Hybrid Active-Passive IRS Assisted Energy-Efficient Wireless Communication

Qiaoyan Peng, Qingqing Wu, Guangji Chen, Ruiqi Liu, Shaodan Ma, Wen Chen

Abstract—Deploying active reflecting elements at the intelligent reflecting surface (IRS) increases signal amplification capability but incurs higher power consumption. Therefore, it remains a challenging and open problem to determine the optimal number of active/passive elements for maximizing energy efficiency (EE). To answer this question, we consider a hybrid active-passive IRS (H-IRS) assisted wireless communication system, where the H-IRS consists of both active and passive reflecting elements. Specifically, we study the optimization of the number of active/passive elements at the H-IRS to maximize EE. To this end, we first derive the closed-form expression for a near-optimal solution under the lineof-sight (LoS) channel case and obtain its optimal solution under the Rayleigh fading channel case. Then, an efficient algorithm is employed to obtain a high-quality sub-optimal solution for EE maximization under the general Rician channel case. Simulation results demonstrate the effectiveness of the H-IRS for maximizing EE under different Rician factors and IRS locations.

Index Terms—Intelligent reflecting surface (IRS), hybrid activepassive IRS (H-IRS), energy efficiency (EE), the number of active/passive elements.

I. INTRODUCTION

Intelligent reflective surface (IRS) has emerged as a revolution for future sixth-generation (6G) networks, which can be generally classified into two categories, i.e., the fully passive IRS and the fully active IRS [1]–[3]. Specifically, the fully passive IRS can provide an asymptotic squared-power beamforming gain with low hardware cost [4], whereas it suffers from the "multiplicative fading" effect [5], [6]. To overcome this issue, the fully active IRS has been proposed and investigated in [7]. Since the fully active IRS requires refection-type amplifiers for each element, its total power consumption is much higher than its passive counterpart of the same size [8]. In summary, previous studies have shown that the conventional IRSs, i.e., the fully passive and active IRS, have complementary advantages.

A hybrid active-passive IRS (H-IRS) composed of both passive and active reflecting elements, has been recently proposed for further improving the performance beyond what can be achieved by using an active or passive IRS alone [9]. The motivation for the need for H-IRS arises from the advantages and limitations of the conventional IRSs. As such, the H-IRS is a promising solution for enhancing various wireless systems, such as integrated sensing and communication (ISAC) systems [10] and unmanned aerial vehicle (UAV) communication [11]. In [9], the transmit precoder and H-IRS parameters were designed to maximize the sum rate of a multi-user system. The optimal



Fig. 1. An H-IRS assisted wireless communication system.

elements allocation of the H-IRS for spectral efficiency (SE) maximization was explored in [12] under the given deployment budget. In addition to SE, energy efficiency (EE) is also a major requirement of future 6G networks, which characterizes the fundamental trade-off between SE and system power consumption. Note that active elements introduce the new function of signal amplification, which is beneficial for improving SE, while they also require higher power consumption and hardware cost. As such, the operating region for the active IRS outperforms the passive IRS regarding EE, is not clear. Moreover, the H-IRS has the potential to balance the SE-cost trade-off by flexibly determining the number of active/passive elements. To this end, how to determine the number of active/passive elements for EE maximization is critical to make the H-IRS feasible for practical scenarios.

Motivated by the above considerations, we investigate the EE maximization problem in an H-IRS assisted wireless communication system. Specifically, we aim to determine the number of active/passive elements at the H-IRS to balance the trade-off between the power consumption incurred by active elements and the ergodic SE. The main contributions of this letter are summarized as follows: 1) To obtain useful insights, we first consider two special cases, i.e., the line-of-sight (LoS) and Rayleigh fading channel cases. Under the LoS channel case, we derive the closed-form expression for a near-optimal solution. Furthermore, we show that at most one active element is required under the Rayleigh fading channel case. 2) We then propose an efficient algorithm to maximize the EE under the Rician fading channel case. 3) Our numerical results demonstrate the EE of H-IRS with the optimized number of active/passive elements outperforms that of the fully active/passive IRSs under different Rician factors and IRS locations.

Notation: For a vector \mathbf{x} , $\|\mathbf{x}\|$, $[\mathbf{x}]_n$ and $\arg(\mathbf{x})$ denote its Euclidean norm, *n*-th entry and phase vector, respectively. \otimes denotes the Kronecker product. The distribution of a circularly symmetric complex Gaussian (CSCG) random variable with mean μ and variance σ^2 is denoted by $\mathcal{CN}(\mu, \sigma^2)$. $[\cdot]$ and $\lfloor \cdot \rfloor$ denote the ceiling and floor operations, respectively. $\arg \max(\cdot)$ denotes the arguments at which the function value is maximized.

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II. SYSTEM MODEL AND PROBLEM FORMULATION

As shown in Fig. 1, we consider an H-IRS assisted wireless communication system composed of a single-antenna base station (BS), a cluster of single-antenna users and an H-IRS with N_{pas} passive elements and N_{act} active elements. To guarantee the overall system performance, we take the performance of the worst-case user into account.

