Closed-loop extended orthogonal space-time block codes for three and four transmit antennas

Yi Yu, Sylvie Kerouedan, Jinhong Yuan

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Yi Yu, *Student Member, IEEE*, Sylvie Keroueden, and Jinhong Yuan, *Member, IEEE*

***Abstract*—Complex orthogonal space–time block codes (STBCs) with linear processing for more than two transmit antennas cannot achieve a full rate in multiple-input and multiple-output (MIMO) channels. However, at the expense of losing some degree of diversity advantage, it is possible to achieve a full rate or even rates higher than one for any number of transmit antennas. In this letter, we propose new closed-loop extended orthogonal STBCs (EO-STBCs) for three and four transmit antennas. We show that the schemes can achieve a full transmit diversity with linear processing, and they outperform the previous closed-loop STBCs. In addition, we investigate the closed-loop EO-STBCs with receive antenna selection. It is shown that for four transmit antennas and up to three receive antennas, performance gains of up to 5.8 dB are obtained over the closed-loop EO-STBCs without antenna selection.**

***Index Terms*—Extended orthogonal space–time block codes (EO-STBC), linear processing, partial feedback.**

I. Introduction

TRANSMIT diversity (TD) is one of the key techniques adopted by 3G and beyond 3G standards to provide high-speed reliable communications [1]. One of the most common TD techniques is the Alamouti code, which is an orthogonal space–time block code (STBC) [2]. However, when three or four transmit antennas were considered, the maximum symbol transmission rate of STBCs from complex orthogonal designs with linear processing was 3/4 [3]. In [4], the quasi-orthogonal STBC (QO-STBC) was proposed to achieve a full rate for wireless systems with more than two transmit antennas at the expense of losing some degree of diversity advantage and increasing the decoding complexity.

Recently, a lot of effort has been put into designing STBC schemes with a full rate and full diversity for three and four transmit antennas [5]–[7]. For open-loop communication systems, the constellation rotation was proposed for the QO-STBC with different modulation schemes [5]. Although the QO-STBC with the optimum constellation rotation achieves full diversity, this method increases the system complexity because of constellation expansions. For closed-loop communication systems, the channel state information (CSI) can be utilized to further improve the system performance. Especially, the CSI can be made available to all transmit antennas through a separate feedback channel. The group-coherent STBC (GC-STBC) with one-bit feedback was proposed in [6]. From the simulation, GC-STBC with full diversity and the limited array gain can outperform the Alamouti code (2 Tx,1 Rx) and the ideal 4-path diversity (4 Tx,1 Rx). Inspired by the idea of Akhtar and Gesbert, Badic *et al.* presented QO-STBCs in the correlated fading channels with partial feedback [7]. It is pointed out that channel-adaptive QO-STBCs using two feedback bits per code block match the ideal 4-path diversity based on the maximum likelihood (ML) decoder and zero forcing (ZF) decoder.

Although a lot of partial feedback methods can be adopted to improve the closed-loop system performance, the major problems of such systems are high cost and high complexity due to more than one RF chain at both link ends. A promising technique to reduce this overhead is to allow the transmitter or the receiver to select the optimum antenna subset according to some optimization criteria [8]–[11]. Based on the above discussion, for practical interests of the design of the closed-loop transmission schemes, it is desirable to have features such as a limited amount of feedback information (e.g., 1–2 bits), low decoding delay, low cost, and simple decoding processing.

In this letter, we present the new extended orthogonal STBCs (EO-STBCs) for the quasi-static flat fading channels with three and four transmit antennas. A simple partial feedback scheme is proposed for EO-STBCs. We show that our scheme can achieve a full transmit diversity with a simple detection. Furthermore, receive antenna selection is combined with the closed-loop STBCs. Based on the instantaneous fading coefficients, the receiver selects the best antenna to maximize the received signal-to-noise ratio (SNR), which can result in a maximum diversity.

