Design of THz-NOMA in the Presence of Beam Misalignment

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***Abstract*—In this letter, the application of non-orthogonal multiple access (NOMA) to Terahertz (THz) transmission is proposed in order to mitigate the harmful effect of beam alignment errors. In particular, two types of cognitive radio inspired NOMA (CR-NOMA) schemes are developed to realize different tradeoffs between system performance and user fairness. Both analytical and simulation results are presented to demonstrate the superior performance gain of THz-NOMA over conventional orthogonal multiple access based THz transmission.**

***Index Terms*—Terahertz (THz) communications, non-orthogonal multiple access (NOMA), beam misalignment, outage probability.**

I. Introduction

BECAUSE of the severe congestion in the sub-6 GHz bands, using the Terahertz (THz) band has been envisioned as a promising solution to support emerging applications requiring super-fast broadband speeds and ultra-low latencies, such as immersive virtual reality (VR) and augmented reality (AR) [1], [2]. However, THz transmission suffers inevitable beam misalignment errors, which are due to the fact that THz transceivers are expected to be equipped with highly directional antenna arrays and the narrow beams generated by these arrays make beam alignment and tracking very challenging [3], [4]. As shown in [5], the presence of beam misalignment can significantly reduce the performance of THz communications.

This letter focuses on a THz network with one primary user and multiple secondary users. On the one hand, the primary user is assumed to be a mobile user and hence suffer from potential beam misalignment. On the other hand, the secondary users are assumed to be static users, e.g., stationary Internet of Things (IoT) devices, which means that it is unlikely for these secondary users to suffer from beam misalignment. Without using non-orthogonal multiple access (NOMA), the bandwidth resources allocated to the primary user cannot be efficiently utilized due to the beam misalignment effect, which can severely degrade the overall system throughput. The aim of this letter is to show how the overall system throughput of THz systems can be improved in the presence of this detrimental misalignment effect by using NOMA, which has not been investigated in existing THz-NOMA works [6], [7]. In particular, the concept of cognitive radio inspired NOMA (CR-NOMA) is used to ensure cooperative use of the spectrum between the primary and secondary users [8]. Analytical results are developed to show that the use of conventional CR-NOMA can yield a moderate gain in system throughput over orthogonal multiple access (OMA) based THz transmission; however, the obtained performance gain is greatly dependent on the primary user's channel condition. To further improve the system performance, a modified CR-NOMA scheme is developed to ensure that the use of NOMA can yield a significant performance gain over OMA-THz, while still guaranteeing the primary user's quality of service (QoS) requirements.

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II. System Model

Consider a THz multi-user network, where one base station employs a directional antenna array and communicates with a primary user, denoted by U_0, which is assumed to be a mobile user and hence suffer from potential beam misalignment due to its movement. In particular, the sectored antenna model is used [5], i.e., within its 3 dB beam-width, the main-lobe beam formed by the base station covers a wedge-shaped sector, denoted by $\mathcal{D}_\theta$, with its sector angle denoted by θ and its radius denoted by $\mathcal{R}$, where the antenna gain for a user located in the main-lobe beam is denoted by G.

This letter considers the use of NOMA to connect additional secondary users in $\mathcal{D}_\theta$ while guaranteeing U_0's QoS requirements. In particular, we assume that these secondary users are randomly deployed following a homogeneous Poisson point process (HPPP) with density λ [9]. Denote the secondary users located in $\mathcal{D}_\theta$ by U_k, $1 \leq k \leq K$, where K is the number of secondary users in $\mathcal{D}_\theta$.

