

# Optimal Multicast of Tiled 360 VR Video in OFDMA Systems

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**Abstract**—In this letter, we study optimal multicast of tiled 360 virtual reality (VR) video from one server (base station or access point) to multiple users in an orthogonal frequency division multiple access (OFDMA) system. For given video quality, we optimize the subcarrier, transmission power and transmission rate allocation to minimize the total transmission power. For given transmission power budget, we optimize the subcarrier, transmission power and transmission rate allocation to maximize the received video quality. These two optimization problems are non-convex problems. We obtain a globally optimal closed-form solution and a near optimal solution of the two problems, separately, both revealing important design insights for multicast of tiled 360 VR video in OFDMA systems.

**Index Terms**—virtual reality, 360 video, multicast, OFDMA.

## I. INTRODUCTION

A virtual reality (VR) video is generated by capturing a scene of interest from all directions at the same time. A user wearing a VR headset can freely watch the scene of interest in any viewing direction at any time, hence enjoying immersive viewing experience. VR has vast applications in entertainment, education, medicine, etc. Transmitting an entire 360 VR video which is of a much larger size than a traditional video brings a heavy burden to wireless networks. To improve transmission efficiency and avoid view switch delay, a 360 VR video is divided into smaller rectangular segments of the same size, referred to as tiles, and the set of tiles covering a user's current *field-of-view* (FoV) and the FoVs that may be watched shortly should be transmitted simultaneously [1].

In [2]–[5], the authors consider 360 VR video transmission in single-user [2], [3] and multi-user [4], [5] wireless networks, and optimize video encoding parameters [2], [3] as well as resource allocation [4], [5] to maximize the received 360 VR video quality. The optimization problems in [2]–[5] are discrete, and the obtained solutions do not offer many design insights. In [4], multicast opportunities are ignored, and hence the resulting solution may not be efficient for multi-user wireless networks. In [5], multicast opportunities are considered, but the tiles are treated separately in the optimization. This

leads to prohibitively high computational complexity, as the number of tiles to be transmitted is usually quite large. Therefore, it is still not known how the required FoVs and channel conditions of all users affect optimal resource allocation and how to obtain low-complexity resource allocation for 360 VR video transmission in multi-user wireless networks.

In this letter, we would like to address the aforementioned issues. We aim to design optimal multicast of tiled 360 VR video from one server (base station or access point) to multiple users in an orthogonal frequency division multiple access (OFDMA) system. We formulate optimal multicast of tiled 360 VR video as multi-group multicast optimization problems. Specifically, for given video quality, we optimize the subcarrier, power and rate allocation to minimize the total transmission power. We obtain a globally optimal closed-form solution of this problem (under a mild condition), which reveals that the minimum transmission power increases exponentially with the total number of tiles that need to be transmitted. For given transmission power budget, we optimize the subcarrier, power and rate allocation to maximize the received video quality. We obtain a near optimal solution of this problem, which reveals that the maximum video quality is inversely proportional to the maximum number of tiles that need to be transmitted for all viewing directions. To the best of our knowledge, these important design insights have never been analytically verified in existing literature. Finally, numerical results demonstrate the advantage of the proposed solutions.

## II. SYSTEM MODEL

As illustrated in Fig. 1, we consider multicast of a 360 VR video from a single-antenna server (base station or access point) to  $K$  ( $\geq 1$ ) single-antenna users, denoted by  $\mathcal{K} \triangleq \{1, \dots, K\}$ , in an OFDMA system.<sup>1</sup> Consider  $M_h \times M_v$  viewing directions, where  $M_h$  and  $M_v$  represent the numbers of horizontal and vertical viewing directions. The  $(m_h, m_v)$ -th viewing direction refers to the viewing direction in the  $m_h$ -th row and  $m_v$ -th column. When a VR user is interested in one viewing direction, he can view a rectangular FoV of size  $F_h \times F_v$  (in rad $\times$ rad) with the viewing direction as its center. The viewing direction of each user can be captured by sensors in his VR headset.

To improve transmission efficiency, we consider tiling. In particular, the 360 VR video is divided into  $V_h \times V_v$  rectangular segments of the same size, referred to as tiles, where  $V_h$  and  $V_v$  represent the numbers of segments in each row and each

<sup>1</sup>Note that the setup is similar to that considered in our previous work [6], except that in this paper, we consider an OFDMA system. We present the details of the setup here for completeness.

