# Index and Composition Modulation

Ferhat Yarkin Graduate Student Member, IEEE and Justin P. Coon Senior Member, IEEE

Abstract—In this paper, we propose a novel modulation concept which we call *index and composition modulation (ICM)*. In the proposed concept, we use indices of active/deactive codeword elements and compositions of an integer to encode information. In this regard, we first determine the activated codeword elements, then we exploit energy levels of these elements to identify the compositions. We depict a practical scheme for using ICM with orthogonal frequency division multiplexing (OFDM) and show that OFDM with ICM (OFDM-ICM) can enhance the spectral efficiency (SE) and error performance of OFDM-IM. We design an efficient low-complexity detector for the proposed technique. Moreover, we analyze the error and SE performance of the OFDM-ICM technique and show that it is capable of outperforming existing OFDM benchmarks in terms of error and SE performance.

Index Terms—Composition modulation (CM), index modulation (IM), orthogonal frequency division multiplexing (OFDM).

## I. INTRODUCTION

Index modulation (IM) that encodes data into the combinations of active/deactive codeword elements offers a variety of attractive advantages including better error performance and improved energy/spectral efficiency over conventional modulation and multiplexing schemes. Hence, the adaptation of IM to orthogonal frequency division multiplexing (OFDM) has attracted several researchers attentions [1]–[6] and it has been shown by these studies that OFDM with IM (OFDM-IM) can achieve better error performance, higher data rate and higher energy efficiency than conventional OFDM. Apart from IM, in [7], we proposed two new modulation concepts, *weak composition modulation (WCM)* and *composition modulation (CM)*, that embed data using integer compositions, and showed that the applications of these concepts to OFDM bring noteworthy improvements in error performance.

5G's key enabling technologies are expected to satisfy the user demands for high data rate, ultra-reliable transmission, and very low latency. However, it is anticipated that 6G and the following next-generation networks will require even higher data rates, better error performance, and lower latency due to the proliferation of a variety of new applications, including extended reality services, telemedicine, haptics, flying vehicles, brain-computer interfaces, and connected autonomous systems [8]. In this regard, the existing OFDM and OFDM-IM schemes require further improvement in error performance and spectral efficiency (SE) to be deployed efficiently in next-generation networks.

Motivated by the advantages of the IM and CM techniques as well as the requirements of next-generation networks, we propose a novel concept that we call *index and composition modulation (ICM)*. In this context, we encode information using the combinations of active/deactive codewords elements as in IM and the combinatorial framework of integer compositions as in CM. We depict a practical model based on OFDM with ICM (OFDM-ICM). Moreover, we design an efficient low-complexity detector to overcome the high complexity arising from maximum-likelihood (ML) detector. We also investigate the bit error rate (BER) and SE of the proposed scheme in this paper. Our analytical, as well as numerical, findings indicate that our novel design can achieve a substantially better SE and BER performance than OFDM, OFDM-IM, OFDM-WCM and OFDM-CM.

## II. INDEX AND COMPOSITION MODULATION

An ICM codebook consists of  $L_{ICM}$  codewords and each codeword is an N-tuple of nonnegative real numbers which can be regarded as a vector  $\mathbf{x}_l = \{x_{l1}, x_{l2}, \dots, x_{lN}\}, l =$  $1, 2, \ldots, L_{ICM}$ , in an Euclidean space S of N dimensions where  $x_{ln} \in \mathbb{R}_{>0}$  and n = 1, 2, ..., N. In an ICM codeword, K elements are positive real numbers where  $K \leq N$ , whereas the remaining N - K elements are zeros. Here, we call the K positive real elements and N - K zero elements as activated and deactivated elements, respectively, since no energy is used to form the zero elements. All other codewords of the codebook can be obtained by permuting the order of the Nelements in the codeword. The values of K activated elements in each codeword are chosen according to the integers that form the compositions of an integer I with K parts. Hence, one can obtain different codewords by using different compositions of an integer. More explicitly, when we form  $\mathbf{x}_l$ , we first determine the indices of K activated elements in an N-tuple and denote the set of these indices as  $\mathcal{I} \coloneqq \{\alpha_1, \alpha_2, \dots, \alpha_K\}$ where  $\alpha_k \in \{1, 2, ..., N\}$  and k = 1, 2, ..., K. Then, the *k*th activated element  $x_{l\alpha_k}$  of the N-tuple is chosen as  $x_{l\alpha_k} = \sqrt{\nu_k}$ where  $\nu_k \in \{1, \ldots, I - K + 1\}$  is the kth summand of the composition and  $\nu_1 + \nu_2 + \ldots + \nu_K = I$ . Note that one needs to pick I > K to be able to construct the ICM codewords. Since the number of permutations regarding the order of K activated and N - K deactivated elements is  $\binom{N}{K}$  and the number of integer compositions of I with K parts is  $\binom{I-1}{K-1}$ , the number of codewords in an ICM codebook is  $L_{ICM} = \binom{N}{K}\binom{I-1}{K-1}$ . In Table I, we give a codebook generation example for an ICM scheme when N = I = 3 and K = 2. As seen from the leftmost column of the table, we first determine the indices of K = 2 activated elements, then we determine the compositions of an integer I = 3 with K = 2 parts and map the summands in those compositions to the elements of the ICM codewords according to the indices as shown in the rightmost column.