A. THz Channel Model

The k^{th} user receives the following signal [3]–[5]:

$$y_k = \sqrt{h_k^{\mathrm{PL}} G h_k^{\mathrm{ML}}} g_k x + n_k, \quad 0 \leq k \leq K, \tag{1}$$

where x denotes the signal sent by the base station, h_k^{PL} denotes the effects of both path loss and molecular absorption, i.e., $h_k^{\mathrm{PL}} = \left(\frac{c}{4\pi f_c}\right)^2 \frac{e^{-\zeta r_k}}{r_k^{\alpha}+1}$, r_k denotes the distance between the base station and U_k, c denotes the speed of light, f_c is the carrier frequency, α denotes the path loss exponent, ζ denotes the molecular absorption coefficient, g_k denotes the small scale Rayleigh fading, n_k denotes additive white Gaussian noise, and h_k^{ML} denotes the beam misalignment error. The following truncated Gaussian model is used in this letter to model the beam misalignment error [5]:

$$h_0^{\mathrm{ML}} = \begin{cases} 1 & \text{with probability} \quad (1-p_e) \\ \xi & \text{with probability} \quad p_e, \end{cases} \tag{2}$$

where ξ denotes the sidelobe to mainlobe ratio, p_e denotes the misalignment probability

$$p_e = 1 - \left(1 - 2Q\left(\frac{\theta}{\sqrt{2\sigma^2}}\right)\right)\left(1 - 2Q\left(\frac{\pi}{\sqrt{2\sigma^2}}\right)\right)^{-1},$$

$Q(\cdot)$ denotes the Q-function, and σ denotes the corresponding error variance. It is assumed that the secondary users do not suffer from this misalignment error, i.e, $h_k^{\mathrm{ML}} = 1$, for $1 \leq k \leq K$. This assumption can be justified if the secondary users are static users, e.g., stationary IoT sensors and devices. In addition to the aforementioned propagation effects, THz transmission also suffers from blockage, whose probability can be modelled as follows: $p_b(r_k) = e^{-\phi r_k}$, $1 \leq k \leq K$, where ϕ denotes the effective blockage density [5]. In this letter, it is assumed that the primary user U_0 is free from blockage.

B. Application of NOMA to THz Transmission

To reduce the system complexity, assume that a single secondary user is scheduled, where the scheduling criterion will be described later. Provided that U_k is scheduled, following the principle of NOMA transmission, x can be expressed as follows: $\sqrt{\alpha}_0 s_0 + \sqrt{\alpha_s} s_k$, where s_i, $0 \leq i \leq K$, denotes the signal for U_i, α_i denotes the power allocation coefficient, $i \in \{0, s\}$, and $\alpha_0 + \alpha_s = 1$ [8]. The details for the choices of α_0 and α_s will be provided in the next section.

It is assumed that all users decode U_0's signal first, which ensures that admitting additional users does not cause any change to U_0's detection strategy. Denote by $\mathcal{S}$ the set including the secondary users that can decode U_0's signal:

$$\mathcal{S} = \left\{k : \log\left(1 + \frac{\rho|h_k|^2\alpha_0}{\rho|h_k|^2\alpha_s + 1}\right) \geq R_0\right\}, \tag{3}$$

where R_0 denotes U_0's target data rate, ρ denotes the transmit signal-to-noise ratio (SNR), $|h_i|^2 = h_i^{\mathrm{PL}} G h_i^{\mathrm{ML}} |g_i|^2$, $0 \leq i \leq K$, and the noise power is assumed to be normalized. For any user in $\mathcal{S}$, the following data rate, $\log(1 + |h_k|^2\alpha_s)$, $k \in \mathcal{S}$, is achievable, and therefore, the following criterion is used to schedule the best secondary user:

$$k^* = \arg\max_{k\in\mathcal{S}} \log(1 + \rho|h_k|^2\alpha_s). \tag{4}$$

III. Performance Analysis

Depending on the choice of α_s, the performance of THz-NOMA can be analyzed differently as follows.

A. Conventional Cognitive Radio Inspired NOMA

The use of conventional CR-NOMA can strictly ensure that serving additional secondary users does not cause any performance degradation to the primary user, i.e, $\log\left(1 + \frac{\rho|h_0|^2\alpha_0}{\rho|h_0|^2\alpha_s+1}\right) \geq R_0$, which leads to the following choice for α_s [8]:

$$\alpha_s = \max\left\{0, \frac{\rho|h_0|^2 - \epsilon_0}{\rho(1+\epsilon_0)|h_0|^2}\right\}, \tag{5}$$

where $\epsilon_0 = 2^{R_0} - 1$.

