# Reduced-Complexity Reed-Solomon Decoders Based on Cyclotomic FFTs 

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#### Abstract

In this paper, we reduce the computational complexities of partial and dual partial cyclotomic FFTs (CFFTs), which are discrete Fourier transforms where spectral and temporal components are constrained, based on their properties as well as a common subexpression elimination algorithm. Our partial CFFTs achieve smaller computational complexities than previously proposed partial CFFTs. Utilizing our CFFTs in both transform- and time-domain Reed-Solomon decoders, we achieve significant complexity reductions.


## Index Terms

Common subexpression elimination (CSE), Decoding, Discrete Fourier transforms, Galois fields, Reed-Solomon codes.

## I. Introduction

Due to the widespread applications of Reed-Solomon (RS) codes [1] in various digital communication and storage systems, efficient RS decoding has been an important research topic (see, for example, [2]-[8]). Since all syndrome-based hard-decision decoding methods for RS codes involve discrete Fourier transforms (DFTs) over finite fields [1], fast Fourier transform (FFT) algorithms can be used to reduce the complexity of RS decoders (see, for example, [3], [4], [6]).

Using an approach similar to those in previous works (see, for example, [9]), cyclotomic FFTs (CFFTs) were recently proposed [10] and two variants were subsequently considered [3], [6]. To avoid confusion, in this paper we refer to the CFFTs proposed in [10] as direct CFFTs (DCFFTs) and those in [3] and [6] as inverse CFFTs (ICFFTs) and symmetric CFFTs (SCFFTs), respectively. Given a primitive element $\alpha \in \operatorname{GF}\left(2^{m}\right)$, the DFT of a vector $\boldsymbol{f}=\left(f_{0}, f_{1}, \ldots, f_{n-1}\right)^{T}$ is defined as $\boldsymbol{F} \triangleq\left(f\left(\alpha^{0}\right), f\left(\alpha^{1}\right), \ldots, f\left(\alpha^{n-1}\right)\right)^{T}$, where $f(x) \triangleq \sum_{i=0}^{n-1} f_{i} x^{i} \in \operatorname{GF}\left(2^{m}\right)[x]$. A DCFFT is given by $\boldsymbol{F}=\boldsymbol{A} \boldsymbol{L} \boldsymbol{f}^{\prime}=$ $\boldsymbol{A} \boldsymbol{Q}\left(\boldsymbol{c} \cdot \boldsymbol{P} \boldsymbol{f}^{\prime}\right)$, where $\boldsymbol{A}$ is an $n \times n$ binary matrix, $\boldsymbol{L}$ is a block diagonal matrix, $\boldsymbol{f}^{\prime}$ is a permutation of the input vector $\boldsymbol{f}, \boldsymbol{c}$ is a pre-computed vector, - stands for pointwise multiplications, and $\boldsymbol{Q}$ and $\boldsymbol{P}$ are both sparse binary matrices. Similarly, an SCFFT is given by $\boldsymbol{F}^{\prime}=\boldsymbol{L}^{T} \boldsymbol{A}^{T} \boldsymbol{f}^{\prime}=\boldsymbol{P}^{T}\left(\boldsymbol{c} \cdot\left(\boldsymbol{A}^{\prime} \boldsymbol{Q}\right)^{T} \boldsymbol{f}^{\prime}\right)$, where both $\boldsymbol{F}^{\prime}$ and $\boldsymbol{f}^{\prime}$ are permuted by the same permutation matrix, and $\boldsymbol{L}^{T} \boldsymbol{A}^{\prime T}$ is symmetric. Finally, based on inverse DFTs, an ICFFT is given by $\boldsymbol{F}^{\prime \prime}=\boldsymbol{L}^{-1} \boldsymbol{A}^{-1} \boldsymbol{f}=\boldsymbol{P}^{T}\left(\boldsymbol{c}^{*} \cdot \boldsymbol{Q}^{T} \boldsymbol{A}^{-1} \boldsymbol{f}\right)$, where $\boldsymbol{F}^{\prime \prime}$ is also a permutation of $\boldsymbol{F}$ and $\boldsymbol{c}^{*}$ is a pre-computed vector. Since all CFFTs are in bilinear forms [1], we refer to $\boldsymbol{P},\left(\boldsymbol{A}^{\prime} \boldsymbol{Q}\right)^{T}$, and $\boldsymbol{Q}^{T} \boldsymbol{A}^{-1}$ as pre-addition matrices and $\boldsymbol{A} \boldsymbol{Q}, \boldsymbol{P}^{T}$, and $\boldsymbol{P}^{T}$ as post-addition matrices for DCFFTs, SCFFTs, and ICFFTs, respectively. The numbers of non-one elements in $c$ or $c^{*}$ are the number of multiplications required, and the pre- and post-addition matrices determine the additive complexities of CFFTs. Though CFFTs in [3], [6], [10] achieve low multiplicative complexities, their additive complexities (numbers of additions required) are very high, with or without the various methods used in [3], [6], [10] to reduce the additive complexities. In our previous work [11], we proposed a novel common subexpression elimination (CSE) algorithm, and then used it to reduce the additive complexities of full CFFTs.

This paper has three main contributions. First, we reduce both multiplicative and additive complexities of partial CFFTs, which compute only part of the spectral components [3], [6], based on their properties as well as our CSE algorithm in [11]. Our partial CFFTs have smaller complexities than those in [3]. Second, we propose dual partial CFFTs, where only a subset of temporal components are nonzero, and reduce their complexities. Finally, applying our partial and dual partial CFFTs, we reduce the complexities of time- and transform-domain RS decoders significantly.

## II. Partial and Dual Partial CFFTs

We now consider CFFTs in two special cases. One special case is when only a subset of frequency components are needed, and we refer to such CFFTs as partial CFFTs following the convention in [3], [6]. The other special case is when a subset of temporal components are all zeros. The two special cases can be viewed as dual to each other; Thus, for the lack of a better term, we refer to CFFTs in the second special case as dual partial CFFTs.

In a partial CFFT, some frequency components are not needed. Thus, we first eliminate the rows corresponding to the unnecessary frequency components from the post-addition matrices, possibly resulting in all-zero columns. We then remove the all-zero columns from the reduced post-addition matrices, as well as the corresponding entries in $c$ or $c^{*}$ and the corresponding

[^0]rows from the pre-addition matrices. For dual partial CFFTs, some temporal components are zeros. Thus, we first remove the corresponding columns in the pre-addition matrices, leading to all-zero rows. We then remove the all-zero rows from the reduced pre-addition matrices and the corresponding entries in $\boldsymbol{c}$ or $\boldsymbol{c}^{*}$ as well as the corresponding columns from the post-addition matrices.

It was shown [11] full SCFFTs and ICFFTs are equivalent in terms of complexities; Using a similar argument we can show that SCFFTs and ICFFTs are also equivalent in partial and dual partial DFTs. In both special cases of CFFTs, removing rows or columns from pre- and post-addition matrices leads to reduced additive complexities, and eliminating entries in $c$ or $c^{*}$ results in reduced multiplicative complexities. Both multiplicative and additive complexity reductions depend on the type of CFFTs. Note that $\boldsymbol{P}^{T}$ and $\boldsymbol{P}$ are sparse, while $\boldsymbol{A} \boldsymbol{Q},\left(\boldsymbol{A}^{\prime} \boldsymbol{Q}\right)^{T}$, and $\boldsymbol{Q}^{T} \boldsymbol{A}^{-1}$ are not. Thus, removing a certain number of rows or columns from $\boldsymbol{P}^{T}$ ( $\boldsymbol{P}$, respectively) leads to less significant reductions in additive complexities than from $\boldsymbol{A} \boldsymbol{Q}$ $\left(\left(\boldsymbol{A}^{\prime} \boldsymbol{Q}\right)^{T}\right.$ and $\boldsymbol{Q}^{T} \boldsymbol{A}^{-1}$, respectively). On the other hand, after removing some rows (columns, respectively), a reduced $\boldsymbol{P}^{T}$ ( $\boldsymbol{P}$, respectively) is more likely to have all-zero columns (rows, respectively) that eliminate entries in $\boldsymbol{c}$ or $\boldsymbol{c}^{*}$ than $\boldsymbol{A} \boldsymbol{Q}\left(\left(\boldsymbol{A}^{\prime} \boldsymbol{Q}\right)^{T}\right.$ and $\boldsymbol{Q}^{T} \boldsymbol{A}^{-1}$, respectively). Thus, partial DCFFTs have higher multiplicative complexities but lower additive complexities than partial SCFFTs/ICFFTs; similarly, dual partial DCFFTs lead to lower multiplicative complexities but higher additive complexities than dual partial SCFFTs/ICFFTs.

The savings in multiplicative complexities by partial SCFFTs/ICFFTs (dual partial DCFFTs, respectively) are improved by considering different permutations of $\boldsymbol{F}^{\prime}$ and $\boldsymbol{F}^{\prime \prime}\left(\boldsymbol{f}^{\prime}\right.$, respectively) while preserving all cyclotomic cosets. These permutations do not impact $\boldsymbol{P}^{T}\left(\boldsymbol{P}\right.$, respectively) in a full CFFT, but by permuting $\boldsymbol{F}^{\prime}$ and $\boldsymbol{F}^{\prime \prime}\left(\boldsymbol{f}^{\prime}\right.$, respectively) the removed rows (columns, respectively) in $\boldsymbol{P}^{T}$ ( $\boldsymbol{P}$, respectively) result in more all-zero columns (rows, respectively) and thus achieve greater savings in multiplicative complexities. This technique is equivalent to the rotation of normal bases in [3].

In addition to the complexity reduction techniques discussed above, which utilizes only the properties of the DFTs, we also apply our CSE algorithm [11] to further reduce the additive complexities of both partial and dual partial CFFTs.

Partial CFFTs and their applications in syndrome computation were considered in [3], [6], while dual partial CFFTs have not been considered in the literature to the best of our knowledge. In Section III-A, we compare the complexities of syndrome computation based on a variety of approaches, including our partial CFFTs and those in [3]. We do not compare to [6] because no details were provided.

## III. Reduced-Complexity RS Decoders

Using full CFFTs [11] as well as partial and dual partial CFFTs described above, we propose both time- and transform-domain RS decoders with reduced complexities.

TABLE I
Complexity of Syndrome Computation

| ( $n, k$ ) | Horner's rule |  | [9] |  | ICFFT [3] |  | [2] |  | [5] |  | Prime-factor [4] |  |  | Our SCFFT/ICFFT |  |  | Our DCFFT |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mult. | Add. | Mult. | Add. | Mult. | Add. | Mult. | Add. | Mult. | Add. | Mult. | Add. | Total | Mult. | Add. | Total | Mult. | Add. | Total |
| $(255,223)$ | 7874 | 8128 | 167 | 5440 | 149 | 5046 | 8160 | 8128 | 3060 | 4998 | 852 | 1804 | 14584 | 149 | 3970 | 6205 | 586 | 2850 | 11640 |
| $(511,447)$ | 32130 | 32640 | - | - | - | - | 32704 | 32640 | 9888 | 17819 | 5265 | 7309 | 35496 | 345 | 16471 | 22336 | 1014 | 7904 | 25142 |
| $(1023,895)$ | 129794 | 130816 | - | - | - | - | 130944 | 130816 | 33620 | 73185 | 6785 | 15775 | 144690 | 824 | 60741 | 76397 | 2827 | 25118 | 78831 |

## A. Syndrome Computation

We implement syndrome computation, which is used in both time- and transform-domain decoders, with partial CFFTs. For $(255,223),(511,447),(1023,895)$ RS codes, which are selected due to their widespread applications [4], we compare the complexities of syndrome computation based on our partial SCFFTs/ICFFTs with the complexities of syndrome computation based on partial CFFTs in [3] and other approaches such as Horner's rule, Zakharova's algorithm [9], and the prime-factor FFT [4] in Table [ The results for the length- 255 RS code using Horner's rule as well as the algorithms in [9] and [3] are obtained from [3]; The results for RS codes of lengths 511 and 1023 using Horner's rule and the algorithm in [5] are reproduced from [5]; the numbers of multiplications and additions for the prime-factor FFT [4] are reproduced from [4]. In comparison to these approaches except the prime-factor FFT [4], our partial SCFFTs/ICFFTs apparently require smaller complexities for syndrome computation. To compare to the prime-factor FFT [4], we use the metric for the total complexities $N_{\text {total }}=(2 m-1) N_{\text {mult }}+N_{\text {add }}$ as in [11]. Syndrome computation based on our partial SCFFTs/ICFFTs requires smaller total complexities than those based on the prime-factor FFT [4]. We provide the details of the syndrome computation for the $(255,223)$ RS code based on our partial SCFFT in the appendix.

## B. Chien Search and Forney's Formula

For errors-only (errors-and-erasures, respectively) decoders, the Chien search evaluates the error locator polynomial of degree at most $t$ (errata locator polynomial of degree at most $2 t$, respectively) over all the elements of the underlying field; each
root leads to one error (errata) location. This evaluation is essentially a DFT of a vector for which only first $t+1(2 t+1$, respectively) temporal components are not zeros. Note that the Chien search in errors-and-erasures decoders needs to evaluate only the error locator polynomial if it is available. For errors-only (errors-and-erasures, respectively) decoders, Forney's formula evaluates two polynomials: one is the error (errata, respectively) evaluator polynomial $A(x)$ and the other polynomial $x \tau^{\prime}(x)$ is based on the derivative of error (errata, respectively) locator polynomial $\tau(x)$. The degree of the error (errata, respectively) evaluator polynomial is less than $t$ ( $2 t$, respectively), while the degree of $x \tau^{\prime}(x)$ is no more than $t$ ( $2 t$, respectively). Roughly half of the coefficients in $x \tau^{\prime}(x)$ are zero. Given these information, the techniques explained above for dual partial CFFTs are again applicable. For simplicity, we assume errors-and-erasures decoders henceforth, and our results can be easily extended to errors-only decoders.

The errata locator polynomial $\tau(x)$ satisfies $\tau(x)=\hat{\tau}_{e}\left(x^{2}\right)+x \hat{\tau}_{o}\left(x^{2}\right)$, where $\hat{\tau}_{e}\left(x^{2}\right)$ and $x \hat{\tau}_{o}\left(x^{2}\right)$ consist of terms with even and odd degrees, respectively. Note that $\hat{\tau}_{e}(x)$ and $\hat{\tau}_{o}(x)$ have degrees at most $t$ and $t-1$, respectively. It is easily verified that $\hat{\tau}_{o}\left(x^{2}\right)=\tau^{\prime}(x)$ for characteristic-2 fields.