We assume that the direct links between the BS and the users are blocked due to dense obstacles and IRS involved links follow the practical Rician fading model similar to [8], [12]. The Rician fading model is able to capture the generalized channel environment with different LoS and non-LoS (NLoS) components by adjusting the Rician factor. As such, the equivalent baseband channel from the BS to the active IRS sub-surface is modeled as $\mathbf{h}_{BI}^{act} = \sqrt{K_1/(K_1+1)} \bar{\mathbf{h}}_{BI}^{act} +$ $\sqrt{1/(K_1+1)}\tilde{\mathbf{h}}_{\rm BI}^{\rm act}$, where K_1 denotes the corresponding Rician fading factor. Specifically, the LoS component is expressed as $\mathbf{h}_{BI}^{act} = \beta_{BI} \mathbf{a}_r(\theta_{BI}^r, \vartheta_{BI}^r, N_{act})$, where $\mathbf{a}_r(\theta_{BI}^r, \vartheta_{BI}^r, N_{act}) =$ $\mathbf{u}(\frac{2d_{\mathrm{I}}}{\lambda}\mathrm{sin}(\theta_{\mathrm{BI}}^{\mathrm{r}})\mathrm{sin}(\vartheta_{\mathrm{BI}}^{\mathrm{r}})N_{x}) \otimes \mathbf{u}(\frac{2d_{\mathrm{I}}}{\lambda}\mathrm{cos}(\vartheta_{\mathrm{BI}}^{\mathrm{r}}),N_{y}), \ N = N_{x}N_{y},$ and $\mathbf{u}(\upsilon,M) = [1,\ldots,e^{-(M-1)j\pi\upsilon}]^{T}. \ d_{\mathrm{I}}, \ \lambda \text{ and } \beta_{\mathrm{BI}}^{2}$ denote the element spacing of active elements, the wavelength and the path loss, respectively. θ_{BI}^r and ϑ_{BI}^r are the azimuth and elevation angles of arrival at the IRS, respectively. The NLoS component is given by $[\mathbf{h}_{BI}^{act}]_n \sim \mathcal{CN}(0, \beta_{BI}^2), \forall n \in \mathcal{N}_{act} \triangleq \{1, \dots, N_{act}\}.$ The equivalent baseband channel from the active IRS sub-surface to the worst-case user is modeled as $\mathbf{h}_{\text{III}}^{\text{act}} = \sqrt{K_2/(K_2+1)} \bar{\mathbf{h}}_{\text{III}}^{\text{act}} +$ $\sqrt{1/(K_2+1)}\tilde{\mathbf{h}}_{\text{IU}}^{\text{act}}$ with the LoS component $\bar{\mathbf{h}}_{\text{IU}}^{\text{act}} \in \mathbb{C}^{N_{\text{act}} \times 1}$, the NLoS component $\tilde{\mathbf{h}}_{\text{IU}}^{\text{act}} \in \mathbb{C}^{N_{\text{act}} \times 1}$ and the corresponding Rician fading factor K_2 . The equivalent baseband channel from the BS to the passive IRS sub-surface and from the passive IRS sub-surface to the worst-case user are denoted by $\mathbf{h}_{\mathsf{BI}}^{\mathsf{pas}} \in \mathbb{C}^{N_{\mathsf{pas}} \times 1}$ and $\mathbf{h}_{IU}^{pas} \in \mathbb{C}^{N_{pas} \times 1}$, which are defined similarly as \mathbf{h}_{BI}^{act} and \mathbf{h}_{IU}^{act} .

Let $\Psi^{\text{pas}} \triangleq \text{diag}(e^{j\varphi_1^{\text{pas}}}, \cdots, e^{j\varphi_{N_{\text{pas}}}^{\text{pas}}})$ denote the reflection matrix of the passive sub-surface, where $\varphi_n^{\rm pas}$ represents the corresponding phase shift with $n \in \mathcal{N}_{\text{pas}} \stackrel{\Delta}{=} \{1, \cdots, N_{\text{pas}}\}.$ The reflection matrix of the active sub-surface is denoted by $\Psi^{\text{act}} \triangleq \mathbf{A}^{\text{act}} \Phi^{\text{act}}$, where $\mathbf{A}^{\text{act}} \triangleq \text{diag}(\alpha_1, \cdots, \alpha_{N_{\text{act}}})$ and $\Phi^{\text{act}} \triangleq \text{diag}(e^{j\varphi_1^{\text{act}}}, \cdots, e^{j\varphi_{N_{\text{act}}}^{\text{act}}})$ denote its reflection amplification matrix and phase-shift matrix, respectively, with the amplification factor α_n and phase shift φ_n^{act} , $n \in \mathcal{N}_{\text{act}}$. Then, the signal received at the worst-case user is given by $y = (\mathbf{h}_{\mathrm{IU}}^{\mathrm{act}})^{H} \boldsymbol{\Psi}^{\mathrm{act}} \mathbf{h}_{\mathrm{BI}}^{\mathrm{act}} s + (\mathbf{h}_{\mathrm{IU}}^{\mathrm{pas}})^{H} \boldsymbol{\Psi}^{\mathrm{pas}} \mathbf{h}_{\mathrm{BI}}^{\mathrm{pas}} s + (\mathbf{h}_{\mathrm{IU}}^{\mathrm{act}})^{H} \boldsymbol{\Psi}^{\mathrm{act}} \mathbf{n}_{\mathrm{r}} + n_{0},$ where $s \in \mathbb{C}$ denotes the transmitted data, which satisfies $\mathbb{E}\{|s|^2\} = P_B$ with P_B denoting the transmit power of the BS. $\mathbf{n}_{\rm r} \sim \mathcal{CN}(\mathbf{0}_{N_{\rm act}}, \sigma_{\rm r}^2 \mathbf{I}_{N_{\rm act}})$ is the thermal noise introduced by active elements with the amplification noise power σ_r^2 , and $n_0 \sim \mathcal{CN}(0, \sigma_0^2)$ is the additive white Gaussian noise (AWGN). Accordingly, the signal-to-noise-ratio (SNR) of the worst-case user is expressed as

$$\gamma = \frac{P_{\rm B} |(\mathbf{h}_{\rm IU}^{\rm act})^H \mathbf{\Psi}^{\rm act} \mathbf{h}_{\rm BI}^{\rm act} + (\mathbf{h}_{\rm IU}^{\rm pas})^H \mathbf{\Psi}^{\rm pas} \mathbf{h}_{\rm BI}^{\rm pas}|^2}{\sigma_{\rm r}^2 \|(\mathbf{h}_{\rm IU}^{\rm act})^H \mathbf{\Psi}^{\rm act}\|^2 + \sigma_0^2}.$$
 (1)

As such, the ergodic achievable rate is given by

$$R = \mathbb{E}\left\{\log_2\left(1+\gamma\right)\right\}.$$
 (2)

The average power consumption of the passive elements is given by $P_{\text{pas}} = N_{\text{pas}}P_{\text{c}}$ [6], where P_{c} represents the switch and control circuit power consumption at each reflecting element. The average power consumption of the active elements is given

by $P_{\text{act}} = N_{\text{act}}(P_{\text{DC}} + P_{\text{c}}) + \xi \mathbb{E}\{P_{\text{out}}\} = N_{\text{act}}(P_{\text{DC}} + P_{\text{c}}) + \xi P_{\text{I}},$ where ξ is the inverse of energy conversion coefficient and P_{DC} is the DC biasing power consumption at each active element [6]. $P_{\text{out}} = P_{\text{B}} \| \Psi^{\text{act}} \mathbf{h}_{\text{BI}}^{\text{act}} \|^{2} + \sigma_{\text{r}}^{2} \| \Psi^{\text{act}} \|^{2}$ denotes the output power of the active elements, which satisfies $\mathbb{E}\{P_{\text{out}}\} = P_{\text{I}}$. As such, the average total power consumption is given by [13]

$$P_{\text{total}} = P_{\text{pas}} + P_{\text{act}} + P_{\text{BS}} + \varsigma P_{\text{B}},\tag{3}$$

where ς and P_{BS} are the inverse of energy conversion coefficient and the dissipated power consumed at the BS, respectively.

