

Stochastic Geometry-Based Analysis of LEO Satellite Communication Systems

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(Invited Paper)

Abstract—This letter studies the performance of a low-earth-orbit (LEO) satellite communication system where the locations of the LEO satellites are modeled as a binomial point process (BPP) on a spherical surface. In particular, we study the user coverage probability for a scenario where satellite gateways (GWs) are deployed on the ground to act as a relay between the users and the LEO satellites. We use tools from stochastic geometry to derive the coverage probability for the described setup assuming that LEO satellites are placed at n different altitudes, given that the number of satellites at each altitude a_k is N_k where $1 \leq k \leq n$. To resemble practical scenarios where satellite communication can play an important role in coverage enhancement, we compare the performance of the considered setup with a scenario where the users are solely covered by a fiber-connected base station (referred to as anchored base station or ABS in the rest of the letter) at a relatively far distance, which is a common challenge in rural and remote areas. Using numerical results, we show the performance gain, in terms of coverage probability, at rural and remote areas when LEO satellite communication systems are adopted. Finally, we draw multiple system-level insights regarding the density of GWs required to outperform the ABS, as well as the number of LEO satellites and their altitudes.

Index Terms—Stochastic geometry, binomial point process, distance distribution, coverage probability.

I. INTRODUCTION

SATELLITE communications have a great potential to achieve the ultimate goal of providing wireless coverage worldwide, including rural and remote areas which are still lacking proper service around the globe [1]–[4]. Particular attention has been recently given to LEO satellites for providing wireless coverage, due to their relatively low latency and cheaper launching costs [5]. This has motivated various companies to invest in launching large number of LEO satellites with the purpose of providing satellite-based cellular service, such as SpaceX, Amazon, and OneWeb [6].

In this letter, using tools from stochastic geometry, we study the performance of LEO satellite-based communication systems given the altitudes of the satellites, the number of satellites at each altitude, and the density of the GWs. For that setup, we derive the coverage probability as a function of the aforementioned system parameters. In addition, we consider a practical use case where satellite communication systems are deployed to enhance coverage in remote and rural areas.

Manuscript received August 14, 2020; revised September 20, 2020; accepted October 6, 2020. Date of publication October 9, 2020; date of current version August 12, 2021. The associate editor coordinating the review of this letter and approving it for publication was Y. Deng. (Corresponding author: Mustafa A. Kishk.)

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Digital Object Identifier 10.1109/LCOMM.2020.3029808

For that purpose, we compare the coverage probability of the satellite-based communication system in such regions with the coverage probability in case of relying on the nearest ABS, which is typically located at far distances from rural and remote areas. Before providing more details regarding the contributions of this letter, we first discuss the related literature in the next subsection.

A. Related Work

Performance analysis of satellite communication systems is essential for efficient implementation and design of such systems. Various aspects of such systems are investigated in literature such as the influence of the elevation angle and the altitude on the coverage area [7], the influence of adopting non-orthogonal multiple access (NOMA) [8], secrecy in UAV-aided satellite-terrestrial systems [9], and overlay spectrum sharing [10]. Furthermore, many works have focused on deriving an accurate channel model for the communication links between satellites and ground stations [11], [12], where it was shown that shadowed-Rician (SR) model is the most accurate. One of the closest works in literature is [13]. In this work, authors studied a setup where the locations of the LEO satellites are modeled as a BPP on a spherical surface. Compared to the work in [13], this letter has the following differences: (i) we consider a more general setup where satellites are deployed at various altitudes (multiple concentric spherical surfaces), which enables more general results, (ii) we adopt SR fading for the channel between the satellite and the GW, instead of Rayleigh fading adopted in [13], and (iii) we provide useful insights for a specific use case of satellite communications in rural and remote areas. More details about the contributions of this letter are provided next.

B. Contributions

We consider a system setup where the locations of the satellite GWs are modeled as a Poisson point process (PPP) on the ground and the LEO satellites are modeled as BPP on a set of spherical surfaces. In particular, to resemble practical scenarios, we assume that satellites are deployed on n spherical surfaces S_k where the number of satellites on S_k is N_k for $1 \leq k \leq n$. For that setup, we study the coverage probability of a typical user on the ground as the joint coverage probability of the user-GW link and the GW-satellite link. Furthermore, we compare the coverage of such setup with the scenario where coverage is provided by a terrestrial base station (referred to as anchored base station or ABS in the rest of the letter) that is located far from the user, which resembles typical challenges in rural and remote areas. Unlike ABS, the GW stations do not require underground fiber-optic connections that need many infrastructures, making

TABLE I
TABLE OF NOTATIONS AND ACRONYMS.

