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Change Detections from Heterogeneous Remote Sensing Images based on Multi-dimension Evidential Reasoning

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Abstract-

In classical evidential reasoning of Shfer's model, all the sources of evidence over the same frame of discernment are underlying considered referring to the invariable decision target, and the conflicting beliefs produced by the conjunctive combination are redistributed by the combination rules to get a specific result which still lies in the same frame. Nevertheless, the sources of evidence to be combined possibly associate with different real targets in some applications, even if they are defined on the same frame. For example, in the data association in multi-target detection, the evidence from the different sensors may refer to different real-targets, although the evidence are defined in the same frame containing various targets. In such case, the fusion results should take care of the joint information of the different exclusive elements, but it is considered as conflicting beliefs in classical way. Besides, the classical frame, whose elements all refer to the single exclusive target separately, is not sufficient to represent such joint hypothesis. So the multi-dimension evidential reasoning (MDER) approach is proposed in this work. The classical frame is extended to multiple dimension denoted by M-D taking into account of the availability of the joint hypothesis of the exclusive elements called joint-element. M-D frame is composed by the joint-element, whose dimensions is coherent with the number of the independent sources. Belief functions are defined in M-D frame for decision-making. MDER is appropriate for the combination of the sources of evidence which don't necessarily arise from the same invariable decision-target. Two kinds of combination rules working in free model and constrained model are proposed for coping with different cases. Several examples are given to illustrate the use of the proposed approach and the difference between the new approach and DSmt.

Keywords: evidence theory, belief functions, change detection, multi-temporal satellite images fusion, DST, DSmt.

I. INTRODUCTION

Change detections from heterogenous multi-temporal remote sensing images is still an important and open problem. The remote sensing images may be acquired from different kinds of sensors, and the parameters of acquisitions may differ from one image to another even when the acquisitions are issued from the same sensor. So the images can be quite different in their modality. That is why, for change detection purpose, the use of the difference image is not an appropriate point of view due to the number of false alarms it induces. Moreover, classification of the change feature is also very important in some applications. Hence the comparison of the classified images seems to be more appropriated. Nevertheless, this yields to deal with uncertain, imprecise and even conflicting information. Evidence theories including Dempster-Shafer Theory (DST) [10], Dezert-Smarandache Theory (DSmt) [11] and Transferable Belief Model (TBM) [13] are good for dealing with such information, and they have been applied for remote sensing applications.

The classification results of remote sensing images can share a common frame of discernment if the images are obtained by the same kind of sensors, but they may lie in the different frames once the images are acquired from the different kinds of sensors. In the classical evidential reasoning, if the sources of evidence to be combined are defined over a common frame, their fusion results will be considered still in the same invariable frame. So it is suitable for the fusion of the images without change occurrences to get more specific classification results. Nevertheless, both content classifications and change detections are involved in the fusion of the multi-temporal remote sensing images. The classical evidential reasoning doesn't work well in such case, since the hypothesis about change occurrences through different sources can't be represented in the classical frame. In the combination of sources of evidence associated with distinct frames, Cartesian product frame is always used, but in the past works, it mainly focus on the representation of joint information from various aspects referring to the same thing instead of change detections.

In this paper, the multi-dimension evidential reasoning (MDER) is proposed for the change detections through the multi-temporal remote sensing images no matter issued from the same or different kinds of sensors. The classification results of each image corresponding the sources of evidence lie in the separate frame, which can be the same or various depending on the properties of images, but their fusion results will work in the multi-dimension frame composed by the Cartesian product of these separate frames. The number of dimensions of the frame depends on the number of multi-temporal images. The

multi-dimension elements can well represent the joint state of the fusion of different images, and it provides more refined information than classical evidential reasoning for the combination of sources of evidence in the common frame, since it takes into account of the sequence of sources. If an area remains the same in multi-temporal images, it will be considered as coherent joint state, whereas the change occurrences corresponds to the incoherent joint state. Combination rules working in free model and constrained model are given in MDER for dealing with different cases. In free model, there is no prior information and all the possible joint hypothesis are considered available, whereas some constraints about the impossible joint hypothesis will be given according to the prior knowledge/experience in constrained model. Belief function $Bel(\cdot)$, plausibility function $Pl(\cdot)$ and pignistic probability $BetP(\cdot)$ [12] are defined in MDER similar as in DST for decision-making support. Two experiments with the real data are given to show interests of the proposed approach.

II. A BRIEF REVIEW OF DSMT

We need to introduce briefly DSMT framework [4], [11] because the MDER approach developed in the sequel shares some common ideas with DSMT, in particular the necessity to deal with hybrid models of the frames in some applications once parts of the constraints information are known. The purpose of DSMT [11] is to overcome the limitations of DST [10] mainly by proposing new underlying models for the frames of discernment in order to fit better with the nature of real problems, and proposing new efficient combination and conditioning rules. In DSMT framework, the elements θ_i , $i = 1, 2, \dots, n$ of a given frame Θ are not necessarily exclusive, and there is no restriction on θ_i but their exhaustivity. The hyper-power set D^Θ in DSMT [4] is defined as the set of all composite propositions built from elements of Θ with operators $\cup$ and $\cap$. For instance, if $\Theta = \{\theta_1, \theta_2\}$, then $D^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cap \theta_2, \theta_1 \cup \theta_2\}$. A (generalized) basic belief assignment (bba for short) is defined as the mapping $m : D^\Theta \rightarrow [0, 1]$. The generalized belief and plausibility functions are defined in almost the same manner as in DST.