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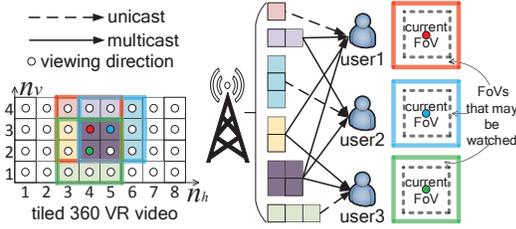


Fig. 1. Illustration of tiled 360 VR video multicast with  $K = 3$ ,  $M_h \times M_v = 8 \times 4$ ,  $V_h \times V_v = 8 \times 4$ .

column, respectively. We consider that all tiles have the same encoding rate, denoted by  $D$  (in bit/s). Note that the encoding rate reflects the video quality. To avoid view switch delay, for each user, the set of tiles that cover its current FoV and the FoVs that may be watched shortly will be delivered.<sup>2</sup>

Let  $\mathbf{X}_k \in \mathcal{X}$  denote the viewing direction of user  $k$ , where  $\mathcal{X} \triangleq \{(m_h, m_v) | m_h = 1, \dots, M_h, m_v = 1, \dots, M_v\}$  represents the set of all possible viewing directions of each user. Let  $\mathbf{X} \triangleq (\mathbf{X}_k)_{k \in \mathcal{K}} \in \mathcal{X} \triangleq \mathcal{X}^K$  denote the system viewing direction state, and let  $\Phi(\mathbf{X})$  denote the corresponding set of tiles that need to be transmitted to all users. In order to utilize multicast opportunities for improving transmission efficiency, we divide  $\Phi(\mathbf{X})$  into  $I(\mathbf{X})$  disjoint non-empty sets  $\mathcal{S}_i(\mathbf{X})$ ,  $i \in I(\mathbf{X}) \triangleq \{1, \dots, I(\mathbf{X})\}$ . For all  $i, j \in I(\mathbf{X})$ ,  $i \neq j$ ,  $\mathcal{S}_i(\mathbf{X})$  and  $\mathcal{S}_j(\mathbf{X})$  are for different groups of users. Let  $S_i(\mathbf{X})$  denote the number of tiles in set  $\mathcal{S}_i(\mathbf{X})$ . We jointly consider the tiles in each set, instead of treating them separately (as in [5]).<sup>3</sup> Let  $\mathcal{K}_i(\mathbf{X})$  denote the set of users that need to receive the tiles in  $\mathcal{S}_i(\mathbf{X})$ . Then, multicast of tiled 360 VR video can be viewed as multi-group multicast.<sup>4</sup>

Consider  $N$  subcarriers, denoted by  $\mathcal{N} \triangleq \{1, \dots, N\}$ . Each subcarrier has a bandwidth  $B$  (in Hz). Consider one frame. Assume block fading, i.e., the channel condition on each subcarrier does not change within one frame [4]. Let  $H_{n,k} \in \mathcal{H}$  denote the power of the channel (i.e., channel state) on subcarrier  $n$  between the server and user  $k$ , where  $\mathcal{H}$  denotes the finite channel state space. Let  $\mathbf{H} \triangleq (H_{n,k})_{n \in \mathcal{N}, k \in \mathcal{K}} \in \mathcal{H} \triangleq \mathcal{H}^{NK}$  denote the system channel state. The system state consists of  $\mathbf{X}$  and  $\mathbf{H}$ , denoted by  $(\mathbf{X}, \mathbf{H}) \in \mathcal{X} \times \mathcal{H}$ . We assume that the server is aware of the system state  $(\mathbf{X}, \mathbf{H})$ , e.g., by explicit feedbacks from all users.

Let  $\mu_{n,i}(\mathbf{X}, \mathbf{H}) \in \{0, 1\}$  denote the subcarrier assignment indicator for subcarrier  $n$  and the tiles in  $\mathcal{S}_i(\mathbf{X})$ , where  $\mu_{n,i}(\mathbf{X}, \mathbf{H}) = 1$  indicates that subcarrier  $n$  is assigned to transmit the symbols for the tiles in  $\mathcal{S}_i(\mathbf{X})$ , and  $\mu_{n,i}(\mathbf{X}, \mathbf{H}) = 0$  otherwise. For ease of implementation, assume each subcarrier is assigned to transmit symbols for only one set of tiles. Thus, we have the following subcarrier allocation constraints:

$$\mu_{n,i}(\mathbf{X}, \mathbf{H}) \in \{0, 1\}, \quad i \in I(\mathbf{X}), n \in \mathcal{N}, \quad (1)$$

$$\sum_{i \in I(\mathbf{X})} \mu_{n,i}(\mathbf{X}, \mathbf{H}) = 1, \quad n \in \mathcal{N}. \quad (2)$$

Let  $p_{n,i}(\mathbf{X}, \mathbf{H})$  and  $c_{n,i}(\mathbf{X}, \mathbf{H})$  denote the transmission power and transmission rate for the symbols for the tiles in  $\mathcal{S}_i(\mathbf{X})$  on subcarrier  $n$ , respectively, where

$$p_{n,i}(\mathbf{X}, \mathbf{H}) \geq 0, \quad i \in I(\mathbf{X}), n \in \mathcal{N}, \quad (3)$$

<sup>2</sup>Note that the proposed framework does not rely on any particular method for determining the set of tiles to be transmitted to each user [1].