We maximize the ergordic EE of the worst-case user by optimizing the number of active and passive elements, N_{act} and N_{pas} , the IRS phase shifts, $\{\Phi^{\text{act}}, \Psi^{\text{pas}}\}$, and the active-elements amplification matrix \mathbf{A}^{act} , which is defined as $\text{EE} = R/P_{\text{total}}$. Given \mathbf{A}^{act} , N_{act} and N_{pas} , i.e., the total power consumption is fixed, the EE maximization problem is equivalent to the SE maximization problem in [12]. Accordingly, the optimal IRS phase shifts can be expressed as [12]

$$\varphi_{n}^{\text{act}} = \arg\left(\left[\bar{\mathbf{h}}_{\text{IU}}^{\text{act}}\right]_{n}\right) - \arg\left(\left[\bar{\mathbf{h}}_{\text{BI}}^{\text{act}}\right]_{n}\right), \forall n \in \mathcal{N}_{\text{act}}, \quad (4)$$
$$\varphi_{n}^{\text{pas}} = \arg\left(\left[\bar{\mathbf{h}}_{\text{IU}}^{\text{pas}}\right]_{n}\right) - \arg\left(\left[\bar{\mathbf{h}}_{\text{BI}}^{\text{pas}}\right]_{n}\right), \forall n \in \mathcal{N}_{\text{pas}}. \quad (5)$$

Similarly, given Φ^{act} , Ψ^{pas} , N_{act} and N_{pas} , the optimal amplification factor for the *n*-th active element is expressed as [12]

$$\alpha_n = \alpha^* \triangleq \sqrt{P_{\rm I} / (N_{\rm act} (P_{\rm B} \beta_{\rm BI}^2 + \sigma_{\rm r}^2))}, \forall n \in \mathcal{N}_{\rm act}.$$
 (6)

Given Φ^{act} , Ψ^{pas} , \mathbf{A}^{act} , the ergodic SE can be expressed as $\tilde{R} = \log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}N_{\text{act}}+N_{\text{pas}}})^2 + \gamma_2(A_{\text{sum}}+N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2})$ [12]. Then, the ergodic EE is given by

$$\eta(N_{\text{act}}, N_{\text{pas}}) = R/P_{\text{total}} = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}N_{\text{act}}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}) = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}N_{\text{act}}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}) = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}N_{\text{act}}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}) = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}N_{\text{act}}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}) = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}N_{\text{act}}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}}) = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}N_{\text{act}}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}}) = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}N_{\text{act}}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}}) = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}} = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}} = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{IU}}^2 + \gamma_2(A_{\text{sum}} + N_{\text{pas}})}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}}) = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{IU}}^2 + \gamma_2(A_{\text{sum}} + N_{\text{sum}} + N_{\text{sum}}))}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}} = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{IU}}^2 + \gamma_2(A_{\text{sum}} + N_{\text{sum}} + N_{\text{sum}})}{A_{\text{sum}}\sigma_r^2\beta_{\text{IU}}^2 + \sigma_0^2}} = \frac{\log_2(1 + \frac{P_{\text{B}}\beta_{\text{IU}}^2 + \gamma_2(A_{\text{sum}} + N_{\text{sum}} + N_{\text$$

where $A_{\text{sum}} \triangleq \mathcal{I}_{\mathbb{R}^+}(N_{\text{act}})P_{\text{I}}/(P_{\text{B}}\beta_{\text{BI}}^2 + \sigma_{\text{r}}^2)$, and

$$\gamma_1 \triangleq \frac{K_1 K_2}{(K_1 + 1) (K_2 + 1)}, \gamma_2 \triangleq \frac{K_1 + K_2 + 1}{(K_1 + 1) (K_2 + 1)}.$$
 (8)

The indicator function $\mathcal{I}_{\mathbb{R}^+}(N_{\rm act}) = 1$ if $N_{\rm act} > 0$, otherwise $\mathcal{I}_{\mathbb{R}^+}(N_{\rm act}) = 0$.

Our objective is to maximize the ergodic EE of the worst-case user by optimizing the number of active/passive elements. Under the Rician fading channel case, the optimization problem is formulated as

$$\max_{\mathbf{N}_{act}, N_{pas}} \eta \left(N_{act}, N_{pas} \right) \quad \text{s.t.} \ N_{act} \in \mathbb{N}, N_{pas} \in \mathbb{N}.$$
(9)

Problem (9) is intractable because the discrete integer variables $\{N_{\text{act}}, N_{\text{pas}}\}$ are coupled in the non-concave objective function. When N_{pas} (N_{act}) is zero, problem (9) is reduced to the optimization problem with the fully active (passive) IRS. Thus, the proposed model generalizes the fully active and passive IRSs as two special cases.

III. PROPOSED SOLUTIONS

In this section, we first investigate the active/passive elements allocation problem via two special cases, i.e., the LoS and Rayleigh fading channel cases, to draw important insights. In particular, the corresponding operating regions for active, passive, and hybrid IRS for EE maximization are characterized. Then, we propose an efficient algorithm to obtain its highquality sub-optimal solution.

A. LoS Channel Case

We first consider the LoS channel case with $K_1 \to \infty$ and $K_2 \to \infty$, which implies $\gamma_1 \to 1$ and $\gamma_2 \to 0$ from (8). By relaxing the integer values $N_{\rm act}$ and $N_{\rm pas}$ into the continuous values $x_{\rm act}$ and $x_{\rm pas}$, the EE is given by

$$\eta_{\rm L}(x_{\rm act}, x_{\rm pas}) = \frac{\log_2(1 + \frac{P_{\rm B}\beta_{\rm BI}^2\beta_{\rm IU}^2(\sqrt{A_{\rm sum}x_{\rm act}} + x_{\rm pas})^2)}{A_{\rm sum}\sigma_{\rm r}^2\beta_{\rm IU}^2 + \sigma_0^2)}}{x_{\rm pas}P_{\rm c} + x_{\rm act}(P_{\rm DC} + P_{\rm c}) + \xi \mathcal{I}_{\mathbb{R}} + (x_{\rm act})P_{\rm I} + P_{\rm BS} + \varsigma P_{\rm B}}.$$
 (10)

Then, problem (9) is transformed to

$$\max_{x_{\text{act}}, x_{\text{pas}}} \eta_{\text{L}} \left(x_{\text{act}}, x_{\text{pas}} \right) \quad \text{s.t.} \ x_{\text{act}} \in \mathbb{R}, x_{\text{pas}} \in \mathbb{R}.$$
(11)

We solve problem (11) by considering three cases, namely the fully active, the fully passive, and the hybrid IRSs. By comparing the achievable EEs of the three cases, we obtain its near-optimal solution to problem (11).