Notation	Description
LEO; GW; ABS	Low Earth Orbit; Gateway; Anchored Base Station
BPP; PPP	Binomial Point Process; Poisson Point Process
N_i	number of satellites on i^{th} sphere
$\sigma_g^2; \sigma_u^2$	Noise power at the GW; at the user
$\gamma_g; \gamma_u$	SNR threshold at the GW; at the user
$\rho_s; \rho_g; \rho_a$	Transmit power of the satellite; the GW; the ABS
$r_e; r_i; a_i$	Radius of the Earth; radius of i^{th} sphere; altitude of i^{th} sphere to the surface of the Earth.
$D; R_{GW-U}; R$	Length of S-GW link; length of GW-U link; length of ABS-U link
$P_{cov}^{S-GW}; P_{cov}^{GW-U}; P_{cov}^{ABS-U};$	Coverage probability for S-GW link; GW-U link; ABS-U link
$SR(\Omega, b_0, m)$	Shadowed-Rician (SR) fading with components: Ω -the line-of-sight ; $2b_0$ -the scatter; m -the Nakagami parameter.
$W_s^2; W_g^2$	SR fading power for the S-GW link; Rayleigh fading power for the GW-U link

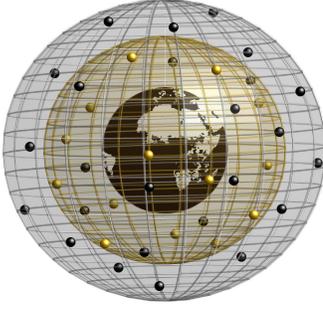


Fig. 1. System model for n level of spheres concentric with the Earth.

it easier and cheaper to deploy GWs in rural areas. Using numerical results, we show the deployment of GWs to enable satellite communications enhances coverage probability when the distance to the nearest ABS is beyond a specific threshold. Finally, we show how this distance threshold is affected by various system parameters such as the density of the GWs, the altitudes of the satellites, and their numbers.

II. SYSTEM MODEL

We consider a setup as represented in Fig. 1 where LEO satellites are deployed at n different altitudes. Each altitude a_k constitutes a sphere $S_k, \forall 1 \leq k \leq n$ over which N_k LEO satellites are uniformly distributed. The radius of each sphere S_k is $r_k = r_e + a_k$, where r_e is the radius of the earth. The communication between the LEO satellites and the ground users is relayed through GWs located on the ground. The locations of the GWs are modeled as a PPP with density λ_{GW} . The transmit power of the satellite is denoted by ρ_s while the transmit power of the GW is ρ_g . In the rest of the letter, we will refer to the link between the satellite and GW as the S-GW link while the link between the GW and the user will be referred to as the GW-U link. Under these links which can be shown as in Fig. 2, we study the network performance at a typical user in terms of the coverage probability.

A. S-GW Link

For the S-GW link, the received signal power at the GW is

$$\rho_r^g = \rho_s |H_s|^2, \quad (1)$$

where H_s represents the channel fading for the S-GW link and is represented as

$$H_s = A \times W_s, \quad (2)$$

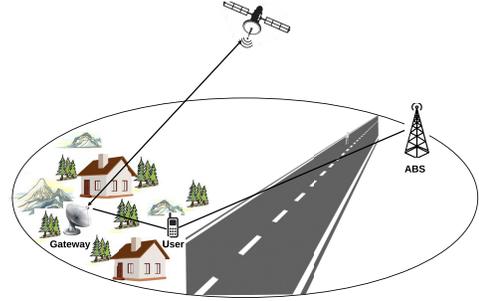


Fig. 2. Satellite communication systems can highly enhance coverage for under-served remote and rural areas.

where A and W_s represent the propagation loss and the SR fading, respectively. The propagation loss equation for the S-GW link is motivated by [14]

$$A = \frac{\lambda G_R s e^{j\phi} \xi}{4\pi D}, \quad (3)$$

where λ denotes the carrier wavelength, D is the distance between the GW and the nearest satellite which is random variable derived in the [15], G_R^2 represents the GW receiver antenna gain directed towards the LEO satellite, s denotes the rain attenuation, ϕ represents the phase due to the beam radiation pattern and radio propagation, and ξ is a function of the maximum satellite antenna gain and the antenna bandwidth.