<sup>3</sup>This will lead to a great computational complexity reduction for optimal multicast of tiled 360 VR video, compared with [5].

<sup>4</sup>Note that the proposed optimization framework is general and can be applied to optimally multicast multiple messages to different groups of users, i.e., multi-group multicast, in OFDMA systems.

$$c_{n,i}(\mathbf{X}, \mathbf{H}) \geq 0, \quad i \in I(\mathbf{X}), n \in \mathcal{N}. \quad (4)$$

Thus, the total transmission power is  $P(\mu(\mathbf{X}, \mathbf{H}), \mathbf{p}(\mathbf{X}, \mathbf{H})) = \sum_{n \in \mathcal{N}} \sum_{i \in I(\mathbf{X})} \mu_{n,i}(\mathbf{X}, \mathbf{H}) p_{n,i}(\mathbf{X}, \mathbf{H})$ . To obtain design insights, we consider capacity achieving code. Consequently, to guarantee that all users in  $\mathcal{K}_i(\mathbf{X})$  can successfully receive the tiles in  $\mathcal{S}_i(\mathbf{X})$ , we have the following transmission rate constraints:

$$\sum_{n \in \mathcal{N}} \frac{\mu_{n,i}(\mathbf{X}, \mathbf{H}) c_{n,i}(\mathbf{X}, \mathbf{H})}{S_i(\mathbf{X})} \geq D, \quad i \in I(\mathbf{X}), \quad (5)$$

$$B \log_2 \left( 1 + \frac{p_{n,i}(\mathbf{X}, \mathbf{H}) H_{n,k}}{n_0} \right) \geq c_{n,i}(\mathbf{X}, \mathbf{H}), \quad k \in \mathcal{K}_i(\mathbf{X}), i \in I(\mathbf{X}), \quad (6)$$

where  $n_0$  is the power of the complex additive white Gaussian noise on each subcarrier at each receiver. Denote  $\mu(\mathbf{X}, \mathbf{H}) \triangleq (\mu_{n,i}(\mathbf{X}, \mathbf{H}))_{n \in \mathcal{N}, i \in I(\mathbf{X})}$ ,  $\mathbf{p}(\mathbf{X}, \mathbf{H}) \triangleq (p_{n,i}(\mathbf{X}, \mathbf{H}))_{n \in \mathcal{N}, i \in I(\mathbf{X})}$  and  $\mathbf{c}(\mathbf{X}, \mathbf{H}) \triangleq (c_{n,i}(\mathbf{X}, \mathbf{H}))_{n \in \mathcal{N}, i \in I(\mathbf{X})}$ .

### III. TRANSMISSION POWER MINIMIZATION

#### A. Problem Formulation

Given the video quality (i.e., encoding rate of each tile  $D$ ), we would like to minimize the transmission power.

*Problem 1 (Transmission Power Minimization):* For all  $(\mathbf{X}, \mathbf{H}) \in \mathcal{X} \times \mathcal{H}$ ,

$$P^*(\mathbf{X}, \mathbf{H}) \triangleq \min_{\mu(\mathbf{X}, \mathbf{H}), \mathbf{p}(\mathbf{X}, \mathbf{H}), \mathbf{c}(\mathbf{X}, \mathbf{H})} P(\mu(\mathbf{X}, \mathbf{H}), \mathbf{p}(\mathbf{X}, \mathbf{H}))$$

$$\text{s.t. (1), (2), (3), (4), (5), (6).}$$

Let  $(\mu_c^*(\mathbf{X}, \mathbf{H}), \mathbf{p}_c^*(\mathbf{X}, \mathbf{H}), \mathbf{c}_c^*(\mathbf{X}, \mathbf{H}))$  denote an optimal solution.

Problem 1 is a mixed discrete-continuous optimization problem. The number of variables of Problem 1 (proportional to the number of sets of tiles  $I(\mathbf{X})$ ) is much smaller than that in [5] (proportional to the number of tiles to be transmitted to all users  $|\Phi(\mathbf{X})|$ ), as  $I(\mathbf{X})$  is much smaller than  $|\Phi(\mathbf{X})|$ .