Because the choice of α_s in (5) guarantees that the primary user experiences the same as in OMA-THz, only the secondary users' performance is focused on in the following. In particular, the outage probability is used as the metric for performance analysis, and the outage probability achieved by THz-NOMA is shown in the following lemma.

Lemma 1: The outage probability achieved by CR-NOMA with α_s in (5) *is given by*

$$\begin{aligned} P^o = {} & e^{-\mu_s(\mathcal{D})} + \sum_{i=1}^{\infty} \frac{(\mu_s(\mathcal{D}))^i}{i!} e^{-\mu_s(\mathcal{D})} \left[F_{|h_0|^2}\left(\frac{\epsilon_0}{\rho}\right)\right. \\ & + \eta_2^i \theta^i \int_{\frac{\epsilon_0}{\rho}}^{\eta_4} g_y^i\left(\frac{\epsilon_s(1+\epsilon_0)y}{\rho y - \epsilon_0}\right) f_{|h_0|^2}(y)dy \\ & \left. + \eta_2^i \theta^i \int_{\eta_4}^{\infty} g_y^i(y) f_{|h_0|^2}(y)dy\right], \end{aligned} \tag{6}$$

where R_s denotes the secondary user's targeted data rate, $\epsilon_s = 2^{R_s} - 1$, $\eta_1 = \left(\frac{c}{4\pi f_c}\right)^2$ and $\eta_2 = \frac{\phi^2}{\theta\gamma(2,\mathcal{R}\phi)}$, $\eta_4 = \frac{\epsilon_0 + \epsilon_s 2^{R_0}}{\rho}$, $f_{|h_0|^2}(y) = \frac{p_e}{h_0^{\mathrm{PL}} G\xi} e^{-\frac{y}{h_0^{\mathrm{PL}} G\xi}} + \frac{1-p_e}{h_0^{\mathrm{PL}} G} e^{-\frac{y}{h_0^{\mathrm{PL}} G}}$, $F_{|h_0|^2}(y) = p_e\left(1 - e^{-\frac{y}{h_0^{\mathrm{PL}} G\xi}}\right) + (1 - p_e)\left(1 - e^{-\frac{y}{h_0^{\mathrm{PL}} G}}\right)$, $\mu_s(\mathcal{D}) = \theta\lambda\phi^{-2}\gamma(2, \mathcal{R}\phi)$, $\gamma(\cdot,\cdot)$ denotes the lower incomplete gamma function, and $g_y(y) = \int_0^{\mathcal{R}}\left[1 - e^{-y\frac{(r^\alpha+1)e^{\zeta r}}{G\eta_1}}\right] e^{-\phi r} r dr$.

Remark 1: By using Lemma 1, it is straightforward to show that error floors exist for both the overall outage probability, P^o, and the outage probability conditioned on a fixed K, $P_{O|K}$ defined in (11). This is because the choice of the power coefficient in (5) leads to the drawback that the secondary user's performance depends significantly on the primary user's channel condition. In particular, if $|h_0|^2 \to 0$, i.e., U_0's channel is too weak, $\alpha_s \to 0$, i.e., the secondary user is in outage. If U_0's channel is very strong, i.e., $|h_0|^2 \to \infty$, the event that $|h_{k^*}|^2 \leq |h_0|^2$ becomes likely, which causes an error floor at high SNR for $P_{O|K}$, as shown in (17).

B. Modified Cognitive Radio Inspired NOMA

To avoid the limitation discussed in Remark 1, in this section, the following modified choice for α_s is used. In particular, α_s is designed to ensure that the outage probability experienced by the primary user is capped, i.e.,

$$\underbrace{F_{|h_0|^2}\left(\frac{\epsilon_0}{\rho(1-(1+\epsilon_0)\alpha_s)}\right)}_{\mathrm{U}_0\text{'s outage probability in NOMA}} \leq \tau \times \underbrace{F_{|h_0|^2}\left(\frac{\epsilon}{\rho}\right)}_{\mathrm{U}_0\text{'s outage probability in OMA}}, \tag{7}$$

where τ denotes the tolerable performance degradation coefficient. Therefore, designing the modified CR-NOMA scheme is to maximize the secondary user's data rate, i.e., α_s, while statistically guaranteeing the primary user's QoS requirement, as formulated in the following optimization problem:

$$\max_{\alpha_s \geq 0} \; \alpha_s \quad \text{s.t. } 1-(1+\epsilon_0)\alpha_{\rm s} \geq 0 \tag{P1a}$$

$$F_{|h_0|^2}\left(\frac{\epsilon_0}{\rho\left(1-(1+\epsilon_0)\alpha_s\right)}\right) \leq \tau F_{|h_0|^2}\left(\frac{\epsilon}{\rho}\right). \tag{P1b}$$