1) Fully Active IRS with $x_{pas} = 0$ and $x_{act} > 0$: We define $\eta_a(x_{act}) = \log_2(1 + \beta_0 x_{act})/(\beta_1 x_{act} + \beta_2)$ and reformulate problem (11) as

$$\max_{\alpha} \eta_{a} \left(x_{act} \right) \quad \text{s.t.} \ x_{act} \in \mathbb{R}^{+}, \tag{12}$$

where $\beta_0 = P_{\rm B}\beta_{\rm BI}^2\beta_{\rm IU}^2/(\sigma_{\rm r}^2\beta_{\rm IU}^2 + \sigma_0^2/A_{\rm sum}')$, $\beta_1 = P_{\rm DC} + P_{\rm c}$, $\beta_2 = \xi P_{\rm I} + P_{\rm BS} + \varsigma P_{\rm B}$ and $A_{\rm sum} = P_{\rm I}/(P_{\rm B}\beta_{\rm BI}^2 + \sigma_{\rm r}^2)$.

Proposition 1. $\eta_a(x_{act})$ first increases with $x_{act} \in (0, x_{act}^*]$ and then decreases with $x_{act} \in [x_{act}^*, \infty)$. The optimal solution to problem (12) is given by $x_{act}^* = -1/\beta_0((\beta_1 - \beta_0\beta_2)/(\beta_1L) + 1)$, where $L = \mathcal{W}(e^{-1}(\beta_0\beta_2 - \beta_1)/\beta_1)$ with the Lambert W function $\mathcal{W}(\cdot)$.

Proof: The first-order derivative of $\eta_a(x_{act})$ with respect to (w.r.t.) x_{act} is given by $\frac{d(\eta_a(x_{act}))}{d(x_{act})} = \frac{\eta_a(x_{act})}{\ln 2(\beta_1 x_{act} + \beta_2)^2(1 + \beta_0 x_{act})}$, where

$$d_1(x_{act}) = \beta_0(\beta_1 x_{act} + \beta_2) - (1 + \beta_0 x_{act})\beta_1 \ln(1 + \beta_0 x_{act}).$$
(13)

Since $\frac{d(d_1(x_{act}))}{d(x_{act})} < 0$, $d_1(x_{act})$ monotonically decreases with x_{act} . There exists one and only one root $x_{act}^{rt} = -1/\beta_0((\beta_1 - \beta_0\beta_2)/(\beta_1L) + 1)$ for (13). When $0 < x_{act} < x_{act}^{rt}$, we have $d_1(x_{act}) > 0$ and $\frac{d(\eta_a(x_{act}))}{d(x_{act})} > 0$, i.e., $\eta_a(x_{act})$ monotonically increases with x_{act} . When $x_{act} > x_{act}^{rt}$, we have $d_1(x_{act}) < 0$ and $\frac{d(\eta_a(x_{act}))}{d(x_{act})} < 0$, i.e., $\eta_a(x_{act})$ monotonically decreases with x_{act} . Accordingly, $\eta_a(x_{act})$ is maximized at $x_{act}^* = x_{act}^{rt}$, which completes the proof.

2) Fully Passive IRS with $x_{act} = 0$ and $x_{pas} > 0$: Under the practical scenarios, the number of IRS elements is very large, i.e., $x_{pas} \gg 1$. We define $\eta_p(x_{pas}) = \log_2 (\beta_3 x_{pas})/(\beta_4 x_{pas} + \beta_5)$ and reformulate problem (11) as

$$\max_{x_{\text{pas}}} \eta_{\text{p}}(x_{\text{pas}}) \quad \text{s.t.} \ x_{\text{pas}} \in \mathbb{R}^+, \tag{14}$$

where $\beta_3 = \sqrt{P_B \beta_{BI}^2 \beta_{IU}^2 / \sigma_0^2}$, $\beta_4 = \frac{1}{2} P_c$ and $\beta_5 = \frac{1}{2} (P_{BS} + \varsigma P_B)$. Then, we have the following results.

Proposition 2. $\eta_p(x_{pas})$ first increases with $x_{pas} \in (0, x_{pas}^*)$ and then decreases with $x_{pas} \in [x_{pas}^*, \infty)$. The optimal solution to problem (14) is given by $x_{pas}^* = \beta_5/(\beta_4 Q)$, where $Q = \omega(-\ln(\beta_4/(\beta_3\beta_5))-1)$ with the Wright omega function $\omega(\cdot)$.

Proof: The first-order derivative of $\eta_p(x_{pas})$ w.r.t. x_{pas} is given by $\frac{d(\eta_p(x_{pas}))}{d(x_{pas})} = \frac{d_2(x_{pas})}{\ln 2(\beta_4 x_{pas} + \beta_5)^2 \beta_3 x_{pas}}$, where

$$d_2\left(x_{\text{pas}}\right) = \beta_3\left(\beta_4 x_{\text{pas}} + \beta_5\right) - \beta_4 \beta_3 x_{\text{pas}} \ln\left(\beta_3 x_{\text{pas}}\right).$$
(15)

Since $\frac{d(d_2(x_{\text{pas}}))}{d(x_{\text{pas}})} < 0$, $d_2(x_{\text{pas}})$ monotonically decreases with x_{pas} . There must exist one and only one root $x_{\text{pas}}^{\text{rt}} = \beta_5/(\beta_4 Q)$ for (15). When $0 < x_{\text{pas}} < x_{\text{pas}}^{\text{rt}}$, we have $d_2(x_{\text{pas}}) > 0$ and $\frac{d(\eta_p(x_{\text{pas}}))}{d(x_{\text{pas}})} > 0$, i.e., $\eta_p(x_{\text{pas}})$ monotonically increases with x_{pas} . When $x_{\text{pas}} > x_{\text{pas}}^{\text{rt}}$, we have $d_2(x_{\text{pas}}) < 0$, i.e., $\eta_p(x_{\text{pas}}) < 0$ and $\frac{d(\eta_p(x_{\text{pas}}))}{d(x_{\text{pas}})} < 0$, i.e., $\eta_p(x_{\text{pas}}) < 0$ and $\frac{d(\eta_p(x_{\text{pas}}))}{d(x_{\text{pas}})} < 0$, i.e., $\eta_p(x_{\text{pas}})$ monotonically decreases with x_{pas} . Accordingly, $\eta_p(x_{\text{pas}})$ is maximized at $x_{\text{pas}}^* = x_{\text{pas}}^{\text{rt}}$, which completes the proof.