#### B. Optimal Solution

In this part, we obtain a globally optimal solution of Problem 1 (under a mild condition). First, to reduce computational complexity, we eliminate  $\mathbf{c}(\mathbf{X}, \mathbf{H})$  and simplify the constraints in (4), (5) and (6) to

$$\sum_{n \in \mathcal{N}} \frac{\mu_{n,i}(\mathbf{X}, \mathbf{H}) B}{S_i(\mathbf{X})} \log_2 \left( 1 + \frac{p_{n,i}(\mathbf{X}, \mathbf{H}) H_{n,i}^{\min}(\mathbf{X}, \mathbf{H})}{n_0} \right) \geq D, \quad i \in I(\mathbf{X}), \quad (7)$$

where  $H_{n,i}^{\min}(\mathbf{X}, \mathbf{H}) \triangleq \min_{k \in \mathcal{K}_i(\mathbf{X})} H_{n,k}$ . Next, we relax the constraints in (1) to

$$\mu_{n,i}(\mathbf{X}, \mathbf{H}) \geq 0, \quad n \in \mathcal{N}, \quad i \in I(\mathbf{X}). \quad (8)$$

Then, let  $P_{n,i}(\mathbf{X}, \mathbf{H}) \triangleq \mu_{n,i}(\mathbf{X}, \mathbf{H}) p_{n,i}(\mathbf{X}, \mathbf{H})$  and  $\mathbf{P}(\mathbf{X}, \mathbf{H}) \triangleq (P_{n,i}(\mathbf{X}, \mathbf{H}))_{n \in \mathcal{N}, i \in I(\mathbf{X})}$ . Thus, Problem 1 can be transformed to the following problem.

*Problem 2 (Relaxation of Problem 1):*

$$\min_{\mu(\mathbf{X}, \mathbf{H}), \mathbf{P}(\mathbf{X}, \mathbf{H})} \sum_{n \in \mathcal{N}} \sum_{i \in I(\mathbf{X})} P_{n,i}(\mathbf{X}, \mathbf{H})$$

s.t. (2), (8),

$$P_{n,i}(\mathbf{X}, \mathbf{H}) \geq 0, \quad i \in I(\mathbf{X}), n \in \mathcal{N}, \quad (9)$$

$$\sum_{n \in \mathcal{N}} \frac{\mu_{n,i}(\mathbf{X}, \mathbf{H}) B}{S_i(\mathbf{X})} \log_2 \left( 1 + \frac{P_{n,i}(\mathbf{X}, \mathbf{H}) H_{n,i}^{\min}(\mathbf{X}, \mathbf{H})}{\mu_{n,i}(\mathbf{X}, \mathbf{H}) n_0} \right) \geq D, \quad i \in I(\mathbf{X}). \quad (10)$$

Problem 2 is convex and can be solved using KKT conditions [7]. Let  $\lambda_i(\mathbf{X}, \mathbf{H}), i \in \mathcal{I}(\mathbf{X})$  denote the lagrange multipliers with respect to the constraints in (10). Define

$$f_{n,i}(\mathbf{X}, \mathbf{H}, \lambda_i) \triangleq \left[ \frac{B\lambda_i}{S_i(\mathbf{X}) \ln 2} - \frac{n_0}{H_{n,i}^{\min}(\mathbf{X}, \mathbf{H})} \right]^+,$$

$$W_{n,i}(\mathbf{X}, \mathbf{H}, \lambda_i) \triangleq \frac{\lambda_i B}{S_i(\mathbf{X})} \left( \log_2 \left( 1 + \frac{H_{n,i}^{\min}(\mathbf{X}, \mathbf{H}) f_{n,i}(\mathbf{X}, \mathbf{H}, \lambda_i)}{n_0} \right) - \frac{H_{n,i}^{\min}(\mathbf{X}, \mathbf{H}) f_{n,i}(\mathbf{X}, \mathbf{H}, \lambda_i)}{(n_0 + H_{n,i}^{\min}(\mathbf{X}, \mathbf{H}) f_{n,i}(\mathbf{X}, \mathbf{H}, \lambda_i)) \ln 2} \right).$$