The authors wish to acknowledge the support of the Bristol Innovation & Research Laboratory of Toshiba Research Europe Ltd.

F. Yarkin and J. P. Coon are with the Department of Engineering Science, University of Oxford, Parks Road, Oxford, OX1 3PJ, U.K. E-mail: {ferhat.yarkin and justin.coon}@eng.ox.ac.uk

TABLE I: Codebook generation example for ICM when N = I = 3 and K = 2.

Indices of Activated Elements, $\mathcal{I}$	Compositions	ICM Codeword	
$\{1, 2\}$	3=1+2	$\mathbf{x}_1 = \{1, \sqrt{2}, 0\}$	
$\{1, 3\}$		$\mathbf{x}_2 = \{1, 0, \sqrt{2}\}$	
$\{2,3\}$		$\mathbf{x}_3 = \{0, 1, \sqrt{2}\}$	
$\{1, 2\}$	3=2+1	$\mathbf{x}_4 = \{\sqrt{2}, 1, 0\}$	
$\{1, 3\}$		$\mathbf{x}_5 = \{\sqrt{2}, 0, 1\}$	
$\{2,3\}$		$\mathbf{x}_6 = \{0, \sqrt{2}, 1\}$	

# III. OFDM WITH INDEX AND COMPOSITION MODULATION

In this section, we present a practical system model in which we apply the ICM concept to OFDM transmissions.

## A. Transmitter

Analogously to OFDM-IM, m input bits enter the transmitter and these bits are divided into B = m/f blocks, each of them having f input bits. The total number of the subcarriers,  $N_T$ , is also split into  $B = N_T/N$  blocks whose size is N. We will focus on the *b*th block in what follows. f information bits are further divided into three parts, having  $f_1$ ,  $f_2$  and  $f_3$ bits with  $f_1 + f_2 + f_3 = f$  in the *b*th block.

The first  $f_1 = \lfloor \log_2 {N \choose K} \rfloor$  bits are used to determine the activated K subcarriers. Then, the  $f_2 = \lfloor \log_2 {I-1 \choose K-1} \rfloor$ bits are used to determine the specific composition of an integer I with K parts where  $I \ge K$ . The energies of the symbols on the activated subcarriers are chosen according to the specific composition of an integer I with K parts. Let us denote the sets that comprise the indices of the activated subcarriers and the energies of the activated subcarriers in the *b*th block, respectively, as  $\mathcal{I}^b := \{\alpha_1, \alpha_2, \ldots, \alpha_K\}$ and  $\beta^b := \{\nu_1 E_T / I, \nu_2 E_T / I, \ldots, \nu_K E_T / I\}$  where  $\alpha_k \in$  $\{1, 2, \ldots, N\}, k = 1, 2, \ldots, K, \nu_k \in \{1, 2, \ldots, I - K + 1\},$ and  $\nu_1 + \nu_2 + \ldots + \nu_K = I$ . Once we decide  $\mathcal{I}^b$  and  $\beta^b$ according to the  $f_1$  and  $f_2$  bits, respectively,  $f_3 = K \log_2 M$ bits are used to determine the M-PSK constellation symbols carried by the activated subcarriers. Hence, the SE of the OFDM-ICM scheme per subcarrier can be given as

$$\eta = \frac{f_1 + f_2 + f_3}{N} = \frac{\lfloor \log_2 \binom{N}{K} \rfloor + \lfloor \log_2 \binom{I-1}{K-1} \rfloor + K \log_2 M}{N}$$
(1)