The cumulative distribution function (CDF) of the SR fading power W_s^2 is given as follows:

$$F_{W_s^2}(t) = \left(\frac{2b_0 m}{2b_0 m + \Omega} \right)^m \sum_{z=0}^{\infty} \frac{(m)_z}{z! \Gamma(z+1)} \left(\frac{\Omega}{2b_0 m + \Omega} \right)^z \times \gamma \left(z+1, \frac{1}{2b_0} t \right), \quad (4)$$

where $\Gamma(\cdot)$ denotes the gamma function, $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function, $(m)_z$ is the Pochhammer symbol, while m, b_0 and Ω are the parameters of the SR fading.

B. GW-U Link

For the GW-U link, the received signal power at the user is

$$\rho_r^u = \rho_g W_g^2 R_{GW-U}^{-\alpha}, \quad (5)$$

where W_g^2 is exponentially distributed with unity mean, R_{GW-U} is the distance between the GW and the user, and α is the path-loss exponent.

C. Association Policy

We consider an association policy where the user associates with its nearest GW, while the GW associates with its nearest LEO satellite. Hence, the user is considered in coverage if the following conditions are satisfied: (i) the signal-to-noise-ratio (SNR) for the S-GW link is above a predefined threshold γ_g and (ii) the SNR for the GW-U link is above a predefined threshold γ_u . Hence, the coverage probability is defined as follows

$$P_{\text{cov}} = P_{\text{cov}}^{S-GW} P_{\text{cov}}^{GW-U}, \quad (6)$$

where

$$P_{\text{cov}}^{S-GW} = \mathbb{P}\left(\frac{\rho_r^g}{\sigma_g^2} \geq \gamma_g\right), \quad (7)$$

$$P_{\text{cov}}^{GW-U} = \mathbb{P}\left(\frac{\rho_r^u}{\sigma_u^2} \geq \gamma_u\right), \quad (8)$$

σ_g^2 , and σ_u^2 are the noise powers at the GW and the user, respectively.

III. COVERAGE PROBABILITY

A. Distance Distribution

Given that the satellites are randomly located on the set of spheres $\{S_k\}$, the distance D is a random variable. In the authors' work [15], the distribution of D was derived as shown in the below lemma.

Lemma 1 (Contact distance distribution):

$$F_D(d) \triangleq P(D < d) = 1 - \prod_{i=1}^n P(D_i \geq d), \quad (9)$$

where the complementary CDF (CCDF) of D_i is

$$P(D_i \geq d) = \begin{cases} 1, & d < a_i \\ \left[1 - \frac{1}{\pi} \arccos\left(1 - \frac{d^2 - a_i^2}{2r_e r_i}\right)\right]^{N_i}, & a_i \leq d \leq d_{\max}(i, 0) \\ \left[1 - \frac{1}{\pi} \arccos\left(\frac{r_e}{r_i}\right)\right]^{N_i}, & d > d_{\max}(i, 0), \end{cases}$$

where $d_{\max}(i, 0) = \sqrt{2r_e a_i + a_i^2}$. The PDF of D is

$$f_D(d) = \left[\prod_{i=1}^n \bar{F}_{D_i}(d) \right] \left[\sum_{i=1}^n \frac{f_{D_i}(d)}{\bar{F}_{D_i}(d)} \right], \quad (10)$$

where

$$f_{D_i}(d) = \frac{dN_i}{\pi r_e r_i} \left[1 - \frac{1}{\pi} \arccos\left(1 - \frac{d^2 - a_i^2}{2r_e r_i}\right)\right]^{N_i-1} \times \frac{1}{\sqrt{1 - \left(1 - \frac{d^2 - a_i^2}{2r_e r_i}\right)^2}},$$

for $a_i \leq d \leq d_{\max}(i, 0)$ and $f_{D_i}(d) = 0$ otherwise.

B. Coverage Analysis

As described in (6), in order to compute the coverage probability, it is required to derive each of P_{cov}^{S-GW} and P_{cov}^{GW-U} . Firstly, P_{cov}^{GW-U} is a well-established result in literature and is provided in the following lemma for completeness.