While the Chien search evaluates $\tau(x)$ at all $n=2^{m}-1$ points, Forney's formula evaluates $A(x)$ and $\tau^{\prime}(x)$ at up to $2 t$ errata locations. Thus, given the errata locations, the evaluations of $A(x)$ and $\tau^{\prime}(x)$ are DFTs, in which not only part of temporal components are zeros but also only part of frequency components are needed. Thus, the complexity reduction techniques for both partial and dual partial CFFTs are applicable, and our CSE algorithm can be applied. However, since the errata locations vary, it is infeasible to minimize the computational complexities "on the fly." Thus, we also evaluate $A(x)$ and $\tau^{\prime}(x)$ over all $n=2^{m}-1$ points. Since $\tau^{\prime}(x)$ is evaluated over all the points, its evaluation is useful for both the Chien search and Forney's formula. Thus, the Chien search and Forney's formula are carried out jointly.

The evaluation of the $A(x)$ is directly implemented as a dual partial CFFT. For any $\alpha \neq 0$ in $\operatorname{GF}\left(2^{m}\right)$, we can either obtain $\left.\hat{\tau}_{e}\left(x^{2}\right)\right|_{x=\alpha}$ by dual partial CFFTs, or first evaluate $\left.\hat{\tau}_{e}(x)\right|_{x=\alpha}$ by dual partial CFFTs and then obtain $\left.\hat{\tau}_{e}\left(x^{2}\right)\right|_{x=\alpha}$ by properly permuting the frequency components. Although $\hat{\tau}_{e}\left(x^{2}\right)$ and $\hat{\tau}_{e}(x)$ have the same number of non-zero terms, the non-zero terms of $\hat{\tau}_{e}(x)$ fall into fewer cosets than those of $\tau_{o}(x)$, so its evaluation based on dual partial CFFTs has smaller multiplicative and additive complexities. Similar to our approach for $\hat{\tau}_{e}\left(x^{2}\right)$, we have two options to obtain $\left.x \hat{\tau}_{o}\left(x^{2}\right)\right|_{x=\alpha}$. The first option is treat $x \hat{\tau}_{o}\left(x^{2}\right)$ as a polynomial of degree at most $2 t-1$ and obtain $\left.x \hat{\tau}_{o}\left(x^{2}\right)\right|_{x=\alpha}$ using dual partial CFFTs. The other option is to first compute $\left.\hat{\tau}_{o}(x)\right|_{x=\alpha}$ using dual partial CFFTs, then obtain $\left.\hat{\tau}_{o}\left(x^{2}\right)\right|_{x=\alpha}$ by permutation, and finally compute $\left.x \hat{\tau}_{o}\left(x^{2}\right)\right|_{x=\alpha}$. For the latter option, similar to the reason given above, the evaluation of $\hat{\tau}_{o}(x)$ based on dual partial CFFTs requires fewer multiplications and additions than that of $\hat{\tau}_{o}\left(x^{2}\right)$, although they have the same number of non-zero terms. However, the latter option also requires $n$ extra multiplications. Thus, the latter option has higher multiplicative complexities but lower additive complexities as opposed to the former option.

We present the computational complexities of combined Chien search and Forney's formula for errors-and-erasures decoders based on our dual partial CFFTs in Table II Note that to evaluate $\tau(x)$ at all $2 t$ errata locations, $2 t$ additions are needed; Also, $2 t$ divisions are needed to compute the errata values in Forney's formula; Both are accounted for in the rows marked by "Misc." The rows marked by "Sum" sum up the numbers of field operations required to evaluate $A(x), \hat{\tau}_{e}\left(x^{2}\right)$, and $x \hat{\tau}_{o}\left(x^{2}\right)$, as well as $2 t$ additions and $2 t$ divisions mentioned above. As in Section III-A the total complexities of each individual step and the sum are measured by the metric in [11], and we assume division has the same complexity as multiplication. They are presented in the columns marked by "Total." The complexities of the two options for evaluating $x \hat{\tau}_{o}\left(x^{2}\right)$ are both given; the $n$ extra multiplications in the second option are shown in the second terms of the summations. Due to the $n$ extra multiplications, for lengths 255 and 511 the first option has a smaller total complexity; for length 1023, the second option has a smaller total complexity. For evaluating $x \hat{\tau}_{o}\left(x^{2}\right)$, the option with smaller total complexity is used.

TABLE II
Complexity of Combined Chien Search and Forney's Formula for errors-and-erasures decoders


The computational complexities based on our dual partial DCFFTs are compared to the complexities based on Horner's rule in Table II. We also reproduce the complexities of the Chien search and Forney's formula in [2] from [4]. The combined Chien search and Forney's formula based on our partial dual CFFTs achieves significantly smaller computational complexities than those based on Horner's rule and in [2]. We do not compare with the approaches in [5], [7], [8] because their computational complexities are not available.

## C. Example

We provide a simple example to illustrate syndrome computation and Chien search based on our CFFTs. For simplicity, let us consider errors-and-erasures decoding of a $(31,25)$ cyclic RS code over $\mathrm{GF}\left(2^{5}\right)$ defined by the primitive polynomial $p(x)=x^{5}+x^{2}+1$. The generator polynomial for the RS code is given by $g(x)=(x-1)(x-\alpha) \cdots\left(x-\alpha^{5}\right)$, where $\alpha$ is a root of $p(x)$. There are seven cyclotomic cosets over this field.

Suppose the received vector is $\boldsymbol{r}=\left(r_{0}, r_{1}, \ldots, r_{n-1}\right)$. To compute the syndromes $S_{i}=\sum_{j=0}^{n-1} r_{j} \alpha^{i j}$ for $0 \leq i \leq 5$, it involves only four cyclotomic cosets: $\{0\},\{2,4,8,16,1\},\{6,12,24,17,3\},\{10,20,9,18,5\}$. As explained in Section III we have rotated the cosets in this order to reduce multiplicative complexity. We do not specify the other cosets since they are irrelevant to our purpose. Using the normal basis ( $\alpha^{3}, \alpha^{6}, \alpha^{12}, \alpha^{24}, \alpha^{17}$ ) and the length-5 convolution algorithm in [1], we first construct a full SCFFT $\boldsymbol{S}^{\prime}=\boldsymbol{P}^{T}\left(\boldsymbol{c} \cdot\left(\boldsymbol{A}^{\prime} \boldsymbol{Q}\right)^{T} \boldsymbol{r}^{\prime}\right)$, in which $\boldsymbol{S}^{\prime}$ and $\boldsymbol{r}^{\prime}$ are permuted versions of $\boldsymbol{S}=\left(S_{0}, S_{1}, \ldots, S_{30}\right)^{T}$ and $\boldsymbol{r}=\left(r_{0}, r_{1}, \ldots, r_{30}\right)^{T}$, both ordered in the chosen cosets. For these four cosets, their $\boldsymbol{P}_{i}^{T}$ and $\boldsymbol{c}_{i}$ are given by $\boldsymbol{P}_{0}^{T}=[1]$, $\boldsymbol{c}_{0}=(1), \boldsymbol{c}_{1}=\boldsymbol{c}_{2}=\boldsymbol{c}_{3}=\left(1, \alpha, \alpha^{25}, \alpha^{7}, \alpha^{2}, \alpha^{16}, \alpha^{4}, \alpha^{28}, \alpha^{14}, \alpha^{27}\right)^{T}$, and

$$
\boldsymbol{P}_{1}^{T}=\boldsymbol{P}_{2}^{T}=\boldsymbol{P}_{3}^{T}=\left[\begin{array}{cccccccccc}
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1
\end{array}\right]
$$

Let $\boldsymbol{A}^{\prime}=\left[\boldsymbol{A}_{0}^{\prime}\left|\boldsymbol{A}_{1}^{\prime}\right| \cdots \mid \boldsymbol{A}_{6}^{\prime}\right], \boldsymbol{A}^{\prime} \boldsymbol{Q}=\left[\boldsymbol{A}_{0}^{\prime} \boldsymbol{Q}_{0}\left|\boldsymbol{A}_{1}^{\prime} \boldsymbol{Q}_{1}\right| \cdots \mid \boldsymbol{A}_{6}^{\prime} \boldsymbol{Q}_{6}\right]$.
In the coset $\{2,4,8,16,1\}$, we need only $\{2,4,1\}$, and thus we remove the third and fourth rows in $\boldsymbol{P}_{1}^{T}$, resulting in the eighth column being all-zero. So we strike out the column and save one more multiplication. Let $\boldsymbol{P}_{1}^{\prime T}$ denote the reduced matrix. Note that other orders of the coset cannot produce all-zero columns. This can also be achieved by rotating the normal basis. Since we only need $\{3\}$ and $\{5\}$ in the cosets $\{6,12,24,17,3\}$ and $\{10,20,9,18,5\}$, respectively, we obtain $\boldsymbol{P}_{2}^{\prime T}=\boldsymbol{P}_{3}^{\prime T}$ by keeping only the last row in $\boldsymbol{P}_{2}^{T}$ and striking out the all-zero fourth, seventh, eighth and ninth columns. Correspondingly, we remove the eighth row from $\boldsymbol{Q}_{1}^{T}$ and $\boldsymbol{c}_{1} \cdot \boldsymbol{Q}_{2}^{\prime T}=\boldsymbol{Q}_{3}^{\prime T}$ and $\boldsymbol{c}_{2}^{\prime}=\boldsymbol{c}_{3}^{\prime}$ are given by removing the fourth, seventh, eighth, and ninth rows from $\boldsymbol{Q}_{2}^{T}$ and $\boldsymbol{c}_{2}$, respectively. Hence the syndromes can be computed by a partial SCFFT as

$$
\left.\left[\begin{array}{l}
S_{0} \\
S_{2} \\
S_{4} \\
S_{1} \\
S_{3} \\
S_{5}
\end{array}\right]=\left[\begin{array}{llll}
\boldsymbol{P}_{0}^{T} & & & \\
& \boldsymbol{P}_{1}^{\prime T} & & \\
& & \boldsymbol{P}_{2}^{\prime T} & \\
& & & \boldsymbol{P}_{3}^{\prime T}
\end{array}\right]\left(\left[\begin{array}{l}
\boldsymbol{c}_{0} \\
\boldsymbol{c}_{1}^{\prime} \\
\boldsymbol{c}_{2}^{\prime} \\
\boldsymbol{c}_{3}^{\prime}
\end{array}\right] \cdot\left[\begin{array}{l}
\left(\boldsymbol{A}_{0}^{\prime} \boldsymbol{Q}_{0}\right)^{T} \\
\left(\boldsymbol{A}_{1}^{\prime} \boldsymbol{Q}_{1}^{\prime}\right)^{T} \\
\left(\boldsymbol{A}_{2}^{\prime} \boldsymbol{Q}_{2}^{\prime}\right)^{T} \\
\left(\boldsymbol{A}_{3}^{\prime} \boldsymbol{Q}_{3}^{\prime}\right)^{T}
\end{array}\right]\right) \boldsymbol{r}^{\prime}\right)
$$

If there are all-zero columns in $\boldsymbol{Q}_{1}^{\prime T}, \boldsymbol{Q}_{2}^{\prime T}, \boldsymbol{Q}_{3}^{\prime T}$, we can strike out those columns and further remove corresponding rows from $\boldsymbol{A}_{i}^{\prime T}$,s.

In the Chien search, the errata locator polynomial $\tau(x)=\sum_{i=0}^{6} \tau_{i} x^{i}$ has degree up to six. So we need to use $\{6,3\}$ for the third coset. Thus the Chien search can by done by a dual partial DCFFT

$$
\left[\boldsymbol{A}_{0}^{\prime} \boldsymbol{Q}_{0}\left|\boldsymbol{A}_{1}^{\prime} \boldsymbol{Q}_{1}^{\prime}\right| \boldsymbol{A}_{2}^{\prime} \boldsymbol{Q}_{2}^{\prime \prime} \mid \boldsymbol{A}_{3}^{\prime} \boldsymbol{Q}_{3}^{\prime}\right]\left(\left[\begin{array}{c}
\boldsymbol{c}_{0} \\
\boldsymbol{c}_{1}^{\prime} \\
\boldsymbol{c}_{2}^{\prime \prime} \\
\boldsymbol{c}_{3}^{\prime}
\end{array}\right] \cdot\left[\begin{array}{llll}
\boldsymbol{P}_{0} & & & \\
& \boldsymbol{P}_{1}^{\prime} & & \\
& & \boldsymbol{P}_{2}^{\prime \prime} & \\
& & & \boldsymbol{P}_{3}^{\prime}
\end{array}\right]\left[\begin{array}{c}
\tau_{0} \\
\tau_{2} \\
\tau_{4} \\
\tau_{1} \\
\tau_{6} \\
\tau_{3} \\
\tau_{5}
\end{array}\right]\right)
$$

where $\boldsymbol{P}_{2}^{\prime \prime}$ is obtained by keeping only the first and last columns of $\boldsymbol{P}_{2}, \boldsymbol{c}_{2}^{\prime \prime}$ and $\boldsymbol{Q}_{2}^{\prime \prime}$ are obtained by removing the eighth and ninth rows from $c_{2}$ and the corresponding columns from $\boldsymbol{Q}_{2}$.

