Exact Solution of the Full RMSA Problem in Elastic Optical Networks

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Abstract—Exact solutions of the Routing, Modulation, and Spectrum Allocation (RMSA) problem in Elastic Optical Networks (EONs), so that the number of admitted demands is maximized while those of regenerators and frequency slots used are minimized, require a complex ILP formulation taking into account frequency-slot continuity and contiguity. We introduce the first such formulation, ending a hiatus of some years since the last ILP formulation for a much simpler RMSA variation was introduced. By exploiting a number of problem and solver specificities, we use the NSFNET topology to illustrate the practicality and importance of obtaining exact solutions.

Index Terms—Elastic optical networks, RMSA problem, continuity and contiguity constraints, ILP problem.

I. INTRODUCTION

Even though data transmission through optical fibers has been a reality for a few decades, the constantly growing demand for higher capacity and greater maximum reach has led research and development in the field to be always evolving. One of the most promising technologies that emerged in the last decade or so and is now particularly well poised to play an important role in the coming years is based on the concept of an Elastic Optical Network (EON) [1]. Like networks based on the more mature and still widely used Dense Wavelength Division Multiplexing (DWDM) technology [2], EONs provide for the sharing of each link's spectrum between end-toend demands, and also for the use of so-called regenerators at certain nodes so that a signal nearing its modulation's maximum reach can be regenerated and continue on its designated end-to-end route. In both respects, however, EONs improve on DWDM networks substantially. First, spectrum sharing in EONs is based on relatively narrow, fixed-width frequency slots (FSs) that can be concatenated to provide the demand with higher, reduced-waste capacity. Second, by employing modern optical transceivers, the same link can carry demands using different modulations (hence with different numbers of FSs if the demands have the same bandwidth). This means that an EON regenerator can change the modulation used to serve a demand and along with it the number of FSs used.

Regenerators are expensive and must therefore be used sparingly. The problem of selecting the nodes at which to install them in a DWDM network is already a difficult problem in its own right [3]. In EONs, however, this gets considerably more complicated by the need to select not only which modulation to use for each demand going through each regenerator but also which FSs to allocate to it. In this case, an important concept is that of a segment, which is a path between two nodes without a regenerator at any intermediate node. Therefore, a path containing R regenerators at intermediate nodes comprises R+1 segments. While traversing any given segment, a demand uses the same modulation and FSs on all the segment's links. Given the network topology and a set of available modulations, each characterized by a bandwidth and a maximum reach, one can readily enumerate all possible segments by considering all paths on the graph. Given a set of demands, deciding how to route them using these segments, and consequently how many regenerators to deploy and where, is the NP-hard problem we deal with in this letter, known as the Routing, Modulation, and Spectrum Allocation (RMSA) problem [4].

We continue by describing the relevant state of the art and our contribution in Section II, then in Sections III and IV introduce two ILP formulations for RMSA that for the first time consider some of the most relevant objectives in EONs while abiding by every constraint they impose. We present computational results and conclude in Section V.

II. STATE OF THE ART AND CONTRIBUTION

In EONs, FS concatenation inside a segment requires that the FSs be perfectly aligned between successive links and, on each link, that they be contiguous. These constitute a continuity and contiguity (CC) criterion that must translate into constraints in any RMSA formulation. With these and other constraints in place, ideally multiple objectives should be pursued, including admitting as many demands as possible while globally using as few regenerators and FSs as possible. To the best of our knowledge, the single previous attempt to provide the RMSA problem with a formulation for exact solution is the one in [5], which is an ILP formulation without CC constraints that targets essentially the minimization of the total number of regenerators to be deployed.

The formulation we introduce, called RMSA-BP to emphasize the user-centric goal of minimizing the chance that a demand is blocked, targets all three objectives and includes CC constraints. We use the NSFNET topology to demonstrate the practical feasibility of obtaining exact solutions, as well as

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to demonstrate potential benefits in analyzing critical network properties.

III. RMSA-BP FORMULATION

We represent the network by an undirected graph G of vertex set N and edge set E, where N is the set of network nodes and E is the set of network links. Every link has the same link capacity (LC), given by how many frequency slots (FSs) it has. We use P to denote the set of segments, with segment $p \in P$ beginning at node s_p and ending at node t_p , and I_p to denote the first link on segment p. The set of demands is denoted by D, with demand $d \in D$ being characterized by its node of origin S_d , its node of destination T_d , and a bandwidth B_d . The number of FSs corresponding to demand d on segment p is denoted by ${\cal F}_p^d$ and given by the ratio of B_d to the bandwidth provided by each FS for the modulation used in p. We use R_{\max} to denote the maximum number of regenerators that can be used per demand. Notably, even though links in E are undirected, each segment in P is inherently directed. Thus, a link can be traversed by a segment in either of the link's two directions.

