Distortion-free Golden-Hadamard Codebook Design for MISO Systems

Md. Abdul Latif Sarker, Md. Fazlul Kader, Moon Ho Lee, and Dong Seog Han

Abstract—In this letter, a novel Golden-Hadamard codebook (GHC) scheme is proposed to improve the performance of the traditional precoded Alamouti coding for multiple-input and single-output systems. Although the traditional discrete Fourier transform codebook (DFTC) performs satisfactorily with Alamouti coding and offers numerous benefits for the Rayleigh fading channel, this scheme inherently generates huge codeword distortion, which leads to a lower minimum chordal distance (MCD). Furthermore, the uncertain format of all prior versions of codebooks results in poorer minimum determinant (MD) values. Hence, the proposed GHC scheme successfully deals with the issues of traditional DFTC to achieve a better codebook format that completely overcome both MCD and MD problems. The effectiveness of the proposed GHC scheme is confirmed, in terms of bit-error-rate through Monte Carlo simulations.

Index Terms—Codeword distortion, minimum chordal distance and minimum determinant, bit-error-rate performance.

I. INTRODUCTION

PACE-TIME block coding (STBC) is an important technique used in wireless communications to improve diversity gain [1] and data rates [2]. Orthogonal STBC (OSTBC) allows low-complexity decoding [3] and does not provide array gain. Thus, precoded OSTBC (POSTBC), such as the discrete Fourier transform codebook with Alamouti (DFTC-A) coding, has been proposed [4] to improve array gain and attain low average codeword distortion using a matrix search method. In contrast, in another study, a vector search method with unitary and non-unitary POSTBC [5] to minimize codebook selection complexity. Additionally, Suh et al. proposed using the DFTC [6] to achieve high eigenvectors with a high resolution. In [7], Hai et al. compared the performance of the discrete fractional sine transforms codebook (DFRSTC) with that of the DFTC, demonstrating that they are equal. The Hadamard codebook (HC) [8] and the diagonal codebook (DC) [9] has been proposed along with OSTBC to improve error performance. Furthermore, Grag et al. showed their use of a DC in a non-OSTBC scheme [10]. Unfortunately, the uncertain format of all previous version of POSTBC schemes produce lower minimum chordal distances (MCDs) and minimum determinant (MD) values. Based on this problem, we propose a Golden-Hadamard (GH) codebook with Alamouti coding

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M. H. Lee is with the Department of Electronics Engineering, Chonbuk National University, Jeonju 54896, South Korea. (e-mail: moonho@jbnu.ac.kr). Digital Object Identifier: 10.1109/LCOMM.2018.2864124 scheme (GHC-A) in this letter. The Golden code has been proposed along with STBC [11]. Although the Golden code has a higher decoding complexity [11], such schemes show a better bit-error-rate (BER) performance [10]. Based on the literature [4] and [9]–[11], a newly design GHC generator can lead an identical BER performance.

In this letter, we present a novel GHC-A scheme to achieve a higher MCD and MD values, which are key factors to demonstrate good performance for any precoded codeword. Developing a better codebook generator is the principal subject of this letter. However, a limited perfect and imperfect feedback scheme is considered in computer simulations. The superiority of the proposed GHC-A scheme over the traditional POSTBC schemes for multiple-input single-output (MISO) systems, in terms of BER is verified through computer simulations.