Two models¹ (the free model and hybrid model) in DSMT can be used to define the bba's to combine. In the free DSMT model, the sources of evidence are combined without taking into account integrity constraints. When the free DSMT model does not hold because the true nature of the fusion problem under consideration, we can take into account some known integrity constraints and define bba's to combine using the proper hybrid DSMT model. All details of DSMT with many examples can be easily found in [11] available freely on the web.

A. The space of multi-dimension evidential reasoning

The sources of evidence acquired from each piece of images can be defined in separate single-dimension frame, but their fusion results will be considered in multi-dimension frame to well represent the joint state of the multi-temporal images. In the M-D frame, all the elements can be called joint hypothesis, and the subelements in each dimension of joint hypothesis corresponds to different sources of evidence. Let's consider n pieces sources of evidence respectively defined over n single-dimension frames as $\Theta_1, \Theta_2, \dots, \Theta_n$, which can be the same or distinct. Then their associated M-D frame of their fusion results is given by

$$\begin{aligned}\Theta^n &= \Theta_1 \times \Theta_2 \times \dots \times \Theta_n \\ &= \{(x_1, x_2, \dots, x_n) | x_i \in \Theta_i, i = 1, 2, \dots, n\}\end{aligned}$$

where Θ_i and $\Theta_j, i \neq j$ can be same or different. $\times$ is the Cartesian product operator. The cardinality of $|\Theta^n| = |\Theta_1| \times |\Theta_2| \times \dots \times |\Theta_n|$. The joint hypothesis $(x_1, x_2, \dots, x_n)$ can be considered as joint-state through different images with the interpretation that in the same area of n pieces of multi-temporal images, the content classification is x_i in the image No.i, $i = 1, \dots, n$.

The space of MDER will be denoted and defined by G^{Θ^n} . In this paper, we assume to work in a more simple case where $G^\Theta = 2^\Theta$, and it will be defined by

$$2^{\Theta^n} = 2^{\Theta_1 \times \Theta_2 \times \dots \times \Theta_n}$$

2^{Θ^n} can be called M-D power-set, which is composed by all the subsets in Θ^n with the union operator $\cup$. The use of the operators $\cup$ and $\cap$ in MDER should satisfy the following reasonable conditions:

(C1) Distributivity of $\cup$ w.r.t. $\rightarrow$

$$(A \cup B, C) = (A, C) \cup (B, C)$$

(C2) Distributivity of $\rightarrow$ w.r.t. $\cup$

$$(A, B \cup C) = (A, B) \cup (A, C)$$

(C3) Associativity of state-transition

$$(A, B) \cap (C, D) = (A \cap C, B \cap D).$$

¹ Actually, Shafer's model, considering all elements of the frame as truly exclusive, can be viewed as a special case of hybrid model.

Please note that in most cases

$$(x_1, \dots, x_n) \cup (y_1, \dots, y_n) \neq (x_1 \cup y_1, \dots, x_n \cup y_n)$$

If $\forall x_i \neq y_i$; x_i, y_i are singletons, $(x_1, \dots, X_n) \cup (y_1, \dots, y_n)$ indicates only two possible hypothesis, whereas the element $(x_1 \cup y_1, \dots, x_n \cup y_n)$ represents $2 \times 2 \times \dots \times 2 = 2^n$ possible hypothesis. It is obvious that they are quite different, and $(x_1 \cup y_1, \dots, x_n \cup y_n)$ is much more imprecise than $(x_1, \dots, x_n) \cup (y_1, \dots, y_n)$. For instance, $(x_1, x_2) \cup (y_1, y_2) \neq (x_1 \cup y_1, x_2 \cup y_2) = (x_1, x_2) \cup (x_1, y_2) \cup (y_1, x_2) \cup (y_1, y_2)$ following C1 and C2.

As we see, the important and major difference between the classical approaches (DST, TBM, DSMT) and MDER approach is the choice of the fusion space we are working with. With DST, TBM or DSMT, the fusion space we work with is always the same (independent of the number of sources) as soon as the sources are defined with respect to same frame Θ , whereas with MDER approach the fusion space is always increasing with the number of sources. This of course increases the complexity of DER approach, but this is the "price to pay" to acquire the more efficient and interesting information as it will be shown in following sections of this paper.