RMSA-BP is stated in terms of one set of constants, three sets of variables, and also some shorthands that allow for a cleaner exposition. We give the full formulation next, but divide the subsequent discussion into two separate parts, depending on which variables, shorthands, and constraints relate to the problem's CC requirement. We use weights $w_1 \gg w_2 \gg w_3$ in the objective function only to symbolize that the problem's three goals are to be prioritized in the following order: 1) Maximize the number of admitted demands; 2) Minimize the total number of regenerators used; 3) Minimize the total number of FSs used. The specific technique we use to enforce this prioritization is discussed in Section V.

$$\max \sum_{d \in D} (w_1 a_d - w_2 R_d - w_3 F_d)$$

s.t.
$$x_p^d \le a_d \qquad \qquad \forall d \in D, p \in P \quad (C1)$$
$$C_n^d = \begin{cases} a_d & \text{if } n = S_d \\ -a_d & \text{if } n = T_d \\ 0 & \text{otherwise} \end{cases} \quad \forall d \in D, n \in N \quad (C2)$$

 $R_d \le R_{\max}$ $\forall d \in D$ (C3)

$$z_d^e < X_d^e \, \mathrm{LC} \qquad \qquad \forall d \in D, e \in E \quad (\mathrm{C4})$$

$$z_d^e + F_d^e - X_d^e \le \text{LC} \qquad \qquad \forall d \in D, e \in E \quad (C5)$$

$$X_d^e Y_p^e z_d^e - X_d^{I_p} Y_p^{I_p} z_d^{I_p} = 0 \qquad \forall d \in D, p \in P, \quad (C6)$$
$$e \in p, e \neq I_p$$

$$\sum_{\substack{d,d'\in D\\d\neq d'}} O^e_{d,d'} = 0 \qquad \qquad \forall e \in E \quad (C7)$$

For each $p \in P$ and each $e \in E$, we use Y_p^e to indicate whether link e is part of segment p. In the affirmative case, the direction of traversal of e by p is given implicitly by nodes s_p and t_p . The Y_p^{e} 's are binary constants whose values (from $\{0, 1\}$) are assigned along with the determination of the network's set of segments P (see Section I). The essential variables for use when the CC requirement is disregarded are all binary as well and are grouped into two sets: a_d , for each $d \in D$, indicating whether demand d is admitted; and x_p^d , for each $d \in D$ and each $p \in P$, indicating whether demand duses segment p. These variables are sometimes used directly in the above formulation, and also sometimes indirectly through the following shorthands:

$$\begin{split} R_d &= \sum_{p \in P} x_p^d - a_d, \qquad F_d^e = \sum_{p \in P} F_p^d x_p^d Y_p^e, \\ F_d &= \sum_{e \in E} F_d^e, \qquad \qquad C_n^d = \sum_{p \in P \atop s_p = n} x_p^d - \sum_{p \in P \atop t_p = n} x_p^d. \end{split}$$

In these equations, R_d is the number of regenerators used by demand d, F_d^e is the number of FSs used by demand d on link e, F_d is the total number of FSs used by demand d, and C_n^d is the flow deficit at node n for demand d (that is, the number of segments used by d that are outgoing from n in excess of those that are incoming to n). If $x_p^d = 1$ for at most one segment p, then $C_n^d \in \{-1, 0, 1\}$.

Only Constraints (C1)–(C3), along with $\sum_{d\in D} F_d^e \leq LC$ for each $e \in E$, henceforth called Constraint (Cx) for the sake of the argument, are needed if CC need not hold. Of these, Constraints (C1) and (C2) take care of how the a_d 's and the x_p^d 's relate to one another, as follows. A blocked demand d ($a_d = 0$) uses no segments ($x_p^d = 0$ for every $p \in P$). An admitted demand d ($a_d = 1$), on the other hand, must imply a positive-unit flow deficit at its node of origin ($C_{S_d}^d = 1$), an egative-unit flow deficit at its destination node ($C_{T_d}^d = -1$), and no flow deficit at any other node. Additionally, Constraints (C3) and (Cx) work, respectively, to ensure that no demand uses more than R_{\max} regenerators and that, taken together, the demands using link e (d such that $x_p^d Y_p^e = 1$ for some $p \in P$) use no more than the link's available capacity (LC).

