Information transmission bounds between moving terminals

Omar J. Faqir, Eric C. Kerrigan, Deniz Gündüz

Abstract—In networks of mobile autonomous agents, e.g. for data acquisition, we may wish to maximize data transfer or to reliably transfer a minimum amount of data, subject to quality of service or energy constraints. These requirements can be guaranteed through both offline node design/specifications and online trajectory/communications design. Regardless of the distance between them, for a stationary point-to-point transmitter-receiver pair communicating across a single link under average power constraints, the total data transfer is unbounded as time tends to infinity. In contrast, we show that if the transmitter/receiver is moving at any constant speed away from each other, then the maximum transmittable data is bounded. Although general closed-form expressions as a function of communication and mobility profile parameters do not yet exist, we provide closed-form expressions for particular cases, such as ideal free space path loss. Under more general scenarios we instead give lower bounds on the total transmittable information across a single link between mobile nodes.

I. Introduction

As autonomous agents become commonplace, the challenging task of developing control policies for communication between agents must be addressed. For a stationary point-to-point transmitter-receiver pair under an average power constraint, the total transmittable data is unbounded as time tends to infinity, regardless of distance. In contrast, we show that if the nodes move away from each other at a constant speed, then the maximum transmittable data is bounded even as time tends to infinity. Without loss of generality we consider one stationary and one mobile node.

This bound is of relevance for aerial communications, where channel gain is dominated by path loss exponents and typically high speeds result in rapid growth in distances. Examples include unmanned aerial vehicles (UAVs) or spacecraft. We consider dynamic deployment scenarios (e.g. [1]–[4]) where UAVs, spacecraft, or other agents are transmitting while moving, and are interested in characterising

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the total data transfer as a function of mobility dynamics and design parameters, such as transmit power. We do not consider transmitter power control as a function of distance as this is challenging to implement in practice, especially at higher speeds. Linear constant speed trajectories, as considered in [1], [2], have practical importance for fixedwing aircraft, since minimum propulsion energy operation occurs at a constant velocity, and result in structured timevarying communication channels. For dense UAV deployment scenarios with random mobility, works such as [5] determine capacity bounds and outage probabilities. Even for a simple linear trajectory in 3D space, the expression for total transmittable data formulated in Section II is nontrivial to evaluate due to the structure of the argument of the logarithm. We instead derive closed-form expressions for specific cases. In Section III we derive a bound for total transmittable data in 1D and show the bound to be tight for reasonable parameter values. In Section IV we consider arbitrary linear agent trajectories deriving an expression for total data transmission, assuming ideal free space path loss.

II. PROBLEM DEFINITION

The Shannon capacity is an information-theoretic limit on the achievable communication rate for reliable transmission across a noisy channel. The corresponding limit on the total amount of transmittable data over time T is

$$D_T := B \int_0^T \ln\left(1 + \text{SNR}(t)\right) dt, \tag{1}$$

where B is the bandwidth and $SNR(\cdot)$ the time-varying signal-to-noise ratio at the receiver. D_T is measured in nats (natural units of information). Neglecting fast-fading dynamics, the channel gain is dominated by line-of-sight fading, typical of aerial channels [1]. Then, the SNR is

$$SNR(P,t) = \frac{PG}{\sigma^2} \left(\frac{d_0}{d(t)}\right)^{\alpha},$$
 (2)

where σ^2 is the receiver noise power, $\alpha \ge 2$ (but typically not much greater than 2) is the path loss exponent, and $d(\cdot)$ is the time-varying distance between transmitter and receiver, satisfying $d(t) > d_0$, $\forall t \in \mathbb{R}^+$, for reference distance d_0 . P is the average transmission power in Watts and G is a unitless antenna parameter representing gain and path loss at the distance d_0 . For simplicity we define $S := PG\sigma^{-2}$.

Consider wireless transmission between a stationary node at the origin and a mobile transmitter at constant longitudinal displacement $z_0 \ge 0$. The transmitter follows a

linear trajectory at constant speed v. At time $t \in [0, T]$ the lateral position of the transmitter relative to the receiver is

$$x(t) = x_0 + vt, \tag{3}$$

and x_0 the initial lateral position. The distance is

$$d(t) = \sqrt{z_0^2 + (x_0 + vt)^2}. (4)$$

Simplifying the distance operator through removing the $\sqrt{(\cdot)}$, either by assuming $z_0 = 0$ (Section III) or setting $\alpha = 2$ (Section IV), allows us to determine closed-form expressions for the total amount of data that can be reliably communicated in these special cases.

