Corrigendum for "Actuator Placement for Symmetric Structural Controllability With Heterogeneous Costs"

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***Abstract*—In this corrigendum, we provide counterexample for Theorem 3 and 4. The lack of correctness of these results arises from the characterization provided in Lemma 1 in the original manuscript that hereafter properly restated. Lastly, we point how the solution to the heterogeneous cost problem can be obtained.**

***Index Terms*—Control of networks, large-scale systems, networked control systems, linear systems.**

Consider the state graph $\mathcal{G}(\bar{A})$ with heterogeneous costs as described in Fig. 1. It readily follows that actuating the central node, which we denote by x_1, suffices to attain symmetric structural controllability (SSC) (i.e., take x_1 as a (degenerate) base path, and, subsequently, append the left and right 3-node cycles to for a state cactus that spans $\mathcal{G}(\bar{A})$). Furthermore, it leads to a minimum actuation cost of $c_1 = 1$, since any other node rooting a cactus anywhere but x_1 will clearly lead to a higher actuation cost of $c_i = 2$. Hence, $B^\star = \mathbb{I}_7(\{1\})$ is the unique solution to Problem 1.

However, as seen in Figure 2, the central node x_1 is not exposed in any possible maximum matching of $\mathcal{G}(\bar{A})$, and, thus, cannot be an exposed node in any possible minimum-cost maximum matching. Thus, the graph in Fig. 1 provides a counterexample for [1, Ths. 3 and 4].

We notice that both Theorem 3 and 4 incorrectness arises from the misinterpreting the second part of [1, Lemma 1]. Lemma 1 should have been stated as follows:

Lemma 1: Let $\mathcal{G}(\bar{A})$ be a connected graph, $\mathcal{G}_1, \ldots, \mathcal{G}_p$ a collection of disjoint state cacti that span $\mathcal{G}(\bar{A})$, where p is the minimum number of such possible collections of cacti, and p' the number of exposed nodes in any maximum matching M of $\mathcal{G}(\bar{A})$. Then, the following dichotomy characterizes the p disjoint state cacti that span $\mathcal{G}(\bar{A})$:

(*i*) if $p' = 0$, then $p = 1$ and $\mathcal{G}_1$ is spanned by the cacti constructed by either choosing an arbitrary matched edge (in M) as the base state path and by recursively appending the edges in M as the subsequent (degenerate) cycles, or by choosing an arbitrary unmatched edge as the base state path and by recursively appending unmatched edges as the subsequent (degenerate) cycles;

(*ii*) if $p' \geq 1$, then $p = p'$ and $\mathcal{G}_1, \ldots, \mathcal{G}_p$ are spanned by the cacti constructed by taking the exposed nodes in M as the base state paths, and by recursively appending the edges in M as the subsequent (degenerate) cycles.

In other words, if a given arbitrary maximum matching M has $p \geq 1$ exposed nodes $x_{i_1}, \ldots, x_{i_p}$, then not only is $\mathcal{J} = \{i_1, \ldots, i_p\}$ a solution to the homogeneous-cost version of Problem 1, but also the constructed cacti detailed in Lemma 1 will each contain *exactly one* of the actuated state nodes $x_{i_1^\star}, \ldots, x_{i_p^\star}$ in a solution to the heterogeneous-cost

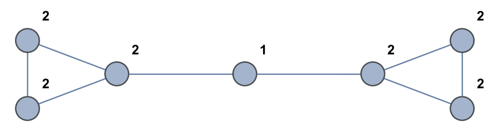

Fig. 1. Connected state graph $\mathcal{G}(\bar{A})$ with heterogeneous costs $c_1 = 1$ and $c_i = 2$ for $i = 2, \ldots, 7$.

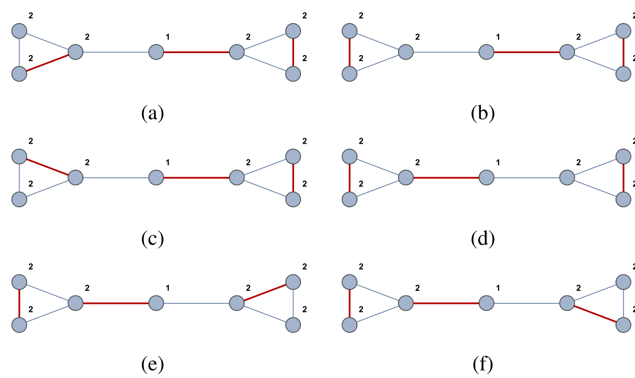

Fig. 2. All maximum matchings of $\mathcal{G}(\bar{A})$.

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version of Problem 1. However, while the constructed cacti from the exposed nodes of M will span disjoint cacti rooted in $\{x_{i_1^\star}, \ldots, x_{i_p^\star}\}$ that themselves span $\mathcal{G}(\bar{A})$, it is not true (in general) that any of the nodes in $\{x_{i_1^\star}, \ldots, x_{i_p^\star}\}$ must coincide with any of the exposed nodes of M, for any M.

Although not as computationally efficient as those algorithms described in [1], the minimum cost algorithm in [2] can used to determine the minimum cost actuation to attain SSC. Specifically, once the minimum cost (direct) input cacti that spans a graph is determined, bidirectionals edges can be readily considered to form undirected input cacti that will attain the minimum cost (undirected) actuator placement by invoking a similar reasoning to that in [2].

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References

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