x$ and the destination receives at least $x + \lceil \frac{c-x}{w} \rceil$ coded packets.

We now provide a lower bound based on the states $(c \geq x, t \geq (x + \lceil \frac{c-x}{w} \rceil)) \in \mathbb{A}$, in order to evaluate the SNC performance in terms of the number of transmissions required to decode a portion of a generation as follows:

$$T_x = \sum_{\forall (c,t) \in \mathbb{A}} \lceil pr_{c,t} \times t \rceil, \quad (4)$$

where t is the number of coded packets received for covering c source packets and T_x represents the expected number of transmissions to decode at least x out of n source packets. Furthermore, to decode the entire of a generation in Eq. (4),

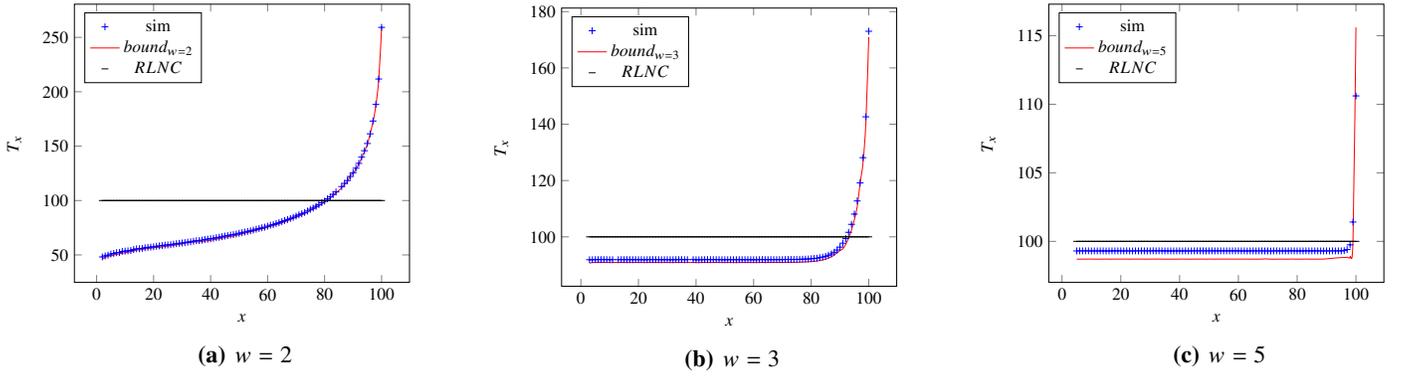


Fig 2: Expected number of transmissions required to decode a portion of a generation, $n = 100$.

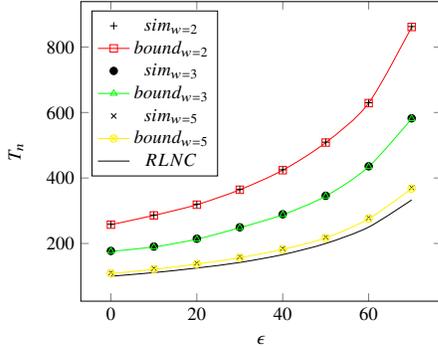


Fig 1: Expected number of transmissions required to recover a generation in loss rate (ϵ). $w = 2, n = 100$.

let $x = n$. In this case, T_n shows the expected number of transmissions required to decode a generation.

B. Lower bound on ADD per packet

We use ADD metric for evaluation of partial decoding in SNC scheme. Let $P = \{p_1, p_2, \dots, p_n\}$ be a set of source packets and the decoding delay of packet p_i is denoted by d_i . More precisely, d_i is the number of transmissions until source packet p_i is decoded. So ADD can be defined as follows:

$$ADD = \frac{\sum_{i=1}^n d_i}{n}. \quad (5)$$

As we mentioned above, T_x is the expected number of transmissions to decode at least x out of n source packets. Thus, for decoding the x -th packet among x source packets, receiving T_x coded packets is required. Also it is enough to receive at most T_x coded packets for decoding the remaining source packets. Hence, T_i can be considered as a lower bound for decoding delay d_i . By replacing T_i with d_i in Eq. (5), a lower bound is obtained for ADD per packet of SNC.

C. The development of proposed model for erasure channel

So far, we have assumed an error-free wireless channel between the sender and the receiver. The model can be easily extended for memory less erasure channel. To this end, we just need to modify the transition probabilities as follows:

$$\widetilde{p_{c,t}^{w,n}}(i) = p_{c,t}^{w,n}(i) \times (1 - \epsilon), \quad (6)$$

where ϵ is the error rate of the wireless link.