3) *H-IRS* with $x_{act} > 0$ and $x_{pas} > 0$: We assume that $g_0^2(\sqrt{A'_{sum}N_{act} + N_{pas}})^2 \gg 1$ and define $\eta_h(x_{act}, x_{pas}) = 2\log_2(g_0(\sqrt{A'_{sum}x_{act} + x_{pas}}))/(x_{pas}P_c + \beta_1x_{act} + \beta_2)$, where $g_0 = \sqrt{P_B\beta_{BI}^2\beta_{IU}^2/(A'_{sum}\sigma_r^2\beta_{IU}^2 + \sigma_0^2)}$. Then, problem (11) is reformulated as

$$\max_{x_{\text{act}}, x_{\text{pas}}} \eta_{\text{h}}\left(x_{\text{act}}, x_{\text{pas}}\right) \quad \text{s.t.} \ x_{\text{act}} \in \mathbb{R}^+, x_{\text{pas}} \in \mathbb{R}^+.$$
(16)

Proposition 3. For any fixed x_{pas} , $\eta_{\text{h}}(x_{\text{act}}, x_{\text{pas}})$ is a quasi-concave function w.r.t. x_{act} . Moreover, for any fixed x_{act} , $\eta_{\text{h}}(x_{\text{act}}, x_{\text{pas}})$ is a quasi-concave function w.r.t. x_{pas} . As such, the optimal solution to problem (16) is given by

$$x_{\text{h-a}}^* = P_{\text{c}}^2 A'_{\text{sum}} / (2\beta_1)^2, \ x_{\text{h-p}}^* = \left(g_2 - (1+G)\,g_1\right) / G, \ (17)$$

where $g_1 = P_c A'_{sum}/(2\beta_1)$, $g_2 = g_1/2 + \beta_2/P_c$ and $G = \mathcal{W}(e^{-1}(g_0g_2 - g_0g_1))$.

Proof: For any fixed x_{pas} , denote the upper contour set of $\eta_h(x_{\text{act}}, x_{\text{pas}})$ as $S_{\alpha'} = \{x_{\text{act}} \in \mathbb{R}^+ | \eta_h(x_{\text{act}}, x_{\text{pas}}) \ge \alpha'\}$. $S_{\alpha'}$ is equivalent to $S_{\alpha'} = \{x_{\text{act}} \in \mathbb{R}^+ | \alpha' U_{\alpha'}(x_{\text{act}}) - V_{\alpha'}(x_{\text{act}}) \le 0\}$, where $U_{\alpha'}(x_{\text{act}}) = x_{\text{pas}}P_c + \beta_1 x_{\text{act}} + \beta_2$ and $V_{\alpha'}(x_{\text{act}}) = 2\log_2(g_0(\sqrt{A'_{\text{sum}}x_{\text{act}} + x_{\text{pas}}}))$. Since $U_{\alpha'}(x_{\text{act}})$ is linear and $V_{\alpha'}(x_{\text{act}})$ is concave, $S_{\alpha'}$ is convex for any $\alpha' \in \mathbb{R}$. For any fixed x_{act} , denote the upper contour set of $\eta_h(x_{\text{act}}, x_{\text{pas}})$ as $S_{\beta'} = \{x_{\text{pas}} \in \mathbb{R}^+ | \eta_h(x_{\text{act}}, x_{\text{pas}}) \ge \beta'\}$. Similarly, $S_{\beta'}$ is convex for any $\beta' \in \mathbb{R}$. $\eta_h(x_{\text{act}}, x_{\text{pas}})$ is quasi-concave if its upper contour set is convex. We set the partial derivative of $\eta_h(x_{\text{act}}, x_{\text{pas}})$ w.r.t. x_{act} and x_{pas} to zero, i.e.,

$$\begin{cases} g_0(P_c x_{pas} + \beta_1 x_{act} + \beta_2) \\ -P_c(g_0 \sqrt{A'_{sum} x_{act}} + x_{pas}) \ln(g_0 \sqrt{A'_{sum} x_{act}} + x_{pas}) = 0, \\ g_0 \sqrt{A'_{sum}}(P_c x_{pas} + \beta_1 x_{act} + \beta_2) \\ -2\beta_1 \sqrt{x_{act}}(g_0 \sqrt{A'_{sum} x_{act}} + x_{pas}) \ln(g_0 \sqrt{A'_{sum} x_{act}} + x_{pas}) = 0. \end{cases}$$
(18)

For any x_{pas} , the root $x_{\text{act}}^{\text{rt}}$ is unique. Given $x_{\text{act}}^{\text{rt}}$, there exists one and only one root $x_{\text{pas}}^{\text{rt}}$. The partial derivative of $\eta_{\text{LoS}}(x_{\text{act}}^{\text{rt}}, x_{\text{pas}})$ w.r.t. x_{pas} is given by $\frac{\partial(\eta_{\text{LoS}}(x_{\text{act}}^{\text{rt}}, x_{\text{pas}}))}{\partial x_{\text{pas}}} = \frac{d_3(x_{\text{pas}})}{2 \ln 2(g_0g_1 + g_0x_{\text{pas}})(P_cx_{\text{pas}} + P_cg_1/2 + \beta_2)^2}$, where $d_3(x_{\text{pas}}) = g_0(P_cx_{\text{pas}} + P_cg_1/2 + \beta_2) - P_c(g_0g_1 + g_0x_{\text{pas}})\ln(g_0g_1 + g_0x_{\text{pas}}).$ (19)