III. Characterization for General α in 1D

We find the total data transfer in a one-dimensional setting, where $z_0 = 0$, and hence $d(t) = x_0 + vt$, as a function of B, v, S and α . The restriction $z_0 = 0$ simplifies the analysis and arises in the context of a mobile UAV communicating with a static aerial base station at the same altitude.

Theorem 1. The total transmittable data as the interval length $T \to \infty$ for $z_0 = 0$, $x_0 = d_0$ and v > 0, i.e.

$$D_{\infty,1} := \lim_{T \to \infty} B \int_0^T \ln\left(1 + S\left(\frac{d_0}{d_0 + \nu t}\right)^{\alpha}\right) \mathrm{d}t,\tag{5}$$

is finite and given by

$$D_{\infty,1} = \frac{Bd_0}{v} \left(\pi \sqrt[\alpha]{S} \csc\left(\frac{\pi}{\alpha}\right) - \ln(1+S) + \alpha \sum_{n=1}^{\infty} \left\{ \frac{(-1)^{n+1}}{S^n(\alpha n+1)} \right\} - \alpha \right), \tag{6}$$

where the sum is positive and convergent for S > 1.

See Appendix A for the proof. Restricting S > 1 implies that the transmit signal power, before undergoing any channel attenuation, is greater than the receiver noise power. In practice we find it reasonable to assume $S \gg 1$. Since the sum is monotonically decreasing in both S and α , we may determine a lower bound $\tilde{D}_{\infty,1}$ on $D_{\infty,1}$.

Corollary 1. A lower bound for $D_{\infty,1}$ is given by

$$\tilde{D}_{\infty,1} := \frac{Bd_0}{\nu} \left(\pi \sqrt[\alpha]{S} \csc\left(\frac{\pi}{\alpha}\right) - \ln(1+S) - \alpha \right),\tag{7}$$

where the error $e_{\infty,1}$ between $D_{\infty,1}$ and the bound $\tilde{D}_{\infty,1}$,

$$e_{\infty,1} := D_{\infty,1} - \tilde{D}_{\infty,1} = \frac{Bd_0}{\nu} \sum_{n=1}^{\infty} \left\{ \frac{(-1)^{n+1}}{S^n(\alpha n + 1)} \right\},\tag{8}$$

is bounded as

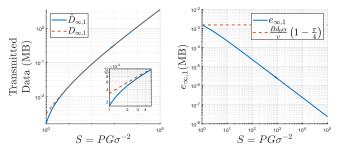
$$e_{\infty,1} \le \frac{Bd_0\alpha}{v} \left(1 - \sqrt{S} \tan^{-1} \left(\frac{1}{\sqrt{S}} \right) \right) \le \frac{Bd_0\alpha}{v} \left(1 - \frac{\pi}{4} \right).$$
 (9)

Furthermore, $e_{\infty,1} \to 0$ as $S \to \infty$.

See Appendix B for a proof. Fig. 1 compares $\tilde{D}_{\infty,1}$ and $D_{\infty,1}$ for the parameters shown in Table I and a range of S^1 . Setting P=1 mW results in $S=10^5$. The corresponding error is shown in Fig. 1b as $e_{\infty,1}=3.33\times 10^{-6}$ MB= 0.278 nats. Fig. 1b also shows that $e_{\infty,1}=\frac{Bd_0\alpha}{\nu}\left(1-\frac{\pi}{4}\right)=1.5\times 10^{-3}$ MB when S=1, but decreases rapidly as S increases.

TABLE I: Table of *default* simulation parameters. Graph axes and labels indicate where parameters differ from defaults.

B(Hz)	$\sigma^2(W)$	$d_0(m)$	G	P(W)	$v(\text{ms}^{-1})$	α
10^{5}	10^{-8}	1	1	1	5	2



(a) Comparison of $D_{\infty,1}$ and $\tilde{D}_{\infty,1}$ (b) Comparison of $e_{\infty,1}$ with bound

Fig. 1: Comparison of $D_{\infty,1}$, $\tilde{D}_{\infty,1}$.