IV. SIMULATION AND MODEL VALIDATION

In this section, we confirm the validity of the proposed model using an extensive simulation campaign. We simulated SNC in Kodo library [20]. This allows us to run a series of measurements

TABLE I: THE EXPECTED NUMBER OF TRANSMISSIONS REQUIRED TO SUCCESSFULLY DECODE A GENERATION FOR DIFFERENT SETTINGS

		$w = 2$	$w = 3$	$w = 5$	RLNC
$n = 20$	Sim	36.00	24.75	21.26	20
	Model	35.36	24.42	20.49	
$n = 40$	Sim	85.82	57.41	43.46	40
	Model	84.73	56.45	42.49	
$n = 60$	Sim	140.92	93.69	65.69	60
	Model	139.46	92.53	64.49	
$n = 80$	Sim	198.99	132.44	87.93	80
	Model	197.62	131.14	86.49	
$n = 100$	Sim	259.57	172.47	110.55	100
	Model	258.31	171.52	108.78	

to characterize the SNC performance. The simulation result is the expected of the 50000 independent runs per configuration, to ensure the statistical tightness of the corresponding results. These experiments were carried out by using $GF(2^{16})$ for encoding/decoding operations. The deviation between two plots is calculated by square average between two vectors. The calculation is carried out using the second vector as reference vector for calculating the square average. For example “the deviation of x and y is 5 %” means that if x and y have n entries, then $\sqrt{\frac{1}{n} \sum_{i=1}^n \left(\frac{x_i - y_i}{x_i}\right)^2}$.

The number of absorbing states in the Markov model is unlimited, due to t parameter. We restrict the number of these states by a constraint. Hence, we only consider the absorbing states that are $pr_{c,t} > \frac{1}{\alpha}$, where $\alpha = 10^5$.

Table I illustrates the simulation (Sim) and the theoretical (Model) results of the expected number of transmissions to decode a generation, for different w and n . The simulation confirms the model and shows a deviation of 6 %. The results in Table I show that, the number of transmitted packet is high in sparse codes. High sparsity increases the probability of transmission of the linear dependent packets. Hence, one drawback of SNC is the additional transmission overhead codes, more specifically in small w . On the contrary, if sparsity is low (w large), the probability of being linear independent packets is higher. Thus, low sparsity helps to reduce the number of required transmissions.

After validating the proposed model over an error-free channel, we extended the evaluation to include erasure scenarios using Eq. (6), in Figure 1. We set $n = 100$, $w = 2, 3$ and 5 , also we have modified error rate for various packet loss rates (ϵ). Figure 1 shows that the difference between the simulation results (+ mark) and the provided lower bound (square mark), are negligible, we can also see better RLNC performance in erasure channel.

Figure 2 (a), (b), (c) considers the expected number of transmission to decode at least x out of n source packets. It depicts that the difference between the simulation results (+ mark) and

TABLE II: ADD PER PACKET FOR DIFFERENT SETTINGS

		w = 2	w = 3	w = 4	w = 5	w = 6	w = 7	w = 8	w = 9	w = 10	w = 11	w = 12	w = 13	rlnc
n = 20	Sim	16.39	18.75	19.65	19.90	19.97	19.99	19.99	19.99	20	20	20	20	20
	Model	15.88	18.28	19.12	19.54	19.66	19.73	19.75	19.75	19.75	19.75	19.75	19.75	19.75
n = 40	Sim	32.84	37.54	39.21	39.76	39.92	39.97	39.99	39.99	39.99	39.99	40	40	40
	Model	32.11	36.91	38.43	39.33	39.43	39.57	39.61	39.63	39.63	39.63	39.63	39.63	39.63
n = 60	Sim	49.23	56.42	58.79	59.61	59.87	59.95	59.98	59.99	59.99	59.99	59.99	60	60
	Model	48.45	55.65	58.05	58.95	59.21	59.35	59.41	59.42	59.43	59.43	59.43	59.43	59.43
n = 80	Sim	65.62	75.34	78.39	79.46	79.82	79.94	79.97	79.99	79.99	79.99	79.99	79.99	80
	Model	64.75	74.46	77.51	78.67	78.97	79.17	79.27	79.31	79.32	79.32	79.32	79.32	79.32
n = 100	Sim	82.076	94.29	97.99	99.31	99.76	99.92	99.97	99.98	99.99	99.99	99.99	99.99	100
	Model	81.11	93.28	96.82	98.26	98.69	98.93	98.32	98.76	98.76	98.76	98.76	98.76	98.76

