

# Intrinsic Mode Cross Correlation: A Novel Technique to Identify Scale-Dependent Lags Between Two Signals and Its Application to Ionospheric Science

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**Abstract**—In this work, we address the following question: can we use modern, cutting-edge techniques conceived for the analysis of nonlinear non-stationary signals to measure scale-wise lags? To this scope, we propose a novel technique, called intrinsic mode cross correlation (IMXC) method, which leverages on the decomposition of nonlinear non-stationary signals by the multivariate fast iterative filtering (MvFIF) technique and the computation of a scale-by-scale cross correlation. We evaluate this technique on artificial signals (whose ground truth is known) and plasma density data provided by the Langmuir probes onboard the Swarm satellites. We show that this technique allows indeed to reconstruct the lag dependence on the involved spatio/temporal scales for the artificial dataset (even in the presence of high levels of noise) and to estimate them in a real-life signal. This can pave the way to future uses of this technique in contexts in which the causation chain can be hidden in a complex, multiscale coupling of the investigated features.

**Index Terms**—I.5.4.l sciences, I.5.4.m signal processing, I.5.4.o waveform analysis.

## I. INTRODUCTION

THE development of modern techniques for the analysis of multiscale systems has become of paramount importance, especially for the investigation of natural phenomena whose complexity manifests on a wide range of spatial and temporal scales. This is the case of the earth's ionosphere, featured

by a complex behavior due to its nonlinear coupling with the solar wind-magnetosphere system from above and with the lower atmosphere from below (see [1]). In this context, we present a novel approach, called intrinsic mode cross correlation (IMXC spelled I-M-cross-C) method, for the scale-wise measurement of lags between two complex and non-stationary signals. We leverage on the multivariate fast iterative filtering (MvFIF) [2] technique, being the multivariate implementation of the fast iterative filtering (FIF) technique [4]. The lags are then identified on a scale-by-scale basis using the maximum cross correlation among homogeneous modes. The scale-wise lags' identification abilities are tested on artificial signals, for which the ground truth is known *a priori*, under different levels of additive noise. The proposed approach is then used on plasma density data provided by the Langmuir probes onboard the Swarm satellites, addressing a simple case of interest.

## II. METHODS

### A. Fast Iterative Filtering (FIF)

FIF [3] is a decomposition method that splits a non-stationary multicomponent signal into simple oscillatory components, named intrinsic mode components (IMCs). Recently, FIF has been extended to handle multivariate signals in what is called multivariate FIF (MvFIF) [2]. When two measurements, assumed to be associated with the “cause” and the “effect” of some physical phenomenon, are analyzed as two channels of the MvFIF technique, the respective IMCs produced are guaranteed to possess the same frequency ranges. This is a fundamental feature required to ensure a suitable scale-wise comparison of the IMCs. The IMCs produced via MvFIF are comparable with the results of Hilbert–Huang transform or other empirical mode decomposition-based techniques, as well as alternative methods. Nevertheless, the MvFIF method proved to have several advantages. In particular, we mention here its low computational complexity which makes it the fastest technique of its kind; the guaranteed uniqueness of the derived decompositions; a complete mathematical framework; and a complete adaptivity to the signal under investigation ensuring that there is no need to set *a priori* neither the number of components to be extracted nor the basis to be used in the process. Interested readers can find more details in [2], [4], and [9].

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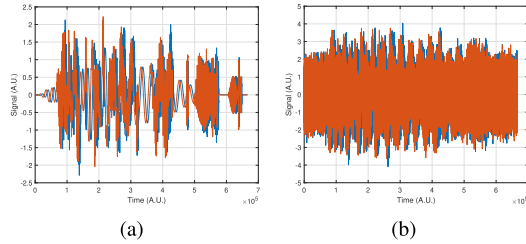


Fig. 1. (a) Clean damped artificial signal. (b) After adding noise with SNR level 13.98 dB.

### B. Lag Estimation

To estimate the lag for every specific frequency (scale) range, in what we call the IMXC technique, we perform the best match filtering using the maximal cross correlation (XC) of the corresponding IMCs of the two signal decompositions. As we mentioned before, the MvFIF guarantees that corresponding IMCs of the two channels correspond to the same frequency range. We are aware that “correlation” does not imply “causation,” and that XC is not the best means to measure it [5]. However, according to our knowledge, this is the first attempt to such a scale-wise lag measurement, and to ease the analysis, we apply the technique on examples in which what is the cause and what is the effect is known in advance. Bearing this in mind, scale-wise lags are thus provided for all the IMC pairs of both “cause” and “effect” signals. Various concepts for the estimation of lags between the same respective frequency ranges signal decompositions were tested. Based on our tests, we observed that most of the XC implementations provide good results. In the following, we opted to use the “normalized” XC, due to its numerical efficiency in MATLAB, especially when the “maxlag” option is used.