Since $\frac{d(d_3(x_{\rm pas}))}{d(x_{\rm pas})} < 0$, $d_3(x_{\rm pas})$ monotonically decreases with $x_{\rm pas}$. If $g_0(P_{\rm c}g_1/4 + \beta_2) - (P_{\rm c}/2)(g_0g_1)\ln(g_0g_1) < 0$, we have $d_3(x_{\rm pas}) < 0$, $\forall x_{\rm pas}$, i.e., $\eta_{\rm LOS}(x_{\rm act}^{\rm rt}, x_{\rm pas})$ monotonically decreases with $x_{\rm pas}$. Accordingly, $\eta_{\rm LOS}(x_{\rm act}^{\rm rt}, x_{\rm pas})$ is maximized at $x_{\rm pas}^* = 0$. Otherwise, there must exist one and only one root $x_{\rm pas}^{\rm rt} = (g_2 - (1+G)g_1)/G$ for (19). When $0 \le x_{\rm pas} < x_{\rm pas}^{\rm rt}$, we have $d_3(x_{\rm pas}) > 0$ and $\frac{d(\eta_{\rm LOS}(x_{\rm act}^{\rm st}, x_{\rm pas}))}{d(x_{\rm pas})} > 0$, i.e., $\eta_{\rm LOS}(x_{\rm act}^{\rm rt}, x_{\rm pas})$ monotonically decreases with $x_{\rm pas}$. When $x_{\rm pas} > x_{\rm pas}^{\rm rt}$, we have $d_3(x_{\rm pas}) < 0$ and $\frac{d(\eta_{\rm LOS}(x_{\rm act}^{\rm st}, x_{\rm pas}))}{d(x_{\rm pas})} < 0$, i.e., $\eta_{\rm LOS}(x_{\rm act}^{\rm rt}, x_{\rm pas})$ monotonically decreases with $x_{\rm pas}$. Accordingly, $\eta_{\rm LOS}(x_{\rm act}^{\rm rt}, x_{\rm pas})$ monotonically decreases $x_{\rm pas} < x_{\rm pas}$. The nontonically decreases $x_{\rm pas} < x_{\rm pas}$. The scale $x_{\rm pas} < x_{\rm pas}$ is maximized at $x_{\rm pas}^* = x_{\rm pas}^{\rm rt}$. In this case, $\eta_{\rm h}(x_{\rm act}, x_{\rm pas})$ is maximized if and only

$$\begin{cases}
N_{\text{pas}}^{*} = 0, N_{\text{act}}^{*} = \arg\max_{x_{\text{act}}} \eta_{a} (x_{\text{act}}), \text{ if } \eta_{\text{LoS}}^{*} = \eta_{\text{act}}^{*} = \max_{x_{\text{act}}} \eta_{a} (x_{\text{act}}), x_{\text{act}} \in \{\lfloor x_{\text{act}}^{*} \rfloor, \lceil x_{\text{act}}^{*} \rceil\}, \\
N_{\text{act}}^{*} = 0, N_{\text{pas}}^{*} = \arg\max_{x_{\text{pas}}} \eta_{p} (x_{\text{pas}}), \text{ if } \eta_{\text{LoS}}^{*} = \eta_{\text{pas}}^{*} = \max_{x_{\text{pas}}} \eta_{p} (x_{\text{pas}}), x_{\text{pas}} \in \{\lfloor x_{\text{pas}}^{*} \rfloor, \lceil x_{\text{pas}}^{*} \rceil\}, \\
N_{\text{act}}^{*}, N_{\text{pas}}^{*} = \arg\max_{x_{\text{pas}}} \eta_{h} (x_{\text{act}}, x_{\text{pas}}), \text{ if } \eta_{\text{LoS}}^{*} = \eta_{\text{pas}}^{*} = \max_{x_{\text{pas}}} \eta_{h} (x_{\text{act}}, x_{\text{pas}}) \in \{(\lceil x_{\text{h-a}}^{*} \rceil, \lceil x_{\text{h-p}}^{*} \rceil), (\lceil x_{\text{h-a}}^{*} \rceil, \lceil x_{\text{h-p}}^{*} \rceil), (\lfloor x_{\text{h-a}}^{*} \rceil, \lfloor x_{\text$$

if $x_{act} = x_{act}^{rt} = x_{h-a}^{*}$ and $x_{pas} = x_{pas}^{rt} = x_{h-p}^{*}$, which are expressed in (17). As such, the proof is completed.

Under the LoS channel case, the optimized number of active/passive elements and the architecture selection for the IRS (i.e., passive, active or hybrid) are given in (20) on the top of this page, where $\eta_{\text{LoS}}^* = \max(\eta_{\text{act}}^*, \eta_{\text{pas}}^*, \eta_{\text{hyb}}^*)$. $\eta_{\text{hyb}}^* > \max(\eta_{\text{act}}^*, \eta_{\text{pas}}^*)$ is the operating region for the EE of H-IRS that outperforms that of the fully active/passive IRSs, where it is determined by the system parameters, i.e., the IRS location, the power consumption, etc. The maximum EE can be obtained at the IRS by flexibly determining the number of active/passive elements according to the system parameters.

B. Rayleigh Fading Channel Case

We next study the Rayleigh fading channel case with $K_1 = K_2 = 0$, which implies $\gamma_1 = 0$ and $\gamma_2 = 1$ from (8). The EE under the Rayleigh fading channel case is given by

$$\eta_{\text{Ray}}(N_{\text{act}}, N_{\text{pas}}) = \frac{\log_2(1 + P_B \beta_{\text{BI}}^2 \beta_{\text{IU}}^2 (A_{\text{sum}} + N_{\text{pas}})/(A_{\text{sum}} \sigma_r^2 \beta_{\text{IU}}^2 + \sigma_0^2))}{N_{\text{pas}} P_c + N_{\text{act}} (P_{\text{DC}} + P_c) + \xi \mathcal{I}_{\mathbb{R}^+} (N_{\text{act}}) P_1 + P_{\text{BS}} + \varsigma P_B}.$$
 (21)

Then, problem (9) is reformulated as

$$\max_{N_{\text{act}},N_{\text{pas}}} \eta_{\text{Ray}} \left(N_{\text{act}}, N_{\text{pas}} \right) \quad \text{s.t.} \quad N_{\text{act}} \in \mathbb{N}, N_{\text{pas}} \in \mathbb{N}.$$
(22)

We solve problem (22) via two cases, i.e., $N_{\rm act} = 0$ and $N_{\rm act}>0$. First, we consider the case of $N_{\rm act} = 0$ and define $\eta_1(N_{\rm pl}) = \log_2(1+\beta_3^2N_{\rm pl})/(N_{\rm pl}P_{\rm c}+P_{\rm BS}+\varsigma P_{\rm B})$. Then, problem (22) is transformed to

$$\max_{N_{\rm pl}} \eta_1(N_{\rm pl}) \quad \text{s.t.} \ N_{\rm pl} \in \mathbb{N}.$$
(23)