The mapping of  $f_1$  bits to the indices of activated subcarriers and  $f_2$  bits to the subcarrier's energies can be implemented by using a look-up table. In Table II, we present an example of how these mappings are performed when N = I = 4 and K =3. Since  $f_1 = \lfloor \log_2 \binom{4}{3} \rfloor = 2$ , the first two bits,  $p_1$ , entering the OFDM-ICM encoder of the corresponding bitstream  $[p_2 \ p_1]$ are used to determine the indices of activated subcarriers. Then, the remaining  $f_2 = \lfloor \log_2 \binom{I-1}{K-1} \rfloor = \lfloor \log_2 \binom{3}{2} \rfloor = 1$ bit,  $p_2$ , is used to determine the specific composition of I = 4with K = 3 parts. For example, when  $[p_2 \ p_1] = [0 \ 1 \ 1]$  bits enter the OFDM-ICM transmitter, the first two bits ' $p_1 = 11$ ' choose the indices of activated subcarriers as  $\mathcal{I}^b := \{2,3,4\}$ , then the remaining bit ' $p_2 = 0$ ' chooses the set  $\beta^b :=$ 

TABLE II: Look-up table implementation example for OFDM-ICM when N = I = 4 and K = 3.

Compositions	$\beta^b$	$\mathcal{I}^b$	OFDM-ICM Symbol Vector	$[p_2 \mid p_1]$
4=1+1+2	$\left\{\frac{E_T}{4}, \frac{E_T}{4}, \frac{2E_T}{4}\right\}$	$\{1, 2, 3\}$	$\left[\frac{E_T}{4}  \frac{E_T}{4}  \frac{2E_T}{4}  0\right]$	[0 0 0]
		$\{1, 2, 4\}$	$\left[\frac{E_T}{4}  \frac{E_T}{4}  0  \frac{2E_T}{4}\right]$	[0 0 1]
		$\{1, 3, 4\}$	$\left[\frac{E_T}{4} \ 0 \ \frac{E_T}{4} \ \frac{2E_T}{4}\right]$	[0 1 0]
		$\{2, 3, 4\}$	$\begin{bmatrix} 0 & \frac{E_T}{4} & \frac{E_T}{4} & \frac{2E_T}{4} \end{bmatrix}$	[0 1 1]
4=1+2+1	$\left\{\frac{E_T}{4}, \frac{2E_T}{4}, \frac{E_T}{4}\right\}$	$\{1, 2, 3\}$	$\left[\frac{E_T}{4} \ \frac{2E_T}{4} \ \frac{E_T}{4} \ 0\right]$	[1 ¦0 0]
		$\{1, 2, 4\}$	$\left[\frac{E_T}{4} \ \frac{2E_T}{4} \ 0 \ \frac{E_T}{4}\right]$	[1 0 1]
		$\{1, 3, 4\}$	$\left[\frac{E_T}{4} \ 0 \ \frac{2E_T}{4} \ \frac{E_T}{4}\right]$	[1 1 0]
		$\{2, 3, 4\}$	$\begin{bmatrix} 0 & \frac{E_T}{4} & \frac{2E_T}{4} & \frac{E_T}{4} \end{bmatrix}$	[1 1 1]
4=2+1+1	$\left\{\frac{2E_T}{4}, \frac{E_T}{4}, \frac{E_T}{4}\right\}$	$\{1, 2, 3\}$	$\begin{bmatrix} \frac{2E_T}{4} & \frac{E_T}{4} & \frac{E_T}{4} & 0 \end{bmatrix}$	unused
		$\{1, 2, 4\}$	$\left[\frac{2E_T}{4}  \frac{E_T}{4}  0  \frac{E_T}{4}\right]$	unused
		$\{1, 3, 4\}$	$\left[\frac{2E_T}{4} \ 0 \ \frac{E_T}{4} \ \frac{E_T}{4}\right]$	unused
		$\{2, 3, 4\}$	$\begin{bmatrix} 0 & \frac{2E_T}{4} & \frac{E_T}{4} & \frac{E_T}{4} \end{bmatrix}$	unused

 $\{E_T/4, E_T/4, 2E_T/4\}$  for the subcarriers' energies. Then, we map the set,  $\beta^b$ , to the energies of the activated subcarriers and obtain the OFDM-ICM symbol vector as shown in the table.