II. Open-Loop EO-STBCs

In the following, an EO-STBC for four transmit antennas is presented. In the case of two transmit antennas, the well-known Alamouti code [2]

$$A_{(2)} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \tag{1}$$

can be used as a building block to design an EO-STBC with four transmit antennas and full rate, given by

$$EA_{(2)} = \xi \begin{bmatrix} x_1 & x_1 & x_2 & x_2 \\ -x_2^* & -x_2^* & x_1^* & x_1^* \end{bmatrix} \tag{2}$$

where ξ is a constant value $\xi = 1/2$. For simplicity, in the following discussions, we ignore this constant value since it does

Y. Yu and S. Keroueden are with the Electronics Department, ENST Bretagne, 29285 BREST Cedex, France (e-mail: yi.yu@enst-bretagne.fr; sylvie.keroueden@enst-bretagne.fr).

J. Yuan is with the School of Electrical Engineering and Telecommunications, The University of New South Wales, New South Wales 2052, Australia (e-mail: jinhong@ee.unsw.edu.au).

not affect the performance. Especially, by deleting one column of $EA_{(2)}$, we can get two EO-STBCs for three transmit antennas as

$$\begin{bmatrix} x_1 & x_1 & x_2 \\ -x_2^* & -x_2^* & x_1^* \end{bmatrix} \text{ and } \begin{bmatrix} x_1 & x_2 & x_2 \\ -x_2^* & x_1^* & x_1^* \end{bmatrix}. \quad (3)$$

When one receive antenna is considered, the received signals r_1 and r_2 at the time intervals 1 and 2, respectively, can be expressed as

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = EH_{(2)} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \quad (4)$$

where $[n_1 \; n_2^*]^T$ is the additive white Gaussian noise (AWGN) noise vector, and $EH_{(2)}$ is an equivalent channel matrix corresponding to $EA_{(2)}$, given by

$$EH_{(2)} = \begin{bmatrix} h_1 + h_2 & h_3 + h_4 \\ h_3^* + h_4^* & -h_1^* - h_2^* \end{bmatrix}. \quad (5)$$

Here the channel coefficients h_1, h_2, h_3, and h_4 are modeled as independent zero-mean complex Gaussian random variables with variance 0.5 per real dimension. Applying the matched filtering at the receiver with the $EH_{(2)}^H$ matrix, we can obtain the Grammian matrix as follows:

$$G_{(2)} = EH_{(2)}^H EH_{(2)} = \begin{bmatrix} \alpha + \beta & 0 \\ 0 & \alpha + \beta \end{bmatrix} \quad (6)$$

where $\alpha = \sum_{i=1}^{4} |h_i|^2$, $\beta = 2Re(h_1 h_2^*) + 2Re(h_3 h_4^*)$. From (6), it is obvious that the Grammian matrix of the EO-STBC is orthogonal, which indicates that the code can be decoded with a simple receiver. In particular, with a linear processing, the detected signals $\tilde{x_1}$ and $\tilde{x_2}$ can be obtained as

$$\begin{bmatrix} \tilde{x_1} \\ \tilde{x_2} \end{bmatrix} = G_{(2)} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} (h_1^* + h_2^*)n_1 + (h_3 + h_4)n_2^* \\ (h_3^* + h_4^*)n_1 - (h_1 + h_2)n_2^* \end{bmatrix}. \quad (7)$$

Although the decoding complexity is low, β may be negative, which leads to some diversity loss. In order to achieve a full diversity, we can use two feedback bits to rotate the phases of the signals for certain antennas to ensure that β is positive during the whole transmission. Similarly, based on the extended code matrices in (3), we can design a scheme with three transmit antennas using one feedback bit. Proposed feedback schemes will be explained in the following section.

It is worth pointing out that the presented EO-STBC in (2) has advantages over other STBCs for more than two transmit antennas. First, a single RF chain can be adopted for two transmit antennas that transmits the first two columns or the last two columns of $EA_{(2)}$, since $EA_{(2)}$ has the same first two columns and the same last two columns. In addition, the full-rate is achieved with the decoding delay equal to two.

III. Closed-Loop EO-STBCs

The block diagram of the proposed closed-loop EO-STBC scheme with four transmit antennas and one receive antenna is depicted in Fig. 1. We assume that the CSI can be estimated at the receiver.