The Chien search can be split into evaluating $\tau_{e}(x)=\hat{\tau}_{e}\left(x^{2}\right)$ and $\tau_{o}(x)=x \hat{\tau}_{o}\left(x^{2}\right)$ to accommodate Forney's formula. The direct evaluation of $\tau_{o}(x)$ can be carried out by

$$
\left[\boldsymbol{A}_{1}^{\prime} \boldsymbol{Q}_{1}^{\prime \prime}\left|\boldsymbol{A}_{2}^{\prime} \boldsymbol{Q}_{2}^{\prime}\right| \boldsymbol{A}_{3}^{\prime} \boldsymbol{Q}_{3}^{\prime}\right]\left(\left[\begin{array}{l}
\boldsymbol{c}_{1}^{\prime \prime} \\
\boldsymbol{c}_{2}^{\prime} \\
\boldsymbol{c}_{3}^{\prime}
\end{array}\right] \cdot\left[\begin{array}{lll}
\boldsymbol{P}_{1}^{\prime \prime} & & \\
& \boldsymbol{P}_{2}^{\prime} & \\
& & \boldsymbol{P}_{3}^{\prime}
\end{array}\right]\left[\begin{array}{c}
\tau_{1} \\
\tau_{3} \\
\tau_{5}
\end{array}\right]\right)
$$

where $\boldsymbol{Q}_{1}^{\prime \prime}=\boldsymbol{Q}_{2}^{\prime}, \boldsymbol{c}_{1}^{\prime \prime}=\boldsymbol{c}_{2}^{\prime}$, and $\boldsymbol{P}_{1}^{\prime \prime}=\boldsymbol{P}_{2}^{\prime}$. Alternatively, the evaluation of $\hat{\tau}_{o}\left(x^{2}\right)$ can be carried out by

$$
\left[\boldsymbol{A}_{0}^{\prime} \boldsymbol{Q}_{0} \mid \boldsymbol{A}_{1}^{\prime} \boldsymbol{Q}_{1}^{\prime \prime \prime}\right]\left(\left[\begin{array}{c}
\boldsymbol{c}_{0} \\
\boldsymbol{c}_{1}^{\prime \prime \prime}
\end{array}\right] \cdot\left[\begin{array}{ll}
\boldsymbol{P}_{0} & \\
& \boldsymbol{P}_{1}^{\prime \prime \prime}
\end{array}\right]\left[\begin{array}{c}
\hat{\tau}_{o, 0} \\
\hat{\tau}_{o, 2} \\
\hat{\tau}_{o, 1}
\end{array}\right]\right)
$$

where $\boldsymbol{Q}_{1}^{\prime \prime \prime}=\boldsymbol{Q}_{2}^{\prime \prime}, \boldsymbol{c}_{1}^{\prime \prime \prime}=\boldsymbol{c}_{2}^{\prime \prime}$, and $\boldsymbol{P}_{1}^{\prime \prime \prime}=\boldsymbol{P}_{2}^{\prime \prime}$. Similarly, the evaluation of $\hat{\tau}_{e}\left(x^{2}\right)$ can be carried out by

$$
\left[\boldsymbol{A}_{0}^{\prime} \boldsymbol{Q}_{0}\left|\boldsymbol{A}_{1}^{\prime} \boldsymbol{Q}_{1}^{\prime \prime \prime}\right| \boldsymbol{A}_{2}^{\prime} \boldsymbol{Q}_{2}^{\prime}\right]\left(\left[\begin{array}{c}
\boldsymbol{c}_{0} \\
\boldsymbol{c}_{1}^{\prime \prime \prime} \\
\boldsymbol{c}_{2}^{\prime}
\end{array}\right] \cdot\left[\begin{array}{ccc}
\boldsymbol{P}_{0} & & \\
& \boldsymbol{P}_{1}^{\prime \prime \prime} & \\
& & \boldsymbol{P}_{2}^{\prime}
\end{array}\right]\left[\begin{array}{c}
\hat{\tau}_{e, 0} \\
\hat{\tau}_{e, 2} \\
\hat{\tau}_{e, 1} \\
\hat{\tau}_{e, 3}
\end{array}\right]\right)
$$

Then we can apply our CSE algorithm to these reduced pre- and post-addition matrices and furthur reduce the numbers of additions, but such details are omitted. It is easy to see that the Chien search based on our partial dual CFFTs achieves significantly smaller computational complexities.

## D. Transform-Domain and Time-Domain RS Decoders

Replacing the prime-factor FFT [4] by our CFFTs proposed above, we propose a transform-domain RS decoder with the following steps: (T.1) Compute the syndromes by our partial SCFFT; (T.2) Use the inverse-free BMA [2] to obtain the errata locator polynomial $\tau(x)$; (T.3) Compute the remaining syndromes by recursive extension using $\tau(x)$; (T.4) Compute the error vector by full CFFT of the syndrome vector. Finally, the corrected codeword is obtained by adding the received vector and the error vector. Similarly, we propose a time-domain RS decoder with the following steps: t. 1 and t. 2 are the same as T. 1 and T.2; (t.3) Compute the errata evaluator polynomial $A(x)$; (t.4) Find the error locations and error values by applying our combined Chien search and Forney's formula based on dual partial DCFFTs to $\tau(x)$ and $A(x)$.

TABLE III
Complexity of Transform-Domain and Time-Domain errors-and-erasures RS Decoders

| $(n, k)$ |  | (255, 223) |  |  |  | (511, 447) |  |  |  | (1023, 895) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mult. | Add. | Div. | Total | Mult. | Add. | Div. | Total | Mult. | Add. | Div. | Total |
| T. 4 (Inverse Transform) | [4] | 1135 | 3887 | 0 | 20912 | 6516 | 17506 | 0 | 128278 | 5915 | 30547 | 0 | 142932 |
|  | [11] | 586 | 6736 | 0 | 15526 | 1014 | 23130 | 0 | 40368 | 2827 | 75360 | 0 | 129073 |
| T.1+T.4 | [4] | 1987 | 5691 | 0 | 35496 | 11781 | 24815 | 0 | 225092 | 12700 | 46322 | 0 | 287622 |
|  | Ours | 735 | 10706 | 0 | 21731 | 1359 | 39601 | 0 | 62704 | 3651 | 136101 | 0 | 205470 |
| t.1+t. 4 | [2] | 23970 | 24703 | 32 | 384733 | 97090 | 98559 | 64 | 1750177 | 390786 | 393727 | 128 | 7821093 |
|  | Ours | 484 | 12375 | 32 | 20115 | 1115 | 50108 | 64 | 70151 | 3530 | 174859 | 128 | 244361 |
| T.2, t. 2 (BMA) | All | 353 | 288 | 0 | 5583 | 1217 | 1088 | 0 | 21777 | 4481 | 4224 | 0 | 89363 |
| T.3 (Remaining syndromes) | Both | 7136 | 6913 | 0 | 113953 | 28608 | 28161 | 0 | 514497 | 114560 | 113665 | 0 | 2290305 |
| T. $1+\mathrm{T} .2+\mathrm{T} .3+\mathrm{T} .4$ | [4] | 9476 | 12892 | 0 | 155032 | 41606 | 54064 | 0 | 761366 | 131741 | 164211 | 0 | 2667290 |
|  | Ours | 8224 | 17907 | 0 | 141267 | 31184 | 68850 | 0 | 598978 | 122692 | 253990 | 0 | 2585138 |
| t. 3 (Errata evaluator poly.) | Both | 1089 | 1024 | 0 | 17359 | 4225 | 4096 | 0 | 75921 | 16641 | 16384 | 0 | 332563 |
| t.1+t.2+t. $3+\mathrm{t} .4$ | [2] | 25412 | 26015 | 32 | 407675 | 102532 | 103743 | 64 | 1847875 | 411908 | 414335 | 128 | 8243019 |
|  | Ours | 1926 | 12679 | 32 | 42049 | 6557 | 55292 | 64 | 167849 | 24652 | 195467 | 128 | 666287 |

We compare the complexities of our time- and transform-domain RS decoders for $(255,223),(511,447)$, and $(1023,895)$ RS codes with those in [2] and [4] respectively in Table III] We are aware of the vast literature on RS decoding, and [2] and [4] are compared here since their data are directly comparable. The computational complexities of the time-domain decoder in [2] and the transform-domain decoder in [4] are all reproduced from [4]. The complexities of T. 4 are reproduced from [11, Table I]. Note that all the computational complexities are for errors-and-erasures decoders. The complexities of T.1/t. 1 and t. 4 are already presented in Tables IT and II.

We first compare the overall complexities of our transform-domain RS decoder with those in [4], presented in the row marked by "T.1+T.2+T.3+T.4." Clearly our transform-domain decoder achieves smaller complexities. However, this comparison is somewhat misleading since our decoder differs from that in [4] only in T. 1 and T.4. We further compare the combined complexities of T. 1 and T. 4 of our transform-domain decoder and that in [4], presented in the row marked by "T.1+T.4." Here, the transform portion of our decoder achieves complexity reductions of $39 \%, 72 \%$, and $29 \%$.

For time-domain RS decoders, in comparison to the decoder considered in [2], the overall complexities of our RS decoder are $90 \%, 91 \%$, and $92 \%$ smaller. Again, since the focus of this paper is on transformation, it is more meaningful to compare only the steps using DFTs: t .1 and t .4 . The sums of the total complexities of t .1 and t .4 are presented separately in the row marked by "t. $1+\mathrm{t} .4$." It can be seen that the transformation portion of our decoder achieves $95 \%, 96 \%$, and $97 \%$ complexity savings over that in [2] for the three RS codes, respectively.

Finally, based on our results, time-domain decoders have smaller complexities than transform-domain decoders. This conclusion is different from that in [4]. However, the conclusion in [4] is based on the comparison of transform-domain decoder using FFT and time-domain decoder without FFT. In our comparison, both decoders use CFFTs.

In this paper, we assume that RS decoders are implemented by integrated circuits, and each CFFT consists of combinational logic and requires no memory. Hence, we consider only the total complexity due to finite field operations above since they directly correspond to combinational logic. The total complexities in Tables II and $\Pi \square$ also assume that the maximum of received symbols are processed concurrently so as to increase throughput. Thus, reduced total complexities by CFFTs translate into smaller areas. However, decoders based on CFFTs have fixed and irregular adder trees for pre- and post-additions, and thus are less conducive to transformations that trade time (throughput) for area than decoders based on other approaches (for example, Horner's rule).

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## Appendix <br> Partial SCFFT for Syndrome Computation in $(255,223)$ RS Codes

We provide the details of the syndrome computation for the $(255,223)$ RS code based on our partial SCFFT.
First, we reorder the received vector $\boldsymbol{r}$ based on cosets:
$\boldsymbol{r}^{\prime}=\left(r_{0}, r_{1}, r_{2}, r_{4}, r_{8}, r_{16}, r_{32}, r_{64}, r_{128}, r_{3}, r_{6}, r_{12}, r_{24}, r_{48}, r_{96}, r_{192}, r_{129}, r_{5}, r_{10}, r_{20}, r_{40}, r_{80}, r_{160}, r_{65}, r_{130}, r_{131}\right.$, $r_{7}, r_{14}, r_{28}, r_{56}, r_{112}, r_{224}, r_{193}, r_{66}, r_{132}, r_{9}, r_{18}, r_{36}, r_{72}, r_{144}, r_{33}, r_{11}, r_{22}, r_{44}, r_{88}, r_{176}, r_{97}, r_{194}, r_{133}, r_{67}, r_{134}, r_{13}$, $r_{26}, r_{52}, r_{104}, r_{208}, r_{161}, r_{195}, r_{135}, r_{15}, r_{30}, r_{60}, r_{120}, r_{240}, r_{225}, r_{34}, r_{68}, r_{136}, r_{17}, r_{98}, r_{196}, r_{137}, r_{19}, r_{38}, r_{76}, r_{152}, r_{49}$, $r_{138}, r_{21}, r_{42}, r_{84}, r_{168}, r_{81}, r_{162}, r_{69}, r_{226}, r_{197}, r_{139}, r_{23}, r_{46}, r_{92}, r_{184}, r_{113}, r_{70}, r_{140}, r_{25}, r_{50}, r_{100}, r_{200}, r_{145}, r_{35}, r_{141}$, $r_{27}, r_{54}, r_{108}, r_{216}, r_{177}, r_{99}, r_{198}, r_{71}, r_{142}, r_{29}, r_{58}, r_{116}, r_{232}, r_{209}, r_{163}, r_{31}, r_{62}, r_{124}, r_{248}, r_{241}, r_{227}, r_{199}, r_{143}, r_{37}$, $r_{74}, r_{148}, r_{41}, r_{82}, r_{164}, r_{73}, r_{146}, r_{39}, r_{78}, r_{156}, r_{57}, r_{114}, r_{228}, r_{201}, r_{147}, r_{43}, r_{86}, r_{172}, r_{89}, r_{178}, r_{101}, r_{202}, r_{149}, r_{45}, r_{90}$, $r_{180}, r_{105}, r_{210}, r_{165}, r_{75}, r_{150}, r_{47}, r_{94}, r_{188}, r_{121}, r_{242}, r_{229}, r_{203}, r_{151}, r_{51}, r_{102}, r_{204}, r_{153}, r_{53}, r_{106}, r_{212}, r_{169}, r_{83}, r_{166}$, $r_{77}, r_{154}, r_{55}, r_{110}, r_{220}, r_{185}, r_{115}, r_{230}, r_{205}, r_{155}, r_{59}, r_{118}, r_{236}, r_{217}, r_{179}, r_{103}, r_{206}, r_{157}, r_{61}, r_{122}, r_{244}, r_{233}, r_{211}$, $r_{167}, r_{79}, r_{158}, r_{63}, r_{126}, r_{252}, r_{249}, r_{243}, r_{231}, r_{207}, r_{159}, r_{85}, r_{170}, r_{87}, r_{174}, r_{93}, r_{186}, r_{117}, r_{234}, r_{213}, r_{171}, r_{91}, r_{182}, r_{109}$, $r_{218}, r_{181}, r_{107}, r_{214}, r_{173}, r_{95}, r_{190}, r_{125}, r_{250}, r_{245}, r_{235}, r_{215}, r_{175}, r_{111}, r_{222}, r_{189}, r_{123}, r_{246}, r_{237}, r_{219}, r_{183}, r_{119}, r_{238}$, $\left.r_{221}, r_{187}, r_{127}, r_{254}, r_{253}, r_{251}, r_{247}, r_{239}, r_{223}, r_{191}\right)$.