*Lemma 1 (Optimal Solution of Problem 1):* Suppose that for all  $n \in \mathcal{N}$ , there exists a unique  $i_n$  such that  $W_{n,i_n}(\mathbf{X}, \mathbf{H}, \lambda_{i_n}) = \max_{j \in \mathcal{I}(\mathbf{X})} W_{n,j}(\mathbf{X}, \mathbf{H}, \lambda_j)$ . Then,  $(\mu_{e,n}^*(\mathbf{X}, \mathbf{H}), p_{e,n}^*(\mathbf{X}, \mathbf{H}), c_{e,n}^*(\mathbf{X}, \mathbf{H}))$  is identical to the optimal solution of Problem 2, where for all  $n \in \mathcal{N}$  and  $i \in \mathcal{I}(\mathbf{X})$ ,

$$\mu_{e,n,i}^*(\mathbf{X}, \mathbf{H}) = \begin{cases} 1, & i = \operatorname{argmax}_{j \in \mathcal{I}(\mathbf{X})} W_{n,j}(\mathbf{X}, \mathbf{H}, \lambda_j^*(\mathbf{X}, \mathbf{H})), \\ 0, & \text{otherwise,} \end{cases}$$

$$p_{e,n,i}^*(\mathbf{X}, \mathbf{H}) = \mu_{e,n,i}^*(\mathbf{X}, \mathbf{H}) f_{n,i}(\mathbf{X}, \mathbf{H}, \lambda_i^*(\mathbf{X}, \mathbf{H})),$$

$$c_{e,n,i}^*(\mathbf{X}, \mathbf{H}) = B \log_2 \left( 1 + \frac{p_{e,n,i}^*(\mathbf{X}, \mathbf{H}) H_{n,i}^{\min}(\mathbf{X}, \mathbf{H})}{n_0} \right).$$

Here,  $\lambda_i^*(\mathbf{X}, \mathbf{H})$  satisfies

$$\sum_{n \in \mathcal{N}} \frac{\mu_{e,n,i}^*(\mathbf{X}, \mathbf{H}) B}{S_i(\mathbf{X})} \log_2 \left( 1 + \frac{p_{e,n,i}^*(\mathbf{X}, \mathbf{H}) H_{n,i}^{\min}(\mathbf{X}, \mathbf{H})}{n_0} \right) = D.$$

$\lambda_i(\mathbf{X}, \mathbf{H}), i \in \mathcal{I}(\mathbf{X})$  can be obtained using the subgradient method. Note that all previous works on power minimization for multicast in OFDMA systems provide only low-complexity suboptimal solutions.

By carefully exploring structural properties of Problem 1 and Problem 2, we have the following result.

*Lemma 2 (Optimal Value of Problem 1):* (i)  $P^*(\mathbf{X}, \mathbf{H}) \in \left[ \frac{n_0 T N}{\max \mathcal{H}} \left( 2^{\frac{D \sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})}{B N}} - 1 \right), \frac{n_0 T I(\mathbf{X})}{\min \mathcal{H}} \left( 2^{\frac{D \sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})}{B}} - 1 \right) \right]$ . (ii)  $g P^*(\mathbf{X}, \mathbf{H}) = P^*(\mathbf{X}, \frac{1}{g} \mathbf{H})$ , for all  $g > 0$ .

Lemma 2 indicates that the minimum transmission power  $P^*(\mathbf{X}, \mathbf{H})$  increases exponentially with the total number of tiles to be transmitted, i.e.,  $\sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})$ , approximately, and is inversely proportional to the channel powers, i.e.,  $H_k, k \in \mathcal{K}$ . Note that  $\sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})$  reflects the concentration of the viewing directions of all users. A smaller value of  $\sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})$  means closer viewing directions of all users.

#### IV. QUALITY MAXIMIZATION

##### A. Problem Formulation

Let  $\bar{P}$  denote the transmission power budget of the system. Consider the maximum transmission power constraint:

$$P(\boldsymbol{\mu}(\mathbf{X}, \mathbf{H}), \mathbf{p}(\mathbf{X}, \mathbf{H})) \leq \bar{P}, (\mathbf{X}, \mathbf{H}) \in \mathcal{X} \times \mathcal{H}. \quad (11)$$

To guarantee user experience, the encoding rate should not change as frequently as the viewing directions and channel states, and should remain constant within a certain time duration. Given the transmission power budget  $\bar{P}$ , we would like to maximize the received video quality (i.e., encoding rate of each tile  $D$ ). Denote  $\boldsymbol{\mu} \triangleq (\boldsymbol{\mu}(\mathbf{X}, \mathbf{H}))_{(\mathbf{X}, \mathbf{H}) \in \mathcal{X} \times \mathcal{H}}$ ,  $\mathbf{p} \triangleq (\mathbf{p}(\mathbf{X}, \mathbf{H}))_{(\mathbf{X}, \mathbf{H}) \in \mathcal{X} \times \mathcal{H}}$  and  $\mathbf{c} \triangleq (\mathbf{c}(\mathbf{X}, \mathbf{H}))_{(\mathbf{X}, \mathbf{H}) \in \mathcal{X} \times \mathcal{H}}$ .

*Problem 3 (Received Video Quality Maximization):*

$$D_q^* \triangleq \max_{D, \boldsymbol{\mu}, \mathbf{p}, \mathbf{c}} D$$

s.t. (1), (2), (3), (4), (5), (6), (11).