Lemma 2: The coverage probability for the GW-U link is

$$P_{\text{cov}}^{GW-U} = \int_0^\infty \exp\left(-\frac{\gamma_u r^\alpha \sigma_u^2}{\rho_g}\right) f_{R_{\text{GW-U}}}(r) dr, \quad (11)$$

where $f_{R_{\text{GW-U}}}(r) = 2\pi\lambda_{\text{GW}} r \exp(-\pi\lambda_{\text{GW}} r)$.

Now, the main result in this letter, which is the derivation of P_{cov}^{S-GW} , is provided in the below theorem.

Theorem 1: The coverage probability for the S-GW link for an arbitrarily located GW under a Shadowed-Rician fading channel is

$$P_{\text{cov}}^{S-GW} = \int_0^\infty \frac{1}{2\sqrt{y}} f_D(\sqrt{y}) - \left(\frac{2b_0 m}{2b_0 m + \Omega}\right)^m \sum_{z=0}^\infty \frac{(m)_z}{z! \Gamma(z+1)} \left(\frac{\Omega}{2b_0 m + \Omega}\right)^z \times \int_0^\infty \gamma \left(z+1, \frac{1}{2b_0} cy\right) \frac{1}{2\sqrt{y}} f_D(\sqrt{y}) dy, \quad (12)$$

where $f_D(\sqrt{y})$ is the PDF of contact distance distribution that is expressed in Lemma 1 and $c = \frac{16\pi^2 \gamma_g \sigma_g^2}{\rho_s |\lambda_{\text{GR}} s e^{j\phi} \xi|^2}$.

Proof: See Appendix A. ■

Remark 1: The lower incomplete gamma function can be approximated at high SNR ($\rho_s \rightarrow \infty$) such as $\gamma\left(z+1, \frac{1}{2b_0} cy\right) \approx \frac{cy}{(z+1)2b_0}$.

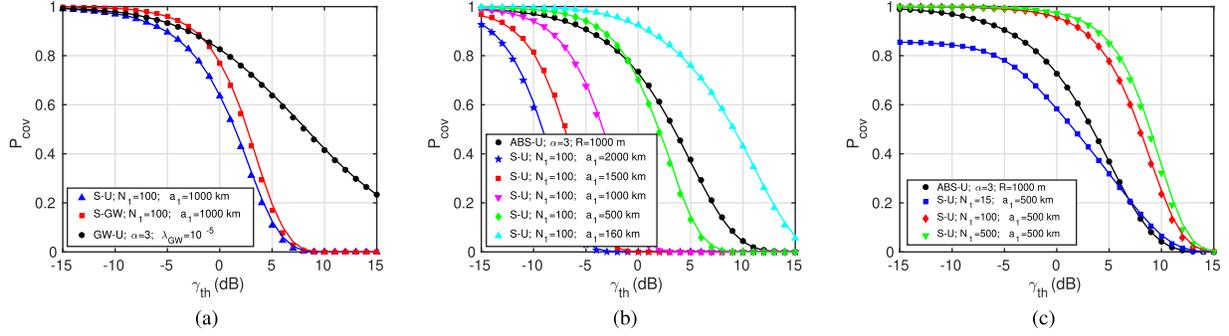
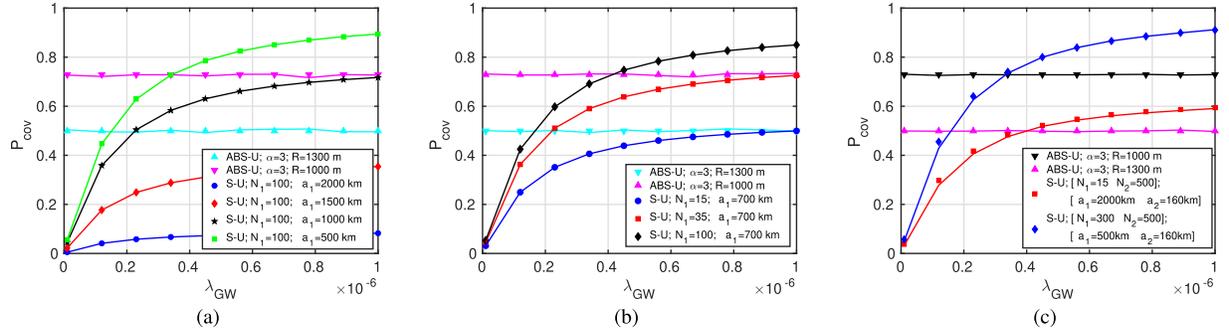
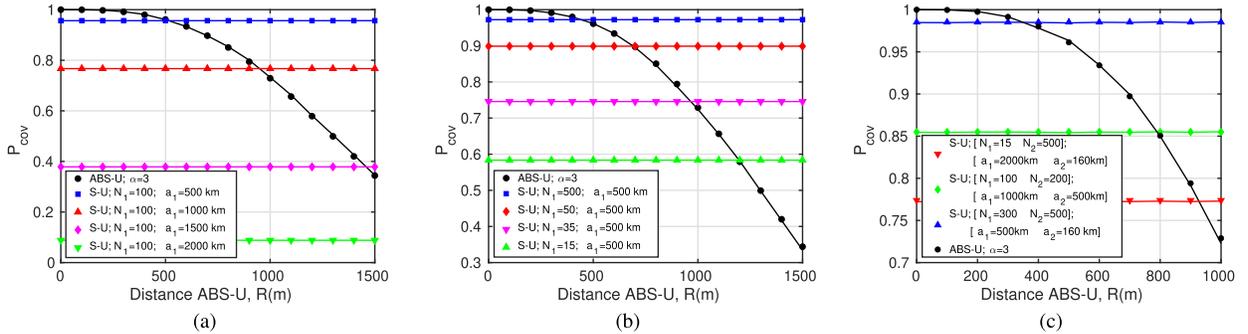
Remark 2: The integral in the theorem can be numerically evaluated by truncating the infinite series.