Once we decided the activated subcarriers and their energies according to the first  $f_1 + f_2$  bits,<sup>1</sup> then the remaining  $f_3$  bits are used to modulate the signals on the activated subcarriers by using an *M*-PSK constellation,  $\mathcal{M}$ . Hence, in the *b*th block, the OFDM-ICM symbol vector can be written as  $\mathbf{x}^b = [x_1^b, x_2^b, \dots, x_N^b]$  where  $x_i^b \in \{\emptyset\} \cup \mathcal{M}$ . The energy of the symbol carried by the *k*th activated subcarrier of the *b*th block is  $E_k^b = |x_k^b|^2 = \nu_k E_T / I$  where  $k = 1, 2, \dots, K$ . After obtaining symbol vectors of all blocks, the overall OFDM-ICM vector is formed as  $\mathbf{x} \coloneqq [x(1), x(2), \dots, x(N_T)]^T = [\mathbf{x}^1, \dots, \mathbf{x}^b, \dots, \mathbf{x}^B]^T \in \mathcal{C}^{N_T \times 1}$ . After this point, exactly the same operations as conventional OFDM are applied.<sup>2</sup>

**Remark.** OFDM-ICM is equivalent to OFDM-IM and OFDM when I = K and I = K = N, respectively. It is clear from (1) that the proposed scheme is capable of providing a higher SE than OFDM-IM and OFDM-CM. It is also important to note that OFDM-ICM is fundamentally different from the OFDM-IM schemes in [3]–[6] due to the use of signal levels to encode information into compositions rather than different signals to encode information into index patterns or permutations.

## B. Receiver

At the receiver, the received signal is down-converted, and the cyclic prefix is then removed from each received baseband symbol vector before processing with an FFT. After employing an  $N_T$ -point FFT operation, the frequency-domain received signal vector can be written as

$$\mathbf{y} \coloneqq [y(1), y(2), \dots, y(N_T)]^T = \mathbf{X}\mathbf{h} + \mathbf{n}$$
(2)

where  $\mathbf{X} = \text{diag}(\mathbf{x})$ . Moreover,  $\mathbf{h}$  and  $\mathbf{n}$  are  $N_T \times 1$  channel and noise vectors, respectively. Elements of  $\mathbf{n}$  follow the complex-valued Gaussian distribution  $\mathcal{CN}(0, N_0)$  where  $N_0$  is the noise variance.

 $^{2}$ We assume that the elements of x are interleaved sufficiently and the maximum spacing is achieved for the subcarriers.

 $<sup>{}^{1}</sup>f_{1} + f_{2}$  bits can be mapped to the indices of activated subcarriers and the compositions without using a look-up table implementation since the mapping of  $f_{1}$  bits to the indices of activated subcarriers and  $f_{2}$  bits to the compositions, thus to the subcarriers' energies, can be performed without a look-up table as discussed in [1] and [7], respectively.

Since the encoding procedure for each block is independent of others, decoding can be performed independently at the receiver. Hence, using ML detection, the detected symbol vector for the bth block can be written as

$$(\hat{\mathcal{I}}^b, \hat{\beta}^b, \hat{\mathbf{x}}^b) = \arg\min_{\mathcal{I}^b, \beta^b, \mathbf{x}^b} ||\mathbf{y}^b - \mathbf{X}^b \mathbf{h}^b||^2$$
(3)

where  $\mathbf{y}^{b} = [y((b-1)N+1), \dots, y(bN)]^{T}$ ,  $\mathbf{X}^{b} = \text{diag}(\mathbf{x}^{b})$ and  $\mathbf{h}^{b} = [h((b-1)N+1), \dots, h(bN)]^{T}$ .

The optimum ML detector in (3) performs  $2^{\lfloor \log_2 \binom{N}{K} \rfloor + \lfloor \log_2 \binom{I-1}{K-1} \rfloor} M^K$  squared Euclidean distance calculations. To overcome the high complexity of the optimum ML detector, we design the following low-complexity ML (LC-ML) detector for the OFDM-ICM technique:

1) Calculate the following log-likelihood ratio (LLR) for the nth subcarrier<sup>3</sup> where  $n \in \{1, 2, ..., N\}$ :

$$\delta(n) = \ln \frac{\sum\limits_{x \in \mathcal{M}} \sum\limits_{E \in \Lambda} P(x(n) = \sqrt{Ex}|y(n))}{P(x(n) = \emptyset|y(n))}$$
(4)  
 
$$\propto \ln \frac{\sum\limits_{x \in \mathcal{M}} \sum\limits_{E \in \Lambda} \exp\left(-|y(n) - h(n)\sqrt{Ex}|^2/N_0\right)}{\exp(-|y(n)|^2/N_0)}$$

