Adaptive Pulse Optimization for Improved Sonar Range Accuracy

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***Abstract*—Using the theory of optimal receivers, the range accuracy of echolocating systems can be expressed as a function of pulse bandwidth and SNR through the well-known Woodward equation. That equation, however, was developed in the limit of very high SNRs and assumes that the correct peak of the cross-correlation function is known *a priori*. In this letter, we show that for increasing levels of noise, the accuracy of the cross-correlation receiver undergoes a sharp transition from the Woodward equation for a coherent receiver to a modified Woodward equation for a semicoherent receiver. Since this transition appears at different SNRs for pulses with different center frequencies and bandwidths, it is possible to choose the optimal pulse for any given SNR. We show that the adaptive method we propose outperforms the classical cross-correlation receiver for low SNRs. The same ideas can be applied to the case of a fixed broadband signal, by performing the cross correlation at the receiver end separately in a set of frequency bands with the appropriate center frequencies and bandwidths.**

***Index Terms*—Adaptive sonar, matched filter, noise, optimal receiver, range accuracy.**

I. INTRODUCTION

THE THEORY OF optimal receivers studies the design of pulses and receivers to obtain optimal detection in the presence of noise. Considerable work on the theoretical accuracy of range measurements has been done in the past, and the Woodward equation has been derived using different methods. A comprehensive description can be found in [1]. However, it appears that interest in the mathematical aspects of the derivation of that equation has faded [2], while its use has become a standard in the field. In this letter, we show that the validity of this equation depends on various assumptions, in particular the assumptions of very low SNRs; therefore, it must be reexamined for the case of low SNRs. The theory of optimal receivers shows that the *matched filter* receiver maximizes the output peak-signal-to-mean-noise (power) ratio [2], [3], and is the optimum method for the detection of signals in noise. Information about the distance of the target is extracted by computing the time at which the cross correlation between the echo and a replica of the pulse is a maximum. This delay is converted into a distance by means of the sound velocity in the particular medium in consideration (e.g., water or air). This type of receiver is generally referred to as a *coherent* receiver.

The classical theory of optimal receivers describes the range accuracy of a sonar system via the well-known Woodward equation, which can be derived by using a variety of methods [1], [4]–[7]. However, all of them rely upon the common crucial assumption of a large SNR, which implies *a priori* knowledge of the location of the central lobe in the cross-correlation function. For small SNRs, one of the parameters in the classical equation—i.e., the bandwidth—has to be modified, and the receiver is then called *semicoherent*. In this letter, we show that the transition between the two types of behaviors occurs at different SNRs, depending on characteristics of the pulses such as bandwidth and center frequency. With this observation, we devise a novel system based on an adaptive choice of the pulse; this can improve accuracy in the case of relatively low SNR, when ambiguity in the choice of the correct peak of the cross-correlation function cannot be avoided. This method can be generalized to the case of a fixed broadband pulse. In this case, both pulse and echo can be passed through a set of filters with appropriate center frequencies and bandwidths, and cross correlation can be performed separately in each frequency band.

II. WOODWARD EQUATION

If we define $\psi_p(t)$ to be the pulse sent by the sonar and $\psi_e(t)$ to be the echo coming from a target at a distance d, then $\psi_e(t) = \psi_p(t+\tau_0)+\eta(t)$, where $\tau_0 = 2d/c$, c is the sound velocity in the particular medium in consideration (e.g., water or air), and $\eta(t)$ is generally white noise. The cross correlation between pulse and echo can be expressed as

$$\begin{aligned}\psi_e \circ \psi_p(\tau) &= \int_{-\infty}^{+\infty} \psi_e(t)\psi_p(t+\tau)\mathrm{d}t = \\ &= \int_{-\infty}^{+\infty} \psi_e(t)\psi_e(t+\tau+\tau_0)\mathrm{d}t \\ &\quad + \int_{-\infty}^{+\infty} \psi_e(t)\eta(t+\tau)\mathrm{d}t \qquad (1)\end{aligned}$$

where the first term in the sum is the autocorrelation function of the pulse centered at τ_0, and the second term is band-limited white noise, with frequency limits defined by the spectrum of the pulse. In the absence of noise, only the first term survives, and the distance from the target can be computed from the delay in time corresponding to the maximum of the cross-correlation function. When the noise level is sufficiently low, its effect is to jitter the position of the maximum around the true