II. SYSTEM MODEL

We consider a MISO system where the transceiver is equipped with N_T transmit antennas and single receive antenna. Let N_T channels remains static over the symbol period, T. Then the received signal $\mathbf{y} \in \mathbb{C}^{1 \times T}$ can be modeled as

$$\mathbf{y} = \mathbf{h}^T \mathbf{X} + \mathbf{z} \tag{1}$$

where **h** is the flat and block fading channel vector between the transmitter and receiver, $(\cdot)^T$ represents transpose operation, and a high-order $N_T \times T$ precoded codeword, **X** is

$$\mathbf{X} = \mathbf{W}_{\mathbf{D}}\mathbf{S} \tag{2}$$

where **S** is a low-order $M \times T$ OSTBC, $M \leq N_T$, $\mathbf{W}_{\mathbf{D}}$ is an $N_T \times M$ tall DFT precoding matrix with $\frac{1}{\sqrt{N_T}}e^{\frac{l2\pi kl}{N_T}}$ at entry (k, l) which is chosen from the codebook generator presented in [4], [5], [7]:

$$\mathcal{F}_D = \{ \Theta_D^{i-1} \mathbf{W}_D \}, i = 1, 2, ..., L - 1$$
(3)

The $N_T \times M$ remaining DFT precoding matrices are given by

$$\mathbf{W}_{D,i} = \mathbf{\Theta}_D^{i-1} \mathbf{W}_{D,1}, i = 2, ..., L$$
(4)

where L is the size of a codebook and $\mathbf{W}_{D,1}$ is the first $N_T \times M$ DFT-based precoding matrix, of which entry (k, l) is given as $\frac{1}{\sqrt{N_T}}e^{\frac{j2\pi(k-1)(l-1)}{N_T}}$, Θ_D is a diagonal matrix [4], and $\mathbf{z} \in \mathbb{C}^{1 \times T}$ is a noise vector with zero mean and unit variance.

III. THE PROBLEM OF THE EXISTING DFTC SCHEMES

The given DFTC entails two main problems in the POSTBC scheme. The first problem is codeword distortion which leads to a lower MCD value, and the second problem is the uncertain format of DFTC causing a smaller MD value.

A. **Codeword distortion**: When a precoded codeword block or blocks contain non-zero diagonal elements, this is denoted as codeword distortion or geometric mean distortion. A geometric mean is a special kind of mean value calculated by multiplying elements and taking their square root (for two elements), their cube root (for three elements), and so on. Let the square root of

the product of two matrices as $\sqrt{\begin{pmatrix} 2 & .08 \\ .08 & 2 \end{pmatrix} \begin{pmatrix} .08 & 2 \\ 2 & .08 \end{pmatrix}} = \begin{pmatrix} \sqrt{.32} & 2 \\ 2 & \sqrt{.32} \end{pmatrix}$, where diagonal element $\sqrt{.32} \neq 0$ which represents a non-zero geometric mean, is directly related to an increase in codeword distortion.

Example 1: Let a 2×2 DFTC-A third codeword matrix be given using by (2-4), [4] as follows:

$$\begin{bmatrix} \mathbf{X}_{3} \end{bmatrix} = \frac{\Theta_{D}^{2}}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & e^{i\pi} \end{bmatrix} \begin{bmatrix} s_{11} & -s_{21}^{*} \\ s_{21} & s_{11}^{*} \end{bmatrix} \\ = \begin{bmatrix} 0.0000 + 0.0000i & 0.8315 - 0.5556i \\ 0.1379 + 0.6935i & 0.1379 + 0.6935i \end{bmatrix}$$
(5)

where i = 3, $N_T = 2$, M = 2, $\mathbf{W}_{D,3} = \Theta_D^2 \mathbf{W}_{D,1}$, the rotation vector $\mathbf{u} = [1 \ 7]$ uses the IEEE 802.16e parameters, $\Theta_D = \text{diag}([e^{j\pi/2}, e^{j7\pi/2}])$, $\mathbf{W}_{D,1}$ is the 2 × 2 DFT first precoding matrix, \mathbf{S}_k is the k-th entry of the 2 × 2 complex Alamouti code, symbols s_{11} and s_{21} belong to a quadrature amplitude modulation (QAM) constellation, and $(\cdot)^*$ is the complex conjugate operator. Now, we increase the size of the DFT precoding in (5). Let i = 3, M = 2, and $N_T = 4$ in (5); and we obtain the 4 × 2 DFTC-A third codeword as follows.