Example 1: Let's consider 2-dimension frame as $\Theta^2 = \Theta \times \Omega$, and $\Theta = \{\theta_1, \theta_2\}$, $\Omega = \{\omega_1, \omega_2\}$. Then, the power set of transitions we want to work with for such very simple example will be given by:

$$\begin{aligned} \Theta^2 &= \{(\theta_1, \omega_1), (\theta_1, \omega_2), (\theta_2, \omega_1), (\theta_2, \omega_2)\} \\ 2^{\Theta^2} &= \{(\emptyset, \emptyset), (\emptyset, \omega_1), (\emptyset, \omega_2), (\theta_1, \emptyset), (\theta_2, \emptyset), \\ &\quad (\theta_1, \omega_1), (\theta_1, \omega_2), (\theta_2, \omega_1), (\theta_2, \omega_2), \\ &\quad (\theta_1, \omega_1) \cup (\theta_2, \omega_2), (\theta_1, \omega_2) \cup (\theta_2, \omega_1), \\ &\quad (\theta_1, \omega_1) \cup (\theta_1, \omega_2) = (\theta_1, \omega_1 \cup \omega_2) = (\theta_1, \Omega) \\ &\quad (\theta_2, \omega_1) \cup (\theta_2, \omega_2) = (\theta_2, \omega_1 \cup \omega_2) = (\theta_2, \Omega) \\ &\quad (\theta_1, \omega_1) \cup (\theta_2, \omega_1) = (\theta_1 \cup \theta_2, \omega_1) = (\Theta, \omega_1) \\ &\quad (\theta_1, \omega_2) \cup (\theta_2, \omega_2) = (\theta_1 \cup \theta_2, \omega_2) = (\Theta, \omega_2) \\ &\quad (\theta_1, \omega_1) \cup (\theta_1, \omega_2) \cup (\theta_2, \omega_1) \\ &\quad (\theta_1, \omega_1) \cup (\theta_1, \omega_2) \cup (\theta_2, \omega_2) \\ &\quad (\theta_2, \omega_1) \cup (\theta_2, \omega_2) \cup (\theta_1, \omega_1) \\ &\quad (\theta_2, \omega_1) \cup (\theta_2, \omega_2) \cup (\theta_1, \omega_2) \\ &\quad (\theta_1, \omega_1) \cup (\theta_1, \omega_2) \cup (\theta_2, \omega_1) \cup (\theta_2, \omega_2) = (\Theta, \Omega)\} \end{aligned}$$

These elements involve with $\emptyset$ of course can be seen as the same with $\emptyset$ in classical way if necessary.

In Example 1, Ω and θ can be equal or not. If Θ and Ω are two distinct frames, it implies the properties of the two pieces of images are total different and their classification results lies in different frames. Then, (θ_i, ω_j) can be considered as the content in the same area belongs to θ_i and ω_j respectively in different images. If Ω is equal to Θ , it indicates the two images possibly issued from the same kind of sensors, and their classification results can be defined in the same frame. $(\theta_i, \omega_j) \stackrel{\Delta}{=} (\theta_i, \theta_j)$ means the content transferred from θ_i to θ_j during the two images.

We recall that the imprecise elements are derived from application of conditions C1 and C2 and not from the componentwise union of n -uples. The cardinality of 2^{Θ^n} is given as

$$|2^{\Theta^n}| = 2^{|\Theta^n|} = 2^{|\Theta_1| \times |\Theta_2| \times \dots \times |\Theta_n|} \quad (1)$$

where all the elements involved with $\emptyset$ are consider same as empty sets.

III. SOME BASIC DEFINITIONS IN MDER

The belief function $Bel(\cdot)$, or plausibility function $Pl(\cdot)$, and pignistic probability $BetP(\cdot)$ ² [12], [13] are important and basic functions in evidential reasoning. They can be used to transfer the belief to probability for the convenience of decision-making. These functions can be also used in MDER approach as well.

All the elements in Θ^n has a specific and unique meaning, and they are considered the singleton elements. All the focal elements in T_n^Θ can be decomposed in the disjunctive canonical form using these singleton elements with the operator $\cup$, and

² $DSmP(\cdot)$ transformation proposed in [11] (Vol.2) which provides of better probabilistic informational content than $BetP(\cdot)$ can also be chosen instead. But $DSmP(\cdot)$ is more complicated to implement than $BetP(\cdot)$ and it has not been tested in our application for now.

we call it canonical focal element. For example, $m((\theta_1 \cup \theta_2), \theta_3) = m((\theta_1, \theta_3) \cup (\theta_2, \theta_3))$ because of the condition (C1). The belief, plausibility functions and the pignistic transformation are defined in MDER similarly as in DST; that is:

$$Bel(A) = \sum_{A, B \in 2^{\Theta^n}; B \subset A} m(B) \quad (2)$$

$$Pl(A) = \sum_{A, B \in 2^{\Theta^n}; A \cap B \neq \emptyset} m(B) \quad (3)$$

The interval $[Bel(A), Pl(A)]$ is then interpreted as the lower and upper bounds of imprecise probability for decision-making support [10] and the pignistic probability $BetP(A)$ commonly used to approximate the unknown probability $P(A)$ in $[Bel(A), Pl(A)]$ is calculated by:

$$BetP(A) = \sum_{A, B \in 2^{\Theta^n}, A \subset B} \frac{|A \cap B|}{|B|} m(B) \quad (4)$$

where $|X|$ is the cardinal of the element X . In MDER, the cardinal of $A \in 2^{\Theta^n}$ is the number of the singleton elements it contains in its canonical form.