C. Coverage Enhancement in Remote Locations

In order to evaluate the performance gains from the deployment of satellite communication systems in rural and remote areas, we consider a scenario where, for a given rural area, the nearest ABS is located at a distance R , as shown in Fig. 2. The value of R is typically large at rural and remote areas, leading to relatively low coverage probability. In such a scenario, the coverage probability when the satellite communication system is absent is

$$P_{\text{cov}}^{\text{ABS-U}} = \exp\left(-\frac{\gamma_u R^\alpha \sigma_u^2}{\rho_a}\right), \quad (13)$$

where ρ_a is the transmission power of the ABS, and assuming Rayleigh fading due to the scatters can be caused by mountains, hill, and trees in rural areas. As we mentioned in the contribution section, extending the fiber connections to rural areas requires a lot of digging, which increases the costs. Therefore, to consider a practical scenario, it is reasonable to assume that the distance to the nearest ABS in a rural area is much larger than the distance to the nearest GW. The above scenario will be used as a benchmark to evaluate the performance enhancement in the numerical results section, which is provided next.

Fig. 3. Coverage probability versus threshold, γ_{th} .Fig. 4. Coverage probability versus density of PPP for GWs, λ_{GW} .Fig. 5. Coverage probability versus distance ABS-U, R .TABLE II
SYSTEM PARAMETERS.

Notation	PARAMETER	VALUE
f_c	S-GW Link frequency band	20GHz(Ka)
ρ_s	Satellite transmit power	15dBW
σ_u^2, σ_a^2	Noise power at the GW and the user	$3.6 \times 10^{-12}; 10^{-8}$
G_R^z	GW antenna gain	41.7dBi
s	Average rain attenuation	2dB
λ_{GW}	Density of GWs	10^{-5}
α	path loss exponent	3
$SR(\Omega, b_0, m)$	SR fading	$SR(1.29, 0.158, 19.4)$

IV. NUMERICAL RESULTS

In this section, we verify the derived expressions using Monte-Carlo simulations. In addition, we study the influence of various system parameters on the performance of the considered system and on the performance gain in rural and remote areas. The system parameters used in the simulations are summarized in Table II. In addition, we assume that $\gamma_g = \gamma_u$ (both are denoted as γ_{th} in the figures) and $\rho_s = \rho_g = \rho_a$. In all the figures, markers represent the derived analytical results while the solid lines represent the Monte-Carlo simulations.

In Fig. 3(a), we show that the satellite-user (S-U) coverage probability is limited by the GW-U coverage at low values of γ_{th} , while at higher values it is limited by the S-U coverage. In Fig. 3(b), we plot the coverage probability for $n = 1$ and fixed number of satellites and study the effect of increasing the altitude. The results show improvement in the coverage probability at lower values of the altitude a_1 . In Fig. 3(c), we fix the altitude and vary the number of satellites for the scenario of $n = 1$. The results show that the improvement in the coverage probability saturates as we increase the number of satellites beyond a specific value.