$$[\mathbf{X}_{3}] = \begin{bmatrix} 0.0000 + 0.0000i & 0.8315 - 0.5556i \\ 0.1379 + 0.6935i & 0.1379 + 0.6935i \\ 0.0000 - 1.0000i & -0.0000 - 0.0000i \\ 0.7071 - 0.0000i & -0.7071 + 0.0000i \end{bmatrix}.$$
 (6)

Using (5) and (6), we plotted in Fig. 1. Fig. 1(a) depicts a low codeword distortion curve at low-order DFTC, while Fig. 1(b) shows how distortion gradually increases for highorder DFTCs. This problem dramatically lowers the MCD value. Thus, MCD is the major issue for subspace packing. The selected precoding matrix $\mathbf{W}_{D,i}^{sel}$ can be treated as the distortion due to non-ideal precoding $\mathbf{W}_{D,i}$. Therefore, a packing can be described by its MCD [4], [5], [7] as following:

$$\delta_{\min}(\mathbf{W}_D) = \min_{2 \le i \le L} d(\mathbf{W}_{D,1}, \mathbf{W}_{D,i})$$
(7)

where $d(\cdot, \cdot)$ is called the chordal distance between two subspaces $\mathcal{P}_{\mathbf{W}_{D,1}}$ and $\mathcal{P}_{\mathbf{W}_{D,i}}$, of which $\mathcal{P}_{\mathbf{W}_{D}}$ is the subspace generated by the columns of matrix \mathbf{W}_{D} .

B. **MD** for Any Codeword: The MD is an important key factor to explain the good performance of any codeword. Thus, the MD of finite code C is given by [11]

$$\delta_{\min}(\mathcal{C}) \ge 2^b \delta_{\min}(\mathcal{C}_{\infty}) \tag{8}$$

where C_{∞} is denoted as the infinite code, b is bits per symbol and $\delta_{\min}(C_{\infty}) = \min_{X \in C_{\infty}, X \neq 0} |det(\mathbf{X})|^2$.

IV. PROPOSED GHC SCHEME

A Golden section [10], [11] with continuous geometric proportion can provide a good OSTBC pattern. Encouraged by this, we first design a GH precoding scheme as follows. A. **GH Precoding Matrices**: Let $N_T = 2^q$ and $\mathbf{W}_{GH}(2^q)$ be a $2^q \times M$ precoding matrix constructed using M columns of

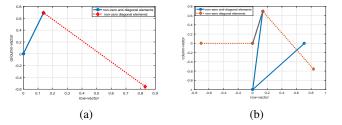


Fig. 1: Line graphs for different sizes of the DFTC-A codeword. (a) Low-distortion of 2×2 DFTC-A codeword in (5) and (b) Highdistortion of 4×2 DFTC-A codeword from (6). Both the red and blue lines represent **non-zero diagonal** and **anti-diagonal** elements.

the $2^q \times 2^q$ recursive GH precoding matrices:

$$\mathbf{W}_{GH}(2^{q}) = \frac{\theta_{G}}{\sqrt{\xi}} \begin{bmatrix} \mathbf{W}_{H}(2^{q-1}) & \mathbf{W}_{H}(2^{q-1}) \\ \mathbf{W}_{H}(2^{q-1}) & (\theta_{G}^{-1} - \theta_{G})\mathbf{W}_{H}(2^{q-1}) \end{bmatrix}$$
(9)

where $2 \leq q \epsilon N_T$ [12], $\mathbf{W}_H(2^{q-1})$ is the $2^{q-1} \times 2^{q-1}$ Hadamard matrix presented in [13], $\xi = n\{(1+n)^q - (1-n)^q\}/2^q$, *n* indicates the root of the geometric number, θ_G is the Golden number [14], which satisfies $\theta_G \theta_G^{-1} = 1$, $(\theta_G^{-1} - \theta_G) = -1$, and its continuous geometric proportion is $\theta_G : 1 : 1/\theta_G$, which indicates that the geometric mean between θ_G and $1/\theta_G$ is unity. Thus, we incorporate (9) in (4) and construct the remaining GH precoding matrices as follows:

$$\mathbf{W}_{GH,i}(2^q) = (e^{j\pi\delta_{\min}(\mathbf{W}_{GH}(2^q))})^{i-1}\mathbf{W}_{GH,1}(2^q), i = 2, .., L (10)$$

where $e^{(\cdot)}$ denotes the exponential function, $\delta_{\min}(\mathbf{W}_{GH}(2^q))$ is the MCD based on (7) and (9), $\mathbf{W}_{GH,1}(2^q)$ is an $N_T \times M$ first GH precoding matrix. Now, we investigate the real and complex Golden numbers with GH precoding as follows: **Case I** (Real Golden Number): We consider that θ_{G_r} is the real-valued root of $\mu_{\theta_{G_r}}(X) = X^2 - X - 1$; $\theta_{G_r} = \frac{1+\sqrt{5}}{2}$ is known as the real Golden number [10], [14], and its algebraic conjugate can be obtained as $\theta_{G_r} = 1 - \theta_{G_r} = \frac{1-\sqrt{5}}{2}$. Let $i = 3, q = 2, n = \sqrt{5}, \xi = 5$ and $\theta_{G_r} = \frac{1+\sqrt{5}}{2}$ in (9) and (10) to obtain the 4×4 real GH precoding matrices as

$$\mathbf{W}_{GH_r,3}(4) = (-1)^2 \mathbf{W}_{GH_r,1}(4), i = 3,$$
(11)

where $e^{j\pi\delta_{\min}(\mathbf{W}_{GH_r}(2^q))} = -1$ and

Case II (Complex Golden Number): Let θ_{G_c} be the complexvalued root of $\mu_{\theta_{G_c}}(jX) = X^2 - jX - 1$; $\theta_{G_c} = \frac{j+\sqrt{3}}{2}$ is known as the complex Golden number, and its algebraic conjugate is obtained as $\theta_{G_c}^- = \frac{j-\sqrt{3}}{2}$. Similarly, let $i = 3, q = 2, n = \sqrt{3}, \xi = 3$ and $\theta_{G_c} = \frac{j+\sqrt{3}}{2}$ values in (9) and (10) to obtain the 4×4 complex GH precoding matrices as

$$\mathbf{W}_{GH_c,3}(4) = (-j)^2 \mathbf{W}_{GH_c,1}(4), i = 3,$$
(13)

MCD, $\sqrt{\delta_{\min}(\mathbf{W})}$ when $i = 2, q = 2, N_T = 4$, and $M = 2$						
DFTC[4, 6]	DC[9, 10]	HC[8]	Proposed GHC Scheme			
			Case $I(15)$	Case $II(15)$		
0.84	0.89	1.00	1.00	1.00		

TABLE I: Computation of the MCD

TABLE II: Computation of the MD

MD, $\sqrt{\delta_{\min}(\mathcal{C})}$ when $i = 2, q = 2, N_T = 4$, and $M = 2$							
b	DFTC[4, 6]	DC[9, 10]	HC[8]	Proposed GHC			
				Case I	Case II		
4	2	2	2	2.89	2.52		
6	4	4	4	5.78	5.05		

where $j e^{j\pi \delta_{\min}(\mathbf{W}_{GH_c}(2^q))} = -j$ and

$$\mathbf{W}_{GH_{c},1}(4) = \frac{j + \sqrt{3}}{2\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -j & -j \\ 1 & -1 & -j & j \end{bmatrix}$$
(14)

Note that (12) and (14) are the exact real and complex GH matrices, respectively. Thus, we can summarize the expression of the designed real and complex GHC generators as

$$\mathcal{F}_{GH} = \begin{cases} \{(-1)^{i-1} \mathbf{W}_{GH_r, i}(2^q)\}, for-\text{Case I} \\ (-j)^{i-1} \mathbf{W}_{GH_c, i}(2^q)\}, for-\text{Case II} \end{cases}$$
(15)

Now, we can calculate the MCD and MD values using (15) in (2) to make Table I and II based on (7) and (8) for both real **CaseI** and complex **CaseII**, respectively.