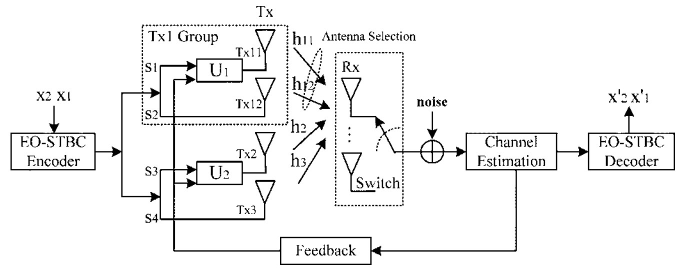

Fig. 1. Baseband representation of the proposed closed-loop system.

At the transmitter, an information sequence is encoded and modulated, and then the modulated symbols are fed into the EO-STBC encoder. At two consecutive symbol intervals t_1 and t_2, we can assume that the modulated inputs of the STBC encoder are x_1 and x_2. The EO-STBC encodes the input data symbols x_1 and x_2 to four data streams, and the outputs of the STBC encoder can be written as

$$\mathbf{s}_1 = \mathbf{s}_2 = \begin{bmatrix} x_1 \\ -x_2^* \end{bmatrix} \text{ and } \mathbf{s}_3 = \mathbf{s}_4 = \begin{bmatrix} x_2 \\ x_1^* \end{bmatrix} \quad (8)$$

where $\mathbf{s}_k$ corresponds to the kth output stream of the STBC encoder, which is transmitted from the kth antenna.

A. EO-STBC for Four Transmit Antennas and One Receive Antenna Using Two Feedback Bits

For the convenience of description, Fig. 1 shows the system for both four (4 Tx) and three (3 Tx) transmit antennas. In the 4 Tx case, the four transmit antennas are denoted by Tx11, Tx12, Tx2, and Tx3. Inthe 3 Tx case, we can group Tx11 and Tx12 to represent a single antenna Tx1, since $\mathbf{s}_1$ equals $\mathbf{s}_2$ in (8). Now let us first consider the 4 Tx case. In order to achieve full diversity, the signals s_1 and s_3 are multiplied by $U_1 = (-1)^i$ and $U_2 = (-1)^k$, where $i, k = 0, 1$, before they are transmitted from antennas Tx11 and Tx2, respectively. Here i and k are two feedback parameters determined by the channel condition. In particular, when i (or k) $= 1$, U_1 (or U_2) $= -1$, which means that s_1 and s_3 in (8) will be phase rotated by 180° before transmission. Otherwise, they can be directly transmitted.

As shown in Fig. 1, for one receive antenna, the received signals at two independent time intervals are

$$\begin{aligned} r_1 &= x_1 U_1 h_{11} + x_1 h_{12} + x_2 U_2 h_2 + x_2 h_3 + n_1 \\ r_2 &= -x_2^* U_1 h_{11} - x_2^* h_{12} + x_1^* U_2 h_2 + x_1^* h_3 + n_2 \end{aligned} \quad (9)$$

where h_{11}, h_{12}, h_2, and h_3 are independent channel coefficients between the corresponding transmit antennas and the receiver. Equation (9) can be rewritten as

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \underbrace{\begin{bmatrix} h_1' & h_2' + h_3 \\ h_2'^* + h_3^* & -h_1'^* \end{bmatrix}}_{NEH_{(2)}} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}}_{\tilde{N}} \quad (10)$$

where h'_1 and h'_2 represent $U_1 h_{11} + h_{12}$ and $U_2 h_2$, respectively. Furthermore, we can calculate the Grammian matrix of $NEH_{(2)}$

$$\begin{aligned} G'_{(2)} &= NEH^H_{(2)} NEH_{(2)} \\ &= \begin{bmatrix} |h'_1|^2 + |h'_2 + h_3|^2 & 0 \\ 0 & |h'_1|^2 + |h'_2 + h_3|^2 \end{bmatrix} \end{aligned} \quad (11)$$

where

$$|h'_1|^2 = |U_1|^2 |h_{11}|^2 + |h_{12}|^2 + 2Re(U_1 h_{11} h^*_{12}) \quad (12)$$

and

$$|h'_2 + h_3|^2 = |U_2|^2 |h_2|^2 + |h_3|^2 + 2Re(U_2 h_2 h^*_3). \quad (13)$$

From (10) and (11), we compute the decision vector $\tilde{X} = [\tilde{x}_1, \tilde{x}_2]^T$ with the receive vector $R = [r_1, r^*_2]^T$ as