It is straightforward to verify that $F_{|h_0|^2}(y)$ is a monotonically increasing function of y, and hence the optimal solution of problem P1, denoted by α_s^*, can be found by using a bisection search between 0 and $\frac{1}{1+\epsilon_0}$.

Because α_s^* is not a function of $|h_0|^2$, by using the steps in the proof for Lemma 1, the outage probability achieved by the modified CR-NOMA scheme when there are K secondary users in $\mathcal{D}_\theta$ can be obtained straightforwardly as follows:

$$P_{O|K} = \eta_2^K \theta^K \left(\int_0^{\mathcal{R}} \left[1-e^{-\frac{\tilde{\epsilon}^*(1+r^\alpha)}{G\eta_1}e^{\zeta r}}\right] e^{-\phi r} r dr\right)^K,$$

where $\tilde{\epsilon}^* = \max\left\{\frac{\epsilon_0}{\rho\left(1-2^{R_0}\alpha_s^*\right)}, \frac{\epsilon_s}{\rho\alpha_s^*}\right\}$.

At high SNR, the constraint in (P1b) can be approximated as follows:

$$\frac{\left(\frac{p_e}{h_0^{\rm PL}G\xi}+\frac{(1-p_e)}{h_0^{\rm PL}G}\right)\epsilon_0}{\rho\left(1-(1+\epsilon_0)\alpha_s\right)} \leq \tau\left(\frac{p_e}{h_0^{\rm PL}G\xi}+\frac{(1-p_e)}{h_0^{\rm PL}G}\right)\frac{\epsilon_0}{\rho}, \tag{8}$$

which means that the high-SNR approximation of α_s^* can be expressed as follows:

$$\hat{\alpha}_s^* = (\tau-1)\tau^{-1}(1+\epsilon_0)^{-1}. \tag{9}$$

By using $\hat{\alpha}_s^*$, applying the approximation $e^x \approx 1-x$ for $x \rightarrow 0$, and with some algebraic manipulations, $P_{O|K}$ can be approximated at high SNR as follows:

$$\begin{aligned} P_{O|K} &\approx \eta_2^K\theta^K\left(\int_0^{\mathcal{R}} \frac{\tilde{\epsilon}^*\left(1+r^\alpha\right)}{G\eta_1} e^{\zeta r} e^{-\phi r} r dr\right)^K \\ &= \frac{\eta_2^K \theta^K (\tilde{\epsilon}^*)^K}{G^K\eta_1^K(\zeta-\phi)^{2K}}\Big(1+e^{(\zeta-\phi)\mathcal{R}}[(\zeta-\phi)\mathcal{R}-1] \\ &\quad + (\phi-\zeta)^{-\alpha}\left[\zeta(\alpha+2)-\Gamma\left(\alpha+2,(\phi-\zeta)\mathcal{R}\right)\right]\Big)^K, \end{aligned} \tag{10}$$

where $\Gamma(\cdot,\cdot)$ denotes the upper incomplete gamma function. By using the fact that $\tilde{\epsilon}^*$ is inversely proportional to ρ, the following corollary can be established straightforwardly.

Corollary 1: When there are K secondary users in $\mathcal{D}_\theta$, the modified CR-NOMA scheme can realize a diversity gain of K.

Remark 2: Corollary 1 clearly shows the benefit of using the modified CR-NOMA scheme, since not only can outage error floors be removed but also a diversity gain of K can be realized. But it is important to point out that the original CR-NOMA scheme can ensure that the primary user's QoS requirement is met instantaneously.