For example, if we have a bba's over $\Theta^2 = \Theta \times \Theta$, and $\Theta = \{\theta_1, \theta_2, \theta_3\}$ as:
 $m(\theta_1, \theta_2) = 0.5, m(\theta_1, \theta_2 \cup \theta_3) = 0.2, m(\Theta, \theta_3) = 0.3$

The cardinal of the imprecise elements is counted by:

$$|(\theta_1, \theta_2 \cup \theta_3)| = |(\theta_1, \theta_2) \cup (\theta_1, \theta_3)| = 2,$$

$$(\Theta, \theta_3) = |(\theta_1, \theta_3) \cup (\theta_2, \theta_3) \cup (\theta_3, \theta_3)| = 3.$$

Then, one gets

$$\begin{aligned} Bel(\theta_1, \theta_2) &= m(\theta_1, \theta_2) = 0.5, \\ Pl(\theta_1, \theta_2) &= m(\theta_1, \theta_2) + m(\theta_1, \theta_2 \cup \theta_3) = 0.7, \\ Pl(\theta_1, \theta_3) &= m(\theta_1, \theta_2 \cup \theta_3) + m(\Theta, \theta_3) = 0.5, \\ Pl(\theta_2, \theta_3) &= m(\Theta, \theta_3) = 0.3, \\ Pl(\theta_3, \theta_3) &= m(\Theta, \theta_3) = 0.3, \\ BetP(\theta_1, \theta_2) &= m(\theta_1, \theta_2) + \frac{m(\theta_1, \theta_2 \cup \theta_3)}{2} = 0.6, \\ BetP(\theta_1, \theta_3) &= \frac{m(\theta_1, \theta_2 \cup \theta_3)}{2} + \frac{m(\Theta, \theta_3)}{3} = 0.2, \\ BetP(\theta_2, \theta_3) &= \frac{m(\Theta, \theta_3)}{3} = 0.1, \\ BetP(\theta_3, \theta_3) &= \frac{m(\Theta, \theta_3)}{3} = 0.1. \end{aligned}$$

IV. COMBINATION RULES IN MULTI-DIMENSION EVIDENTIAL REASONING

The combination rules [6], [10], [14] in classical frame mainly try to get very specific results by various kinds of redistribution of the conflicting belief. Nevertheless, the conflicting belief may represent interesting information in some applications, and they will be totally or partly kept in M-D frame according to the model. The new combination rules working in free model and constrained model in M-D frame are proposed to get the useful information as much as possible through the fusion of the sources of evidence.

A. Combination rule in free model

In the free model, there is no available prior knowledge in the fusion, and all kinds of hypothesis in 2^{Θ^n} are considered possible to happen.

In the fusion of sources of evidence obtained from multi-temporal images, the bba's associated with each source can be extended to M-D format. For example, $m_i(A) \triangleq m_i(\Theta_1, \Theta_2, \dots, \Theta_{i-1}, A, \Theta_{i+1}, \dots, \Theta_n)$, and it can be interpreted as the mass of the hypothesis A in source i without knowing anything in the other sources.

Let's assume n sequential sources of information defined over the frames $\Theta_1, \dots, \Theta_n$ to be combined as $\mathbf{m} = \mathbf{m}_1 \oplus \mathbf{m}_2 \oplus \dots \mathbf{m}_n$.

The conjunctive combination rule is given by $\forall A, X_i \in 2^{\Theta^n}$

$$m(A) = m_1(X_1)m_2(X_2) \dots m_n(X_n), X_1 \cap X_2 \cap \dots X_n = A. \quad (5)$$

This combination rule of the M-D bba's is associative and commutative.

The single dimension bba's can also be directly combined without the extending to M-D format. The combination rule of n sequential sources of evidence represented by single dimension bba's in M-D frame is defined by $\forall x_1, x_2, \dots, x_n \in 2^{\Theta_i}, A \in 2^{\Theta^n}$

$$m(A) = m_1(x_1)m_2(x_2) \cdots m_n(x_n), A = (x_1, x_2, \dots, x_n) \quad (6)$$

obviously, this combination rule is associative but not commutative.

For simplicity, the sequential sources of information can be iteratively combined one by one as

$$m_{1 \rightarrow n}(A) = m_{1 \rightarrow n-1}(X)m_n(x_n), \quad (7)$$

where $X = (x_1, x_2, \dots, x_{n-1}) \in 2^{\Theta^{n-1}}, A = (X, x_n) \in 2^{\Theta^n}$. In this free model, the result of the combination is very specific since all kinds of change occurrences are distinguished in the results.

The following example will show the combination of sources of evidence in different frames.

Example 2: Let's consider two pieces of images obtained by different sensors, and their classification results are defined in two different frames of discernment $\Theta = \{\theta_1, \theta_2\}$ and $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and the bba's are given by

	θ_1	θ_2	Θ
$\mathbf{m}_1$	0.6	0.1	0.3
	ω_1	ω_2	ω_3
$\mathbf{m}_2$	0.3	0.5	0.2

With MDER free rule (MDERf):

$$\begin{aligned} m(\theta_1, \omega_1) &= 0.18, \\ m(\theta_1, \omega_2) &= 0.3, \\ m(\theta_1, \omega_3) &= 0.12, \\ m(\theta_2, \omega_1) &= 0.03, \\ m(\theta_2, \omega_2) &= 0.05, \\ m(\theta_2, \omega_3) &= 0.02, \\ m(\Theta, \omega_1) &= 0.09, \\ m(\Theta, \omega_2) &= 0.15, \\ m(\Theta, \omega_3) &= 0.06, \end{aligned}$$