### III. TESTING OF THE IMXC APPROACH

The proposed IMXC approach was tested using the two-channel signal shown in Fig. 1(a), whose components, resolved using MvFIF, are depicted in Fig. 2. In particular, this signal was constructed combining nine non-stationary components, with stationary frequency and non-stationary amplitudes (providing different patterns for each frequency), and applying nine randomly chosen lags for each component.

To test robustness against noise, the two channels were perturbed using five different levels of noise, measured as SNR in dB using the formula  $20 \log_{10}(A_{\text{signal}}/A_{\text{noise}})$ .

For each level of noise and for each channel, we considered five different realizations of Gaussian noise. In Fig. 1(b), we show, as an example, the signal with a Gaussian noise of 13.98 dB SNR. In Fig. 3, we report statistics summarizing the performance of the IMXC method in reconstructing the scale-by-scale lag under different levels of SNR, including the case of clean signal (SNR = Inf). To take into account small perturbations induced by noise, we set a threshold of 5% accuracy in the reconstructed lag measurements.

### IV. REAL-LIFE EXAMPLE—IONOSPHERIC PLASMA

Multi-satellite measurements in the Earth’s ionosphere provide ample opportunities to identify the lags, among others induced by the spacecraft distance, that are known to exist

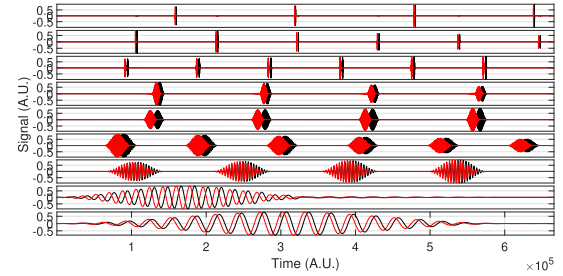


Fig. 2. Components of the two-channel noiseless artificial signal in red and black, respectively.

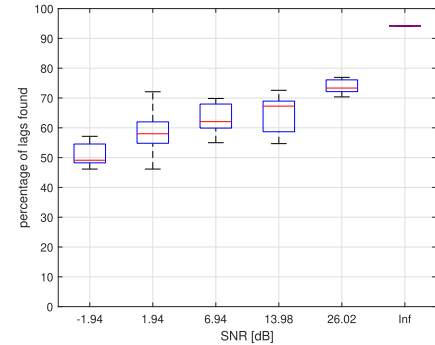


Fig. 3. Box plots summarizing the statistics on the reconstructed lags resolved with 5% accuracy.

but hidden in the data. The only assumption we make here is that the background conditions are stable so that the temporal variation is negligible between the respective satellite measurements above the nearly same spatial location.

In our work, we use the closely separated measurements of the European Space Agency’s Swarm Alpha (A) and Charlie (C) satellites [6] that use identical Langmuir probe instruments to sample the ionospheric plasma density in the topside ionosphere (about at 460 km altitude). At low- and mid-latitudes, Swarm A and Swarm C have longitudinal and latitudinal separations of about 146 and 62 km, respectively. As Swarm A and Swarm C fly at about 7 km/s in those regions, latter separation translates into a lag of about 8.8 s between the two satellites. The selected case is the passage of the two close-by satellites flying over the Japanese longitudinal range on 1 May 2014 around 15 UT. Over Japan, they encounter a plasma enhancement due to the passage of a medium-scale traveling ionospheric disturbance (MSTID), as reported by [7]. Swarm satellites pass fast over the MSTID, as it moves at velocity of the order of a few hundreds of meters per second [8], and therefore, we can consider the MSTID frozen in that frame. Additionally, we consider for the same Swarm A and Swarm C tracks the passage through a peak in the plasma density, due to northern crest of the equatorial ionospheric anomaly (EIA), which reaches the Swarm altitudes (see [10]).

The nighttime MSTIDs, identified by [7], occur during the local midnight and are caused by electro-dynamical forces, such as the Perkins instability, as supported by [11]. What we aim at in this work is the identification on top of the 8.8 s delay between the satellites, the delay associated with the respective ionospheric structure, and the corresponding scales at which this lag is found.