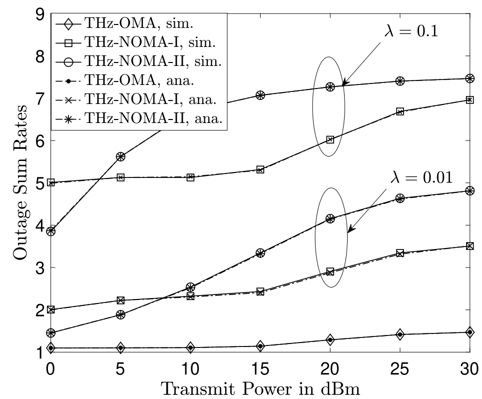
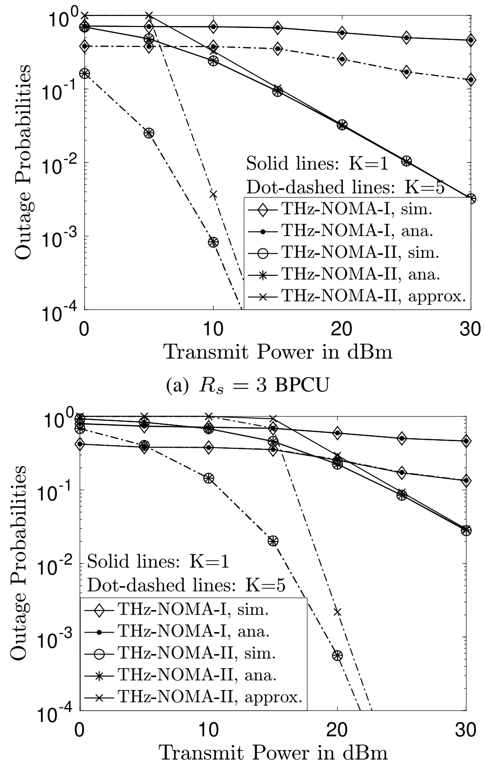


Fig. 1. Performance comparison of the considered transmission schemes. $R_0 = 1.5$ bits per channel use (BPCU), $R_s = 6$ BPCU, $\hat{\alpha}_s^*$ in (9) is used for THz-NOMA-II.



(a) $R_s = 3$ BPCU

(b) $R_s = 6$ BPCU

Fig. 2. The outage performance of the two THz-NOMA schemes with fixed K, where $\hat{\alpha}_s^*$ in (9) is used for THz-NOMA-II.

IV. Simulation Results

In this section, computer simulation results will be provided to demonstrate the performance of THz-NOMA. As in [5], $f_c = 300$ GHz, $\alpha = 2$, $\zeta = 5e^{-3}$, $G = 35.35$ dB, $\mathcal{R} = 10$ m, $\theta = 3.17$ degrees, $d_0 = 5$ m, $\phi = 0.1$, $\tau = 1.1$, $\xi = 10^{-4}$, and the noise power is set as -90 dBm.

In Fig. 1, the performance of the considered transmission schemes is studied by using the outage sum rate as the metric [10]. Note that in the figure, THz-NOMA-I refers to the CR-NOMA scheme with α_s in (5) and THz-NOMA-II refers to the modified CR-NOMA scheme discussed in Section III-B. As can be seen from the figure, the use of NOMA in THz networks yields a

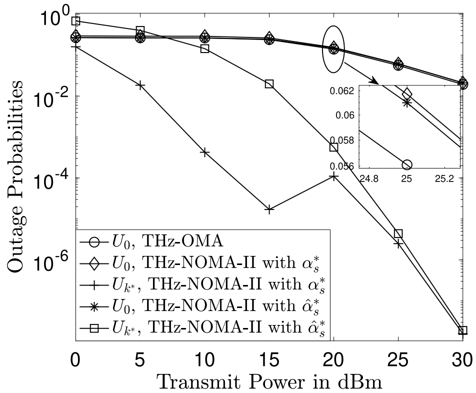


Fig. 3. The impact of different choices of α_s on THz-NOMA-II.

significant performance gain over THz-OMA, particularly in dense deployments. The use of the modified CR-NOMA scheme can further improve the throughput of THz networks, compared to conventional CR-NOMA. In addition, Fig. 1 also verifies the accuracy of the developed analytical results.