Fig. 2 shows $D_{\infty,1}$ as a function of speed $v \in [1,100]$ and power $P \in [10^{-3}, 100]$, calculated using (6). We approximate the infinite sum by the first 100 terms. Since $e_{\infty,1}$ is monotonically decreasing in P and v, the maximum approximation error corresponds to data point $(P, v) = (10^{-3}, 1)$ and is $e_{\infty,1} = 0.333$ nats. Although controlling node mobility is outside of the scope of this paper, Fig. 2 shows the set $\{(P, v) \mid D_{\infty,1} = M\}$ of admissible powers and velocities that successfully transmit M bits of data. (P, v) may then be chosen from this set to achieve alternative operating points, such as energy efficiency [1]. Since many transmitters operate at a single, or finite set of power levels, ν may often be found as an explicit function of (P, M). For comparison, the red and blue lines show $D_{\infty,1}$ calculated from the Matlab quadrature function integral of (1) for infinite T. Fig. 3 shows $D_T/D_{\infty,1}$ as a function of both time and position for a range of speeds. D_T converges faster to $D_{\infty,1}$ with increasing ν . Even for a moderately slow speed of $v = 5 \,\mathrm{m/s}$, over 80% of the data is transmitted

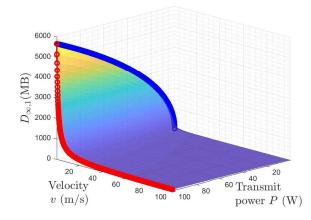


Fig. 2: $D_{\infty,1}$ as a function of speed v and transmit power P in the 1D case ($z_0 = 0$) for parameters from Table I.

¹For the sake of relatability all data is plotted in MB, not nats.

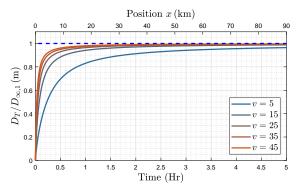


Fig. 3: Finite-time data D_T as a proportion of the bound $D_{\infty,1}$ in the 1-D case for a range of speeds ν in m/s.

within one hour. For the AWGN case here, with fixed power and speed, any SNR type constraint can be directly converted to a maximum allowable distance between the transmitter and receiver. Similarly, if constraints on node endurance (e.g. finite energy) are functions of velocity, one may quantitatively investigate how limiting endurance is, compared to the bound $D_{\infty,1}$. Fig. 3 shows a decreasing marginal gain in transmitted data as endurance increases.

IV. CHARACTERIZATION FOR $\alpha = 2$

We now consider the specific, but not unrealistic case of $\alpha = 2$, corresponding to ideal free space. The maximum transmittable data over time T simplifies to

$$D_T = B \int_0^T \ln\left(1 + \frac{Sd_0^2}{z_0^2 + x(t)^2}\right) dt.$$
 (10)

Theorem 2. The total transmittable data as $T \to \infty$ for a UAV with starting position $x_0 \ge d_0$, v > 0 and $\alpha = 2$, i.e.

$$D_{\infty,2} := \lim_{T \to \infty} B \int_0^T \ln \left(1 + \frac{Sd_0^2}{z_0^2 + x(t)^2} \right) dt, \tag{11}$$

for trajectory (3) is finite and given by

$$D_{\infty,2} = \frac{B}{\nu} \left[-x_0 \ln(1+S) + \pi \left(\sqrt{\epsilon} - z_0 \right) - 2 \left(\sqrt{\epsilon} \tan^{-1} \left(\frac{x_0}{\sqrt{\epsilon}} \right) - x_0 \tan^{-1} \left(\frac{x_0}{z_0} \right) \right) \right].$$
 (12)

where $\epsilon := z_0^2 + Sd_0^2$.

The proof is in Appendix C. Fig. 4 shows how $D_{\infty,2}$ varies with displacement z_0 . In the straightforward case of $\alpha = 2$, $z_0 = 0$ a simpler expression follows.