B. Distortion-free codeword: When a precoded codeword block or blocks contain zero-diagonal elements, they are distortion-free. Using Section III-A, we consider $\sqrt{\begin{pmatrix} 2.0 & 0.08 \\ -0.08 & 2.0 \end{pmatrix} \begin{pmatrix} -0.08 & 2.0 \\ 2.0 & 0.08 \end{pmatrix}} = \begin{pmatrix} \sqrt{0} & 2 \\ 2 & \sqrt{0} \end{pmatrix}$, where diagonal element $\sqrt{0} = 0$ represents a zero value of geometric mean and is directly related to achieving a distortion-free codeword which exhibit as an error-free feedback link [15]. *Example 2*, Let $i = 3, q = 2, N_T = 4, M = 2$, and $n = \sqrt{5}$. Then, by substituting (12) in (6), we obtain the 4×2 GHC-A third codeword matrix using *Case I* as

$$\begin{bmatrix} \mathbf{X}_3 \end{bmatrix} = \begin{bmatrix} 0.0000 + 0.0000i & 1.0233 - 1.0233i \\ 1.0233 + 1.0233i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 1.0233 - 1.0233i \\ 1.0233 + 1.0233i & 0.0000 + 0.0000i \end{bmatrix}$$
(16)

Similarly, by substituting (14) in (6), we obtain the 4×2 GHC-A third codeword matrix using **Case II** as

$$[\mathbf{X}_{3}] = \begin{bmatrix} 0.0000 + 0.0000i \ 1.1153 - 0.2988i \\ 0.2988 + 1.1153i \ 0.0000 + 0.0000i \\ 0.0000 + 0.0000i \ 1.1153 - 0.2988i \\ 0.2988 + 1.1153i \ 0.0000 + 0.0000i \end{bmatrix}.$$
(17)

Using (16) and (17), we plotted Fig. 2 and , by comparison with Fig. 1, we can see distortion-free codeword curve. Thus, we state *Theorem 1* as follows.

Theorem 1: All forms of \mathbf{X} become distortion-free if the precoding matrix \mathbf{W} is a Hadamard or a Golden-Hadamard.

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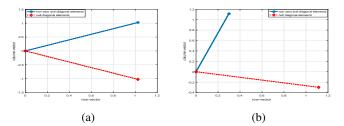


Fig. 2: Line graphs of the proposed GHC-A codeword: (a) distortionfree 4×2 GHC-A codeword for **CaseI** obtained using (16), and (b) distortion-free 4×2 GHC-A codeword for **CaseII** obtained using (17). The red lines represent **null-diagonal** elements and the blue lines represent **non-zero anti-diagonal** elements.

Proof of Theorem 1: DFTC-A schemes are not distortion-free due to the exponential function of DFT precoding, which leads to non-zero diagonal elements in the codewords as shown in (5) and (6). Let the exponential function of a DFT precoding be approximately equal to the Golden-Hadamard function:

$$e^{\frac{j^{2\pi(k-1)(l-1)}}{N_T}} \approx e^{j\pi} = (\theta_G^{-1} - \theta_G) = -1$$
(18)

where Euler's formula is related to the Golden section by $e^{j\pi} = (\theta_G^{-1} - \theta_G)$ as stated in [14]. Thus, we incorporate (18) in (10) and obtain the distortion free third codeword using (2)

$$[\mathbf{X}_3] = \frac{(-1)^2 \theta_G}{\sqrt{\xi}} \begin{bmatrix} 0 & -s_{21}^* + s_{11}^* \\ s_{11} - s_{21} & 0 \\ 0 & -s_{21}^* + s_{11}^* \\ s_{11} - s_{21} & 0 \end{bmatrix}$$
(19)

We can see that (19) is completely distortion-free owing to **null-diagonal** elements in the codeword block or blocks as explained Section IV-B. Thus, *Theorem 1* has been proven.