$$\begin{aligned} \tilde{X} &= NEH^H_{(2)} \cdot R = (|h'_1|^2 + |h'_2 + h_3|^2) \cdot X + V \\ &= (\underbrace{|U_1|^2 |h_{11}|^2 + |h_{12}|^2 + |U_2|^2 |h_2|^2 + |h_3|^2}_{g_c} \\ &\quad + \underbrace{2Re(U_1 h_{11} h^*_{12}) + 2Re(U_2 h_2 h^*_3)}_{g_f}) \cdot X + V \end{aligned} \quad (14)$$

where $V = NEH^H_{(2)} \tilde{N}$ is a noise component with the covariance matrix $(|h'_1|^2 + |h'_2 + h_3|^2) I_2 N_0$. Since $|U_1|^2 = |U_2|^2 = 1$

$$g'_c = |h_{11}|^2 + |h_{12}|^2 + |h_2|^2 + |h_3|^2. \quad (15)$$

Here, g'_c is the conventional channel gain for four transmit antennas. The feedback performance gain is

$$g_f = 2Re(h_{11} h^*_{12})(-1)^i + 2Re(h_2 h^*_3)(-1)^k. \quad (16)$$

It changes with the defined values $U_1 = (-1)^i$ and $U_2 = (-1)^k$, which are determined by the two feedback information bits i and k. Moreover, we calculate the SNR at the receiver as follows:

$$\gamma_{EA_{(2)}} = \frac{g'_c + g_f}{4} \gamma_0, \quad (i, k = 0 \text{ or } 1) \quad (17)$$

where $\gamma_0 = E_s/N_0$ is the SNR without diversity. It is obvious that if $g_f > 0$, the designed closed-loop system can obtain additional performance gain, which leads to an improved SNR at the receiver. According to the above analysis, we can propose the design criterion of the two-bit feedback scheme. That is, each element of the feedback performance gain g_f in (16) should be nonnegative

$$(i,k) = \begin{cases} (0,0), & \text{if } Re(h_{11} h^*_{12}) \geq 0 \text{ and } Re(h_2 h^*_3) \geq 0 \\ (0,1), & \text{if } Re(h_{11} h^*_{12}) \geq 0 \text{ and } Re(h_2 h^*_3) < 0 \\ (1,0), & \text{if } Re(h_{11} h^*_{12}) < 0 \text{ and } Re(h_2 h^*_3) \geq 0 \\ (1,1), & \text{if } Re(h_{11} h^*_{12}) < 0 \text{ and } Re(h_2 h^*_3) < 0. \end{cases} \quad (18)$$

It is worth pointing out that the proposed new scheme is different from [6], in the sense that we choose the feedback bits to maximize the value of g_f in (14). This can lead to a larger received SNR. It turns out that the new scheme achieves better performance than the GC-STBC in [6]. Moreover, our scheme can be generalized to a large number of transmit antennas, e.g., five or six antennas. However, more feedback bits are required for a large number of antennas.

B. EO-STBC for Three Transmit Antennas and One Receive Antenna Using One Feedback Bit

As mentioned before, for the system with three transmit antennas, we can use one transmit antenna Tx1 to replace the group of the antennas Tx11 and Tx12, which is shown in the dotted region in Fig. 1. Accordingly, we adopt a single path gain h_1 to replace the path gains h_{11} and h_{12} in (10). In this case, we set $U_1 = 1$. Therefore, only a single feedback bit k is required to determine $U_2 = (-1)^k$. With this encoding scheme, the SNR at the receiver is expressed as

$$\gamma_{EA'_{(2)}} = \frac{g''_c + g'_f}{3} \gamma_0, \quad (k = 0 \text{ or } 1) \quad (19)$$

where $\gamma_0 = E_s/N_0$, $g''_c = |h_1|^2 + |h_2|^2 + |h_3|^2$ with $|U_2|^2 = 1$, and $g'_f = 2Re(h_2 h^*_3)(-1)^k$. The single feedback bit k is computed to ensure that $g'_f \geq 0$. It is given by

$$k = \begin{cases} 0, & \text{if } Re(h_2 h^*_3) \geq 0 \\ 1, & \text{otherwise.} \end{cases} \quad (20)$$

C. Receive Antenna Selection for the Closed-Loop EO-STBC

In this section, we combine receive antenna selection with the proposed closed-loop EO-STBCs. Here, EO-STBC is one of the nonorthogonal STBCs. We present the selection criterion for the code with four transmit antennas as follows.