Corollary 2. In the 1-D case $(z_0 = 0)$, with $x_0 = d_0$ and v > 0,

$$D_{\infty,3} := \lim_{T \to \infty} B \int_0^T \ln\left(1 + \frac{Sd_0^2}{x(t)^2}\right) dt, \tag{13}$$

the maximal transmittable data as $T \to \infty$ is

$$D_{\infty,3} = \frac{Bd_0}{\nu} \left[2\sqrt{S} \left(\frac{\pi}{2} - \tan^{-1} \left(\frac{1}{\sqrt{S}} \right) \right) - \ln(1+S) \right]. \tag{14}$$

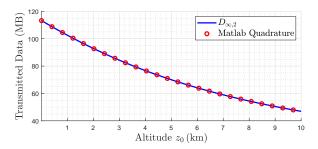


Fig. 4: $D_{\infty,2}$ as a function of altitude z_0 .

Proof. The proof follows by setting $z_0 = 0$, $x_0 = d_0$ in (12) or by setting $\alpha = 2$ in (6) and noting (e.g. using [6, p. 422])

$$\sum_{n=1}^{\infty} \left\{ \frac{(-1)^{n+1}}{S^n (2n+1)} \right\} = 1 - \sqrt{S} \tan^{-1} \left(\frac{1}{\sqrt{S}} \right). \tag{15}$$

V. Conclusions

We have considered the amount of data that can be reliably transmitted between a stationary and a mobile agent on a straight trajectory. We have shown that, as opposed to static channels, the maximum transmittable data is bounded, even as time tends to infinity. For certain special cases of path loss and mobility profiles, we have been able to derive closed-form expressions for these bounds. These bounds have been verified through simulations and may be used for design/specification of mobile communication channels subject to data transmission constraints.

$\begin{array}{c} \text{Appendix A} \\ \text{Proof of Theorem 1} \end{array}$

Note, e.g. from [6, p. 212], the identity

$$\ln(1+z) = \begin{cases} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} z^n}{n}, & \text{if } |z| < 1, z \in \mathbb{R}, \\ \ln(z) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n z^n}, & \text{if } |z| \ge 1, z \in \mathbb{R}, \end{cases}$$
(16)

and

$$\csc(z) = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{(-1)^n}{z^2 - (\pi n)^2} \quad ; \frac{z}{\pi} \notin \mathbb{Z}, z \in \mathbb{R}.$$
 (17)

Starting with (5), we use the known affine trajectory $x(t) = x_0 + vt$ to change the variable of integration, resulting in

$$D_{\infty,1} = \lim_{T \to \infty} \frac{B}{v} \underbrace{\int_{d_0}^{x_T} \ln\left(1 + S\left(\frac{d_0}{x}\right)^{\alpha}\right) dx}_{:=D_a},$$

where $x_T := x(T)$. Define $\gamma := Sd_0^{\alpha}$. Ignoring the limits for now, and in accordance with identity (16),

$$D_{a} = \int_{d_{0}}^{\sqrt[\alpha]{\gamma}} \ln\left(1 + \frac{\gamma}{x^{\alpha}}\right) dx + \int_{\sqrt[\alpha]{\gamma}}^{x_{T}} \ln\left(1 + \frac{\gamma}{x^{\alpha}}\right) dx$$

$$D_{a} = \underbrace{\int_{d_{0}}^{\sqrt[\alpha]{\gamma}} \ln\left(\frac{\gamma}{x^{\alpha}}\right) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{x^{\alpha}}{\gamma}\right)^{n} dx}_{:=D_{a1}} + \underbrace{\int_{\sqrt[\alpha]{\gamma}}^{x_{T}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{\gamma}{x^{\alpha}}\right)^{n} dx}_{:=D_{a2}},$$

where the domain of integration has been split at $\gamma x^{-\alpha} = 1$. Considering just D_{a1} , we are able to switch the order of integration and summation, because the infinite sum is absolutely and uniformly convergent on the domain of integration, as may be shown through the Weierstrass Mtest [7, Theorem 7.10]. Therefore,

$$D_{a1} = \left[x \left(\ln \left(\frac{\gamma}{x^{\alpha}} \right) + \alpha \right) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{x^{\alpha n+1}}{\gamma^n (\alpha n+1)} \right) \right]_{x_0}^{\alpha \sqrt{S}}$$
$$= d_0 \left[\alpha \sqrt[\alpha]{S} - \alpha - \ln(S) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(\alpha n+1)} \left(\sqrt[\alpha]{S} - \frac{1}{S^n} \right) \right].$$