V. PAIRWISE ERROR PROBABILITY (PEP)

Let S_k and S_l be the transmitted and decoded space-time codewords, respectively, and $l \neq k$. Thus, the union bound on the BER can be written as [16]

$$BER \le \sum_{l \ne k} \frac{e(\mathbf{S}_k, \mathbf{S}_l)}{log_2 2^b} \mathbf{P}_r(\mathbf{S}_k \longrightarrow \mathbf{S}_l)$$
(20)

where $e(\mathbf{S}_k, \mathbf{S}_l)$ is the Hamming distance between the binary sequences representing \mathbf{S}_k and \mathbf{S}_l , the codeword PEP for an effective channel $\tilde{\mathbf{h}}$ can be upper bounded by the Chernoff bound [3] as $\mathbf{P}_r(\mathbf{S}_k \longrightarrow \mathbf{S}_l | \tilde{\mathbf{h}}) \leq e^{-\frac{\gamma_0 ||\tilde{\mathbf{h}}||^2}{2 \cdot 2^q}}$, γ_0 is the SNR, $||\tilde{\mathbf{h}}||^2 = \mathbf{h}^T \mathbf{W}_{GH}(2^q) \mathbf{A} \mathbf{W}_{GH}^H(2^q) \mathbf{h}^*$, $\mathbf{A} = \bar{\mathbf{S}} \bar{\mathbf{S}}^H$ as the covariance distance product matrix, $(\cdot)^H$ denotes the Hermitian operator. In the worst case, the average distance, $\overline{\mathbf{A}} = \frac{1}{T} \mathbb{E}[\mathbf{A}] = \frac{1}{T} \sum_{k \neq l} p_{kl} \Xi_{kl} \Xi_{kl}^H$ where p_{kl} is the probability of the pair $(\mathbf{S}_k, \mathbf{S}_l)$ and $\Xi = (\mathbf{X} - \hat{\mathbf{X}})$.

If \mathbf{h}_{max} and \mathbf{h}_{min} represent the elements of \mathbf{h} with the largest and smallest magnitude, respectively as explained in [9], [10], then the obtained norm of the effective channel provides the correct, $\|\mathbf{\tilde{h}}\|_{correct}^2 = 2|\theta_G|^2|\mathbf{h}_{max}|^2 + (1 + |\theta_G|^2 - |\theta_G|^2|\theta_G|^2)|\mathbf{h}_{min}|^2$ and the incorrect, $\|\mathbf{\tilde{h}}\|_{incorrect}^2 =$

 $2|\theta_G|^2|\mathbf{h}_{min}|^2 + (1+|\theta_G|^2-|\theta_G|^2|\theta_G|^2)|\mathbf{h}_{max}|^2$ feedback bits, respectively. Thus, the PEP in (20) is bounded by

$$\mathbf{P}_{r}(\mathbf{S}_{k} \longrightarrow \mathbf{S}_{l} | \widetilde{\mathbf{h}}) \leq e^{-\frac{1.6180\gamma_{0}(|\mathbf{h}_{1}|^{2} + |\mathbf{h}_{2}|^{2})}{2 \cdot 2^{4}}}$$
(21)

where $P_c = 1$ is the probability for the correct feedback bit [9], [10] and $|\theta_G|^2 = 1.6180$ is the optimal choice for **Case II**. Similarly, we can measure the optimal choice for **Case II** based on $|\theta_{G_c}|^2 = 0.8660 + 0.50000i$.