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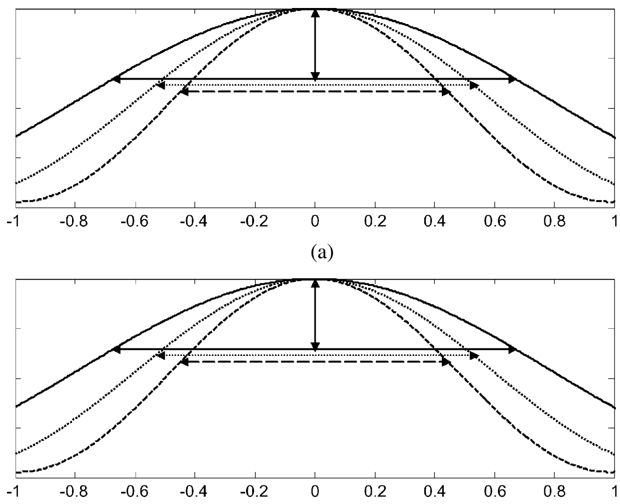

Fig. 1. Effect of noise level on the accuracy of the temporal measurement for pulses with the same bandwidth and different center frequencies. (Solid line) Smallest f_c. (Dotted line) Intermediate f_c. (Dashed line) Largest f_c. (a) Delay estimate error with no peak ambiguity (high SNR). (b) Delay estimate error due to peak ambiguity (low SNR).

value of the delay τ_0. To a first approximation, the jitter can be related to the width of the central peak in the autocorrelation function, which is a function of the signal's bandwidth and center frequency [Fig. 1(a)]. By using a rigorous argument based on the concept of inverse probability due to Woodward [6], it is possible to demonstrate that the standard deviation of the location in time of the maximum around the true value τ_0 is $\sigma_c = (2\pi B_{\mathrm{RMS}} d)^{-1}$. In this formula, B_{RMS} is the root mean square (RMS) bandwidth of the pulse and is defined as $B_{\mathrm{RMS}} = \left(\int_0^{+\infty} f^2 P_{\mathrm{SD}}(f)\mathrm{d}f\right)^{1/2}$, where $P_{\mathrm{SD}}(f)$ is the power spectral density of the pulse, and the SNR $d = \sqrt{2E/N_0}$ is a function of the ratio between the total energy E of the echo (measured in watts), and the spectral density N_0 of the noise [measured in watts per hertz equals watts per second ($\mathrm{W/Hz} = \mathrm{Ws}$)]. In the case of uniform Gaussian noise with variance σ_N^2, the spectral density of the noise sampled at a rate f_s can be expressed as $N_0 = 2\sigma_N^2/f_s$. The SNR is usually expressed in decibels as $\mathrm{SNR}_{\mathrm{dB}} = 20\log_{10} d$. Notice that the RMS bandwidth can be written as $B_{\mathrm{RMS}}^2 = B_{\mathrm{CRMS}}^2 + f_c^2$, where $f_c = \int_0^{+\infty} f \cdot P_{\mathrm{SD}}(f)\mathrm{d}f$ is the center frequency of the signal, and $B_{\mathrm{CRMS}} = \left(\int_0^{+\infty} (f - f_c)^2 P_{\mathrm{SD}}(f)\mathrm{d}f\right)^{1/2}$ is the centralized RMS (CRMS) bandwidth. When the center frequency is much larger than the CRMS bandwidth (a condition which is generally satisfied for radar), then $B_{\mathrm{RMS}} \approx f_c$.