Fig. 2 shows the outage probabilities realized by the two THz-NOMA schemes by fixing the number of secondary users. Fig. 2 demonstrates that the robustness of the two NOMA schemes is improved if there are more secondary users available in $\mathcal{D}_\theta$. However, error floors exist for the outage probabilities realized by conventional CR-NOMA, as discussed in Remark 1, whereas for the modified CR-NOMA scheme, the slope of the outage probability curve can be increased by increasing K, which confirms Corollary 1.

In Fig. 3, the impact of different choices of α_s on the performance of modified CR-NOMA is studied, where α_s^* is obtained from problem P1 and $\hat{\alpha}_s^*$ is shown in (9). In particular, Fig. 3 demonstrates that the use of both choices of α_s does not bring significant performance degradation to U_0, compared to THz-OMA. The reason to have zigzag outage curves for the case with α_s^* is that α_s^* is a function of the transmit power, e.g., $\alpha_s^* = 0.2347$ for the transmit power of 15 dBm, but $\alpha_s^* = 0.0485$ for the transmit power of 20 dBm. $\hat{\alpha}_s^*$ is not a function of the transmit power, which is the reason why U_{k^*}'s outage curves are smooth for the case with $\hat{\alpha}_s^*$.

V. Conclusion

In this letter, a new THz-NOMA transmission scheme has been proposed in order to mitigate the harmful effect of beam alignment errors. In particular, two types of CR-NOMA schemes have been developed to realize different tradeoffs between system performance and user fairness.

Appendix A
Proof for Lemma 1

The outage probability can be expressed as follows:

$$P^o = P(K=0) + \sum_{i=1}^{\infty} P(K=i) P_{O|K}, \tag{11}$$

where $P(K=i)$ denotes the probability of having $K=i$ secondary users in $\mathcal{D}_\theta$ and $P_{O|K}$ denotes the corresponding conditional outage probability.

A. Evaluating $P(K=i)$

Note that the blockage effect thins the original HPPP process to a new PPP with intensity $\lambda_s(x) = \lambda e^{-\phi r_k}$, where x denotes the location of a secondary user free from blockage [10]. Therefore, the mean measure of this thinning process in $\mathcal{D}$ is given by

$$\mu_s(\mathcal{D}) \triangleq \int_{\mathcal{D}} \lambda_s(x) dx = \theta \lambda \phi^{-2} \gamma(2, \mathcal{R}\phi), \tag{12}$$

which means that the probability of having K secondary users free from blockage is given by

$$P(K=i) = \frac{(\mu_s(\mathcal{D}))^i}{i!} e^{-\mu_s(\mathcal{D})}. \tag{13}$$

B. Evaluating $P_{O|K}$

Following steps similar to those in [8] and by using the assumption that the K users' channel gains are independent and identically distributed, $P_{O|K}$ can be expressed as follows:

$$P_{O|K} = P\left(|h_{k^*}|^2 \leq \tilde{\epsilon}\right) = \left[P\left(|h_k|^2 \leq \tilde{\epsilon}\right)\right]^K, \tag{14}$$

where $\tilde{\epsilon} = \max\left\{\frac{\epsilon_0}{\rho(\alpha_0 - \epsilon_0\alpha_s)}, \frac{\epsilon_s}{\rho\alpha_s}\right\}$.

In (14), it is assumed that $\alpha_0 - \epsilon_0\alpha_s \geq 0$. It is important to point out that the use of the CR-NOMA power coefficients in (5) can guarantee that $\alpha_0 - \epsilon_0\alpha_s \geq 0$. In particular, if $\log(1+\rho|h_0|^2) \geq R_0$, $\alpha_0 - \epsilon_0\alpha_s$ can be bounded as follows:

$$\alpha_0 - \epsilon_0\alpha_s = 1 - \frac{\rho|h_0|^2 - \epsilon_0}{\rho|h_0|^2} = \frac{\epsilon_0}{\rho|h_0|^2} > 0. \tag{15}$$

For the case of $\epsilon_s = 0$, $\alpha_0 - \epsilon_0\alpha_s = 1$ which is also positive.