For the n-d singleton elements, one gets

	$Bel(.)$	$BetP(.)$	$Pl(.)$
(θ_1, ω_1)	0.18	0.225	0.27
(θ_1, ω_2)	0.3	0.375	0.45
(θ_1, ω_3)	0.12	0.15	0.18
(θ_2, ω_1)	0.03	0.075	0.12
(θ_2, ω_2)	0.05	0.125	0.2
(θ_2, ω_3)	0.02	0.05	0.08

From the results, we can see the joint state (θ_1, ω_2) is most likely to be true, and it indicates that the associated area belonged to θ_1 in 1^{st} image, and it was ω_2 in the 2^{ed} image. if θ_1 and ω_2 are coherent with each other from different aspects, it implies the content of the area is invariable. Otherwise, it was incoherent joint state, and this represents change occurrence.

In the following examples, we will show how MDER works and its difference from the classical methods.

Example 3: Let's consider two sources of information in the same frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$, and the bba's are given by

	θ_1	θ_2	θ_3	θ_4
$\mathbf{m}_1$	0.45	0	0.25	0.3
$\mathbf{m}_2$	0	0.1	0.55	0.35

With DS_m classic/conjunctive rule (DS_mC):

$$\begin{aligned}
m(\theta_1 \cap \theta_2) &= 0.045, \\
m(\theta_1 \cap \theta_3) &= 0.2475, \\
m(\theta_1 \cap \theta_4) &= 0.1575, \\
m(\theta_2 \cap \theta_3) &= 0.025, \\
m(\theta_2 \cap \theta_4) &= 0.03, \\
m(\theta_3 \cap \theta_4) &= \mathbf{0.2525}, \\
m(\theta_3) &= 0.1375, \\
m(\theta_4) &= 0.105.
\end{aligned}$$

With DERf rule:

$$\begin{aligned}
m(\theta_1, \theta_2) &= m_1(\theta_1)m_2(\theta_2) = 0.045 \\
m(\theta_1, \theta_3) &= m_1(\theta_1)m_2(\theta_3) = 0.2475, \\
m(\theta_1, \theta_4) &= m_1(\theta_1)m_2(\theta_4) = 0.1575, \\
m(\theta_3, \theta_2) &= m_1(\theta_3)m_2(\theta_2) = 0.025, \\
m(\theta_3, \theta_3) &= m_1(\theta_3)m_2(\theta_3) = 0.1375, \\
m(\theta_3, \theta_4) &= m_1(\theta_3)m_2(\theta_4) = \mathbf{0.0875}, \\
m(\theta_4, \theta_2) &= m_1(\theta_4)m_2(\theta_2) = 0.03, \\
m(\theta_4, \theta_3) &= m_1(\theta_4)m_2(\theta_3) = \mathbf{0.165}, \\
m(\theta_4, \theta_4) &= m_1(\theta_4)m_2(\theta_4) = 0.105.
\end{aligned}$$

Note that the main difference between DS_mC and MDERf lies in the ability of MDERf to refine the partial conflicts into several distinct joint hypothesis. More precisely, the the mass 0.2525 of the partial conflict $\theta_3 \cap \theta_4$ computed as $m(\theta_3 \cap \theta_4) = m_1(\theta_3)m_2(\theta_4) + m_1(\theta_4)m_2(\theta_3) = 0.2525$ with classical DS_m free rule becomes clearly split into two joint hypothesis (θ_3, θ_4) and (θ_4, θ_3) with associated masses 0.0875 and 0.165.

One sees that, if DS_mC rule is used to detect the change occurrence in some applications, $\theta_3 \cap \theta_4$ gets the biggest mass which indicates that the hypothesis of change occurrences (θ_3, θ_4) or (θ_4, θ_3) is most likely to happen. Nevertheless, the most mass (or credibility) is actually committed to θ_1 in by the source no.1, and to θ_3 by the source no. 2. So the change occurrence from θ_1 to θ_3 is intuitively most likely to occur, which is consistent with the result provided by MDERf rule.

Example 4: Let's consider three sources of information in the same frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and the bba's are given by

	θ_1	θ_2	θ_3	$\theta_1 \cup \theta_3$	Θ
m₁	0.8	0	0.1	0	0.1
m₂	0	1	0	0	0
m₃	0.6	0	0	0.4	0

With DS_mC rule :

$$\begin{aligned}
m(\theta_1 \cap \theta_2) &= 0.84, \\
m(\theta_2 \cap \theta_3) &= 0.04, \\
m(\theta_1 \cap \theta_2 \cap \theta_3) &= 0.06, \\
m(\theta_2 \cap (\theta_1 \cup \theta_3)) &= 0.06.
\end{aligned}$$

With MDERf rule:

$$\begin{aligned}
m(\theta_1, \theta_2, \theta_1) &= 0.48, \\
m(\theta_3, \theta_2, \theta_1) &= 0.06, \\
m(\Theta, \theta_2, \theta_1) &= 0.04, \\
m(\theta_1, \theta_2, \theta_1 \cup \theta_3) &= 0.32, \\
m(\theta_3, \theta_2, \theta_1 \cup \theta_3) &= 0.04, \\
m(\Theta, \theta_2, \theta_1 \cup \theta_3) &= 0.06.
\end{aligned}$$