By relaxing the integer value $N_{\rm p1}$ into the continuous value $x_{\rm p1}$, problem (23) is converted to a convex problem. The optimal solution is given by $x_{\rm p1}^* = -1/\beta_3^2((P_{\rm c} - 2\beta_3^2\beta_5)/(P_{\rm c}J)+1)$, where $J = \mathcal{W}(e^{-1}(2\beta_3^2\beta_5 - P_{\rm c})/P_{\rm c})$. The optimal solution to problem (23) is given by

$$N_{\mathrm{pl}}^* = \operatorname*{arg\,max}_{x_{\mathrm{pl}}} \eta_1(x_{\mathrm{pl}}), x_{\mathrm{pl}} \in \left\{ \lfloor x_{\mathrm{pl}}^* \rfloor, \lceil x_{\mathrm{pl}}^* \rceil \right\}.$$
(24)

Second, for $N_{\rm act} > 0$, we obtain that $N_{\rm act}^* = 1$ because $\eta_{\rm Ray}(N_{\rm act}, N_{\rm pas})$ monotonically decreases with $N_{\rm act}$. We define $\eta_2(N_{\rm p2}) = \log_2(1+\beta_0(1+A_{\rm sum}'N_{\rm p2}))/(N_{\rm p2}P_{\rm c}+\beta_6)$ and reformulate problem (22) as

$$\max_{N_{p2}} \eta_2(N_{p2}) \quad \text{s.t.} \quad N_{p2} \in \mathbb{N}, \tag{25}$$

where $\beta_6 = \beta_1 + \beta_2$. By relaxing the integer value N_{p2} into the continuous value x_{p2} , problem (25) is converted to a convex problem. The optimal solution is given by $x_{p2}^* = 0$ if $\beta_0\beta_6/A'_{sum} - P_c(1+\beta_0)\ln(1+\beta_0)<0$; otherwise, $x_{p2}^* =$ $A'_{sum}/\beta_0((\beta_0\beta_6/A'_{sum} - (1+\beta_0)P_c)/(P_cL') - (1+\beta_0))$, where $L = \mathcal{W}(e^{-1}(\beta_0\beta_6/A'_{sum} - (1+\beta_0)P_c)/P_c)$. The optimal solution to problem (25) is given by

$$N_{p2}^{*} = \arg\max_{x_{p2}} \eta_{2} (x_{p2}), x_{p2} \in \left\{ \lfloor x_{p2}^{*} \rfloor, \lceil x_{p2}^{*} \rceil \right\}.$$
(26)

Based on the previous discussions, the optimal solution to problem (22) is expressed as

$$\begin{cases} N_{\text{act}}^* = 0, N_{\text{pas}}^* = N_{\text{p1}}^*, & \eta_{\text{p1}}^* \ge \eta_{\text{p2}}^*, \\ N_{\text{act}}^* = 1, N_{\text{pas}}^* = N_{\text{p2}}^*, & \text{Otherwise,} \end{cases}$$
(27)

where $\eta_{p1}^* = \eta_1(N_{p1}^*)$ and $\eta_{p2}^* = \eta_2(N_{p2}^*)$. It is observed from (27) that at most one active element is required under the Rayleigh fading channel case. In this case, the system cannot attain beamforming gain since the design of IRS phase shifts is based only on the LoS components. Moreover, the active elements also cannot reap aperture gain due to the amplified power constraint. Therefore, the ergodic rate is independent of N_{act} and thus deploying more active elements only results in higher power consumption rather than the improvement of rate, which leads to the fact that at most one active element is needed. By contrast, the ergodic rate still scales linearly w.r.t. N_{pas} benefited from the aperture gain provided by passive elements.

C. Rician Fading Channel Case

Finally, we study the general Rician fading channel case. Since the discrete integer variables $\{N_{act}, N_{pas}\}$ are coupled in the objective function, problem (9) is a non-convex optimization problem, which is challenging to be solved optimally. To overcome this issue, we obtain the following proposition.

Proposition 4. For any fixed x_{pas} , $\eta(x_{\text{act}}, x_{\text{pas}})$ is a quasi-concave function w.r.t. $x_{\text{act}} \in \mathbb{R}^+$, where x_{act} and x_{pas} are the continuous values of N_{act} and N_{pas} with integer relaxation, respectively.

Proof: For any fixed x_{pas} , denote the upper contour set of $\eta(x_{\text{act}}, x_{\text{pas}})$ as $S_{\tau} = \{x_{\text{act}} \in \mathbb{R}^+ | \eta(x_{\text{act}}, x_{\text{pas}}) \ge \tau\}$. S_{τ} is equivalent to $S_{\tau} = \{x_{\text{act}} \in \mathbb{R}^+ | \tau U_{\tau}(x_{\text{act}}) - V_{\tau}(x_{\text{act}}) \le 0\}$, where $V_{\tau}(x_{\text{act}}) = \log_2(1 + P_{\text{B}}\beta_{\text{BI}}^2\beta_{\text{IU}}^2(\gamma_1(\sqrt{A_{\text{sum}}x_{\text{act}} + x_{\text{pas}})^2 + \gamma_2(A_{\text{sum}} + x_{\text{pas}}))/(A_{\text{sum}}\sigma_{r}^2\beta_{\text{IU}}^2 + \sigma_0^2))$ and $U_{\tau}(x_{\text{act}}) = x_{\text{pas}}P_{\text{c}} + x_{\text{act}}\beta_1 + \xi P_1 + 2\beta_5$. Since $V_{\tau}(x_{\text{act}})$ is concave and $U_{\tau}(x_{\text{act}})$ is linear, S_{τ} is convex for any $\tau \in \mathbb{R}$. As such, the proof is completed.

Based on Proposition 4, we can apply an efficient algorithm to obtain a high-quality sub-optimal solution under the Rician fading channel case. Given x_{pas} , we can obtain the optimal solution $x_{act}^* \in \mathbb{R}^+$ by the Newton's algorithm or $x_{act}^* = 0$. Given x_{act}^* , one stationary point of x_{pas} can be obtained by adopting the gradient ascent method. By updating x_{pas} and x_{act} iteratively until the convergence is reached, one high-quality solution to the original problem can be obtained.

IV. SIMULATION RESULTS

In this section, numerical results are provided to illustrate the effectiveness of using the H-IRS to improve the ergodic EE. The BS, the H-IRS and the worst-case user are located at (0,0) meter (m), $(x_{IRS},0)$ m, and $(x_{IRS},10)$ m, respectively. For both the BS-IRS and IRS-user links, the Rician factors are



Fig. 2. Comparison on EE and elements allocation of the H-IRS versus K.

considered to be the same, i.e., $K_1 = K_2 = K$, the path-loss factors are set to 2.2 and the signal attenuation at a reference distance of 1 m is set to 30 dB. Other system parameters are set as follows: $\sigma_r^2 = \sigma_0^2 = -80$ dBm, $P_B = 20$ dBm, $P_I = -15$ dBm, $P_C = 1.5$ dBm, $P_{DC} = 10$ dBm, $P_{BS} = 30$ dBm, and $\xi = \varsigma = 1.1$.