The above description corresponds to the case of a *coherent* receiver. Such a receiver computes the cross-correlation function of the pulse and the echo and estimates echo delay as the time corresponding to the maximum peak in the *fine structure* of the cross-correlation function. An alternative type of receiver, the *semicoherent* receiver, estimates echo delay as the time corresponding to the maximum of the *envelope* of the cross-correlation function between the pulse and the echo. For the semicoherent receiver, delay accuracy can be expressed by modifying the Woodward equation by substituting the signal CRMS bandwidth to the RMS bandwidth, so that $\sigma_s = (2\pi B_{\mathrm{CRMS}} d)^{-1}$.

III. SNR Breakpoint

Uncertainty in the delay estimate increases with noise. For relatively low levels of noise, the time jitter falls within the central peak of the autocorrelation function and is inversely proportional to the SNR. However, when the noise level becomes comparable to the difference in amplitude between the center peak and the first side lobe, ambiguity in the choice of the correct peak arises. Fig. 1 illustrates the effect of the noise level on the accuracy of the temporal measurement. Fig. 1(a) shows a detail of the autocorrelation function in the neighborhood of the central peak for three pulses with the same bandwidth and different center frequencies. The jitter in amplitude introduced by the noise is translated into a jitter in time that is controlled by the width of the central peak: the higher the center frequency, the smaller the jitter in time. However, when the noise level is of the order of the difference between the amplitude of the central peak and the first side lobe, the situation is reversed [Fig. 1(b)]. In fact, the difference in amplitude is smaller for higher center frequencies, so that signals with high center frequencies are more susceptible to peak ambiguity.

To study the effect of increasing levels of noise as a function of signal CRMS bandwidth and center frequency, we ran a set of Monte Carlo simulations. The pulses we considered are cosine packets of the form $\psi_{\sigma,\eta}(t) = K_{\sigma,\eta}\exp(-t^2/2\sigma^2)\cos(2\pi\eta t)$, where η is the center frequency, σ controls the spread in time of the pulse and its frequency bandwidth, and $K_{\sigma,\eta}$ is a normalization factor such that $\int_{-\infty}^{+\infty} \psi_{\sigma,\eta}^2(t)dt = 1$. This signal can be used without loss of generality. Analogous results would be obtained using different types of pulses with the same center frequencies and CRMS bandwidths used in our simulations. In each simulation, white noise is added to the pulse to generate an echo, and the delay estimate is computed as the time corresponding to the maximum amplitude in the cross correlation between pulse and echo. In each set of simulations, 200 realizations of the noise were generated. Different sets corresponded to pulses with different center frequencies and CRMS bandwidths. Fig. 2(a) shows the RMS error (RMSE) computed using the Monte Carlo simulations, for a fixed center frequency and CRMS bandwidth. Confidence intervals have been computed through bootstrapping, by sampling with replacement from the empirical distribution of the delay estimates obtained from the simulation. For high SNRs (region IV), performance is in accordance with the standard Woodward equation for the coherent receiver. As the SNR decreases, the performance shows a sharp transition (region III) to the modified version of the Woodward equation, corresponding to the semicoherent receiver (region II). For very low levels of SNR (region I), the intensity of the noise is so high that the accuracy rapidly decreases to zero. This behavior is common to all pulses. However, the transition region is different according to the center frequency and CRMS bandwidth [Fig. 2(b) and (c)]. Fig. 2(b) shows the RMSE of the delay estimates as a function of SNR in decibels ($\mathrm{SNR}_{\mathrm{dB}}$) and CRMS bandwidth, for a fixed center frequency. For high SNRs, all signals follow the standard Woodward equation [Fig. 2(a)]. As the SNR decreases, signals with lower CRMS bandwidths are affected by peak ambiguity first, and their performance degrades to