For the singleton elements, based on MDERf, one gets:

	$Bel(.)$	$BetP(.)$	$Pl(.)$
$(\theta_1, \theta_2, \theta_1)$	0.48	0.663	0.9
$(\theta_3, \theta_2, \theta_1)$	0.06	0.103	0.2
$(\theta_1, \theta_2, \theta_3)$	0	0.17	0.38
$(\theta_2, \theta_2, \theta_1)$	0	0.024	0.1
$(\theta_2, \theta_2, \theta_3)$	0	0.01	0.06
$(\theta_3, \theta_2, \theta_3)$	0	0.03	0.1

One can see that most belief is committed to the joint hypothesis $(\theta_1, \theta_2, \theta_1)$ according to the results of MDERf. It can be interpreted in the application of data association as m_1 and m_3 are associated with the same target θ_1 , whereas m_2 refers to θ_2 . If it is used in the change detection of multi-temporal satellite images, it can be considered as the content transfers from θ_1 to θ_2 , and then back to θ_1 in one area of the images. Nevertheless, the elements in DSmC are all single dimension, and they can't provide the joint or transition information.

B. Combination rule in constrained model

In some applications, part of joint hypothesis in 2^{Θ^n} is known impossible to happen as the prior information, and these hypothesis can be considered as empty sets. The set $\emptyset \triangleq \{\emptyset_{\mathcal{M}}, \emptyset\}$ can be defined in introducing some integrity constraints as done in the hybrid model of DSmT. $\emptyset_{\mathcal{M}}$ includes all the hypothesis in 2^{Θ^n} , which have been forced to be empty because of the chosen integrity constraints in the model $\mathcal{M}$, and $\emptyset$ is the classical empty set. The mass of the empty sets arising from integrity constraints can be distributed to the other focal elements.

The notation $(x_1, x_2, \dots, x_n) \stackrel{\mathcal{M}}{=} X$, means that the hypothesis $(x_1, x_2, \dots, x_n)$ is equivalent to X in the underlying model $\mathcal{M}$ given the integrity constraints.

- **DER_{DS} rule of combination:**

The mass of empty sets can be distributed by a lot of ways like Dempster's rule [10], Yager's rule [14], etc. For simplicity, it is proportionally distributed to the other focal elements similarly to Dempster-Shafer's rule denoted by $MDER_{DS}$ in this work. All the single dimension bba's are extended to M-D at first. Then, the combination rule of n sequential sources of evidence is mathematically defined as follows:

$$\forall A, X, Y \in 2^{\Theta^n}$$

$$m(A) = \frac{\sum_{X \cap Y \stackrel{\mathcal{M}}{=} A} m_1(X)m_2(Y)}{1 - K} \quad (8)$$

where the factor $\frac{1}{1-K}$ is used for the normalization of the results, and K represents the mass of belief committed to the empty sets (i.e. the degree of conflict) which is given by

$$K = \sum_{X \cap Y \in \emptyset} m_1(X)m_2(Y) \quad (9)$$

Please note that this combination rule seems similar to Dempster's rule, but it works in a quite different frame. It is associative and communicative, and it can be used to combine the n sources of evidence one by one.

When considering the direct combination of n sequential single dimension bba's altogether, one has $\forall A \in 2^{\Theta^n}$, and $x_i \in 2^{\Theta_i}, i = 1, 2, \dots, n$

$$m(A) = \frac{\sum_{(x_1, x_2, \dots, x_n) \stackrel{\mathcal{M}}{=} A} m_1(x_1) \cdots m_n(x_n)}{1 - K} \quad (10)$$

where

$$K = \sum_{(x_1, x_2, \dots, x_n) \in \emptyset} m_1(x_1) \cdots m_n(x_n) \quad (11)$$

Remark: The summation introduced in (8) and (10) allows to take into account the integrity constraints of the model of the space of transitions as shown in the next example.

Example 4:(Continued) Let's continue Example 4. Once some constraints about the impossible joint hypothesis are available as

$$\emptyset \stackrel{\mathcal{M}}{=} \{(\Theta, \theta_2, \theta_3), (\theta_3, \Theta, \Theta)\},$$

$MDER_{DS}$ can be used in the constrained model. The constraints mean that θ_2 and θ_3 can't be in the last two dimensions at same time whatever the elements are in the 1st dimension, and θ_3 will never be in the 1st dimension whatever the last two dimensions are.