The order of integration and summation may similarly be swapped in D_{a2} , resulting in

$$\begin{split} D_{a2} &= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(1-\alpha n)} \left[x \left(\frac{\gamma}{x^{\alpha}} \right)^{n} \right]_{\sqrt[q]{\gamma}}^{x_{T}} \\ &= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(1-\alpha n)} \left(x_{T} \left(\frac{\gamma}{x_{T}^{\alpha}} \right)^{n} - \sqrt[q]{S} d_{0} \right). \end{split}$$

The term D_{a1} does not change with x_T , and the limit

$$\lim_{x_T \to \infty} D_{a2} = -\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(1-\alpha n)} \left(\sqrt[\alpha]{S} d_0\right).$$

Therefore, the total transmittable data

$$D_{\infty,1} = \frac{Bd_0}{\nu} \left[\alpha \sqrt[\alpha]{S} - \alpha - \ln(S) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(\alpha n+1)} \left(\sqrt[\alpha]{S} - \frac{1}{S^n} \right) - \sqrt[\alpha]{S} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(1-\alpha n)} \right].$$

Noting that,

$$(n(\alpha n+1))^{-1} - (n(1-\alpha n))^{-1} = -2\alpha(1-(\alpha n)^2)^{-1}, \quad (18)$$

we may rewrite the infinite sums in $D_{\infty,1}$, resulting in

$$D_{\infty,1} = \frac{Bd_0}{v} \left[\alpha \sqrt[\alpha]{S} - \alpha - \ln(S) - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{nS^n} \right] + \alpha \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(\alpha n+1)S^n} - 2\alpha \sqrt[\alpha]{S} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1 - (\alpha n)^2} \right],$$

where the transformation in the first line follows from (16). Restricting S > 1, the transformation in the second line follows from (17). Since $\alpha \ge 2$, the restriction of the domain of (17) to $\frac{Z}{\pi} \notin \mathbb{Z}$ is not prohibitive. Finally,

$$D_{\infty,1} = \frac{Bd_0}{\nu} \left[\pi \sqrt[\alpha]{S} \csc\left(\frac{\pi}{\alpha}\right) - \ln(1+S) + \alpha \sum_{n=1}^{\infty} \left\{ \frac{(-1)^{n+1}}{S^n(\alpha n+1)} \right\} - \alpha \right]. \tag{19}$$

APPENDIX B PROOF OF COROLLARY 1

Simply ignoring the infinite sum results in (7). The infinite sum in (6) is bounded for S > 1 if $\alpha \ge 2$ and is monotonically decreasing in both α and S. Therefore,

$$\sum_{n=1}^{\infty} \left\{ \frac{(-1)^{n+1}}{S^n(\alpha n+1)} \right\} \le \sum_{n=1}^{\infty} \left\{ \frac{(-1)^{n+1}}{S^n(2n+1)} \right\} = 1 - \sqrt{S} \tan^{-1} \left(\frac{1}{\sqrt{S}} \right) \le 1 - \frac{\pi}{4},$$

where the infinite sum is evaluated noting that [6, p. 422]

$$\tan^{-1}(z) = -\sum_{n=0}^{\infty} (-1)^{n+1} \frac{z^{2n+1}}{2n+1}, z \in \mathbb{R}.$$

Therefore,

$$D_{\infty,1} - \tilde{D}_{\infty,1} \le \frac{Bd_0\alpha}{\nu} \left(1 - \sqrt{S} \tan^{-1} \left(\frac{1}{\sqrt{S}} \right) \right) \le \frac{Bd_0\alpha}{\nu} \left(1 - \frac{\pi}{4} \right).$$

Since $\lim_{S\to\infty} \sqrt{S} \tan^{-1} \left(\frac{1}{\sqrt{S}} \right) = 1$, the error $e_{\infty,1} \to 0$ as $S \to \infty$.