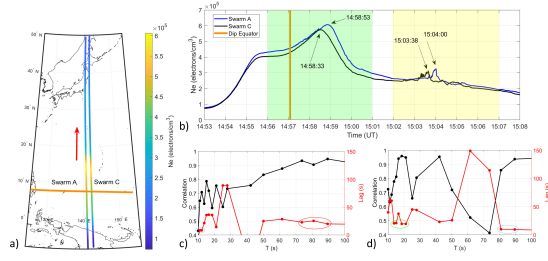


Fig. 4. (a) Geographic distribution of the electron density measured by Swarm A and Swarm C. Orange line indicates the position of the magnetic equator, while the red arrow indicates the flying direction of the satellites. (b) Electron density as a function of time measured by Swarm A (blue) and Swarm C (black). Times of plasma peaks are also indicated. Orange line indicates the time at which the satellites cross the magnetic equator, while green and yellow shaded areas indicate the time windows used to produce the plots in (c) and (d), respectively. (c) Correlation between IMCs and the corresponding measured lags in the EIA crest region [green area in (b)]. (d) Same as (c), but for the MSTID region [yellow area in (b)].

TABLE I

LAGS IDENTIFIED USING IMXC APPROACH ON THE IONOSPHERIC PLASMA DENSITIES MEASURED WITH SWARM A AND SWARM C SATELLITES ON 1 MAY 2014 AROUND 15 UT

	Orbital delay	Crest-like	MSTID
Lag (s)	8.8	22	20
Scale (s)	10–20	50–100	80–100

The electron density (Ne) measurements from Swarm A and Swarm C are shown in Fig. 4(a) and (b). The cross correlation was computed IMC per IMC by considering 5 min time windows, which are highlighted in Fig. 4(b), as the green and yellow shaded areas, respectively. Fig. 4(c) and (d) shows the obtained lags' cross correlation as a function of the IMC temporal scale. In these (c) and (d), we report only positive lags, i.e., when what we consider here as the effect (Swarm A) comes after the cause (Swarm C).

Besides the 8.8 s orbital delay (highlighted in Fig. 4(c) and (d) by blue ellipses), the IMXC approach was also able to recover a 22 s lag, (a crest-like delay, highlighted in Fig. 4(c), by a red ellipse) and a 20 s lag (the MSTID delay, highlighted in Fig. 4(d), by a green ellipse). These last two lags are physically reasonable delays between peaks in Ne latitudinal/time profiles modified by the satellites traversing extensive ionospheric structures, the former by the dip equator and the latter by the MSTID. Identified lags are reported in Table I, together with the corresponding scales at which they are found. The orbital-delay-related lags are generally associated with shorter scales in the range of 10–20 s while the lags associated with the actual ionospheric features are covered at longer scales of 50–100 s. Concerning the orbital delay, if we consider that the satellites are flying at about 7 km/s, the corresponding spatial scale is of the order of 100 km, which is, as expected, at the same order of magnitude of the spatial separation between the satellites.

## V. CONCLUSION AND OUTLOOK

The proposed scale-wise lag reconstruction technique, called IMXC, was found to perform well when applied to couples of artificial and real-life signals, even in the presence of high level of noise. It is important to point out that the correlation analysis alone without a preliminary decomposition

performed via MvFIF is unable to provide any of the correct lags for both the artificial and natural signals, when used on the original datasets (not shown). The key result presented in this work can be summarized as follows. When we deal with a multicomponent signal, which is associated with a multiscale process that has different lags at different scales, one way to reconstruct physically meaningful lags is by first decomposing the signal into well-separated scales and then apply a scale-by-scale analysis.

As reminded in the introduction, to ensure that two signals under study are in a cause–effect relationship, one has to use standard measures such as Granger causality or entropic principles. More advanced concepts such as the fluctuation-response protocol, proposed in [5], are applicable even in the presence of weak nonlinear terms. After this preliminary analysis has confirmed a cause–effect relationship between two measurements, the presented technique allows to investigate quantitatively the information process. In a future work, we plan to apply the proposed approach to the study of physical delays involved in the solar wind–magnetosphere–ionosphere–thermosphere coupling (see [1]).

Finally, we point out that the IMXC method, as it is, does not allow to analyze all the cases in which the cause, to initiate any effect, has to build up to reach over some threshold. An example from the space physics is given by the geomagnetic storm, which starts after some duration of specific level of the geoeffective southward interplanetary magnetic field. We plan to work in the future to extend the IMXC technique to cover also this kind of phenomena.

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