where x is a unit-energy M-PSK symbol.  $P(x(n) = \sqrt{Ex}|y(n))$  stands for the probability that the symbol carried by the nth subcarrier, x(n), is equal to  $\sqrt{Ex}$ given the received signal regarding the nth subcarrier, y(n), whereas  $P(x(n) = \emptyset|y(n))$  is the probability that the nth subcarrier is deactivated given y(n). Moreover,  $\Lambda$  is the set which consists of the possible energy levels, i.e.,  $\Lambda \coloneqq \{E_T/I, 2E_T/I, \dots, (I - K + 1)E_T/I\}$ . Note that the LLR value in (4) can be approximated by [4]

$$\delta(n) \approx \frac{\exp\left(-|y(n) - h(n)\sqrt{\hat{E}(n)\hat{x}(n)}|^2/N_0\right)}{\exp(-|y(n)|^2/N_0)}$$
(5)

where  $(\hat{x}(n), \hat{E}(n)) = \arg \min_{x,E} |y(n) - h(n)\sqrt{E}x|^2$ .

- 2) Sort the LLR values  $(\delta(1), \delta(2), \dots, \delta(N))$  in descending order, i.e.,  $\delta(\rho_1) > \delta(\rho_2) > \dots > \delta(\rho_N)$  where  $\rho_n \in \{1, 2, \dots, N\}$ . Then, determine the indices of the activated subcarriers as  $\hat{\mathcal{I}} = \{\rho_1, \rho_2, \dots, \rho_K\}$ .
- Sort the channel gains of the activated subcarriers in descending order, i.e., |h(γ<sub>1</sub>)|<sup>2</sup> > |h(γ<sub>2</sub>)|<sup>2</sup> > ... > |h(γ<sub>K</sub>)|<sup>2</sup> where γ<sub>k</sub> ∈ Î and Γ := {γ<sub>1</sub>, γ<sub>2</sub>,..., γ<sub>K</sub>}.
- 4) Determine the energy levels and *M*-ary symbols on the activated *K* subcarriers by following the order in Γ and using ML detection. More explicitly, we start with the γ<sub>1</sub>th subcarrier and determine its energy level and *M*-ary symbol by (*x̂*(γ<sub>1</sub>), *Ê*(γ<sub>1</sub>)) = arg min<sub>x∈M,E∈Λ</sub> |*y*(γ<sub>1</sub>) *h*(γ<sub>1</sub>)√*Ex*|<sup>2</sup>. The related composition can be obtained by *v̂*(γ<sub>1</sub>) = *Ê*(γ<sub>1</sub>)*I*/*E<sub>T</sub>*. Then, we update Λ according to previously estimated energy levels and the fact that *v̂*(γ<sub>k</sub>) ≥ 1 as Λ := {*E<sub>T</sub>*/*I*,...,(*I* (*v̂*(γ<sub>1</sub>) + *K* 2))*E<sub>T</sub>*/*I*} and we proceed with the γ<sub>2</sub>th subcarrier and determine its energy level and *M*-ary symbol. For the γ<sub>k</sub>th, 1 < *k* < *K*, and γ<sub>K</sub>th subcarriers, the sets in question can be written

respectively as  $\Lambda \coloneqq \{E_T/I, \dots, (I - (K - k + \Delta_k))E_T/I\}$ and  $\Lambda \coloneqq \{(I - \Delta_K)E_T/I\}$  where  $\Delta_k = \sum_{l=1}^k \hat{\nu}(\gamma_l)$  and  $\Delta_K = \sum_{l=1}^K \hat{\nu}(\gamma_l)$ .

For the proposed detector, one needs to perform (I - K + 1)M squared Euclidean distance calculations in (4). Hence, the proposed algorithm makes N(I - K + 1)M squared Euclidean distance calculations in total, which is considerably smaller than the complexity of the optimum ML detector.

#### **IV. PERFORMANCE ANALYSIS**

## A. Bit-Error Rate

An upper-bound on the average BER is given by the wellknown union bound as follows

$$P_b \le \frac{1}{f2^f} \sum_{i=1}^{2^f} \sum_{j=1}^{2^f} P(\mathbf{X}^i \to \mathbf{X}^j) D(\mathbf{X}^i \to \mathbf{X}^j)$$
(6)

where  $P(\mathbf{X}^i \to \mathbf{X}^j)$  is the pairwise error probability (PEP) regarding the erroneous detection of  $\mathbf{X}^i$  as  $\mathbf{X}^j$  where  $i \neq j$ ,  $i, j \in \{1, ..., L\}$ ,  $\mathbf{X}^i = \text{diag}(\mathbf{x}^i)$ ,  $\mathbf{X}^j = \text{diag}(\mathbf{x}^j)$ , and  $D(\mathbf{X}^i \to \mathbf{X}^j)$  is the number of bits in error for the corresponding pairwise error event. Here, L is the codebook size for the proposed scheme. One can use the same PEP expression as in [1] and substitute the codewords of the proposed scheme to obtain the upper bound on the average BER.