Contemplating the CC requirements depends on one further set of variables, now taking values from $\{0, 1, ..., LC\}$. For $d \in D$ and $e \in E$, the new variable is z_d^e and serves to indicate, if greater than zero, the index of the first FS used by demand d on link e. To use these variables more cleanly in the above formulation, it has once again proven convenient to adopt the following additional shorthands:

$$\begin{split} X^e_d &= 1 \text{ if } \sum_{p \in P} x^d_p Y^e_p > 0; \ 0 \text{ otherwise}, \\ O^e_{d,d'} &= 1 \text{ if } z^e_d \leq z^e_{d'} \leq z^e_d + F^e_d - 1; \ 0 \text{ otherwise}. \end{split}$$

 X_d^e indicates whether demand d uses link e and $O_{d,d'}^e$ indicates whether at least one of link e's FSs is used by more than one demand. Clearly, $X_d^e = 0$ if and only if $F_d^e = 0$. The shorthands X_d^e and $O_{d,d'}^e$ can be used to constrain the values of the z_d^e 's so that the CC requirement is enforced.

Constraints (C4)–(C7) are now needed. Constraint (C4) is used to ensure that z_d^e is not a valid FS index ($z_d^e = 0$) when demand d does not use link e ($X_d^e = 0$). If demand d does use link e ($X_d^e = 1$), then Constraint (C5) ensures that the value of z_d^e is such that the indices of the remaining $F_d^e - X_d^e = F_d^e - 1$ FSs are no higher than LC. When demand d uses link $e \neq I_p$ on segment p ($X_d^e Y_p^e = X_d^{I_p} Y_p^{I_p} = 1$), Constraint (C6) ensures continuity by enforcing $z_d^e = z_d^{I_p}$. Contiguity is ensured by Constraint (C7), according to which no two distinct demands d, d' can be such that $z_{d'}^e \in \{z_d^e, \ldots, z_d^e + F_d^e - 1\}$ for any link *e*. Constraints (C4)–(C7) subsume Constraint (Cx), which is therefore not part of the formulation.

IV. RMSA-BP FORMULATION, REVISITED

The formulation given in Section III is correct in terms of reflecting our understanding of the problem's three concomitant objectives, and also in terms of laying down the constraints that guide the assignment of values to its variables. However, in preliminary experiments it proved excessively time-consuming even in relatively simple cases, owing mainly to the need to comply with the CC requirement, that is, the need to satisfy Constraints (C4)–(C7). In this section we describe two alterations to the formulation that ended up making considerable difference in terms of performance. Together with some fine-tuning of the solver employed, to be described in Section V, these alterations have made it possible to enlarge the range of exactly solvable instances quite widely.

The first alteration is to incorporate a preprocessing step (i.e., a step prior to calling the solver). This step creates, for each $d \in D$, a set S_d comprising all solutions that admit demand d while complying with Constraints (C2), (C3), and (Cx). Each solution $s \in S_d$ is therefore an assignment of values to the x_p^d 's that fully comply with the flow-conservation requirement in Constraint (C2) for $a_d = 1$, as well as with the upper bounds R_{\max} and LC imposed by Constraints (C3) and (Cx), respectively. Such explicit enumeration is in general out of the question, but in the case at hand it has proven feasible for many instances of the problem. This is owed mainly to the pruning effect of R_{\max} , which as mentioned in Section I is in practice already assigned a small value.

For $s \in S_d$, we denote the value assigned to each x_p^d during preprocessing by $x_p^d(s)$. These values give rise to the following useful constants:

$$\begin{split} R^s_d &= \sum_{p \in P} x^d_p(s) - 1, \\ F^s_d &= \sum_{e \in E} \sum_{p \in P} F^d_p x^d_p(s) Y^e_p, \\ X^{e,s}_d &= 1 \text{ if } \sum_{p \in P} x^d_p(s) Y^e_p > 0; \ 0 \text{ otherwise.} \end{split}$$