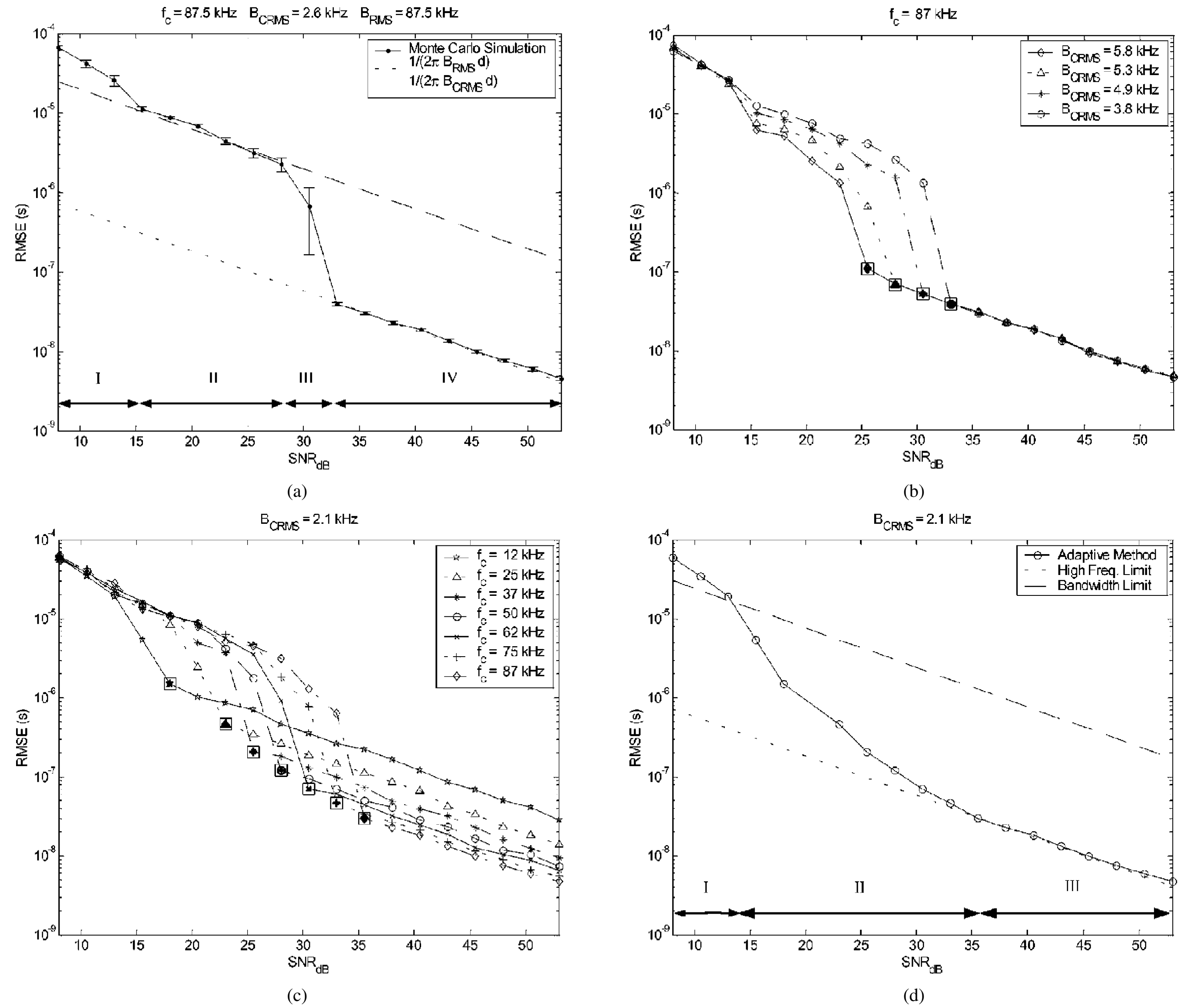

Fig. 2. Results of the Monte Carlo simulations. (a) RMSE of the delay estimates for a fixed center frequency and CRMS bandwidth. (b) RMSE as a function of SNR in decibels and CRMS bandwidth, for a fixed center frequency. (c) RMSE as a function of SNR in decibels and center frequency, for a fixed CRMS bandwidth. (d) Optimal performance curve obtained through this adaptive method.

that of a semicoherent receiver. Signals with larger CRMS bandwidths are more resilient to peak ambiguity and continue to perform according to the standard Woodward equation for even lower SNRs. The breaking point for each signal is marked with a square.