**Remark.** The minimum Hamming distance between the sets that keep the subcarriers' energies is two, just like the index symbols of OFDM-IM. However, the proposed scheme can send conventional modulation symbols together with embedding information into the indices and/or compositions, and the minimum Hamming distance between the conventional modulation symbols is limited to one. This limits the diversity gain of the proposed scheme to one.

As a special case of OFDM-ICM, one can use only the indices and compositions to embed information and obtain a diversity gain.<sup>4</sup> However, in this case, the transmitted symbol vectors should be carefully chosen to get optimum error performance. One simple yet efficient strategy is choosing the transmitted symbols in a way that the angular difference between the different energy symbols is maximized. Hence, in this case, one can choose the signal on the *k*th activated subcarrier of the *b*th block as  $x_k^b = \sqrt{\nu_k E_T/I} \exp(\frac{2j\nu_k \pi}{I-K+1})$ . As will be shown in the Numerical Results section, the error performance of this special case is promising for low SEs.

## B. Spectral Efficiency

By rewriting K and I as  $K = \alpha N$  and  $I = \beta N$ , respectively, the SE maximization problem is formulated as

$$\begin{array}{ll} \underset{\alpha}{\text{maximize}} & \eta = \frac{\log_2 \binom{N}{\alpha N} + \log_2 \binom{\beta N - 1}{\alpha N - 1} + \alpha N \log_2 M}{N} \\ \text{subject to} & 0 < \alpha \le 1, \\ & \alpha \le \beta. \end{array}$$

$$(7)$$

<sup>4</sup>For this special case, the SE is  $\eta = \frac{\lfloor \log_2 {\binom{N}{K}} \rfloor + \lfloor \log_2 {\binom{I-1}{K-1}} \rfloor}{N}$ 

<sup>&</sup>lt;sup>3</sup>Here, we drop the block superscript, b, for convenience since the proposed LC-ML detector can be operated for each block independently.

**Proposition 1.** The value of  $\alpha$  that maximizes (10) is

$$\alpha^* = \frac{M(\beta+1) - \sqrt{M^2(\beta-1)^2 + 4M\beta}}{2(M-1)}.$$
 (8)

Thus, one can pick the optimum number of activated subcarriers in an OFDM block as

$$K^* \in \left\{ \lfloor \alpha^* N \rfloor, \lceil \alpha^* N \rceil \right\}$$
(9)

where  $\lfloor . \rfloor$  and  $\lceil . \rceil$  are floor and ceiling operations, respectively.

*Proof.* For  $0 < \alpha_1 < \alpha_2 \le 1$ , one can write

$$\frac{\Delta\eta}{\Delta\alpha} = \frac{\log_2\binom{N}{\alpha_2 N} + \log_2\binom{\beta N-1}{\alpha_2 N-1} + \alpha_2 N \log_2 M}{N(\alpha_2 - \alpha_1)} - \frac{\log_2\binom{N}{\alpha_1 N} + \log_2\binom{\beta N-1}{\alpha_1 N-1} + \alpha_1 N \log_2 M}{N(\alpha_2 - \alpha_1)}.$$
(10)

Since  $K \in \mathbb{Z}^+$ , the minimum value of  $\Delta \alpha = \alpha_2 - \alpha_2$  is  $\Delta \alpha = \frac{1}{N}$ . By substituting  $\alpha_2 = \alpha_1 + \frac{1}{N}$  into (10), the optimal value of  $\alpha$  can be obtained when  $\frac{\Delta \eta}{\Delta \alpha} = 0$ . By arranging this equation, one can obtain classical quadratic program and this classical program yields the solution in (8).<sup>5</sup>

**Proposition 2.** As  $N \to \infty$ , the asymptotic SE of OFDM-IM can be given as

$$\eta \sim H(\alpha) + \log_2(\beta^\beta / \alpha^\alpha) - (\beta - \alpha) \log_2(\beta - \alpha) + \alpha \log_2 M$$
(11)

where  $H(\alpha)$  is the entropy function of  $\alpha$ .

*Proof.* The proof is simply obtained by using the Stirling's formula<sup>6</sup> in the SE and ignoring the expressions that approach zero as  $N \to \infty$ .