 R_d^s is the number of regenerators used by demand d in solution s, F_d^s is the total number of FSs used by demand d in solution s, and $X_d^{e,s}$ indicates whether demand d uses link e in solution s. The revised RMSA-BP formulation we present uses three sets of variables: x_d^s , a binary variable for each $d \in D$ and each $s \in S_d$, indicating whether demand d uses solution s; the z_d^e 's already used in Section III; and the binary variables $o_{(d,d')}^e$ and $o_{(d',d)}^e$, for each unordered pair (d,d') of distinct demands from D and each link $e \in E$, used in simplifying the enforcement of the contiguity part of the CC requirement. We denote the set of such unordered pairs by D_u^2 . The formulation also uses the shorthand

$$A_d = \sum_{s \in S_d} x_d^s.$$

 A_d is the number of solutions in S_d that demand d uses and, provided $x_d^s = 1$ for at most one solution $s \in S_d$, indicates whether demand d is admitted. The objective function in the first formulation can thus be rewritten as

$$\varphi = \sum_{d \in D} \left(w_1 A_d - w_2 \sum_{s \in S_d} R_d^s x_d^s - w_3 \sum_{s \in S_d} F_d^s x_d^s \right).$$

The revised RMSA-BP formulation is as follows.

max φ s.t.

$$\sum_{s \in S_d} x_d^s \le 1 \qquad \qquad \forall d \in D \quad (\mathbf{R}\mathbf{1})$$

 $s \in S_{I}$

$$\begin{aligned} & (X_d^{e,s}Y_p^e z_d^e) & \quad \forall d \in D, p \in P, \quad (\mathbf{R2}) \\ & -X_d^{I_p,s}Y_n^{I_p} z_d^{I_p}) x_d^s = 0 & \quad e \in p, e \neq I_p, \end{aligned}$$

$$o^{e}_{(d,d')} + o^{e}_{(d',d)} \le 1 \qquad \qquad \forall (d,d') \in D^{2}_{u}, \quad (\mathbf{R3})$$
$$e \in E$$

$$z_d^e + F_d^e \le z_{d'}^e + Mo_{(d,d')}^e \qquad \forall (d,d') \in D_u^2, \quad (\mathbf{R4})$$
$$e \in E$$

$$z_{d'}^e + F_{d'}^e \le z_d^e + Mo_{(d',d)}^e \qquad \forall (d,d') \in D_u^2, \quad (\mathbf{R5})$$
$$e \in E$$

Constraint (R1) ensures that no demand d uses more than one solution from S_d . Constraints (R2)–(R5) target the CC requirement, with Constraint (R2) ensuring that continuity holds and the remaining three taking care of contiguity. Constraints (R3)-(R5) implement the second performanceoriented alteration mentioned earlier in this section. The reason why ensuring contiguity has such negative impact on the performance of the first RMSA-BP formulation is the roundabout way Constraint (C7) approaches it. A much more direct way would be to do something in the style of the M-independent part of Constraints (R4) and (R5), viz., $z_d^e + F_e^d \leq z_{d'}^e$ and $z_{d'}^e + F_{d'}^e \leq z_d^e$ for each unordered pair (d, d') of demands. These, however, can never be concomitantly satisfied, essentially forcing an a priori choice between them. This impossibility is what the M-dependent part of the two constraints helps circumvent. For a sufficiently large value of M, and given that Constraint (R3) disallows the occurrence of $o^e_{(d,d')} = o^e_{(d',d)} = 1$, it is possible, e.g., that Constraint (R4) ends up enforcing $z_d^e + F_e^d \leq z_{d'}^e$ (with $o_{(d,d')}^e = 0$) while Constraint (R5) enforces $z_{d'}^e + F_{d'}^e \leq z_d^e + M$ ($o_{(d',d)}^e = 1$), avoiding the superposition of the two demands' FSs while opening up the possibility of $z_d^e < z_{d'}^e$. Clearly, to ensure that the two constraints function in this way it suffices that we have M > LC, so we use M = LC + 1 throughout. The overall strategy is known as the bigM approach to handle indicatordependent constraints efficiently [6].