In Fig. 4(a), plot the coverage probability for different values of λ_{GW} and a_1 while fixing the number of satellites. The results show that the density of gateways required to outperform the ABS coverage reduces as we reduce the altitude of the satellites. Similarly, in Fig. 4(b), we observe that the required density of gateways reduces as we increase the number of satellites, for a fixed altitude. In Fig. 4(c), we plot the coverage probability for the scenario of $n = 2$ with different values of a_1, a_2, N_1 , and N_2 .

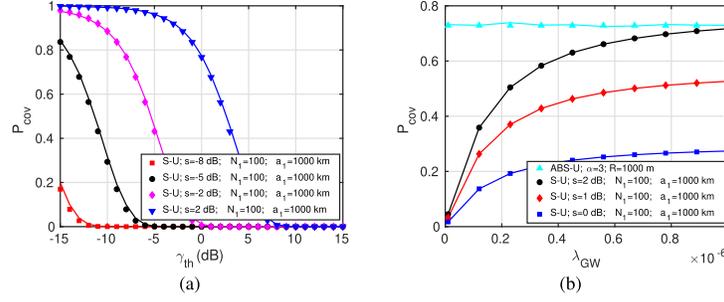


Fig. 6. Coverage probability versus threshold and density of GW for different values of rain fading attenuation.

In Fig. 5, we show how the satellite communication system improves the coverage in a remote location as the distance to the nearest ABS increases.

As we would expect, the performance, in terms of coverage probability, degrades as the rain fading attenuation increases as presented in the Fig. 6. This result can be used to obtain the coverage probability for a given threshold, knowing the average attenuation value for the S-GW link.

V. CONCLUSION

In this letter, we proposed a stochastic geometry-based model for the LEO satellite locations in order to study and analyze the performance of satellite communication systems. Assuming randomly located satellites on spherical surfaces, we derived the coverage probability for a setup where satellite gateways are distributed according to a PPP on the ground. We have verified all the derived expressions using Monte-Carlo simulations and ensured perfect fit. Finally, we have studied the effect of the altitudes of the satellites, their numbers, and the density of the gateways on the performance of the system. The provided framework can be extended in many directions. For instance, the provided framework can be extended to capture more general setups, typically referred to as integrated satellite-aerial-terrestrial networks [16].

APPENDIX

A. Proof of Theorem 1

$$\begin{aligned}
 P_{cov}^{S-GW} &= \mathbb{P}\left(\frac{\rho_r^g}{\sigma_g^2} \geq \gamma_g\right) \\
 &= \mathbb{P}\left(\frac{\rho_s |A|^2 W_s^2}{\sigma_g^2} \geq \gamma_g\right) \\
 &= \mathbb{P}\left(\rho_s \left|s \frac{\lambda G_{R\xi} e^{j\phi}}{4\pi D \sigma_g}\right|^2 W_s^2 \geq \gamma_g\right) \\
 &= \mathbb{P}\left(\frac{W_s^2}{D^2} \geq c\right) \quad \text{let } c = \frac{16\pi^2 \gamma_g \sigma_g^2}{\rho_s |\lambda G_{R\xi} e^{j\phi}|^2} \\
 &= \mathbb{P}\left(\frac{X}{Y} \geq c\right) \quad \text{with } W_s^2 = X \text{ and } D^2 = Y \\
 &= \int_0^\infty \bar{F}_X(cy) f_Y(y) dy \quad \text{with } \bar{F}_X(cy) = 1 - F_X(cy)
 \end{aligned}$$

$$\begin{aligned}
 &= \int_0^\infty f_Y(y) dy - \left(\frac{2 b_0 m}{2 b_0 m + \Omega}\right)^m \sum_{z=0}^\infty \frac{\binom{m}{z}}{z! \Gamma(z+1)} \\
 &\quad \times \left(\frac{\Omega}{2 b_0 m + \Omega}\right)^z \int_0^\infty \gamma\left(z+1, \frac{1}{2 b_0} cy\right) \frac{1}{2\sqrt{y}} f_D(\sqrt{y}) dy.
 \end{aligned}$$

This concludes the proof.

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