Let  $(D_q^*, \boldsymbol{\mu}_q^*, \mathbf{p}_q^*, \mathbf{c}_q^*)$  denote an optimal solution.

Similar to Problem 1, Problem 3 is a mixed discrete-continuous optimization problem.

##### B. Near Optimal Solution

In this part, we obtain a near optimal solution of Problem 3. Let  $\mathbf{H}_{\min}$  denote the vector with all  $K$  elements being  $\min \mathcal{H}$ . First, we consider a related problem.

*Problem 4 (Equivalent Problem of Problem 3):*

$$\min_{\mathbf{X} \in \mathcal{X}} D_q^*(\mathbf{X}, \mathbf{H}_{\min})$$

where  $D_q^*(\mathbf{X}, \mathbf{H}_{\min})$  is given by the following subproblem.

*Problem 5 (Subproblem of Problem 4):* For all  $\mathbf{X} \in \mathcal{X}$ ,

$$D_q^*(\mathbf{X}, \mathbf{H}_{\min}) \triangleq \max_{D, \{N_i(\mathbf{X}, \mathbf{H}_{\min})\}_{i \in \mathcal{I}(\mathbf{X})}} D$$

$$\text{s.t. } N_i(\mathbf{X}, \mathbf{H}_{\min}) \in \mathcal{N}, i \in \mathcal{I}(\mathbf{X}), \quad (12)$$

$$\sum_{i \in \mathcal{I}(\mathbf{X})} N_i(\mathbf{X}, \mathbf{H}_{\min}) \leq N, \quad (13)$$

$$\sum_{i \in \mathcal{I}(\mathbf{X})} \frac{N_i(\mathbf{X}, \mathbf{H}_{\min}) n_0}{\min \mathcal{H}} \left( 2^{\frac{D S_i(\mathbf{X})}{B N_i(\mathbf{X}, \mathbf{H}_{\min})}} - 1 \right) \leq \bar{P}. \quad (14)$$

Let  $(D_q^*(\mathbf{X}, \mathbf{H}_{\min}), (N_i^*(\mathbf{X}, \mathbf{H}_{\min}))_{i \in \mathcal{I}(\mathbf{X})})$  denote an optimal solution.

Note that  $N_i(\mathbf{X}, \mathbf{H}_{\min})$  indicates the number of subcarriers assigned to transmit the symbols for the tiles in  $S_i(\mathbf{X})$  at system channel state  $\mathbf{H}_{\min}$ . By carefully exploring structural properties of Problem 3, we have the following result.

*Lemma 3 (Equivalence between Problem 3 and Problem 4):*

(i) The optimal values of Problem 3 and Problem 4 are equivalent, i.e.,  $D_q^* = \min_{\mathbf{X} \in \mathcal{X}} D_q^*(\mathbf{X}, \mathbf{H}_{\min})$ . (ii) For all  $i \in \mathcal{I}(\mathbf{X}), \mathbf{X} \in \mathcal{X}, N_i^*(\mathbf{X}, \mathbf{H}_{\min}) = \sum_{n=1}^N \mu_{q,n,i}^*(\mathbf{X}, \mathbf{H}_{\min}), p_{q,n,i}^*(\mathbf{X}, \mathbf{H}_{\min}) =$

$$\begin{cases} \frac{n_0}{\min \mathcal{H}} \left( 2^{\frac{S_i(\mathbf{X}) D_q^*(\mathbf{X}, \mathbf{H}_{\min})}{B N_i^*(\mathbf{X}, \mathbf{H}_{\min})}} - 1 \right), & \mu_{q,n,i}^*(\mathbf{X}, \mathbf{H}_{\min}) = 1; \\ 0, & \text{otherwise.} \end{cases} \quad (15)$$

*Proof:* (sketch) We eliminate  $\mathbf{c}$ , and replace (4), (5) and (6) with (7). By (ii) of Lemma 2, it is equivalent to consider only  $\mathbf{H}_{\min}$  instead of all  $\mathbf{H} \in \mathcal{H}$ . By Lemma 1, we have  $p_{e,n,i}^*(\mathbf{X}, \mathbf{H}_{\min}) = p_{e,m,i}^*(\mathbf{X}, \mathbf{H}_{\min})$ , for all  $n, m$  with  $\mu_{e,n,i}^*(\mathbf{X}, \mathbf{H}_{\min}) = \mu_{e,m,i}^*(\mathbf{X}, \mathbf{H}_{\min}) = 1$ . By (7) and (11), we can show (15) by contradiction. Substituting (15) into (11), we can obtain (14). By setting  $N_i(\mathbf{X}, \mathbf{H}_{\min}) = \sum_{n=1}^N \mu_{n,i}(\mathbf{X}, \mathbf{H}_{\min})$ , (1) and (2) can be transformed to (12) and (13). Thus, Problem 3 can be equivalently transformed to Problem 4. ■

Relaxing (12) to  $N_i(\mathbf{X}, \mathbf{H}_{\min}) \in [1, N], i \in \mathcal{I}(\mathbf{X})$ , Problem 5 can be transformed to a convex problem, which can be solved using KKT conditions.