### V. NUMERICAL RESULTS

In this section, we provide numerical BER and SE results. In BER figures, "OFDM-ICM (N, K, I, M)" is the proposed OFDM-ICM scheme having K activated subcarriers out of N in each block, choosing the energies of the activated subcarriers according to the compositions of I into K parts and employing M-PSK modulation on the activated subcarriers, whereas "OFDM-ICM (N, K, I)" stands for the special case of OFDM-ICM that embeds information into only indices of subcarriers and compositions of an integer I with K parts. "OFDM-IM (N, K, M)" signifies the conventional OFDM-IM scheme in which K out of N subcarriers are activated to send M-PSK modulated symbols in each block. Finally, "OFDM-WCM  $(N, I, \lambda)$ , Alg. 1" stands for the OFDM-WCM scheme that employs Algorithm 1 in [7] and uses  $\lambda$  to alter the SE as well as integer compositions of I with N parts to decide the energies of N subcarriers, whereas "OFDM-CM (N, I, M)" is the OFDM-CM scheme that determines the energies of Nsubcarriers according to the composition of an integer I with N parts and carries M-ary PSK symbols on each subcarrier. For the simulated schemes in this section, we pick  $E_T = N$ , thus the average energy per subcarrier is assumed to be one.



Fig. 1: BER comparison of OFDM-ICM with OFDM-WCM, OFDM-CM, OFDM-IM, and OFDM when  $N = 4, K \in \{2,3\}, I \in \{4,5,8\}, M \in \{2,4\}$  and  $\lambda = 1$ .



Fig. 2: BER comparison of OFDM-ICM with OFDM-IM, OFDM-WCM, OFDM-CM, and OFDM when  $N = 4, K = 3, I \in \{4, 7, 8\}, M \in \{4, 8\}$ , and  $\lambda = 2$ .

In Fig. 1, we compare the BER performance of OFDM-ICM with that of OFDM-WCM, OFDM-CM, OFDM-IM, and OFDM. Here, except for OFDM (BPSK), all schemes achieve the same SE, which is 1.5 bits per subcarrier (bps). As seen from the figure, OFDM-ICM (4,3,8) considerably outperforms all other schemes by introducing an additional diversity gain. Moreover, the BER performance of OFDM-ICM (4, 3, 4, 2) is very close to that of OFDM-WCM (4, 3, 1), Alg. 1 and these schemes outperform the OFDM-CM, OFDM-IM, and OFDM schemes, especially at high signal-to-noise ratio (SNR), since they produce more symbols whose minimum Hamming distance is two. However, since the minimum Euclidean distance for the overall codebook of OFDM-ICM and OFDM-WCM is less than that of OFDM-IM, they are outperformed by OFDM-IM at low SNR. Due to the same reason, OFDM-ICM (4, 2, 5, 2) is outperformed by all other schemes except for OFDM-CM (4, 5, 2) at low SNR; however, it outperforms all other schemes except for OFDM-ICM (4,3,8) at high SNR.

Fig. 2 compares the BER performance of OFDM-ICM with

 $<sup>^{5}</sup>$ Note that the quadratic program produces two solutions. However, only the solution in (8) satisfies the constraints in (7).

<sup>&</sup>lt;sup>6</sup>  $N! \sim \sqrt{2\pi N} (N/e)^N$  when  $N \to \infty$ .



Fig. 3: BER comparison of the proposed LC-ML detector with the optimum ML detector for OFDM-ICM.

that of OFDM-IM, OFDM-WCM, OFDM-CM, and OFDM.<sup>7</sup> As observed from the figure, OFDM-ICM (4,3,8,4) and OFDM-CM (4,7,4) exhibit very close BER performance and they outperform all other schemes, especially at high SNR. Compared to Fig. 1, the effectiveness of the OFDM-WCM scheme against the other schemes substantially degrades as it is required to employ higher order modulation in this scheme to achieve 3 bps. However, the OFDM-ICM scheme preserves its effectiveness in both data rates. It is also worth mentioning that OFDM-ICM (4,3,8,4) considerably outperforms OFDM (8-QAM) and OFDM (8-APSK) that use the energies of the constellation symbols for encoding like the proposed scheme.

Fig. 3 demonstrates the BER performance of the proposed LC-ML detector of the OFDM-ICM scheme compared to the optimum ML detector when N = 4,  $K \in \{2,3\}$ ,  $I \in \{5,8\}$ , and  $M \in \{2,4\}$ . As seen from the figure, the performance of the proposed LC-ML detector is very close to that of the optimum ML detector, especially at low and high SNR. On the other hand, curves with extensions "Theory" in the legend relate to the theoretical upper-bound results for the proposed scheme. As observed from the figure, upper-bound curves are consistent with computer simulations, especially at high SNR.