Therefore, these beliefs of the joint hypothesis $(\theta_3, \theta_2, \theta_1)$ and $(\theta_3, \theta_2, \theta_1 \cup \theta_2)$ in free model take part in the conflict. Thus, the mass of belief committed to the total conflict is therefore given by:

$$K = m_f(\theta_3, \theta_2, \theta_1) + m_f(\theta_3, \theta_2, \theta_1 \cup \theta_2) = 0.1$$

Due to the integrity constraints $(\Theta, \theta_2, \theta_3), (\theta_3, \Theta, \Theta) \in \emptyset$ and the conditions given by C1 and C2, one has

$$\begin{aligned} (\theta_1, \theta_2, \theta_1 \cup \theta_3) &= (\theta_1, \theta_2, \theta_1) \cup (\theta_1, \theta_2, \theta_3) \\ &\stackrel{\mathcal{M}}{=} (\theta_1, \theta_2, \theta_1) \cup \emptyset \\ &= (\theta_1, \theta_2, \theta_1); \\ (\Theta, \theta_2, \theta_1) &= (\theta_3, \theta_2, \theta_1) \cup (\theta_1 \cup \theta_2, \theta_2, \theta_1) \\ &\stackrel{\mathcal{M}}{=} \emptyset \cup (\theta_1 \cup \theta_2, \theta_2, \theta_1) \\ &= (\theta_1 \cup \theta_2, \theta_2, \theta_1); \\ (\Theta, \theta_2, \theta_1 \cup \theta_3) &= (\theta_3, \theta_2, \theta_1 \cup \theta_3) \cup (\theta_1 \cup \theta_2, \theta_2, \theta_1) \cup (\theta_1 \cup \theta_2, \theta_2, \theta_3) \\ &\stackrel{\mathcal{M}}{=} \emptyset \cup (\theta_1 \cup \theta_2, \theta_2, \theta_1) \cup \emptyset \\ &= (\theta_1 \cup \theta_2, \theta_2, \theta_1). \end{aligned}$$

Therefore, the mass of $m_f(\theta_1, \theta_2, \theta_1 \cup \theta_3)$ must be transferred to $m(\theta_1, \theta_2, \theta_1)$, and $m_f(\Theta, \theta_2, \theta_1)$ and $m_f(\Theta, \theta_2, \theta_1 \cup \theta_3)$ must be transferred to $m(\theta_1 \cup \theta_2, \theta_2, \theta_1)$.

Finally, the result given by DER_{DS} rule is

$$\begin{aligned} m(\theta_1, \theta_2, \theta_1) &= \frac{m_f(\theta_1, \theta_2, \theta_1) + m_f(\theta_1, \theta_2, \theta_1 \cup \theta_3)}{1 - K} = 0.9, \\ m(\theta_1 \cup \theta_2, \theta_2, \theta_1) &= \frac{m_f(\Theta, \theta_2, \theta_1) + m_f(\Theta, \theta_2, \theta_1 \cup \theta_3)}{1 - K} = 0.1. \end{aligned}$$

In this constrained model, $MDER_{DS}$ gets more specific results than in the free model because of the use of the constraints. In the constrained model, if all transitions among the exclusive elements are not allowed, it will becomes Shafer's model, and the $MDER_{DS}$ combination rule reduces to classical Dempster-Shafer's rule of combination.

V. APPLICATION ON REAL REMOTE SENSING IMAGES

VI. CHANGE DETECTIONS WITH A PAIR OF SPOT XS-ERS AND SPOT IMAGES

In this experiment, we want to show how MDER works for the change detections between a pair of heterogenous images form different sensors. Quartile regression has been applied on this pair of images: SPOT Xs-ERS images and SPOT images corresponding to a flood over Gloucester, U.K. from October and November 2000. The ERS *before* image reflecting the roughness of the ground was acquired on 10/21/1999, and the multi-spectral SPOT HRV *after* image was obtained on 10/21/2000 during a flood that occurs in October 2000, as shown on Fig.1 and 2.

At first, the bba's associated with the mass of belief of classes in each image are determined based on the image classification. In this level, many supervised/unsupervised methods can be applied for the image classification. In this work, the unsupervised clustering method ECM (Evidential C-Means), which is adapted to the classification of uncertain data in belief functions framework, is applied for the images classification using the pixel value, and it results can be directly used as bba's. The ERS image is gray image, and it is not so clear and specific as the colorful SPOT image. The number of clusters for ERS *before* image is given by $k_E = 3$, and its classification results are defined in the frame as

$$\Theta = \{\theta_1 \triangleq \text{Dark area}, \theta_2 \triangleq \text{Gray area}, \theta_3 \triangleq \text{White area}\},$$

whereas SPOT *after* image can be clustered $k_S = 4$ groups as as

$$\Omega = \{\omega_1 \triangleq \text{Red area}, \omega_2 \triangleq \text{Dark-red area}, \omega_3 \triangleq \text{Green area}, \omega_4 \triangleq \text{Dark-green area}\}.$$

The other tuning parameters are defined by: Maximum number of iterations $T = 10$, Termination threshold $\varepsilon = 3$, Termination measure $E = 1$. The normalized membership is used as the mass functions (bba's). As to decision making, we take the criteria that true hypothesis gets the maximum of pignistic probability.

In the fusion process, the constrained model is selected, since we can have some prior knowledge that the change occurrence mainly happened about the flood. It is not necessary to consider all kinds of joint hypothesis, since some incoherent joint states are impossible and they can be forced to be empty set. We can find some links between the two frames that θ_1 mainly corresponding to ω_1 or ω_2 , θ_2 mainly corresponding to ω_4 , and θ_3 mainly corresponding to ω_3 . So we only consider such joint sates are available including the coherent joint states as (θ_1, ω_1) , (θ_1, ω_2) , (θ_2, ω_4) , (θ_3, ω_3) , and the incoherent joint state (θ_1, ω_3) which is considered as change occurrences.