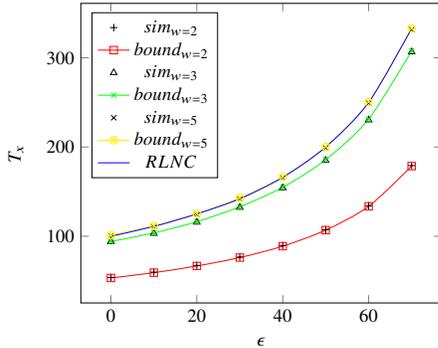


Fig 3: Expected number of transmissions required to recover a portion of generation in loss rate, (ϵ), $w = 2$, $n = 100$ and $x = 10$.

the provided lower bound (line) is very small (a deviation of 7 %). For the case of $w = 2$, the receiver starts to decode the packets after it receives 48 coded packet. This number increased to 90 for $w = 3$, and reached to 97 for $w = 5$. For the case of $w \geq 5$, SNC performance is very close to RLNC, meaning that almost all packets are decoded after receiving n coded packets at once. By decoding a portion of the packets, every next transmission has the probability to include only decoded source packets. These coded packets does not increase the degrees of freedom (independent linear combination of the source packets) at the receiver, but they increase the transmission overhead. Due to this reason, the case of $w = 2$ has the highest number of transmissions. Though $w = 2$ has the highest number of transmissions than other w , the receiver starts to decode in the beginning of its reception. Thus $w = 2$ has small ADD. Figure 3 considers an erasure channel where $n = 100$, $x = 10$, $w = 2, 3$ and 5. It validates the results obtained from the model using simulation results.

Table II depicts ADD per packet to decode a generation for different configurations. We can observe the effect of sparsity level on ADD per packet. So that, for a fixed value of n , the highest sparsity has lower ADD per packet. Also the influence of sparsity level depends on the size of the generation in the sense that, for $n = 40$ ADD per packet of SNC is less than ADD per packet of RLNC up $w = 11$. This number is increased to 12 for $n = 60$, and 13 for $n = 80$. We can finally observe that ADD per packet for RLNC is always equal to n , and the provided lower bound is always less than or equal to n , and it can be concluded that, SNC provides a better or equal ADD per packet compared to RLNC.

V. CONCLUSION AND FUTURE WORK

The provided model is based on an absorbing Markov chain to derive the main metrics performance that are, the expected number of transmissions to recover a generation, the expected number of transmissions to recover a portion of a generation, and ADD per packet to recover a generation. The model is the first analytical model toward the partial decoding in SNC. By

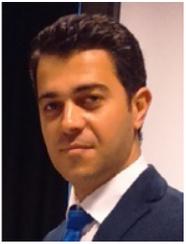
exploiting the proposed model, we showed that SNC provides a better or equal performance compared to RLNC in terms of ADD per packet. We broadened the model for erasure link and validated by an extensive simulation, obtaining a virtually perfect match. There are still some aspects that we would like to address in our future work. First, The model can use to select the most appropriate w to establish the trade-off between the number of transmissions and ADD per packet to decode a generation. Then, it is extending the model so that it is valid for each field size.