In Fig. 4, we compare theoretical, numerical and asymptotic SE results for OFDM-ICM when  $\beta = 0.5$  and  $M \in \{4, 8, 16\}$ .<sup>8</sup> Fig. 4 verifies the validity of the SE expressions in Section IV-B since the theoretical and numerical SE results match perfectly and they approach asymptotic SE as N increases. On the other hand, the SE is considerably improved when we increase N. When N = 4, the OFDM-ICM schemes can achieve only 70.14%, 77.60%, and 82.20% of the asymptotic SE for M = 2, 4, and 8, respectively, whereas



Fig. 4: Comparison of theoretical, numerical and asymptotic SE results for OFDM-ICM when  $\beta = 0.5$  and  $M \in \{4, 8, 16\}$ .

these percentages increase to 81.82%, 87.30%, and 90.41% for M = 2, 4, and 8, respectively, when N = 8.

# VI. CONCLUSION

In this paper, we proposed a novel modulation concept, which we call ICM. We depicted a practical OFDM-ICM scheme that is capable of improving the SE of OFDM-IM by embedding data into the compositions of an integer and using energy levels to identify them. We demonstrated through simulations and theoretical calculations that the proposed scheme can provide noteworthy improvements compared to OFDM, OFDM-IM, OFDM-WCM, and OFDM-CM.

As future work, the proposed scheme could be generalized by utilizing in-phase and quadrature dimensions of the modulation symbols as in [2] and space time coding with coordinate interleaving could be combined with the proposed scheme as in [11] to obtain additional diversity gain.

## REFERENCES

- E. Basar, U. Aygolu, E. Panayirci, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Signal Process.*, vol. 61, no. 22, pp. 5536–5549, Nov. 2013.
- [2] R. Fan, Y. J. Yu, and Y. L. Guan, "Generalization of orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5350–5359, Oct. 2015.
- [3] T. Mao, Z. Wang, Q. Wang, S. Chen, and L. Hanzo, "Dual-mode index modulation aided OFDM," *IEEE Access*, vol. 5, pp. 50–60, Feb. 2017.
- [4] M. Wen, E. Basar, Q. Li, B. Zheng, and M. Zhang, "Multiple-mode orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Commun*, vol. 65, no. 9, pp. 3892–3906, Sep. 2017.
- [5] F. Yarkin and J. P. Coon, "Set partition modulation," *IEEE Trans. Wireless Commun.*, pp. 1–1, 2020.
- [6] —, "Q-ary multi-mode OFDM with index modulation," *IEEE Wireless Commun. Lett.*, vol. 9, no. 7, pp. 1110–1114, 2020.
- [7] —, "Composition modulation," in accepted for presentation in IEEE ICCT 2020, Oct. 2020, pp. 1–5. [Online]. Available: https://arxiv.org/pdf/2006.14400.pdf
- [8] W. Saad, M. Bennis, and M. Chen, "A vision of 6g wireless systems: Applications, trends, technologies, and open research problems," *IEEE Network*, pp. 1–9, 2019.
- [9] R. Lucky and J. Hancock, "On the optimum performance of N-ary systems having two degrees of freedom," *IRE Trans. Commun. Sys.*, vol. 10, no. 2, pp. 185–192, 1962.
- [10] C. Thomas, M. Weidner, and S. Durrani, "Digital amplitude-phase keying with M-ary alphabets," *IEEE Trans. Commun.*, vol. 22, no. 2, pp. 168–180, 1974.

<sup>&</sup>lt;sup>7</sup>For the curve related to OFDM (8-APSK), we use the optimum 8-ary amplitude and phase shift keying (APSK) design in [9], [10]. In this design, constellation consists of two circles (inner and outer) with four signals on each of them. The radii of the inner and outer circles are  $r_1 = 1/\sqrt{2} \simeq 0.707$  and  $r_2 = 1.366$ , respectively.

<sup>&</sup>lt;sup>8</sup>Here, curves with extensions "Theory" in the legend are obtained by substituting (9) into (1), whereas those with with extensions "Numerical" in the legend are obtained by substituting all possible values of K into (1) and choosing the maximum SE. On the other hand, curves with extensions "Asymptotic" in the legend are obtained by substituting (8) into (11).

[11] E. Basar, "OFDM with index modulation using coordinate interleaving," *IEEE Wireless Commun. Lett.*, vol. 4, no. 4, pp. 381–384, Aug. 2015.