Further note that $\tilde{\epsilon}$ can be expressed differently depending on whether the condition $\frac{\epsilon_0}{\rho(\alpha_0-\epsilon_0\alpha_s)} \geq \frac{\epsilon_s}{\rho\alpha_s}$ holds. Recall that the condition $\frac{\epsilon_0}{\rho(\alpha_0-\epsilon_0\alpha_s)} \geq \frac{\epsilon_s}{\rho\alpha_s}$ is equivalent to $\alpha_s \geq \frac{\epsilon_s}{\epsilon_0 + 2^{R_0}\epsilon_s}$, which means that $\tilde{\epsilon}$ can be expressed as follows:

$$\tilde{\epsilon} = \begin{cases} \infty, & \text{if } |h_0|^2 < \frac{\epsilon_0}{\rho} \\ \frac{\epsilon_s(1+\epsilon_0)|h_0|^2}{\rho|h_0|^2 - \epsilon_0}, & \text{if } \frac{\epsilon_0}{\rho} \leq |h_0|^2 \leq \eta_4 \\ |h_0|^2, & \text{if } |h_0|^2 \geq \eta_4, \end{cases} \tag{16}$$

which further means that $P_{O|K}$ can be expressed as follows:

$$P_{O|K} = \int_0^{\frac{\epsilon_0}{\rho}} f_{|h_0|^2}(y)dy + \int_{\frac{\epsilon_0}{\rho}}^{\eta_4} \left[q\left(\frac{\epsilon_s}{\rho\alpha_s}\right)\right]^K f_{|h_0|^2}(y)dy + \int_{\eta_4}^{\infty} [q(y)]^K f_{|h_0|^2}(y)dy, \tag{17}$$

where $q(z) \triangleq P\left(|h_k|^2 \leq z\right)$ is the cumulative distribution function (CDF) of $|h_k|^2$, and $f_{|h_0|^2}(y)$ denotes the probability density function (pdf) of $|h_0|^2$.

In order to find the pdf of $|h_0|^2$, recall that $|h_0|^2 = h_0^{\mathrm{PL}} G h_0^{\mathrm{ML}} |g_0|^2$. By using the facts that $|g_0|^2$ is exponentially distributed and h_0^{ML} is distributed as shown in (2), the CDF of $|h_0|^2$ is given by

$$F_{|h_0|^2}(y) = P\left(h_0^{\mathrm{PL}} G h_0^{\mathrm{ML}} |g_0|^2 \leq y\right) = p_e\left(1 - e^{-\frac{y}{h_0^{\mathrm{PL}} G\xi}}\right) + (1-p_e)\left(1 - e^{-\frac{y}{h_0^{\mathrm{PL}} G}}\right). \tag{18}$$

By using the CDF of $|h_0|^2$, its pdf, denoted by $f_{|h_0|^2}(y)$, can be obtained as shown in the lemma.

To evaluate $q(z)$, the pdf of the location of a secondary user, denoted by x, is needed and can be obtained as follows [10]:

$$p_X(x) = \frac{\lambda_s(x)}{\mu_s(\mathcal{D})} = \frac{\lambda\phi^2 e^{-\phi r(x)}}{\theta\lambda\gamma(2, \mathcal{R}\phi)}. \tag{19}$$

Therefore, $q(z)$ can be expressed as follows:

$$q(z) = P\left(|g_k|^2 < \frac{z}{h_k^{\mathrm{PL}} G}\right) = \int_{\mathcal{D}} \left[1 - e^{-\frac{z}{h_k^{\mathrm{PL}} G}}\right] \frac{\phi^2 e^{-\phi r(x)}}{\theta\gamma(2, \mathcal{R}\phi)} dx. \tag{20}$$

By using the expression for h_k^{PL} and also applying polar coordinates, the probability, $P_{O|K}$, can be expressed as follows:

$$q(z) = \eta_2\theta \int_0^{\mathcal{R}} \left[1 - e^{-\frac{z(1+r^\alpha)}{G\eta_1} e^{\eta r}}\right] e^{-\phi r} r dr. \tag{21}$$

Therefore, the lemma can be proved by applying the pdf of $|h_0|^2$ and also substituting (21) into (17).

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