REFERENCES

- [1] T. Ho , M. Medard , R. Koetter , D.R. Karger , M. Effros , J. Shi and B. Leong "A Random Linear Network Coding Approach to Multicast," *IEEE Trans. on Inf. Theory* , vol. 52, 2006.
- [2] Ralf Koetter and Frank R. Kschischang "Coding for Errors and Erasures in Random Network Coding," in *IEEE Trans. on Inf. Theory*, vol. 54, 2008.
- [3] P. Sadeghi, R. Shams and D. Traskov, "An Optimal Adaptive Network Coding Scheme for Minimizing Decoding Delay in Broadcast Erasure Channels," *EURASIP Journal on Wireless Commun. and Networking*, 2010.
- [4] D. Silva, W. Zeng and F. R. Kschischang, "Sparse network coding with overlapping classes," in *NetCod*, 2009.
- [5] J. Heide, M. V. Pedersen, F. H. P. Fitzek and M. Medard, "On Code Parameters and Coding Vector Representation for Practical RLNC," in *2011 IEEE Int. Conf. on Commun. (ICC)*, 2011.
- [6] S. Yang and R. W. Yeung, "Batched Sparse Codes," *IEEE Trans.on Inf. Theory* , vol. 60, no. 9, pp. 5322 - 5346, 2014.
- [7] P. Pahlavani, S. Crisostomo and D. E. Lucani, "An Analytical Model for Perpetual Network Codes in Packet Erasure Channels," *Int. Workshop on Multiple Access Commun.*, pp. 126-135, 2016.
- [8] P. Garrido, D. E. Lucani and R. Aguero, "A Markov Chain Model for the Decoding Probability of Sparse Network Coding," *IEEE Trans.on Commun.* , vol. 65, 2017.
- [9] D. E. Lucani, M. Medard and M. Stojanovic, "On coding for delay - New approaches based on network coding in networks with large latency," in *Proc. Workshop on Inf. Theory and Applications*, 2009.
- [10] D. E. Lucani, M. Stojanovic, and M. Medard, "Random linear network coding for time division duplexing: Field size considerations," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, 2009.
- [11] D. E. Lucani, M. Medard, and M. Stojanovic, "On coding for delay - Network coding for time-division duplexing," *IEEE Trans. Inf. Theory*, vol. 58, no. 4, pp. 2330-2348, 2012.
- [12] Z. Yan, H. Xie, and B. W. Suter, "Rank deficient decoding of linear network coding," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process.*, 2013.
- [13] M. Gadouleau and A. Goupil, "A matroid framework for noncoherent random network communications," *IEEE Trans. Inf. Theory*, vol. 57, 2011.
- [14] J. Claridge and I. Chatzigeorgiou, "Probability of Partially Decoding Network-Coded Messages," *IEEE commun. letters*, vol. 21, 2017.
- [15] A. Eryilmaz, A. Ozdaglar, and M. Medard, "On delay performance gains from network coding," in *Proc. 40th Conf. on Inf. Sciences and Systems (CISS)*, 2006.
- [16] S. Brown, O. Johnson and A. Tassi, "Reliability of Broadcast Communications Under Sparse Random Linear Network Coding," in *IEEE Trans. on Vehicular Technology*, vol. PP 2018.
- [17] S. Feizi, D. E. Lucani and M. Medard, "Tunable sparse network coding," in *Zurich Seminar on Commun.*, 2012.
- [18] S. Feizi, D. E. Lucani, C. W. Sorensen, A. Makhdomi and M. Medard, "Tunable sparse network coding for multicast networks," in *NetCod*, 2014.
- [19] C. W. Sorensen, A. S. Badr, J. A. Cabrera, D. E. Lucani, J. Heide and F. H. P. Fitzek, "A Practical View on Tunable Sparse Network Coding," in *European Wireless 2015; 21th European Wireless Conf.*, 2015.
- [20] M. V. Pedersen, J. Heide and F. H. P. Fitzek, "Kodo: An Open and Research Oriented Network Coding Library," in *Int. Conf. on Research in Networking*, 2011.



Amir Zarei received BSc degree in Software Engineering from Ilam University, Ilam, Iran in 2014. Currently he is a master student of computer science at Institute for Advanced Studies in Basic Sciences (IASBS) in Zanjan, Iran. His research interests include Wireless Communication, Network coding, Approximation and Randomized Algorithms.



Peyman Pahlevani is an Assistant Professor in the department of Computer Science and Information Technology of Institute for Advanced Studies in Basic Science (IASBS), Iran. He received his PhD in Wireless Communication from Aalborg University, Denmark in 2014. Prof. Pahlevani was a visiting researcher at the Computer Science Department of University of California, Los Angeles (UCLA). Moreover, He has been collaborated with different institutes and universities such as MIT, and Porto universities. Beside his academic backgrounds, he extended his skills to solve practical

challenges in the area of video communication in his previous carrier, as a Researcher Engineer at AIRTAME company. His research interests are in the area of wireless communication, network coding and its applications, vehicular communications, cooperative networking, and video streaming over WiFi links. Dr. Pahlevani has also served as TPC member for international conferences and as reviewer for high impact journals, such as IEEE Transactions on Vehicular Technology.



Mansoor Davoodi Monfared received his B.S. degree in computer science from Vali-Asr University, Rafsanjan, Iran, in 2005, and M.S. Ph.D degrees in Amirkabir (Tehran Polytechnic University) University in 2007 and 2012 respectively. He is currently Assistant Professor in the Department of Computer Science and Information Technology, Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan, Iran. His main research area includes: computational geometry, imprecise data modeling and multi-objective optimization.