VI. SIMULATION RESULTS

This section shows some simulation results to demonstrate the proposed GHC schemes for both real (Case I) and complex (Case II) GH precoding. We implemented the simulations of MISO systems as Monte-Carlo simulations over a block flat Rayleigh fading channel. Moreover, we considered limited feedback cases with an optimal codebook search method at an SNR level of 30 dB. Throughout the simulations, we assumed that q = 2, $N_T = 4$, M = 2, and L = 64 with six-bit codebooks. We assume $P_c = 1$ and $|\theta_{G_r}|^2 = 1.6180$ (Case I) and $|\theta_{G_c}|^2 = 0.8660 + 0.5000i$ (Case II) for the optimal choice. In a fair comparison, our GHC schemes achieved higher MCD and MD values (calculated as shown in Table I and II) for both 16-QAM and 64-QAM signal constellations and outperformed all prior versions of POSTBC schemes. In Table I, it can be seen that the MCD of the GHC and HC scheme is $\delta_{\min}(\mathbf{W}) = 1 = \sin(90^{\circ})$, whereas the MCD of the DFTC and DC schemes are $\delta_{\min}(\mathbf{W}) = 0.84 = \sin(57.14^{\circ})$ and $\delta_{\min}(\mathbf{W}) = 0.89 = \sin(62.87^{\circ})$, respectively. This means that the degree of freedom was increased by almost 27^0 to 33^0 in the GHC and HC scheme compared with that of the DFTC [4], [6] and DC [9], [10] schemes. In contrast, the performance of the HC scheme [8] dramatically worsened than the GHC scheme due to the poorer MD values shown in Table II.

In Fig. 3, a 4×2 system using a POSTBC with 16-QAM and 64QAM constellations is assumed for the perfect feedback case. For the imperfect feedback with 16QAM shown in Fig. 4, we assumed $\mathbf{w} = f(\mathbf{\tilde{h}})$ where $\mathbf{\tilde{h}} = \alpha \mathbf{h} + \sqrt{1 - \alpha^2} \mathbf{h}_{error}$ and $\alpha = J_0(2\pi f_d T_c \Delta)$, $f_d T_c = 0.01$ is the normalized Doppler frequency and $\Delta = 12$ is the feedback delay. Similarly, we expect 64QAM to show a good performance over imperfect feedback. Furthermore, the performance of the DFRSTC [7] is equal to that of the DFTC scheme. The optimal codebook selection method of the GHC scheme provides a 3 to 5 dB array gain compared with all prior versions of POSTBC schemes. Both real **Case I** and complex **Case II** demonstrated the better BER performance. The BER performance of the complex GHC scheme is slightly worsened than that of the real GHC scheme because of the reduction of the Golden ratio.

VII. CONCLUSIONS

We analyzed codeword distortion along with the MCD and MD problems for all versions of POSTBC schemes. The traditional POSTBC schemes are illustrated worse BER performance by at least 10^{-1} steps than the proposed GHC-A scheme due to the smaller MCD and MD values. Moreover, the simulation results confirmed that the proposed GHC scheme

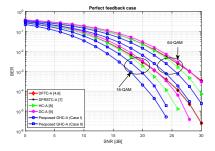


Fig. 3: BER comparison under perfect feedback. q = 2, $N_T = 4$, M = 2, and L = 64.

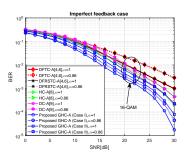


Fig. 4: BER comparison under imperfect feedback. q = 2, $N_T = 4$, M = 2, L = 64, $f_d T_c = 0.01$, $\Delta = 12$, and $\alpha = 0.86$.

outperforms all previous versions of the POSTBC schemes, in terms of BER. Lastly, the GHC scheme can be extended further to be applied in multi-layer wireless networks considering more general cases, which is subjected to future works.

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