Step 1) Calculate $Re(h_{11,i} h^*_{12,i})$ and $Re(h_{2,i} h^*_{3,i})$, $i = 1, \ldots, M$, where M is the number of receive antennas. Use **(18)** to obtain different feedback bits for every receive antenna.
Step 2) Calculate the corresponding values $U_{1,i}$ and $U_{2,i}$ according to the partial feedback information at the receiver.
Step 3) Calculate $g'_{c,i}$ and $g_{f,i}$ using **(15)** and **(16)** to obtain the $\gamma_{EA_{(2)},i}$, $i = 1, \ldots, M$, for each receive antenna.
Step 4) According to the maximum receiver SNR, select the corresponding receive antenna. At the same time, the receiver sends two feedback bits to the transmitter.

In this criterion, steps 1) and 2) ensure that the values of $g_{f,i}$ are positive, $i = 1, \ldots, M$. Step 4) selects the desired receive antenna based on the EO-STBC with full diversity. For the system with three transmit antennas, it is straightforward to apply this scheme accordingly.

IV. Simulation Results

In this section, we evaluate the error performance of the proposed schemes in quasi-static flat fading channels. The fading is constant within a frame and changes independently from frame to frame. For the closed-loop system, we have simulated the BER against E_b/N_0 using QPSK symbols leading to an information rate of 2 bits/s/Hz. Each frame consists of 1000 symbols in our simulation. In Fig. 2, we show the performance of the proposed closed-loop EO-STBC for four transmit antennas. The performance of the O-STBC [3], the QO-STBC

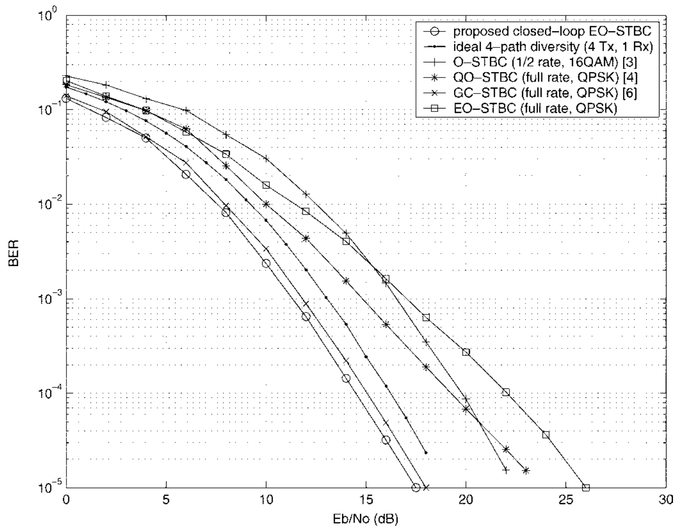

Fig. 2. Performance comparison for various STBCs in 4×1 systems.

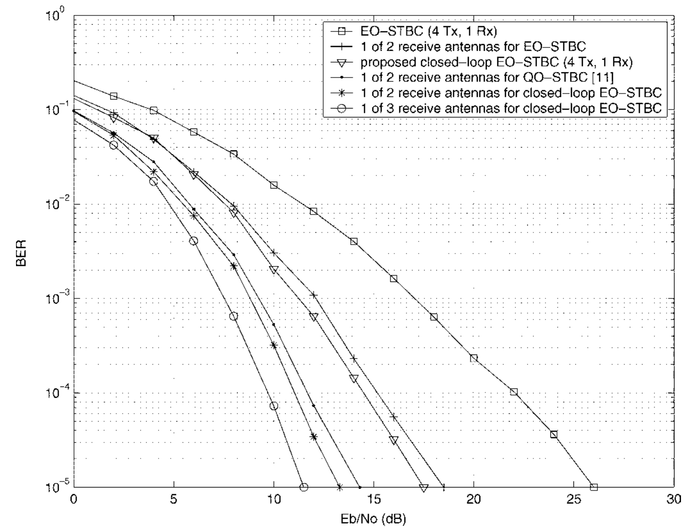

Fig. 3. BER of the EO-STBCs with receive antenna selection for 4 Tx.