In Fig. 2a, we plot the ergodic EE of the worst-case user versus Rician factor when $x_{\text{IRS}} = 40$ m. It is observed that the EE of H-IRS with the active/passive elements optimized by the proposed method is close to that by the exhaustive search algorithm, which outperforms that of the fully active/passive IRS under different Rician factors. The reason is that the H-IRS provides an additional degree of freedom for determining the number of active/passive elements, thereby enabling a flexible balance the trade-off between SE and power consumption. In Fig. 2b, we plot the number of passive and active elements of the H-IRS optimized by the proposed algorithm versus Rician factor K. One can observe that the optimized N_{act} and N_{pas} are 5 and 195, respectively when K = 15 dB, while the optimized $N_{\rm act}$ is one and the optimized $N_{\rm pas}$ is zero when $K \leq 10$ dB, which agrees with our analysis in Section III-B. Then, the EE of the H-IRS is equal to that of the fully active IRS (see Fig. 2a). In addition, it is observed that the required number of active elements first increases and then decreases with the Rician factor. This is because the received power scales proportionally w.r.t. $\gamma_1 N_{\text{pas}}^2$ and thus increases significantly with N_{pas} when K is not very small, i.e., γ_1 approaches 1. Benefiting from the high passive beamforming gain in the LoS-dominated channel case, the system tends to employ fewer active elements to reduce power consumption, which is helpful for maximizing EE.

In Fig. 3a, we plot the ergodic EE of the worst-case user versus x_{IRS} when K = 15 dB. First, it is observed that the EE of the three IRSs decrease with x_{IRS} . Second, we observe that the EE of the fully passive IRS decreases significantly and that of the fully active IRS remains almost unchanged with x_{IRS} . The reason is that the fully passive IRS suffers from severe path loss attenuation while the fully active IRS can amplify the signal attenuated after the transmission via the BS-IRS link. In Fig. 3b, we plot the number of passive and active elements of the H-IRS optimized by the proposed algorithm versus the IRS location x_{IRS} . It is observed that the optimized N_{act} is 7 and the optimized $N_{\rm pas}$ is 190 when $x_{\rm IRS}=50$ m, while the optimized $N_{\rm act}$ and $N_{\rm pas}$ are 20 and 0, respectively when $x_{\text{IRS}} \ge 90$ m. Then, the EE of the H-IRS is equal to that of the fully active IRS (see Fig. 3a). In addition, it is observed that the optimized number of active elements increases and that of passive elements decreases with x_{IRS} . It is because more active elements should be deployed when the system suffers severe



Fig. 3. Comparison on EE and elements allocation of the H-IRS versus x_{IRS} .

path loss attenuation, thereby improving the EE.

V. CONCLUSION

This letter studied the elements allocation problem for maximizing the ergodic EE in an H-IRS assisted wireless communication system. We first derived the closed-form expression for a near-optimal solution under the LoS channel case and unveiled that at most one active element is required under the Rayleigh channel case. Then, we proposed an efficient algorithm under the general Rician fading channel case. Simulation results demonstrated that the H-IRS is a promising architecture for flexibly balancing the SE-cost trade-off. In future work, it is worthy of further investigating the EE of H-IRS assisted optical wireless networks.

REFERENCES

- M. D. Renzo *et al.*, "Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2450–2525, Nov. 2020.
- [2] R. Liu *et al.*, "A path to smart radio environments: An industrial viewpoint on reconfigurable intelligent surfaces," *IEEE Wirel. Commun.*, vol. 29, no. 1, pp. 202–208, Feb. 2022.
- [3] C. Ma *et al.*, "Reconfigurable distributed antennas and reflecting surface (RDARS): A new architecture for wireless communications," 2023.
 [Online]. Available: https://doi.org/10.48550/arXiv.2303.06950
- [4] J. Wang, S. Gong, Q. Wu, and S. Ma, "RIS-aided MIMO systems with hardware impairments: Robust beamforming design and analysis," 2022. [Online]. Available: https://doi.org/10.48550/arXiv.2209.11425
- [5] Z. Zhang, L. Dai, X. Chen, C. Liu, F. Yang, R. Schober, and H. V. Poor, "Active RIS vs. passive RIS: Which will prevail in 6G?" *IEEE Trans. Commun.*, vol. 71, no. 3, pp. 1707–1725, Mar. 2023.
- [6] R. Long, Y. Liang, Y. Pei, and E. G. Larsson, "Active reconfigurable intelligent surface-aided wireless communications," *IEEE Trans. Wirel. Commun.*, vol. 20, no. 8, pp. 4962–4975, Mar. 2021.
- [7] G. Chen, Q. Wu, C. He, W. Chen, J. Tang, and S. Jin, "Active IRS aided multiple access for energy-constrained IoT systems," *IEEE Trans. Wirel. Commun.*, vol. 22, no. 3, pp. 1677–1694, Mar. 2023.
- [8] K. Zhi, C. Pan, H. Ren, K. K. Chai, and M. Elkashlan, "Active RIS versus passive RIS: Which is superior with the same power budget?" *IEEE Commun. Lett.*, vol. 26, no. 5, pp. 1150–1154, May 2022.
- [9] N. T. Nguyen *et al.*, "Hybrid active-passive reconfigurable intelligent surface-assisted multi-user MISO systems," in *Proc. IEEE SPAWC Work-shop*, Jul. 2022, pp. 1–5.
- [10] R. S. P. Sankar and S. P. Chepuri, "Beamforming in hybrid RIS assisted integrated sensing and communication systems," in *Proc. EUSIPCO*, Oct. 2022, pp. 1082–1086.
- [11] N. T. Nguyen *et al.*, "Hybrid active-passive reconfigurable intelligent surface-assisted UAV communications," in *Proc. IEEE GLOBECOM*, Dec. 2022, pp. 3126–3131.
- [12] Z. Kang et al., "Active-passive IRS aided wireless communication: New hybrid architecture and elements allocation optimization," 2022. [Online]. Available: https://doi.org/10.48550/arXiv.2207.01244
- [13] K. Liu, Z. Zhang, L. Dai, S. Xu, and F. Yang, "Active reconfigurable intelligent surface: Fully-connected or sub-connected?" *IEEE Commun. Lett.*, vol. 26, no. 1, pp. 167–171, Jan. 2022.