Fig. 2(c) shows the case of a fixed CRMS bandwidth and different center frequencies. For high SNRs, all signals follow the standard Woodward equation [Fig. 2(a)]. As the noise level increases, signals with higher center frequencies are affected by peak ambiguity first, and their performance degrades to that of a semicoherent receiver. Signals with lower center frequencies are more resilient to peak ambiguity and continue to perform according to the standard Woodward equation for lower SNRs.

IV. Pulse Optimization

The above simulations show that the performance of the receiver degrades from coherent to semicoherent as the SNR decreases. Pulses with different center frequencies and CRMS bandwidths undergo this transition at different levels of noise, such that for each SNR it is possible to find an optimal pulse with respect to the RMSE of the delay estimate. Ideally, the larger the CRMS bandwidth, the higher the accuracy that can be achieved. However, in a realistic system, only a limited range of frequencies is available to both the pulse and the receiver. This limitation introduces an upper bound for both the pulse's center frequency and CRMS bandwidth.

For a given SNR and bandwidth, there exists an optimal signal in terms of RMSE of the delay estimate, i.e., the signal with the highest center frequency that has not yet been affected by ambiguity (its RMSE is enclosed in a square). Fig. 2(d) shows the optimal performance curve obtained through the adaptive method, when the maximal CRMS bandwidth allowed is the same for all pulses. Three regions can be identified. For high SNR (region III), the RMSE follows the Woodward equation corresponding to the maximum center frequency allowed; this limitation is a direct consequence of the limited frequency available to the system (87 kHz in this example). For intermediate SNR values (region II), high accuracy can be achieved adapting the system to use the pulse with the highest center frequency that has not yet been affected by ambiguity. As the SNR decreases, the choice falls onto pulses with decreasing center frequencies, hence decreasing accuracies. When the value of the optimal pulse's center frequency is of the order of the common CRMS bandwidth, the RMSE curve crosses the Woodward equation curve corresponding to the semicoherent receiver. For lower values of SNR (region I), the intensity of the noise is so high that the accuracy rapidly decreases to zero.

The pulse optimization algorithm we propose is based on the adaptation of the pulse center frequency to the operating noise level. The constraints on the pulse's bandwidth and frequency range generate a plot like the one in Fig. 2(c), where the SNR breakpoints for different center frequencies are shown. Then, for a given SNR, we choose the pulse with the highest center frequency that has not yet been affected by ambiguity.

V. CONCLUSION

We show that for increasing levels of noise the accuracy of the range estimate undergoes a sharp transition from the Woodward equation for a coherent receiver to a modified Woodward equation for a semicoherent receiver, due to ambiguity in the choice of the correct peak of the cross correlation between pulse and echo. We find that the breakpoint appears for lower SNRs in pulses with lower center frequencies and larger CRMS bandwidths, so that it is possible to optimize the pulse for a given SNR. The accuracy of the adaptive method we propose is equal to that of a coherent receiver for high SNRs, but significantly better for lower SNRs, since peak ambiguity is removed by the optimal choice of the pulse. The same ideas can be extended to the case of a fixed broadband signal, by performing the cross correlation at the receiver end separately in a set of frequency bands with the appropriate center frequencies and bandwidths. While the Woodward equation is not derived for the case of nonadditive (such as reverberation) noise, the proposed method of estimating the effect on the cross-correlation function should still be applicable. This will be tested in future simulations.

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