APPENDIX C

PROOF OF THEOREM 2

Note that, using integration by parts,

$$\int_{c}^{b} \ln(z^{2} + a) dz = \left[z \left(\ln(z^{2} + a) - 2 \right) + 2\sqrt{a} \tan^{-1} \left(\frac{z}{\sqrt{b}} \right) \right]_{c}^{b}.$$
(20)

We begin by rewriting (11) as

$$D_{\infty,2} = \lim_{x_T \to \infty} B \int_{x_0}^{x_T} \ln\left((z_0^2 + Sd_0^2) + x^2\right) - \ln\left(z_0^2 + x^2\right) dx.$$
 (21)

Integrating the first logarithm, using $\epsilon = z_0^2 + Sd_0^2$ and (20),

$$\int_{x_0}^{x_T} \ln\left(\left(z_0^2 + Sd_0^2\right) + x^2\right) dx = 2(x_0 - x_T) + x_t \ln(x_T^2 + \epsilon)$$
$$- x_0 \ln(x_0^2 + \epsilon) + 2\sqrt{\epsilon} \left(\tan^{-1}\left(\frac{x_T}{\sqrt{\epsilon}}\right) - \tan^{-1}\left(\frac{x_0}{\sqrt{\epsilon}}\right)\right).$$

Similarly, the second logarithm is integrated as

$$\int_{x_0}^{x_T} \ln\left(z_0^2 + x^2\right) dx = 2(x_0 - x_T) + x_T \ln(x_T^2 + z_0^2)$$
$$- x_0 \ln(x_0^2 + z_0^2) + 2z_0 \left(\tan^{-1}\left(\frac{x_T}{z_0}\right) - \tan^{-1}\left(\frac{x_0}{z_0}\right)\right).$$

Therefore.

$$\begin{split} D_{\infty,2} &= \lim_{x_T \to \infty} \frac{B}{v} \left[x_T \ln \left(\frac{x_T^2 + \epsilon}{x_T^2 + z_0^2} \right) + x_0 \ln \left(\frac{x_0^2 + z_0^2}{x_0^2 + \epsilon} \right) \right. \\ &+ 2\sqrt{\epsilon} \left(\tan^{-1} \left(\frac{x_T}{\sqrt{\epsilon}} \right) - \tan^{-1} \left(\frac{x_0}{\sqrt{\epsilon}} \right) \right) \\ &+ 2z_0 \left(\tan^{-1} \left(\frac{x_T}{z_0} \right) - \tan^{-1} \left(\frac{x_0}{z_0} \right) \right) \right] \\ &= \frac{B}{v} \left[x_0 \ln (1 + S) + \pi \left(\sqrt{z_0^2 + Sd_0^2} - z_0 \right) \right. \\ &- 2 \left(\sqrt{z_0^2 + Sd_0^2} \tan^{-1} \left(\frac{x_0}{\sqrt{z_0^2 + Sd_0^2}} \right) - z_0 \tan^{-1} \left(\frac{x_0}{z_0} \right) \right) \right]. \end{split}$$

REFERENCES

- [1] O. J. Faqir, Y. Nie, E. C. Kerrigan, and D. Gündüz, "Energy-efficient communication in mobile aerial relay-assisted networks using predictive control," *IFAC-PapersOnLine*, vol. 51, no. 20, pp. 197–202, 2018.
- [2] Y. Zeng and R. Zhang, "Energy-efficient UAV communication with trajectory optimization," *IEEE Transactions on Wireless Communications*, vol. 16, no. 6, pp. 3747–3760, 2017.
- [3] Q. Wu, L. Liu, and R. Zhang, "Fundamental trade-offs in communication and trajectory design for UAV-enabled wireless network," *IEEE Wireless Communications*, vol. 26, no. 1, pp. 36–44, 2019.

- [4] Y. Zeng, R. Zhang, and T. J. Lim, "Throughput maximization for mobile relaying systems," in 2016 IEEE Globecom Workshops (GC Wkshps), pp. 1–6, IEEE, 2016.
 [5] X. Yuan, Z. Feng, W. Xu, W. Ni, J. A. Zhang, Z. Wei, and R. P.
- [5] X. Yuan, Z. Feng, W. Xu, W. Ni, J. A. Zhang, Z. Wei, and R. P. Liu, "Capacity analysis of UAV communications: Cases of random trajectories," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 8, pp. 7564–7576, 2018.
- pp. 7564–7576, 2018.
 [6] K. Knopp, *Theory and Application of Infinite Series*. Dover Books on Mathematics, Dover Publications, 1990.
- [7] W. Rudin, *Principles of mathematical analysis*. McGraw-hill New York, 3 ed., 1964.