*Lemma 4 (Optimal Solution of Relaxation of Problem 5):*

The optimal solution of the relaxation of Problem 5 is

$$N_i^\dagger(\mathbf{X}, \mathbf{H}_{\min}) = \frac{S_i(\mathbf{X}) N}{\sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})}, i \in \mathcal{I}(\mathbf{X}),$$

$$D^\dagger(\mathbf{X}, \mathbf{H}_{\min}) = \frac{B N \ln \left( \frac{\bar{P} \min \mathcal{H}}{N n_0} + 1 \right)}{\ln 2 \sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})}.$$

Lemma 4 indicates that  $N_i^\dagger(\mathbf{X}, \mathbf{H}_{\min})$  is proportional to  $S_i(\mathbf{X})$ . By exploring properties of Problem 3, Problem 4 and Problem 5 and by Lemma 4, we have the following result.

**Lemma 5 (Optimal Value of Problem 3):**  $D_q^* \in \left[ \frac{B \ln(\frac{\bar{P} \min \mathcal{H}}{\max_{\mathbf{X} \in \mathcal{X}} \sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})} + 1)}{\ln 2 \max_{\mathbf{X} \in \mathcal{X}} \sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})}, \frac{BN \ln(\frac{\bar{P} \min \mathcal{H}}{N n_0} + 1)}{\ln 2 \max_{\mathbf{X} \in \mathcal{X}} \sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})} \right]$ .

Lemma 5 indicates that approximately,  $D_q^*$  is affected by the smallest channel power  $\min \mathcal{H}$  among all channel powers, and is inversely proportional to  $\max_{\mathbf{X} \in \mathcal{X}} \sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})$  which represents the maximum number of tiles that need to be transmitted for all viewing directions.

Now, we propose a low complexity algorithm, i.e., Algorithm 1, to obtain a near optimal encoding rate of each tile of Problem 3, denoted by  $D_q^\diamond$ , by constructing a feasible solution based on the solution in Lemma 4 in a greedy manner.

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#### Algorithm 1 Near Optimal Solution of Problem 3

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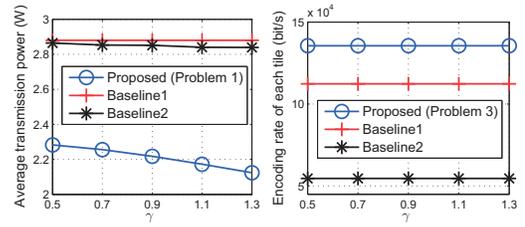
- 1: For all  $i \in \mathcal{I}(\mathbf{X})$ , set  $N_i^\diamond(\mathbf{X}, \mathbf{H}_{\min}) = \lfloor N_i^\dagger(\mathbf{X}, \mathbf{H}_{\min}) \rfloor$ ;
  - 2: **while**  $\sum_{i \in \mathcal{I}(\mathbf{X})} N_i^\diamond(\mathbf{X}, \mathbf{H}_{\min}) < N$  **do**
  - 3: Set  $N_{i^*}^\diamond(\mathbf{X}, \mathbf{H}_{\min}) = \frac{N_i^\diamond(\mathbf{X}, \mathbf{H}_{\min})}{S_i(\mathbf{X})} + 1$ , where  $i^* = \operatorname{argmax}_{i \in \mathcal{I}(\mathbf{X})} \frac{S_i(\mathbf{X}) \ln 2}{BN_i^\diamond(\mathbf{X}, \mathbf{H}_{\min})}$ ;
  - 4: **end while**
  - 5: For all  $\mathbf{X} \in \mathcal{X}$ , obtain  $D_q^\diamond(\mathbf{X}, \mathbf{H}_{\min})$  by solving 
$$\sum_{i \in \mathcal{I}(\mathbf{X})} \frac{N_i^\diamond(\mathbf{X}, \mathbf{H}_{\min}) n_0}{\min \mathcal{H}} \left( 2^{\frac{D_q^\diamond(\mathbf{X}, \mathbf{H}_{\min}) S_i(\mathbf{X})}{BN_i^\diamond(\mathbf{X}, \mathbf{H}_{\min})}} - 1 \right) = \bar{P}$$
 using bisection search;
  - 6: Set  $D_q^\diamond = \min_{\mathbf{X} \in \mathcal{X}} D_q^\diamond(\mathbf{X}, \mathbf{H}_{\min})$ .
- 