The fusion results by the combination rules DER_{DS} are shown as Fig.[], and the comparison results between the change occurrences extracted from Fig.[] and the ground truth are shown in Fig.[].

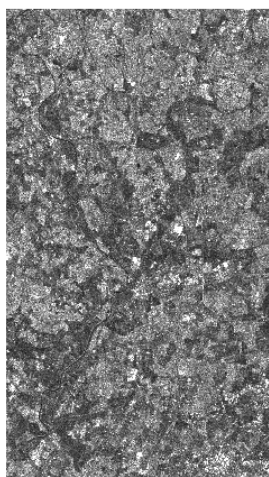


Figure 1. before image: 11/16/1999

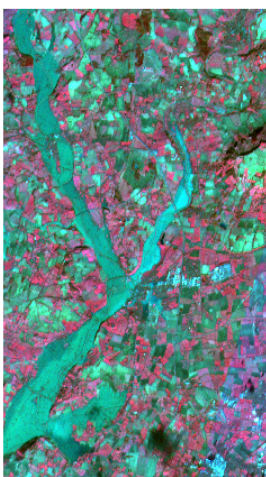


Figure 2. after image: 10/21/2000



Figure 3. Ground truth of the flood

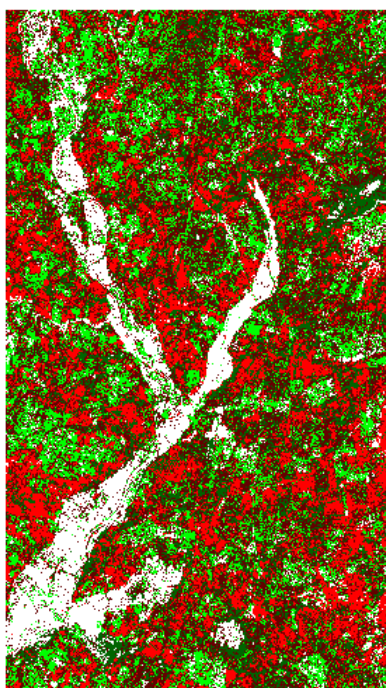


Figure 4. Fusion result by DER_{DS} with all transitions

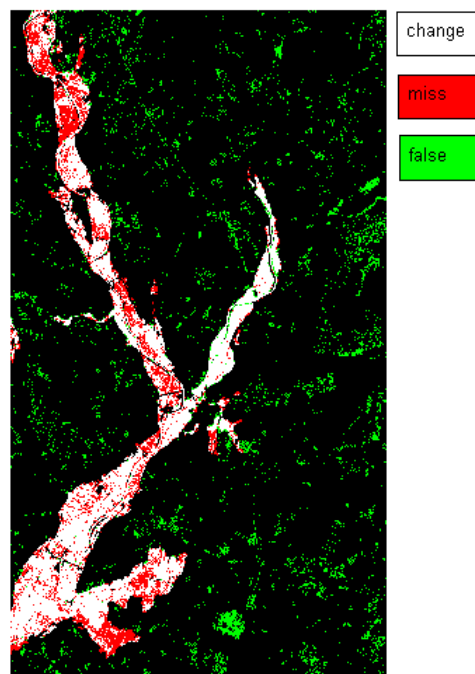


Figure 5. Results of change detections

As we can see, the fusion results not only present the change occurrences but also give some indication of the content classification, which can be helpful in some applications. Nevertheless, there are still some miss detections and false alarms. Some parts along the river is gray or even white color before flood, and they are green after the flood. As our above analysis, gray and white are mainly coherent with green between the two images. So they are not considered as change occurrences, and it mainly leads to the miss detections. The reason of the false alarms mainly lies that the changing of pixels in some small areas similarly to the changing in the flood area during the images. As we know, the flood mainly happened along the river. So if some more prior information about location of the river is known, false alarms would be reduced using the prior probability about change occurrences.

VII. CONCLUSIONS

Multi-dimension evidential reasoning (MDER) is proposed in this work, and it is shown how to use MDER in data association in multi-target detection. MDER is appropriate to combine the sources of evidence which are not necessarily refers to the same thing by introducing the multi-dimension (M-D) frame of belief functions. The joint hypothesis in M-D frame can well represent the joint information from the different sources of evidence. The belief functions $bel(\cdot)$, plausibility function $Pl(\cdot)$ and pignistic probability $BetP(\cdot)$ in M-D frame are defined similarly as in DST for the convenience of decision-making. The free model of MDER is designed for the combination of sources of evidence in case that no prior knowledge about the joint hypothesis is available. If some constraints on the impossible joint hypothesis are acquired, the constrained model will be adopted to get better fusion results with less computational complexity. $MDER_{DS}$ rule is given in the constrained model as the direct extensions of Dempster-Shafer's rule in M-D frame. Several simple numerical examples were given to show how to use MDER and to show its difference with classical fusion approaches. Finally, the application of MDER for data association in multi-target detection is introduced by a simple example, and it is compared with the method using conflicting beliefs. The comparison indicates that MDER is more appropriate in this application, especially in case of the similar bba's.

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