V. COMPUTATIONAL RESULTS AND CONCLUSION

Our computational experiments involved solving RMSA-BP, as formulated in Section IV, on a computer with two AMD EPYC 7763 64-core processors and 512 GB RAM. We used the Gurobi 9.5.2 solver along with the NetworkX 3.1 Python

 TABLE I

 Average Blocking and Usage Results, 30 Instances

D	$R_{\rm max}$	BD	TR	TFS	$R_{\rm max}$	BD	TR	TFS
100	1	1.0	43.4	1 265	2	0.6	42.4	1 263
110	1	4.4	55.1	1 2 9 2	2	2.9	59.0	1 2 8 3
120	1	7.5	55.7	1 3 2 9	2	5.1	64.8	1 3 1 9

TABLE II TIMEOUTS AND EXECUTION TIMES (H:M:S), 30 INSTANCES

D	$R_{\rm max}$	ТО	Minimum	Maximum	80th Percentile
100	1	2	00:03:06	15:48:10	00:50:44
110	1	3	00:04:15	14:03:06	01:28:03
120	1	8	00:14:24	14:24:14	01:57:48
100	2	4	00:07:52	29:39:28	04:42:32
110	2	1	00:03:39	12:27:29	03:51:46
120	2	4	00:08:26	14:03:24	08:34:09

package on the Debian 11 operating system. All the RMSA-BP instances we considered are relative to a network with the NSFNET topology and modulation, per-slot efficiency, and maximum reach as in [5]. The experiment for each instance consisted in preprocessing for solution enumeration followed by optimization, and was allowed to run for no more than 30 hours before timeout. Gurobi was allowed no more than 32 parallel threads per instance.

Our use of the Gurobi solver was predicated on the adoption of three of the features it offers that turned out to be of crucial importance. First, to address the prioritization symbolized by weights w_1, w_2, w_3 in our formulation's objective function, we used the solver's multi-objective hierarchical mode, which allows a priority to be set for each individual objective and ensures that they are optimized from highest to lowest priority. Solving for an objective does not affect any solution found when solving for higher-priority objectives. Second, to ensure that the problem's formulation is indeed of the ILP type, we used the solver's indicator-type constraints. These are instrumental in view of the otherwise quadratic nature of Constraint (R2). Third, we used the solver's parameter tuning tool (grbtune) prior to any actual experiment. This tool is given one or more instances to solve and analyzes them automatically to fine-tune the solver's internal parameters. Relying on the parameter values it outputs is based on the expectation that they will likewise lead to good performance on the instances it has not analyzed. We found this to be generally true.

We used 90 instances for each value of R_{max} , 30 of them with |D| = 100, 30 with |D| = 110, and 30 with |D| = 120. Each demand d had bandwidth $B_d = 100$ Gbps, and nodes of origin S_d and destination T_d chosen uniformly at random. We initially used LC = 160 but this never resulted in any demand being blocked. We then turned to LC = 80, which made it possible for links to saturate and more interesting results to be observed. We report on the LC = 80 cases exclusively.

For $R_{\text{max}} = 1, 2$, in Table I we show the average number of blocked demands (BD $\leq |D|$), the average total number of regenerators used (TR $\leq |D| R_{\text{max}}$), and the average total number of FSs used per demand (TFS $\leq 21 \text{ LC} = 1680$, since the NSFNET has 21 links). In Table II, we see that the



Fig. 1. Link-usage heat map for |D| = 30 (A), 60 (B), 90 (C), 120 (D).

number of timeouts (TO \leq 30) was consistently low, however with considerable variation in execution times for successful instances. This notwithstanding, for 24 (80%) of the instances they fell below a small number of hours, though substantially more for $R_{\rm max} = 2$ than for $R_{\rm max} = 1$. Supplementing the information in the tables, we note that the average numbers of CPU cores used were between 20 and 22.

We also conducted one further set of experiments to help in understanding how network topology influences link usage as the number of demands increases. The setup is still mostly the same as in the previous experiments, except that now four demand sets are used, with |D| = 30, 60, 90, 120. A link's usage is the ratio of the number of FSs used on it to LC. A heat map with averages over each demand set's 30 instances is shown in Figure 1. Even though the increase in link usage does, as expected, grow with |D|, somewhat unexpectedly we also see that link (8,9) is always one of the most used in all cases, while (1,2) is one of the least used. These seem like inherent structural properties of the NSFNET topology, which in principle might not have come up if some heuristic had been used instead. To conclude, we then note that therein lies the importance of exact approaches like RMSA-BP, since they can provide crucial aid in the analysis of network topology and usage, and through such analysis can influence design and deployment policies. Further research should concentrate on generalizing the objective function we have used to meet other goals, as well as on seeking additional improvement opportunities to both problem formulation and solver-feature exploitation, so that larger networks can be handled as well.

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