### V. SIMULATION

In this section, we compare the proposed solutions in Section III and Section IV with two baselines using numerical results. Baseline 1 considers serving each user separately using unicast in an optimal way, similar to the proposed solutions. Baseline 2 considers multicast but with equal subcarrier allocation for each transmitted tile and optimal transmission power as well as transmission rate allocation based on the equal subcarrier allocation. In the simulation, we use Kvazaar as the 360 VR video encoder and video sequence *Boxing* as the video source. To avoid view switch delay, we transmit extra  $15^\circ$  in the four directions of each requested FoV [1]. Different 360 VR videos in general have different popularity distributions for viewing directions. To illustrate the importance of the concentration of the viewing directions, we assume all users randomly and independently select viewing directions according to Zipf distribution<sup>5</sup> for the  $M_h \times M_v$  viewing directions. In particular, suppose viewing direction  $(m_h, m_v)$  is of rank  $(m_h - 1)M_v + m_v$  and  $\Pr[\mathbf{X}_k = (m_h, m_v)] = \frac{((m_h - 1)M_v + m_v)^{-\gamma}}{\sum_{i=1, \dots, M_h M_v} i^{-\gamma}}$ , where  $\gamma$  is the Zipf exponent. In addition, assume  $\sqrt{H_{n,k}}$ ,  $n \in \mathcal{N}$ ,  $k \in \mathcal{K}$  are randomly and independently distributed according to  $\mathcal{CN}(0, \frac{1}{d})$ , where  $d$  reflects the path loss. We randomly choose 100 global channel states to form  $\mathcal{H}$ , and evaluate the average transmission power over  $\mathcal{H}$ .

Fig. 2 (a) illustrates the average transmission power versus the Zipf exponent  $\gamma$ . We can see that the average transmission

<sup>5</sup>Zipf distribution is widely used to model content popularity in Internet and wireless networks. For any popularity rank, a larger Zipf exponent  $\gamma$  indicates a smaller tail of the popularity distribution, i.e., higher concentration of requests for contents. Here, we adopt Zipf distribution for ease of exposition.



(a) Average transmission (b) Encoding rate of each power versus  $\gamma$ .  $K = 3$ , tile versus  $\gamma$ .  $K = 4$ ,  $d = D = 30$  kbit/s,  $d = 10^3$ ,  $600$ ,  $\bar{P} = 10^4$  W.

Fig. 2. Performance comparison.  $F_h = F_v = 100^\circ$ ,  $M_h \times M_v = 30 \times 2$ ,  $V_h \times V_v = 30 \times 15$ ,  $B = 39$  kHz,  $N = 128$ ,  $n_0 = 10^{-9}$  W.

power of each multicast scheme decreases with  $\gamma$ , as multicast opportunities increase with  $\gamma$ ; the average transmission power of the unicast scheme almost does not change with  $\gamma$ , as it does not capture multicast opportunities. Fig. 2 (b) illustrates the encoding rate of each tile versus  $\gamma$ . We can see that the encoding rate of each tile of each scheme does not change with  $\gamma$ , as that of each multicast scheme is determined by  $\operatorname{argmax}_{\mathbf{X} \in \mathcal{X}} \sum_{i \in \mathcal{I}(\mathbf{X})} S_i(\mathbf{X})$  corresponding to the case with the fewest multicast opportunities, and that of the unicast scheme does not depend on  $\mathbf{X}$ . From Fig. 2, we can also observe that the proposed solutions outperform the two baselines. Specifically, the gains of the proposed solutions over Baseline 1 arise from the fact that the proposed solutions utilize multicast. The gains of the proposed solutions over Baseline 2 are due to the fact that the proposed solutions carefully allocate subcarrier, transmission power and transmission rate.

### VI. CONCLUSION

In this letter, we studied optimal multicast of tiled 360 VR video in an OFDMA system, and formulated two non-convex optimization problems, i.e., the minimization of the average transmission power for given video quality, and the maximization of the received video quality for given transmission power budget. We obtained a globally optimal closed-form solution and a near optimal solution of the two non-convex problems, separately, and revealed important design insights for tiled 360 VR multicast. This letter opens up several directions for future research. For instance, the proposed multicast mechanism and optimization framework can be extended to design optimal multi-quality multicast of tiled 360 VR video in OFDMA systems. In addition, a possible direction for future research is to design optimal single-quality or multi-quality multicast of tiled 360 VR video in different wireless systems.

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