[4], and the GC-STBC [6] is also shown in Fig. 2 for comparison. All schemes have the same spectral efficiency. We can see that the new EO-STBC with two-bit feedback is better than the O-STBC, the QO-STBC, and the GC-STBC. It is obvious that in the high SNR region, the slope of our codes is parallel to that of O-STBC. Especially, the closed-loop EO-STBC can outperform the ideal four-path diversity. As shown in Fig. 2, when the BER is 10^{-4}, it can get a 2.1-dB gain over the ideal diversity. Moreover, our scheme outperforms the GC-STBC [6] by 0.6 dB at the BER of 10^{-4}. Furthermore, the performance of the closed-loop EO-STBC with the imperfect CSI is investigated. A pilot sequence with a length of eight symbols is inserted at the beginning of each frame for the channel estimation. The simulation results show that due to the imperfect channel estimation, the scheme performance is degraded by about 1.2 dB compared to the case with the ideal CSI at the BER of 10^{-5}.

Fig. 3 shows the simulation results of the EO-STBCs with receive antenna selection for four transmit antennas. Simulation results for four transmit antennas and up to three receive antennas indicate that when the BER is 10^{-5}, performance gains of up to 5.8 dB are obtained over the closed-loop EO-STBCs without antenna selection. In Fig. 3, at the BER of 10^{-5}, increasing the number of receive antennas from one to two, we achieve an SNR gain of 4.2 dB, whereas only an additional gain of 1.6 dB is obtained to increase the number of the receive antennas from two to three. In addition, our scheme can get 1 dB over the scheme for the QO-STBC in [11] with receive antenna selection.

V. Conclusion

A simple closed-loop EO-STBC is proposed in this letter. For systems with three and four transmit antennas, the amount of the feedback information is as few as 1 bit and 2 bits, respectively. Receive antenna selection is combined with the closed-loop EO-STBCs to achieve the additional performance gain. Simulation results show that the proposed closed-loop EO-STBC outperforms not only the open-loop diversity techniques but also previous closed-loop STBCs. It represents a good candidate for the future space–time multiple-input and multiple-output (MIMO) systems.

References

[1] R. T. Derryberry, S. D. Gray, and D. M. Ionescu, "Transmit diversity in 3G CDMA systems," *IEEE Commun. Mag.*, vol. 40, no. 4, pp. 68–75, Apr. 2002.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.

[4] H. Jafarkhani, "A quasiorthogonal space-time codes," *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 1–4, Jan. 2001.

[5] W. Su and X. G. Xia, "Signal constellations for quasiorthogonal space-time block codes with full diversity," *IEEE Trans. Inf. Theory*, vol. 50, no. 10, pp. 2331–2347, Oct. 2004.

[6] J. Akhtar and D. Gesbert, "Extending orthogonal block codes with partial feedback," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1959–1962, Nov. 2004.

[7] B. Badic, M. Rupp, and H. Weinrichter, "Extended Alamouti codes in correlated channels using partial feedback," in *Proc. IEEE Int. Conf. Communications*, vol. 2, Jun. 2004, pp. 896–900.

[8] D. Gore and A. Paulraj, "Space-time block coding with optimal antenna selection," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing*, vol. 4, May 2001, pp. 2441–2444.

[9] Z. Chen, J. Yuan, B. Vucetic, and Z. Zhou, "Performance of Alamouti scheme with transmit antenna selection," *Electron. Lett.*, vol. 39, no. 23, pp. 1666–1668, Nov. 2003.

[10] S. Sanayei and A. Nostratinia, "Antenna selection in MIMO systems," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 68–73, Oct. 2004.

[11] B. Badic, P. Fuxjaeger, and H. Weinrichter, "Performance of quasiorthogonal space-time code with antenna selection," *Electron. Lett.*, vol. 40, no. 20, pp. 1282–1284, Sep. 2004.