Pre-additions ( 3793 additions): $\boldsymbol{p}=(\boldsymbol{A} \boldsymbol{Q})^{T} \boldsymbol{r}^{\prime}$.
$t_{2399}=r_{150}^{\prime}+r_{197}^{\prime}, t_{2404}=r_{228}^{\prime}+t_{2399}, t_{2247}=r_{189}^{\prime}+r_{190}^{\prime}, t_{2085}=r_{92}^{\prime}+r_{234}^{\prime}, t_{2056}=r_{78}^{\prime}+r_{153}^{\prime}, t_{1917}=r_{33}^{\prime}+r_{122}^{\prime}, t_{1853}=$ $r_{20}^{\prime}+r_{219}^{\prime}, t_{1788}=r_{34}^{\prime}+r_{70}^{\prime}, t_{1707}=r_{83}^{\prime}+r_{203}^{\prime}, t_{2397}=r_{152}^{\prime}+t_{1707}, t_{1662}=r_{35}^{\prime}+r_{92}^{\prime}, t_{1572}=r_{8}^{\prime}+r_{233}^{\prime}, t_{1571}=$ $r_{1}^{\prime}+r_{193}^{\prime}, t_{1561}=r_{19}^{\prime}+r_{41}^{\prime}, t_{1553}=r_{227}^{\prime}+r_{245}^{\prime}, t_{1516}=r_{82}^{\prime}+r_{113}^{\prime}, t_{1455}=r_{54}^{\prime}+r_{121}^{\prime}, t_{1617}=r_{153}^{\prime}+t_{1455}, t_{1425}=$ $r_{50}^{\prime}+r_{223}^{\prime}, t_{1371}=r_{222}^{\prime}+r_{242}^{\prime}, t_{1326}=r_{29}^{\prime}+r_{114}^{\prime}, t_{1313}=r_{152}^{\prime}+r_{199}^{\prime}, t_{1308}=r_{122}^{\prime}+r_{220}^{\prime}, t_{1277}=r_{25}^{\prime}+r_{35}^{\prime}, t_{1244}=$ $r_{232}^{\prime}+r_{246}^{\prime}, t_{1240}=r_{168}^{\prime}+r_{191}^{\prime}, t_{1221}=r_{18}^{\prime}+r_{166}^{\prime}, t_{1218}=r_{105}^{\prime}+r_{133}^{\prime}, t_{1177}=r_{21}^{\prime}+r_{49}^{\prime}, t_{1150}=r_{7}^{\prime}+r_{176}^{\prime}, t_{1368}=$ $r_{151}^{\prime}+t_{1150}, t_{1134}=r_{9}^{\prime}+r_{179}^{\prime}, t_{1127}=r_{23}^{\prime}+r_{219}^{\prime}, t_{1118}=r_{3}^{\prime}+r_{85}^{\prime}, t_{1101}=r_{60}^{\prime}+r_{63}^{\prime}, t_{1092}=r_{172}^{\prime}+r_{239}^{\prime}, t_{1085}=$ $r_{183}^{\prime}+r_{214}^{\prime}, t_{1082}=r_{37}^{\prime}+r_{89}^{\prime}, t_{1066}=r_{2}^{\prime}+r_{184}^{\prime}, t_{1060}=r_{83}^{\prime}+r_{119}^{\prime}, t_{1059}=r_{210}^{\prime}+r_{226}^{\prime}, t_{1054}=r_{169}^{\prime}+r_{182}^{\prime}, t_{1420}=$ $r_{122}^{\prime}+t_{1054}, t_{1043}=r_{141}^{\prime}+r_{195}^{\prime}, t_{1024}=r_{97}^{\prime}+r_{178}^{\prime}, t_{1022}=r_{22}^{\prime}+r_{194}^{\prime}, t_{1676}=r_{249}^{\prime}+t_{1022}, t_{1008}=r_{17}^{\prime}+r_{162}^{\prime}, t_{1532}=$ $r_{145}^{\prime}+t_{1008}, t_{1365}=r_{160}^{\prime}+t_{1008}, t_{1547}=r_{243}^{\prime}+t_{1365}, t_{999}=r_{177}^{\prime}+r_{181}^{\prime}, t_{993}=r_{188}^{\prime}+r_{236}^{\prime}, t_{983}=r_{125}^{\prime}+r_{131}^{\prime}, t_{970}=$ $r_{47}^{\prime}+r_{139}^{\prime}, t_{957}=r_{33}^{\prime}+r_{34}^{\prime}, t_{1310}=r_{205}^{\prime}+t_{957}, t_{1438}=r_{129}^{\prime}+t_{1310}, t_{950}=r_{158}^{\prime}+r_{217}^{\prime}, t_{1041}=r_{241}^{\prime}+t_{950}, t_{948}=$ $r_{81}^{\prime}+r_{248}^{\prime}, t_{946}=r_{48}^{\prime}+r_{160}^{\prime}, t_{940}=r_{199}^{\prime}+r_{254}^{\prime}, t_{931}=r_{197}^{\prime}+r_{221}^{\prime}, t_{1457}=r_{178}^{\prime}+t_{931}, t_{913}=r_{39}^{\prime}+r_{216}^{\prime}, t_{2122}=$ $r_{199}^{\prime}+t_{913}, t_{1263}=r_{112}^{\prime}+t_{913}, t_{908}=r_{127}^{\prime}+r_{149}^{\prime}, t_{1020}=r_{59}^{\prime}+t_{908}, t_{907}=r_{4}^{\prime}+r_{171}^{\prime}, t_{905}=r_{55}^{\prime}+r_{56}^{\prime}, t_{1052}=r_{118}^{\prime}+$ $t_{905}, t_{903}=r_{150}^{\prime}+r_{252}^{\prime}, t_{987}=r_{140}^{\prime}+t_{903}, t_{901}=r_{109}^{\prime}+r_{121}^{\prime}, t_{1179}=r_{76}^{\prime}+t_{901}, t_{892}=r_{62}^{\prime}+r_{84}^{\prime}, t_{864}=r_{32}^{\prime}+r_{123}^{\prime}, t_{862}=$ $r_{51}^{\prime}+r_{180}^{\prime}, t_{2150}=r_{83}^{\prime}+t_{862}, t_{1343}=t_{862}+t_{1024}, t_{1039}=r_{87}^{\prime}+t_{862}, t_{858}=r_{25}^{\prime}+r_{89}^{\prime}, t_{817}=r_{157}^{\prime}+r_{208}^{\prime}, t_{1387}=$ $r_{14}^{\prime}+t_{817}, t_{812}=r_{129}^{\prime}+r_{152}^{\prime}, t_{811}=r_{13}^{\prime}+r_{222}^{\prime}, t_{795}=r_{91}^{\prime}+r_{192}^{\prime}, t_{1446}=r_{224}^{\prime}+t_{795}, t_{1276}=r_{7}^{\prime}+t_{795}, t_{934}=r_{66}^{\prime}+t_{795}, t_{793}=$ $r_{0}^{\prime}+r_{155}^{\prime}, t_{1193}=r_{99}^{\prime}+t_{793}, t_{788}=r_{16}^{\prime}+r_{101}^{\prime}, t_{1230}=r_{77}^{\prime}+t_{788}, t_{785}=r_{15}^{\prime}+r_{217}^{\prime}, t_{874}=r_{77}^{\prime}+t_{785}, t_{1189}=r_{21}^{\prime}+t_{874}, t_{783}=$ $r_{74}^{\prime}+r_{115}^{\prime}, t_{1204}=r_{234}^{\prime}+t_{783}, t_{779}=r_{168}^{\prime}+r_{210}^{\prime}, t_{1056}=r_{165}^{\prime}+t_{779}, t_{774}=r_{161}^{\prime}+r_{196}^{\prime}, t_{754}=r_{46}^{\prime}+r_{220}^{\prime}, t_{1622}=$ $r_{164}^{\prime}+t_{754}, t_{936}=r_{162}^{\prime}+t_{754}, t_{2082}=r_{178}^{\prime}+t_{936}, t_{748}=r_{52}^{\prime}+r_{141}^{\prime}, t_{1564}=r_{66}^{\prime}+t_{748}, t_{977}=r_{128}^{\prime}+t_{748}, t_{742}=$ $r_{86}^{\prime}+r_{178}^{\prime}, t_{1133}=r_{43}^{\prime}+t_{742}, t_{1470}=r_{174}^{\prime}+t_{1133}, t_{737}=r_{115}^{\prime}+r_{119}^{\prime}, t_{1996}=r_{32}^{\prime}+t_{737}, t_{1376}=r_{135}^{\prime}+t_{737}, t_{1149}=$ $r_{161}^{\prime}+t_{737}, t_{735}=r_{65}^{\prime}+r_{243}^{\prime}, t_{1488}=r_{88}^{\prime}+t_{735}, t_{846}=r_{246}^{\prime}+t_{735}, t_{732}=r_{215}^{\prime}+r_{223}^{\prime}, t_{731}=r_{49}^{\prime}+r_{240}^{\prime}, t_{965}=r_{15}^{\prime}+t_{731}, t_{1158}=$ $r_{198}^{\prime}+t_{965}, t_{1669}=r_{99}^{\prime}+t_{1158}, t_{728}=r_{185}^{\prime}+r_{189}^{\prime}, t_{720}=r_{14}^{\prime}+r_{108}^{\prime}, t_{973}=r_{204}^{\prime}+t_{720}, t_{718}=r_{92}^{\prime}+r_{118}^{\prime}, t_{1131}=$ $r_{145}^{\prime}+t_{718}, t_{1690}=t_{1131}+t_{1572}, t_{769}=r_{130}^{\prime}+t_{718}, t_{1620}=t_{769}+t_{858}, t_{716}=r_{146}^{\prime}+r_{236}^{\prime}, t_{2370}=r_{160}^{\prime}+t_{716}, t_{1050}=r_{96}^{\prime}+$ $t_{716}, t_{2406}=r_{55}^{\prime}+t_{1050}, t_{713}=r_{101}^{\prime}+r_{212}^{\prime}, t_{797}=r_{150}^{\prime}+t_{713}, t_{709}=r_{3}^{\prime}+r_{144}^{\prime}, t_{841}=t_{709}+t_{774}, t_{1272}=r_{169}^{\prime}+t_{841}, t_{707}=$ $r_{28}^{\prime}+r_{114}^{\prime}, t_{1412}=r_{215}^{\prime}+t_{707}, t_{1110}=r_{225}^{\prime}+t_{707}, t_{706}=r_{165}^{\prime}+r_{245}^{\prime}, t_{1069}=t_{706}+t_{735}, t_{2149}=r_{61}^{\prime}+t_{1069}, t_{705}=$ $r_{17}^{\prime}+r_{234}^{\prime}, t_{704}=r_{25}^{\prime}+r_{201}^{\prime}, t_{825}=r_{175}^{\prime}+t_{704}, t_{1028}=t_{825}+t_{864}, t_{1145}=t_{1028}+t_{1092}, t_{701}=r_{142}^{\prime}+r_{198}^{\prime}, t_{954}=$ $r_{44}^{\prime}+t_{701}, t_{976}=r_{172}^{\prime}+t_{954}, t_{696}=r_{203}^{\prime}+r_{207}^{\prime}, t_{1288}=r_{251}^{\prime}+t_{696}, t_{695}=r_{24}^{\prime}+r_{233}^{\prime}, t_{1459}=r_{252}^{\prime}+t_{695}, t_{693}=$ $r_{69}^{\prime}+r_{169}^{\prime}, t_{889}=r_{253}^{\prime}+t_{693}, t_{688}=r_{29}^{\prime}+r_{85}^{\prime}, t_{1036}=t_{688}+t_{705}, t_{1114}=t_{709}+t_{1036}, t_{2170}=r_{48}^{\prime}+t_{1114}, t_{1226}=$
$t_{797}+t_{1114}, t_{679}=r_{120}^{\prime}+r_{134}^{\prime}, t_{678}=r_{19}^{\prime}+r_{81}^{\prime}, t_{1713}=r_{33}^{\prime}+t_{678}, t_{2398}=t_{1713}+t_{2397}, t_{1383}=t_{678}+t_{1376}, t_{1528}=$ $r_{214}^{\prime}+t_{1383}, t_{677}=r_{228}^{\prime}+r_{237}^{\prime}, t_{1520}=r_{213}^{\prime}+t_{677}, t_{861}=r_{100}^{\prime}+t_{677}, t_{674}=r_{90}^{\prime}+r_{124}^{\prime}, t_{842}=r_{26}^{\prime}+t_{674}, t_{664}=$ $r_{80}^{\prime}+r_{172}^{\prime}, t_{663}=r_{157}^{\prime}+r_{160}^{\prime}, t_{1078}=r_{144}^{\prime}+t_{663}, t_{1672}=r_{18}^{\prime}+t_{1078}, t_{1299}=r_{93}^{\prime}+t_{1078}, t_{1402}=t_{1299}+t_{1326}, t_{660}=$ $r_{10}^{\prime}+r_{104}^{\prime}, t_{1574}=t_{660}+t_{905}, t_{1038}=t_{660}+t_{993}, t_{1403}=r_{12}^{\prime}+t_{1038}, t_{759}=r_{221}^{\prime}+t_{660}, t_{659}=r_{74}^{\prime}+r_{239}^{\prime}, t_{1344}=$ $r_{4}^{\prime}+t_{659}, t_{1526}=r_{236}^{\prime}+t_{1344}, t_{658}=r_{8}^{\prime}+r_{175}^{\prime}, t_{1021}=t_{658}+t_{679}, t_{651}=r_{84}^{\prime}+r_{176}^{\prime}, t_{647}=r_{5}^{\prime}+r_{214}^{\prime}, t_{865}=t_{647}+t_{679}, t_{1422}=$ $r_{146}^{\prime}+t_{865}, t_{646}=r_{9}^{\prime}+r_{44}^{\prime}, t_{1874}=t_{646}+t_{1272}, t_{656}=r_{159}^{\prime}+t_{646}, t_{1530}=t_{656}+t_{1221}, t_{644}=r_{91}^{\prime}+r_{117}^{\prime}, t_{2363}=$ $t_{644}+t_{1343}, t_{1130}=r_{107}^{\prime}+t_{644}, t_{1926}=r_{168}^{\prime}+t_{1130}, t_{755}=r_{102}^{\prime}+t_{644}, t_{643}=r_{38}^{\prime}+r_{238}^{\prime}, t_{828}=r_{133}^{\prime}+t_{643}, t_{1197}=$ $r_{14}^{\prime}+t_{828}, t_{638}=r_{11}^{\prime}+r_{211}^{\prime}, t_{2280}=t_{638}+t_{1056}, t_{634}=r_{158}^{\prime}+r_{162}^{\prime}, t_{632}=r_{113}^{\prime}+r_{235}^{\prime}, t_{981}=r_{251}^{\prime}+t_{632}, t_{866}=$ $r_{68}^{\prime}+t_{632}, t_{1664}=t_{866}+t_{1060}, t_{627}=r_{173}^{\prime}+r_{249}^{\prime}, t_{623}=r_{30}^{\prime}+r_{193}^{\prime}, t_{804}=t_{623}+t_{674}, t_{1517}=t_{804}+t_{1101}, t_{2019}=$ $t_{1517}+t_{1564}, t_{1492}=r_{147}^{\prime}+t_{804}, t_{1503}=t_{977}+t_{1492}, t_{1860}=r_{11}^{\prime}+t_{1503}, t_{622}=r_{56}^{\prime}+r_{73}^{\prime}, t_{909}=r_{170}^{\prime}+t_{622}, t_{1760}=$ $t_{731}+t_{909}, t_{1049}=r_{57}^{\prime}+t_{909}, t_{1784}=r_{6}^{\prime}+t_{1049}, t_{770}=t_{622}+t_{705}, t_{1037}=r_{83}^{\prime}+t_{770}, t_{1102}=r_{141}^{\prime}+t_{1037}, t_{1300}=$ $t_{647}+t_{1102}, t_{621}=r_{40}^{\prime}+r_{186}^{\prime}, t_{1167}=t_{621}+t_{861}, t_{914}=t_{621}+t_{664}, t_{1529}=t_{647}+t_{914}, t_{1769}=t_{1529}+t_{1574}, t_{1227}=r_{56}^{\prime}+$ $t_{914}, t_{619}=r_{51}^{\prime}+r_{148}^{\prime}, t_{1400}=t_{619}+t_{720}, t_{1007}=r_{142}^{\prime}+t_{619}, t_{618}=r_{174}^{\prime}+r_{250}^{\prime}, t_{751}=r_{137}^{\prime}+t_{618}, t_{1033}=r_{231}^{\prime}+t_{751}, t_{615}=$ $r_{67}^{\prime}+r_{167}^{\prime}, t_{1274}=r_{28}^{\prime}+t_{615}, t_{614}=r_{88}^{\prime}+r_{126}^{\prime}, t_{953}=r_{31}^{\prime}+t_{614}, t_{1494}=r_{177}^{\prime}+t_{953}, t_{745}=r_{242}^{\prime}+t_{614}, t_{1095}=r_{77}^{\prime}+t_{745}, t_{613}=$ $r_{54}^{\prime}+r_{191}^{\prime}, t_{951}=t_{613}+t_{658}, t_{1443}=r_{75}^{\prime}+t_{951}, t_{768}=r_{72}^{\prime}+t_{613}, t_{1120}=t_{768}+t_{828}, t_{607}=r_{151}^{\prime}+r_{155}^{\prime}, t_{604}=r_{39}^{\prime}+r_{57}^{\prime}, t_{2361}=$ $r_{242}^{\prime}+t_{604}, t_{1144}=r_{72}^{\prime}+t_{604}, t_{1534}=t_{1144}+t_{1455}, t_{2003}=t_{618}+t_{1534}, t_{2010}=t_{1024}+t_{2003}, t_{727}=r_{79}^{\prime}+t_{604}, t_{986}=$ $t_{659}+t_{727}, t_{603}=r_{145}^{\prime}+r_{213}^{\prime}, t_{2083}=r_{156}^{\prime}+t_{603}, t_{654}=t_{603}+t_{627}, t_{1139}=t_{654}+t_{948}, t_{602}=r_{105}^{\prime}+r_{154}^{\prime}, t_{2291}=$ $t_{602}+t_{643}, t_{1014}=r_{208}^{\prime}+t_{602}, t_{781}=r_{209}^{\prime}+t_{602}, t_{1486}=t_{732}+t_{781}, t_{1569}=r_{38}^{\prime}+t_{1486}, t_{1654}=r_{100}^{\prime}+t_{1569}, t_{601}=$ $r_{70}^{\prime}+r_{248}^{\prime}, t_{897}=r_{40}^{\prime}+t_{601}, t_{1576}=r_{55}^{\prime}+t_{897}, t_{683}=r_{96}^{\prime}+t_{601}, t_{1186}=r_{48}^{\prime}+t_{683}, t_{600}=r_{103}^{\prime}+r_{107}^{\prime}, t_{598}=r_{170}^{\prime}+r_{189}^{\prime}, t_{853}=$ $r_{235}^{\prime}+t_{598}, t_{1053}=r_{42}^{\prime}+t_{853}, t_{595}=r_{122}^{\prime}+r_{136}^{\prime}, t_{1548}=t_{595}+t_{812}, t_{762}=r_{200}^{\prime}+t_{595}, t_{1235}=r_{73}^{\prime}+t_{762}, t_{1137}=$ $r_{35}^{\prime}+t_{762}, t_{1626}=t_{660}+t_{1137}, t_{933}=r_{137}^{\prime}+t_{762}, t_{594}=r_{132}^{\prime}+r_{206}^{\prime}, t_{1169}=r_{244}^{\prime}+t_{594}, t_{593}=r_{109}^{\prime}+r_{226}^{\prime}, t_{1661}=$ $t_{593}+t_{601}, t_{1718}=r_{247}^{\prime}+t_{1661}, t_{612}=r_{19}^{\prime}+t_{593}, t_{1416}=t_{612}+t_{1134}, t_{1790}=r_{27}^{\prime}+t_{1416}, t_{721}=r_{171}^{\prime}+t_{612}, t_{927}=$ $r_{143}^{\prime}+t_{721}, t_{1575}=t_{927}+t_{1470}, t_{1382}=r_{107}^{\prime}+t_{927}, t_{592}=r_{22}^{\prime}+r_{231}^{\prime}, t_{670}=r_{47}^{\prime}+t_{592}, t_{1076}=t_{670}+t_{769}, t_{1605}=$ $t_{934}+t_{1076}, t_{836}=t_{595}+t_{670}, t_{1621}=r_{198}^{\prime}+t_{836}, t_{949}=r_{237}^{\prime}+t_{836}, t_{2052}=t_{949}+t_{1457}, t_{591}=r_{45}^{\prime}+r_{219}^{\prime}, t_{1969}=$ $t_{591}+t_{1095}, t_{815}=r_{10}^{\prime}+t_{591}, t_{1081}=r_{153}^{\prime}+t_{815}, t_{1493}=r_{228}^{\prime}+t_{1081}, t_{2027}=r_{217}^{\prime}+t_{1493}, t_{590}=r_{49}^{\prime}+r_{53}^{\prime}, t_{588}=$ $r_{149}^{\prime}+r_{195}^{\prime}, t_{1121}=t_{588}+t_{817}, t_{1225}=t_{618}+t_{1121}, t_{1974}=t_{1225}+t_{1969}, t_{673}=r_{140}^{\prime}+t_{588}, t_{587}=r_{43}^{\prime}+r_{225}^{\prime}, t_{1862}=$ $t_{587}+t_{1860}, t_{1863}=t_{1066}+t_{1862}, t_{1100}=r_{78}^{\prime}+t_{587}, t_{616}=r_{59}^{\prime}+t_{587}, t_{1338}=r_{226}^{\prime}+t_{616}, t_{810}=t_{616}+t_{688}, t_{1433}=$ $r_{28}^{\prime}+t_{810}, t_{583}=r_{135}^{\prime}+r_{139}^{\prime}, t_{1129}=r_{64}^{\prime}+t_{583}, t_{582}=r_{111}^{\prime}+r_{123}^{\prime}, t_{2156}=t_{582}+t_{751}, t_{2158}=r_{15}^{\prime}+t_{2156}, t_{2161}=$ $r_{175}^{\prime}+t_{2158}, t_{1108}=r_{95}^{\prime}+t_{582}, t_{1454}=t_{1014}+t_{1108}, t_{712}=r_{50}^{\prime}+t_{582}, t_{2344}=r_{34}^{\prime}+t_{712}, t_{764}=r_{76}^{\prime}+t_{712}, t_{920}=r_{6}^{\prime}+$ $t_{764}, t_{1533}=r_{47}^{\prime}+t_{920}, t_{581}=r_{62}^{\prime}+r_{183}^{\prime}, t_{2276}=t_{581}+t_{1707}, t_{682}=r_{100}^{\prime}+t_{581}, t_{888}=t_{656}+t_{682}, t_{1501}=t_{788}+t_{888}, t_{580}=$ $r_{94}^{\prime}+r_{98}^{\prime}, t_{1906}=t_{580}+t_{897}, t_{576}=r_{20}^{\prime}+r_{229}^{\prime}, t_{1685}=r_{106}^{\prime}+t_{576}, t_{633}=r_{8}^{\prime}+t_{576}, t_{1212}=t_{633}+t_{1053}, t_{574}=r_{1}^{\prime}+r_{218}^{\prime}, t_{1340}=$ $t_{574}+t_{701}, t_{1414}=t_{682}+t_{1340}, t_{1198}=t_{574}+t_{1007}, t_{1307}=t_{1198}+t_{1276}, t_{1638}=t_{1149}+t_{1307}, t_{572}=r_{7}^{\prime}+r_{197}^{\prime}, t_{599}=$ $r_{188}^{\prime}+t_{572}, t_{702}=r_{216}^{\prime}+t_{599}, t_{884}=r_{34}^{\prime}+t_{702}, t_{1023}=t_{663}+t_{884}, t_{571}=r_{93}^{\prime}+r_{184}^{\prime}, t_{890}=t_{571}+t_{768}, t_{1164}=r_{57}^{\prime}+t_{890}, t_{681}=$ $r_{209}^{\prime}+t_{571}, t_{947}=t_{681}+t_{940}, t_{570}=r_{58}^{\prime}+r_{179}^{\prime}, t_{1580}=t_{570}+t_{858}, t_{859}=r_{230}^{\prime}+t_{570}, t_{1909}=t_{859}+t_{1561}, t_{1248}=$ $t_{859}+t_{1145}, t_{569}=r_{71}^{\prime}+r_{75}^{\prime}, t_{1877}=r_{149}^{\prime}+t_{569}, t_{1879}=t_{1874}+t_{1877}, t_{1881}=t_{664}+t_{1879}, t_{1105}=t_{569}+t_{634}, t_{2339}=r_{39}^{\prime}+$ $t_{1105}, t_{567}=r_{36}^{\prime}+r_{190}^{\prime}, t_{2152}=t_{567}+t_{901}, t_{730}=t_{567}+t_{651}, t_{2086}=t_{730}+t_{755}, t_{1124}=r_{237}^{\prime}+t_{730}, t_{1994}=r_{59}^{\prime}+t_{1124}, t_{564}=$ $r_{112}^{\prime}+r_{138}^{\prime}, t_{609}=r_{164}^{\prime}+t_{564}, t_{1637}=t_{609}+t_{949}, t_{772}=r_{131}^{\prime}+t_{609}, t_{561}=r_{12}^{\prime}+r_{16}^{\prime}, t_{845}=t_{561}+t_{600}, t_{559}=r_{78}^{\prime}+r_{82}^{\prime}, t_{558}=$ $r_{2}^{\prime}+r_{143}^{\prime}, t_{665}=r_{194}^{\prime}+t_{558}, t_{1087}=r_{45}^{\prime}+t_{665}, t_{1432}=t_{947}+t_{1087}, t_{2215}=t_{1212}+t_{1432}, t_{2218}=t_{764}+t_{2215}, t_{1354}=r_{60}^{\prime}+$ $t_{1087}, t_{1682}=r_{65}^{\prime}+t_{1354}, t_{2337}=t_{1020}+t_{1682}, t_{771}=t_{665}+t_{678}, t_{556}=r_{95}^{\prime}+r_{99}^{\prime}, t_{553}=r_{27}^{\prime}+r_{31}^{\prime}, t_{837}=t_{553}+t_{556}, t_{552}=$ $r_{106}^{\prime}+r_{128}^{\prime}, t_{1040}=r_{156}^{\prime}+t_{552}, t_{1295}=r_{244}^{\prime}+t_{1040}, t_{630}=r_{202}^{\prime}+t_{552}, t_{1765}=r_{21}^{\prime}+t_{630}, t_{1772}=t_{908}+t_{1765}, t_{1775}=$ $r_{47}^{\prime}+t_{1772}, t_{767}=t_{619}+t_{630}, t_{1972}=t_{558}+t_{767}, t_{550}=r_{60}^{\prime}+r_{64}^{\prime}, t_{2023}=t_{550}+t_{1414}, t_{547}=r_{32}^{\prime}+r_{110}^{\prime}, t_{1489}=t_{547}+$ $t_{889}, t_{1968}=t_{842}+t_{1489}, t_{650}=r_{205}^{\prime}+t_{547}, t_{1849}=r_{195}^{\prime}+t_{650}, t_{860}=t_{650}+t_{673}, t_{1170}=t_{860}+t_{1082}, t_{519}=r_{60}^{\prime}+r_{82}^{\prime}, t_{672}=$ $r_{37}^{\prime}+t_{519}, t_{799}=r_{63}^{\prime}+t_{672}, t_{1332}=t_{664}+t_{799}, t_{1560}=r_{11}^{\prime}+t_{1332}, t_{1607}=t_{1164}+t_{1560}, t_{2062}=t_{1607}+t_{1622}, t_{1080}=$ $t_{759}+t_{799}, t_{1581}=t_{594}+t_{1080}, t_{1521}=t_{884}+t_{1080}, t_{515}=r_{34}^{\prime}+r_{38}^{\prime}, t_{514}=r_{99}^{\prime}+r_{251}^{\prime}, t_{1257}=t_{514}+t_{934}, t_{715}=$ $r_{163}^{\prime}+t_{514}, t_{814}=t_{715}+t_{716}, t_{510}=r_{49}^{\prime}+r_{75}^{\prime}, t_{967}=t_{510}+t_{772}, t_{1334}=t_{651}+t_{967}, t_{1409}=t_{1167}+t_{1334}, t_{1796}=$ $t_{1409}+t_{1784}, t_{509}=r_{158}^{\prime}+r_{189}^{\prime}, t_{1667}=r_{238}^{\prime}+t_{509}, t_{1088}=r_{116}^{\prime}+t_{509}, t_{1419}=t_{591}+t_{1088}, t_{508}=r_{27}^{\prime}+r_{107}^{\prime}, t_{585}=$ $r_{224}^{\prime}+t_{508}, t_{675}=r_{129}^{\prime}+t_{585}, t_{906}=t_{638}+t_{675}, t_{740}=t_{598}+t_{675}, t_{1562}=r_{154}^{\prime}+t_{740}, t_{944}=t_{740}+t_{815}, t_{2173}=$ $t_{944}+t_{1149}, t_{506}=r_{133}^{\prime}+r_{137}^{\prime}, t_{504}=r_{236}^{\prime}+r_{240}^{\prime}, t_{502}=r_{12}^{\prime}+r_{155}^{\prime}, t_{1044}=r_{76}^{\prime}+t_{502}, t_{1405}=r_{87}^{\prime}+t_{1044}, t_{499}=$ $r_{167}^{\prime}+r_{168}^{\prime}, t_{2091}=t_{499}+t_{613}, t_{2093}=t_{1085}+t_{2091}, t_{893}=r_{26}^{\prime}+t_{499}, t_{2270}=t_{893}+t_{1204}, t_{1266}=t_{677}+t_{893}, t_{498}=$ $r_{58}^{\prime}+r_{62}^{\prime}, t_{497}=r_{149}^{\prime}+r_{153}^{\prime}, t_{494}=r_{128}^{\prime}+r_{132}^{\prime}, t_{493}=r_{136}^{\prime}+r_{140}^{\prime}, t_{492}=r_{162}^{\prime}+r_{185}^{\prime}, t_{2318}=t_{492}+t_{1718}, t_{2265}=$ $t_{492}+t_{600}, t_{807}=r_{41}^{\prime}+t_{492}, t_{1084}=t_{634}+t_{807}, t_{549}=t_{492}+t_{509}, t_{491}=r_{59}^{\prime}+r_{63}^{\prime}, t_{982}=t_{491}+t_{846}, t_{2125}=r_{132}^{\prime}+t_{982}, t_{490}=$ $r_{150}^{\prime}+r_{154}^{\prime}, t_{1384}=r_{104}^{\prime}+t_{490}, t_{1254}=t_{490}+t_{561}, t_{577}=t_{490}+t_{506}, t_{489}=r_{204}^{\prime}+r_{208}^{\prime}, t_{488}=r_{102}^{\prime}+r_{106}^{\prime}, t_{919}=$ $r_{205}^{\prime}+t_{488}, t_{1122}=t_{919}+t_{1050}, t_{1640}=t_{1122}+t_{1400}, t_{2124}=t_{553}+t_{1640}, t_{1062}=t_{643}+t_{919}, t_{1703}=t_{1062}+t_{1526}, t_{628}=$ $t_{488}+t_{494}, t_{1807}=t_{628}+t_{728}, t_{487}=r_{10}^{\prime}+r_{14}^{\prime}, t_{998}=t_{487}+t_{556}, t_{773}=t_{487}+t_{489}, t_{486}=r_{180}^{\prime}+r_{184}^{\prime}, t_{1190}=r_{24}^{\prime}+t_{486}, t_{485}=$ $r_{95}^{\prime}+r_{247}^{\prime}, t_{1171}=r_{52}^{\prime}+t_{485}, t_{1885}=r_{180}^{\prime}+t_{1171}, t_{955}=t_{485}+t_{603}, t_{1140}=t_{955}+t_{986}, t_{1392}=r_{223}^{\prime}+t_{1140}, t_{1406}=$ $t_{707}+t_{1392}, t_{1551}=t_{502}+t_{1406}, t_{791}=r_{186}^{\prime}+t_{485}, t_{1485}=t_{705}+t_{791}, t_{1297}=r_{50}^{\prime}+t_{791}, t_{2135}=t_{489}+t_{1297}, t_{2137}=$ $t_{2125}+t_{2135}, t_{2141}=t_{1085}+t_{2137}, t_{2142}=t_{2124}+t_{2141}, t_{596}=t_{485}+t_{514}, t_{484}=r_{141}^{\prime}+r_{145}^{\prime}, t_{482}=r_{171}^{\prime}+r_{175}^{\prime}, t_{1243}=$ $t_{482}+t_{576}, t_{2095}=t_{1243}+t_{2093}, t_{1971}=t_{1243}+t_{1637}, t_{972}=t_{482}+t_{572}, t_{481}=r_{52}^{\prime}+r_{56}^{\prime}, t_{818}=t_{481}+t_{559}, t_{2244}=$
$t_{818}+t_{1534}, t_{480}=r_{227}^{\prime}+r_{231}^{\prime}, t_{1265}=t_{480}+t_{970}, t_{938}=t_{480}+t_{683}, t_{1270}=t_{892}+t_{938}, t_{624}=t_{480}+t_{486}, t_{1293}=$ $t_{624}+t_{1105}, t_{479}=r_{109}^{\prime}+r_{113}^{\prime}, t_{2268}=t_{479}+t_{1257}, t_{1011}=t_{479}+t_{588}, t_{1496}=t_{1011}+t_{1382}, t_{478}=r_{36}^{\prime}+r_{40}^{\prime}, t_{1236}=$ $r_{200}^{\prime}+t_{478}, t_{1395}=t_{1069}+t_{1236}, t_{548}=t_{478}+t_{498}, t_{1634}=t_{548}+t_{728}, t_{1214}=t_{548}+t_{845}, t_{1694}=t_{491}+t_{1214}, t_{477}=$ $r_{213}^{\prime}+r_{217}^{\prime}, t_{575}=t_{477}+t_{484}, t_{1658}=t_{575}+t_{1419}, t_{476}=r_{152}^{\prime}+r_{156}^{\prime}, t_{1468}=t_{476}+t_{506}, t_{475}=r_{67}^{\prime}+r_{246}^{\prime}, t_{2269}=$ $r_{224}^{\prime}+t_{475}, t_{2274}=r_{23}^{\prime}+t_{2269}, t_{2275}=r_{136}^{\prime}+t_{2274}, t_{1602}=t_{475}+t_{781}, t_{1336}=t_{475}+t_{1124}, t_{803}=t_{475}+t_{754}, t_{474}=$ $r_{193}^{\prime}+r_{197}^{\prime}, t_{473}=r_{188}^{\prime}+r_{192}^{\prime}, t_{472}=r_{16}^{\prime}+r_{151}^{\prime}, t_{962}=r_{177}^{\prime}+t_{472}, t_{1478}=t_{731}+t_{962}, t_{645}=t_{472}+t_{594}, t_{2335}=$ $r_{86}^{\prime}+t_{645}, t_{2347}=t_{1520}+t_{2335}, t_{2153}=t_{645}+t_{1414}, t_{2162}=t_{2153}+t_{2161}, t_{733}=r_{227}^{\prime}+t_{645}, t_{1715}=t_{733}+t_{1127}, t_{922}=$ $t_{650}+t_{733}, t_{2073}=r_{29}^{\prime}+t_{922}, t_{1341}=t_{477}+t_{922}, t_{565}=t_{472}+t_{502}, t_{824}=t_{480}+t_{565}, t_{471}=r_{238}^{\prime}+r_{242}^{\prime}, t_{1347}=$ $t_{471}+t_{508}, t_{1566}=t_{903}+t_{1347}, t_{1232}=t_{471}+t_{515}, t_{2061}=t_{645}+t_{1232}, t_{2063}=t_{2061}+t_{2062}, t_{2065}=r_{116}^{\prime}+t_{2063}, t_{470}=$ $r_{219}^{\prime}+r_{223}^{\prime}, t_{469}=r_{77}^{\prime}+r_{81}^{\prime}, t_{1161}=t_{469}+t_{715}, t_{847}=t_{469}+t_{550}, t_{694}=t_{469}+t_{583}, t_{1219}=t_{638}+t_{694}, t_{2189}=$ $t_{1219}+t_{1626}, t_{1222}=t_{672}+t_{1219}, t_{468}=r_{93}^{\prime}+r_{97}^{\prime}, t_{467}=r_{17}^{\prime}+r_{21}^{\prime}, t_{2241}=t_{467}+t_{774}, t_{1482}=r_{5}^{\prime}+t_{467}, t_{2261}=$ $t_{634}+t_{1482}, t_{466}=r_{214}^{\prime}+r_{218}^{\prime}, t_{1508}=t_{466}+t_{580}, t_{538}=t_{466}+t_{471}, t_{714}=t_{504}+t_{538}, t_{465}=r_{110}^{\prime}+r_{114}^{\prime}, t_{464}=r_{9}^{\prime}+r_{13}^{\prime}, t_{698}=$ $r_{181}^{\prime}+t_{464}, t_{1187}=r_{173}^{\prime}+t_{698}, t_{1677}=t_{1187}+t_{1266}, t_{463}=r_{65}^{\prime}+r_{244}^{\prime}, t_{1851}=t_{463}+t_{1602}, t_{1614}=r_{42}^{\prime}+t_{463}, t_{1911}=$ $t_{1614}+t_{1906}, t_{1913}=t_{616}+t_{1911}, t_{462}=r_{33}^{\prime}+r_{37}^{\prime}, t_{1539}=t_{462}+t_{1226}, t_{1237}=t_{462}+t_{538}, t_{461}=r_{51}^{\prime}+r_{55}^{\prime}, t_{881}=$ $r_{194}^{\prime}+t_{461}, t_{1380}=t_{881}+t_{1218}, t_{1540}=t_{1300}+t_{1380}, t_{1280}=r_{68}^{\prime}+t_{881}, t_{1910}=t_{1280}+t_{1402}, t_{1912}=r_{215}^{\prime}+t_{1910}, t_{1920}=$ $t_{1912}+t_{1913}, t_{460}=r_{139}^{\prime}+r_{203}^{\prime}, t_{1601}=r_{13}^{\prime}+t_{460}, t_{1372}=t_{460}+t_{906}, t_{459}=r_{98}^{\prime}+r_{181}^{\prime}, t_{964}=r_{232}^{\prime}+t_{459}, t_{1194}=$ $r_{249}^{\prime}+t_{964}, t_{458}=r_{237}^{\prime}+r_{241}^{\prime}, t_{1287}=t_{458}+t_{466}, t_{925}=t_{458}+t_{481}, t_{544}=t_{458}+t_{462}, t_{2176}=t_{544}+t_{561}, t_{1785}=$ $t_{538}+t_{544}, t_{952}=t_{471}+t_{544}, t_{1635}=t_{952}+t_{1197}, t_{457}=r_{31}^{\prime}+r_{103}^{\prime}, t_{1479}=t_{457}+t_{1110}, t_{830}=t_{457}+t_{755}, t_{1570}=$ $t_{751}+t_{830}, t_{1720}=t_{815}+t_{1570}, t_{912}=t_{812}+t_{830}, t_{1498}=t_{460}+t_{912}, t_{1373}=t_{912}+t_{993}, t_{1702}=r_{202}^{\prime}+t_{1373}, t_{739}=$ $t_{457}+t_{508}, t_{1051}=t_{482}+t_{739}, t_{589}=r_{97}^{\prime}+t_{457}, t_{456}=r_{4}^{\prime}+r_{8}^{\prime}, t_{1148}=t_{456}+t_{706}, t_{760}=t_{456}+t_{575}, t_{1424}=r_{39}^{\prime}+t_{760}, t_{2271}=$ $t_{1424}+t_{2265}, t_{2273}=t_{1517}+t_{2271}, t_{455}=r_{64}^{\prime}+r_{78}^{\prime}, t_{1599}=r_{110}^{\prime}+t_{455}, t_{736}=t_{455}+t_{519}, t_{597}=r_{133}^{\prime}+t_{455}, t_{1286}=$ $t_{597}+t_{797}, t_{988}=t_{597}+t_{811}, t_{1999}=t_{614}+t_{988}, t_{2000}=r_{104}^{\prime}+t_{1999}, t_{667}=r_{254}^{\prime}+t_{597}, t_{819}=r_{71}^{\prime}+t_{667}, t_{1172}=$ $t_{819}+t_{1041}, t_{1697}=t_{1172}+t_{1248}, t_{935}=t_{623}+t_{819}, t_{2130}=t_{732}+t_{935}, t_{2131}=t_{772}+t_{2130}, t_{1491}=r_{36}^{\prime}+t_{935}, t_{454}=r_{70}^{\prime}+$ $r_{74}^{\prime}, t_{843}=r_{177}^{\prime}+t_{454}, t_{1260}=r_{27}^{\prime}+t_{843}, t_{1116}=t_{698}+t_{843}, t_{537}=t_{454}+t_{481}, t_{766}=t_{537}+t_{596}, t_{1656}=t_{766}+t_{1494}, t_{1290}=$ $t_{760}+t_{766}, t_{453}=r_{170}^{\prime}+r_{174}^{\prime}, t_{1659}=r_{3}^{\prime}+t_{453}, t_{586}=t_{453}+t_{473}, t_{452}=r_{80}^{\prime}+r_{84}^{\prime}, t_{610}=t_{452}+t_{487}, t_{2031}=t_{610}+t_{952}, t_{963}=$ $t_{490}+t_{610}, t_{451}=r_{211}^{\prime}+r_{215}^{\prime}, t_{1027}=t_{451}+t_{714}, t_{1947}=t_{963}+t_{1027}, t_{566}=t_{451}+t_{491}, t_{871}=t_{506}+t_{566}, t_{1241}=$ $t_{498}+t_{871}, t_{746}=t_{544}+t_{566}, t_{450}=r_{50}^{\prime}+r_{54}^{\prime}, t_{1609}=t_{450}+t_{553}, t_{857}=t_{450}+t_{783}, t_{1604}=r_{23}^{\prime}+t_{857}, t_{1436}=r_{53}^{\prime}+t_{857}, t_{449}=$ $r_{26}^{\prime}+r_{30}^{\prime}, t_{1401}=t_{449}+t_{739}, t_{711}=r_{251}^{\prime}+t_{449}, t_{1797}=t_{711}+t_{785}, t_{448}=r_{18}^{\prime}+r_{22}^{\prime}, t_{975}=t_{448}+t_{537}, t_{1418}=r_{200}^{\prime}+t_{975}, t_{573}=$ $t_{448}+t_{474}, t_{838}=t_{573}+t_{624}, t_{447}=r_{144}^{\prime}+r_{148}^{\prime}, t_{1176}=t_{447}+t_{519}, t_{446}=r_{79}^{\prime}+r_{83}^{\prime}, t_{1946}=t_{446}+t_{550}, t_{1958}=$ $t_{1241}+t_{1946}, t_{1321}=r_{188}^{\prime}+t_{446}, t_{2243}=t_{1321}+t_{1540}, t_{445}=r_{43}^{\prime}+r_{47}^{\prime}, t_{1348}=r_{125}^{\prime}+t_{445}, t_{1359}=t_{590}+t_{1348}, t_{1636}=$ $t_{1359}+t_{1528}, t_{910}=t_{445}+t_{586}, t_{1477}=r_{51}^{\prime}+t_{910}, t_{1259}=t_{480}+t_{910}, t_{2123}=t_{1259}+t_{2122}, t_{2128}=r_{2}^{\prime}+t_{2123}, t_{444}=r_{57}^{\prime}+$ $r_{61}^{\prime}, t_{959}=t_{444}+t_{847}, t_{1421}=r_{0}^{\prime}+t_{959}, t_{443}=r_{117}^{\prime}+r_{121}^{\prime}, t_{994}=t_{443}+t_{506}, t_{1047}=r_{60}^{\prime}+t_{994}, t_{1070}=r_{103}^{\prime}+t_{1047}, t_{1378}=$ $r_{182}^{\prime}+t_{1070}, t_{539}=t_{443}+t_{479}, t_{442}=r_{94}^{\prime}+r_{177}^{\prime}, t_{1546}=t_{442}+t_{497}, t_{1495}=t_{442}+t_{1120}, t_{1652}=t_{1129}+t_{1495}, t_{848}=$ $t_{442}+t_{793}, t_{1519}=t_{848}+t_{1169}, t_{1063}=t_{602}+t_{848}, t_{1870}=r_{94}^{\prime}+t_{1063}, t_{655}=t_{442}+t_{589}, t_{1030}=t_{655}+t_{771}, t_{551}=$ $t_{442}+t_{459}, t_{1973}=t_{551}+t_{1546}, t_{1675}=t_{551}+t_{695}, t_{441}=r_{87}^{\prime}+r_{91}^{\prime}, t_{2421}=t_{441}+t_{468}, t_{776}=t_{441}+t_{633}, t_{2340}=$ $r_{154}^{\prime}+t_{776}, t_{2345}=r_{94}^{\prime}+t_{2340}, t_{1565}=t_{681}+t_{776}, t_{1042}=r_{171}^{\prime}+t_{776}, t_{440}=r_{72}^{\prime}+r_{76}^{\prime}, t_{1251}=t_{440}+t_{897}, t_{2353}=$ $r_{254}^{\prime}+t_{1251}, t_{1183}=t_{440}+t_{575}, t_{560}=t_{440}+t_{468}, t_{1178}=t_{560}+t_{998}, t_{439}=r_{186}^{\prime}+r_{190}^{\prime}, t_{438}=r_{228}^{\prime}+r_{232}^{\prime}, t_{1721}=$ $t_{438}+t_{1295}, t_{1871}=t_{1297}+t_{1721}, t_{437}=r_{53}^{\prime}+r_{71}^{\prime}, t_{1168}=t_{437}+t_{612}, t_{1147}=t_{437}+t_{767}, t_{1732}=r_{67}^{\prime}+t_{1147}, t_{642}=$ $t_{437}+t_{574}, t_{2172}=t_{642}+t_{1037}, t_{1094}=t_{630}+t_{642}, t_{1317}=t_{842}+t_{1094}, t_{1000}=t_{638}+t_{642}, t_{541}=t_{437}+t_{510}, t_{436}=$ $r_{104}^{\prime}+r_{108}^{\prime}, t_{1031}=t_{436}+t_{444}, t_{1731}=t_{1031}+t_{1056}, t_{780}=t_{436}+t_{497}, t_{1717}=t_{549}+t_{780}, t_{535}=t_{436}+t_{450}, t_{1837}=$ $t_{535}+t_{714}, t_{939}=t_{497}+t_{535}, t_{763}=t_{445}+t_{535}, t_{1349}=t_{650}+t_{763}, t_{1585}=t_{1230}+t_{1349}, t_{435}=r_{248}^{\prime}+r_{252}^{\prime}, t_{1071}=$ $r_{126}^{\prime}+t_{435}, t_{2002}=t_{1071}+t_{1996}, t_{2004}=t_{2000}+t_{2002}, t_{2007}=t_{1994}+t_{2004}, t_{652}=t_{435}+t_{452}, t_{1573}=t_{652}+t_{983}, t_{2320}=$ $t_{538}+t_{1573}, t_{434}=r_{19}^{\prime}+r_{23}^{\prime}, t_{2105}=t_{434}+t_{1237}, t_{527}=t_{434}+t_{438}, t_{877}=t_{494}+t_{527}, t_{1233}=t_{581}+t_{877}, t_{700}=$ $t_{527}+t_{539}, t_{433}=r_{202}^{\prime}+r_{206}^{\prime}, t_{1255}=t_{433}+t_{474}, t_{2171}=t_{1095}+t_{1255}, t_{2175}=t_{1601}+t_{2171}, t_{2177}=t_{1043}+t_{2175}, t_{657}=$ $r_{156}^{\prime}+t_{433}, t_{1117}=t_{656}+t_{657}, t_{832}=t_{510}+t_{657}, t_{1115}=r_{218}^{\prime}+t_{832}, t_{584}=t_{433}+t_{493}, t_{790}=t_{548}+t_{584}, t_{1397}=$ $t_{790}+t_{999}, t_{432}=r_{212}^{\prime}+r_{216}^{\prime}, t_{1017}=t_{432}+t_{535}, t_{729}=t_{432}+t_{451}, t_{2178}=r_{250}^{\prime}+t_{729}, t_{2179}=t_{2173}+t_{2178}, t_{1502}=$ $t_{552}+t_{729}, t_{1098}=t_{466}+t_{729}, t_{666}=t_{432}+t_{434}, t_{1579}=t_{666}+t_{773}, t_{606}=t_{432}+t_{515}, t_{1026}=t_{606}+t_{736}, t_{974}=$ $t_{498}+t_{606}, t_{431}=r_{159}^{\prime}+r_{163}^{\prime}, t_{1234}=r_{250}^{\prime}+t_{431}, t_{2090}=t_{1234}+t_{2085}, t_{1032}=r_{61}^{\prime}+t_{431}, t_{1350}=t_{1032}+t_{1038}, t_{1445}=$ $t_{1115}+t_{1350}, t_{1559}=t_{1063}+t_{1445}, t_{1716}=t_{982}+t_{1559}, t_{896}=t_{431}+t_{489}, t_{531}=t_{431}+t_{439}, t_{1915}=r_{235}^{\prime}+t_{531}, t_{1919}=$ $t_{1915}+t_{1917}, t_{1922}=t_{455}+t_{1919}, t_{747}=t_{531}+t_{541}, t_{1535}=t_{739}+t_{747}, t_{430}=r_{111}^{\prime}+r_{115}^{\prime}, t_{995}=t_{430}+t_{700}, t_{1610}=$ $t_{476}+t_{995}, t_{2364}=t_{1610}+t_{2152}, t_{1984}=t_{1255}+t_{1610}, t_{429}=r_{165}^{\prime}+r_{166}^{\prime}, t_{2134}=t_{429}+t_{848}, t_{937}=r_{66}^{\prime}+t_{429}, t_{1374}=$ $t_{462}+t_{937}, t_{511}=r_{0}^{\prime}+t_{429}, t_{1123}=r_{138}^{\prime}+t_{511}, t_{1588}=r_{192}^{\prime}+t_{1123}, t_{1758}=t_{858}+t_{1588}, t_{1388}=t_{458}+t_{1123}, t_{1886}=$ $r_{96}^{\prime}+t_{1388}, t_{794}=r_{116}^{\prime}+t_{511}, t_{2220}=t_{794}+t_{962}, t_{2228}=t_{1000}+t_{2220}, t_{1089}=r_{159}^{\prime}+t_{794}, t_{1304}=t_{972}+t_{1089}, t_{428}=r_{160}^{\prime}+$ $r_{164}^{\prime}, t_{1215}=t_{428}+t_{539}, t_{1029}=t_{428}+t_{435}, t_{427}=r_{229}^{\prime}+r_{233}^{\prime}, t_{426}=r_{3}^{\prime}+r_{7}^{\prime}, t_{1597}=t_{426}+t_{694}, t_{2346}=t_{1597}+t_{2337}, t_{2348}=$ $t_{2345}+t_{2346}, t_{2349}=t_{2347}+t_{2348}, t_{2355}=t_{1571}+t_{2349}, t_{787}=t_{426}+t_{476}, t_{1009}=t_{494}+t_{787}, t_{516}=t_{426}+t_{447}, t_{2430}=$ $t_{516}+t_{714}, t_{1487}=t_{516}+t_{1017}, t_{826}=t_{516}+t_{803}, t_{425}=r_{222}^{\prime}+r_{226}^{\prime}, t_{800}=t_{425}+t_{477}, t_{1464}=t_{800}+t_{1387}, t_{1426}=$ $t_{800}+t_{1116}, t_{424}=r_{119}^{\prime}+r_{123}^{\prime}, t_{1698}=r_{180}^{\prime}+t_{424}, t_{1447}=t_{424}+t_{539}, t_{1057}=t_{424}+t_{810}, t_{806}=t_{424}+t_{440}, t_{423}=$ $r_{1}^{\prime}+r_{5}^{\prime}, t_{2107}=t_{423}+t_{2105}, t_{1281}=t_{423}+t_{1084}, t_{1318}=r_{31}^{\prime}+t_{1281}, t_{1469}=r_{164}^{\prime}+t_{1318}, t_{738}=t_{423}+t_{590}, t_{822}=$
$t_{439}+t_{738}, t_{422}=r_{143}^{\prime}+r_{147}^{\prime}, t_{421}=r_{221}^{\prime}+r_{225}^{\prime}, t_{2342}=t_{421}+t_{2339}, t_{2350}=t_{1676}+t_{2342}, t_{2351}=t_{2344}+t_{2350}, t_{1386}=$ $t_{421}+t_{874}, t_{758}=t_{421}+t_{573}, t_{1608}=t_{454}+t_{758}, t_{1192}=r_{9}^{\prime}+t_{758}, t_{1497}=r_{207}^{\prime}+t_{1192}, t_{2216}=t_{1497}+t_{1547}, t_{2224}=$ $t_{1130}+t_{2216}, t_{2225}=t_{727}+t_{2224}, t_{529}=t_{421}+t_{445}, t_{1954}=t_{489}+t_{529}, t_{1512}=t_{529}+t_{700}, t_{926}=t_{516}+t_{529}, t_{1964}=$ $t_{458}+t_{926}, t_{420}=r_{96}^{\prime}+r_{100}^{\prime}, t_{1003}=t_{420}+t_{654}, t_{1199}=t_{1003}+t_{1060}, t_{1206}=t_{1089}+t_{1199}, t_{662}=t_{420}+t_{431}, t_{902}=$ $t_{504}+t_{662}, t_{2192}=t_{425}+t_{902}, t_{419}=r_{86}^{\prime}+r_{90}^{\prime}, t_{2016}=t_{419}+t_{1519}, t_{2018}=t_{592}+t_{2016}, t_{2021}=t_{779}+t_{2018}, t_{2022}=$ $t_{1498}+t_{2021}, t_{2024}=t_{2022}+t_{2023}, t_{2028}=t_{2024}+t_{2027}, t_{856}=r_{139}^{\prime}+t_{419}, t_{536}=t_{419}+t_{449}, t_{1841}=t_{467}+t_{536}, t_{418}=$ $r_{20}^{\prime}+r_{24}^{\prime}, t_{1333}=t_{418}+t_{936}, t_{1356}=t_{674}+t_{1333}, t_{1568}=r_{231}^{\prime}+t_{1356}, t_{1763}=t_{1551}+t_{1568}, t_{1766}=t_{1546}+t_{1763}, t_{699}=$ $t_{418}+t_{443}, t_{854}=t_{699}+t_{780}, t_{2072}=t_{854}+t_{1421}, t_{2078}=t_{806}+t_{2072}, t_{671}=r_{247}^{\prime}+t_{418}, t_{1335}=t_{671}+t_{711}, t_{534}=$ $t_{418}+t_{427}, t_{417}=r_{134}^{\prime}+r_{138}^{\prime}, t_{1724}=r_{26}^{\prime}+t_{417}, t_{697}=t_{417}+t_{565}, t_{1239}=t_{612}+t_{697}, t_{1471}=t_{1089}+t_{1239}, t_{530}=$ $t_{417}+t_{476}, t_{1945}=t_{530}+t_{696}, t_{1951}=t_{577}+t_{1945}, t_{611}=t_{464}+t_{530}, t_{1513}=t_{577}+t_{611}, t_{416}=r_{172}^{\prime}+r_{176}^{\prime}, t_{1525}=$ $t_{416}+t_{1485}, t_{543}=t_{416}+t_{435}, t_{882}=t_{452}+t_{543}, t_{415}=r_{194}^{\prime}+r_{198}^{\prime}, t_{1413}=t_{415}+t_{549}, t_{1004}=t_{415}+t_{441}, t_{2217}=$ $t_{1004}+t_{1635}, t_{1352}=r_{98}^{\prime}+t_{1004}, t_{414}=r_{69}^{\prime}+r_{73}^{\prime}, t_{2264}=t_{414}+t_{1118}, t_{2272}=t_{1352}+t_{2264}, t_{2281}=t_{2272}+t_{2275}, t_{2286}=$ $t_{567}+t_{2281}, t_{761}=t_{414}+t_{447}, t_{1411}=t_{596}+t_{761}, t_{918}=t_{580}+t_{761}, t_{533}=t_{414}+t_{461}, t_{2309}=t_{480}+t_{533}, t_{1107}=$ $r_{110}^{\prime}+t_{533}, t_{1342}=r_{130}^{\prime}+t_{1107}, t_{687}=t_{533}+t_{551}, t_{1714}=t_{687}+t_{747}, t_{929}=t_{465}+t_{687}, t_{413}=r_{157}^{\prime}+r_{161}^{\prime}, t_{1282}=$ $t_{413}+t_{634}, t_{1162}=t_{413}+t_{584}, t_{775}=t_{413}+t_{470}, t_{412}=r_{66}^{\prime}+r_{243}^{\prime}, t_{1441}=t_{412}+t_{589}, t_{1655}=r_{186}^{\prime}+t_{1441}, t_{1002}=$ $r_{65}^{\prime}+t_{412}, t_{1249}=r_{230}^{\prime}+t_{1002}, t_{1630}=t_{1249}+t_{1446}, t_{1789}=r_{135}^{\prime}+t_{1630}, t_{802}=t_{412}+t_{732}, t_{1309}=r_{33}^{\prime}+t_{802}, t_{1005}=$ $t_{567}+t_{802}, t_{1370}=t_{572}+t_{1005}, t_{503}=t_{412}+t_{463}, t_{1699}=t_{503}+t_{826}, t_{1223}=r_{185}^{\prime}+t_{503}, t_{528}=t_{499}+t_{503}, t_{891}=$ $t_{459}+t_{528}, t_{1247}=t_{771}+t_{891}, t_{1484}=t_{882}+t_{1247}, t_{2379}=r_{95}^{\prime}+t_{1484}, t_{411}=r_{35}^{\prime}+r_{39}^{\prime}, t_{1631}=t_{411}+t_{444}, t_{629}=$ $t_{411}+t_{497}, t_{542}=t_{411}+t_{446}, t_{1692}=t_{452}+t_{542}, t_{1558}=t_{443}+t_{542}, t_{1673}=r_{12}^{\prime}+t_{1558}, t_{1997}=t_{736}+t_{1673}, t_{2001}=$ $t_{1186}+t_{1997}, t_{2009}=t_{2001}+t_{2007}, t_{2011}=t_{2009}+t_{2010}, t_{1557}=t_{542}+t_{1031}, t_{2316}=t_{457}+t_{1557}, t_{2324}=t_{425}+t_{2316}, t_{2326}=$ $t_{2241}+t_{2324}, t_{2328}=t_{1131}+t_{2326}, t_{835}=t_{542}+t_{549}, t_{1977}=t_{835}+t_{1290}, t_{410}=r_{42}^{\prime}+r_{46}^{\prime}, t_{869}=r_{147}^{\prime}+t_{410}, t_{1680}=$ $r_{218}^{\prime}+t_{869}, t_{1859}=t_{1443}+t_{1680}, t_{1323}=t_{869}+t_{1161}, t_{2242}=t_{1323}+t_{1519}, t_{2252}=t_{907}+t_{2242}, t_{1275}=t_{470}+t_{869}, t_{409}=$ $r_{187}^{\prime}+r_{191}^{\prime}, t_{2266}=t_{409}+t_{1110}, t_{2279}=t_{2266}+t_{2268}, t_{2283}=t_{432}+t_{2279}, t_{1181}=t_{409}+t_{533}, t_{2168}=t_{761}+t_{1181}, t_{668}=$ $t_{409}+t_{416}, t_{2336}=r_{108}^{\prime}+t_{668}, t_{1632}=t_{543}+t_{668}, t_{792}=t_{542}+t_{668}, t_{900}=t_{488}+t_{792}, t_{1267}=t_{763}+t_{900}, t_{540}=$ $t_{409}+t_{428}, t_{1213}=r_{70}^{\prime}+t_{540}, t_{1283}=t_{678}+t_{1213}, t_{2020}=t_{1283}+t_{2019}, t_{778}=t_{540}+t_{560}, t_{408}=r_{44}^{\prime}+r_{48}^{\prime}, t_{1143}=$ $t_{408}+t_{671}, t_{2352}=t_{460}+t_{1143}, t_{2356}=t_{2351}+t_{2352}, t_{1128}=t_{408}+t_{599}, t_{2222}=r_{169}^{\prime}+t_{1128}, t_{1360}=t_{1021}+t_{1128}, t_{1481}=$ $r_{73}^{\prime}+t_{1360}, t_{1908}=r_{237}^{\prime}+t_{1481}, t_{684}=t_{408}+t_{438}, t_{1152}=t_{684}+t_{715}, t_{1061}=t_{611}+t_{684}, t_{899}=t_{684}+t_{775}, t_{1366}=$ $t_{854}+t_{899}, t_{532}=t_{408}+t_{425}, t_{1523}=t_{532}+t_{582}, t_{839}=t_{532}+t_{611}, t_{407}=r_{249}^{\prime}+r_{253}^{\prime}, t_{1678}=t_{407}+t_{468}, t_{1914}=$ $r_{196}^{\prime}+t_{1678}, t_{710}=t_{407}+t_{615}, t_{2239}=r_{227}^{\prime}+t_{710}, t_{523}=t_{407}+t_{422}, t_{978}=t_{461}+t_{523}, t_{1933}=t_{530}+t_{978}, t_{1428}=$ $t_{978}+t_{1259}, t_{945}=t_{450}+t_{523}, t_{1653}=t_{945}+t_{1183}, t_{637}=t_{469}+t_{523}, t_{2079}=t_{490}+t_{637}, t_{1200}=r_{241}^{\prime}+t_{637}, t_{984}=$ $r_{159}^{\prime}+t_{637}, t_{1995}=t_{970}+t_{984}, t_{2005}=r_{221}^{\prime}+t_{1995}, t_{1550}=t_{450}+t_{984}, t_{2180}=t_{1386}+t_{1550}, t_{406}=r_{101}^{\prime}+r_{105}^{\prime}, t_{1068}=$ $t_{406}+t_{695}, t_{405}=r_{127}^{\prime}+r_{131}^{\prime}, t_{1385}=r_{69}^{\prime}+t_{405}, t_{546}=t_{405}+t_{430}, t_{404}=r_{118}^{\prime}+r_{122}^{\prime}, t_{1627}=t_{404}+t_{1051}, t_{1112}=$ $t_{404}+t_{701}, t_{1725}=t_{412}+t_{1112}, t_{1556}=t_{679}+t_{1112}, t_{725}=t_{404}+t_{465}, t_{1567}=t_{725}+t_{939}, t_{1111}=t_{405}+t_{725}, t_{1591}=$ $r_{90}^{\prime}+t_{1111}, t_{2136}=t_{1562}+t_{1591}, t_{2139}=t_{2134}+t_{2136}, t_{2144}=t_{2139}+t_{2142}, t_{2145}=r_{72}^{\prime}+t_{2144}, t_{403}=r_{41}^{\prime}+r_{45}^{\prime}, t_{1268}=$ $t_{403}+t_{1040}, t_{2219}=r_{223}^{\prime}+t_{1268}, t_{921}=t_{403}+t_{430}, t_{1650}=t_{921}+t_{963}, t_{2103}=t_{445}+t_{1650}, t_{518}=t_{403}+t_{404}, t_{887}=$ $t_{470}+t_{518}, t_{2385}=t_{486}+t_{887}, t_{915}=t_{471}+t_{887}, t_{741}=t_{427}+t_{518}, t_{1767}=r_{136}^{\prime}+t_{741}, t_{1774}=t_{1766}+t_{1767}, t_{402}=$ $r_{169}^{\prime}+r_{173}^{\prime}, t_{956}=t_{402}+t_{421}, t_{813}=t_{402}+t_{417}, t_{1612}=r_{166}^{\prime}+t_{813}, t_{886}=t_{619}+t_{813}, t_{1125}=t_{616}+t_{886}, t_{1301}=$ $r_{58}^{\prime}+t_{1125}, t_{401}=r_{135}^{\prime}+r_{207}^{\prime}, t_{1577}=t_{401}+t_{1068}, t_{863}=t_{401}+t_{463}, t_{1693}=t_{803}+t_{863}, t_{1434}=r_{245}^{\prime}+t_{863}, t_{703}=$ $r_{187}^{\prime}+t_{401}, t_{1582}=t_{621}+t_{703}, t_{1660}=r_{163}^{\prime}+t_{1582}, t_{2392}=t_{1317}+t_{1660}, t_{2395}=r_{88}^{\prime}+t_{2392}, t_{2410}=t_{2395}+t_{2406}, t_{1119}=$ $t_{703}+t_{1042}, t_{1216}=r_{108}^{\prime}+t_{1119}, t_{1444}=t_{946}+t_{1216}, t_{2278}=t_{1444}+t_{2273}, t_{2284}=t_{2278}+t_{2280}, t_{507}=t_{401}+t_{460}, t_{966}=$ $t_{456}+t_{507}, t_{1104}=t_{409}+t_{966}, t_{2195}=t_{1104}+t_{2192}, t_{2201}=t_{1374}+t_{2195}, t_{2054}=t_{1104}+t_{1139}, t_{2055}=t_{1107}+t_{2054}, t_{2060}=$ $t_{1422}+t_{2055}, t_{400}=r_{230}^{\prime}+r_{234}^{\prime}, t_{620}=t_{400}+t_{467}, t_{840}=t_{417}+t_{620}, t_{517}=t_{400}+t_{424}, t_{852}=t_{439}+t_{517}, t_{1155}=$ $t_{407}+t_{852}, t_{649}=t_{467}+t_{517}, t_{1642}=t_{649}+t_{760}, t_{1987}=t_{1487}+t_{1642}, t_{1229}=t_{404}+t_{649}, t_{1511}=t_{546}+t_{1229}, t_{399}=$ $r_{125}^{\prime}+r_{129}^{\prime}, t_{1339}=t_{399}+t_{806}, t_{1930}=t_{852}+t_{1339}, t_{1013}=t_{399}+t_{704}, t_{1475}=t_{623}+t_{1013}, t_{1315}=t_{1013}+t_{1095}, t_{1362}=$ $t_{933}+t_{1315}, t_{717}=t_{399}+t_{478}, t_{1555}=t_{696}+t_{717}, t_{1935}=t_{780}+t_{1555}, t_{1034}=t_{694}+t_{717}, t_{2412}=t_{772}+t_{1034}, t_{1515}=$ $t_{532}+t_{1034}, t_{680}=t_{399}+t_{415}, t_{1381}=r_{63}^{\prime}+t_{680}, t_{1035}=t_{539}+t_{680}, t_{2053}=t_{474}+t_{1035}, t_{875}=t_{680}+t_{711}, t_{557}=$ $t_{399}+t_{465}, t_{1246}=t_{557}+t_{1176}, t_{1647}=t_{420}+t_{1246}, t_{398}=r_{28}^{\prime}+r_{32}^{\prime}, t_{1331}=r_{218}^{\prime}+t_{398}, t_{397}=r_{68}^{\prime}+r_{245}^{\prime}, t_{923}=$ $t_{397}+t_{429}, t_{1404}=r_{113}^{\prime}+t_{923}, t_{1704}=r_{143}^{\prime}+t_{1404}, t_{1305}=t_{923}+t_{1052}, t_{1648}=t_{1117}+t_{1305}, t_{483}=r_{210}^{\prime}+t_{397}, t_{1430}=$ $t_{483}+t_{484}, t_{831}=r_{182}^{\prime}+t_{483}, t_{1584}=t_{569}+t_{831}, t_{1067}=t_{831}+t_{864}, t_{1273}=t_{693}+t_{1067}, t_{2126}=t_{1273}+t_{1530}, t_{2129}=$ $t_{1516}+t_{2126}, t_{2133}=t_{2129}+t_{2131}, t_{2138}=t_{1119}+t_{2133}, t_{2143}=t_{698}+t_{2138}, t_{524}=r_{209}^{\prime}+t_{483}, t_{1872}=t_{524}+t_{720}, t_{545}=$ $t_{475}+t_{524}, t_{1316}=t_{545}+t_{1194}, t_{1611}=t_{1316}+t_{1418}, t_{579}=t_{511}+t_{545}, t_{608}=t_{528}+t_{579}, t_{396}=r_{195}^{\prime}+r_{199}^{\prime}, t_{1449}=$ $t_{396}+t_{499}, t_{941}=t_{396}+t_{414}, t_{1554}=t_{557}+t_{941}, t_{1086}=t_{482}+t_{941}, t_{1681}=t_{704}+t_{1086}, t_{2338}=t_{1681}+t_{2336}, t_{2354}=$ $t_{1342}+t_{2338}, t_{2359}=t_{2354}+t_{2355}, t_{2226}=t_{1653}+t_{1681}, t_{2227}=t_{2218}+t_{2226}, t_{1361}=t_{1086}+t_{1098}, t_{1473}=t_{800}+t_{1361}, t_{395}=$ $r_{250}^{\prime}+r_{254}^{\prime}, t_{2240}=t_{395}+t_{745}, t_{2251}=t_{1632}+t_{2240}, t_{2253}=t_{2251}+t_{2252}, t_{2256}=t_{1664}+t_{2253}, t_{1184}=t_{395}+t_{406}, t_{1453}=$ $t_{537}+t_{1184}, t_{1156}=t_{395}+t_{416}, t_{648}=t_{395}+t_{453}, t_{1510}=t_{474}+t_{648}, t_{1146}=t_{410}+t_{648}, t_{1351}=r_{188}^{\prime}+t_{1146}, t_{2322}=$ $t_{1351}+t_{2320}, t_{2323}=r_{51}^{\prime}+t_{2322}, t_{928}=t_{540}+t_{648}, t_{520}=t_{395}+t_{413}, t_{1423}=r_{75}^{\prime}+t_{520}, t_{821}=t_{520}+t_{586}, t_{1467}=$ $t_{821}+t_{1026}, t_{1458}=r_{137}^{\prime}+t_{821}, t_{2058}=t_{1458}+t_{2053}, t_{2066}=t_{2058}+t_{2065}, t_{2067}=t_{2052}+t_{2066}, t_{1850}=t_{1270}+t_{1458}, t_{1865}=$ $t_{1850}+t_{1863}, t_{394}=r_{178}^{\prime}+r_{182}^{\prime}, t_{1979}=t_{394}+t_{1977}, t_{782}=t_{394}+t_{532}, t_{992}=t_{484}+t_{782}, t_{1875}=r_{27}^{\prime}+t_{992}, t_{1882}=$
$t_{1871}+t_{1875}, t_{1883}=t_{1631}+t_{1882}, t_{724}=t_{394}+t_{550}, t_{1328}=t_{724}+t_{761}, t_{961}=t_{672}+t_{724}, t_{1587}=t_{961}+t_{987}, t_{555}=$ $t_{394}+t_{441}, t_{979}=t_{555}+t_{837}, t_{1624}=t_{450}+t_{979}, t_{809}=t_{474}+t_{555}, t_{1522}=t_{809}+t_{835}, t_{2191}=t_{666}+t_{1522}, t_{2199}=$ $t_{548}+t_{2191}, t_{1019}=t_{454}+t_{809}, t_{393}=r_{235}^{\prime}+r_{239}^{\prime}, t_{1729}=t_{393}+t_{456}, t_{1103}=t_{393}+t_{856}, t_{1298}=t_{558}+t_{1103}, t_{878}=$ $t_{393}+t_{629}, t_{689}=t_{393}+t_{468}, t_{796}=t_{529}+t_{689}, t_{2197}=t_{524}+t_{796}, t_{2202}=t_{2197}+t_{2201}, t_{2203}=r_{244}^{\prime}+t_{2202}, t_{1595}=$ $t_{488}+t_{796}, t_{1898}=t_{845}+t_{1595}, t_{1835}=t_{544}+t_{1595}, t_{521}=t_{393}+t_{444}, t_{911}=t_{428}+t_{521}, t_{1719}=t_{911}+t_{1521}, t_{1970}=$ $r_{211}^{\prime}+t_{1719}, t_{1975}=t_{502}+t_{1970}, t_{1976}=t_{1972}+t_{1975}, t_{1978}=t_{1971}+t_{1976}, t_{1980}=t_{1974}+t_{1978}, t_{1981}=t_{1968}+t_{1980}, t_{1982}=$ $t_{1979}+t_{1981}, t_{1983}=t_{1973}+t_{1982}, t_{60}=t_{890}+t_{1983}, t_{808}=t_{60}+t_{570}, t_{930}=t_{462}+t_{911}, t_{1065}=t_{652}+t_{930}, t_{669}=$ $t_{507}+t_{521}, t_{2325}=t_{669}+t_{1699}, t_{2331}=t_{1130}+t_{2325}, t_{1641}=t_{669}+t_{1039}, t_{1079}=t_{479}+t_{669}, t_{883}=t_{491}+t_{669}, t_{392}=$ $r_{179}^{\prime}+r_{183}^{\prime}, t_{1330}=t_{392}+t_{552}, t_{851}=t_{392}+t_{504}, t_{1895}=t_{851}+t_{1513}, t_{1596}=t_{560}+t_{851}, t_{1957}=t_{1426}+t_{1596}, t_{1083}=$ $t_{668}+t_{851}, t_{1880}=t_{1083}+t_{1331}, t_{1888}=t_{1605}+t_{1880}, t_{1889}=t_{1883}+t_{1888}, t_{526}=t_{392}+t_{420}, t_{868}=t_{449}+t_{526}, t_{1452}=$ $t_{443}+t_{868}, t_{719}=t_{526}+t_{543}, t_{1753}=t_{719}+t_{1065}, t_{1537}=t_{548}+t_{719}, t_{1182}=t_{418}+t_{719}, t_{391}=r_{201}^{\prime}+r_{205}^{\prime}, t_{1174}=$ $t_{391}+t_{847}, t_{872}=t_{391}+t_{464}, t_{2394}=t_{872}+t_{901}, t_{2405}=t_{2394}+t_{2398}, t_{1531}=t_{714}+t_{872}, t_{1153}=r_{216}^{\prime}+t_{872}, t_{1099}=$ $t_{493}+t_{872}, t_{743}=t_{391}+t_{448}, t_{1450}=t_{609}+t_{743}, t_{2254}=t_{1450}+t_{2247}, t_{2258}=t_{1153}+t_{2254}, t_{1357}=t_{666}+t_{743}, t_{1686}=$ $t_{775}+t_{1357}, t_{1097}=t_{743}+t_{814}, t_{1852}=t_{1097}+t_{1849}, t_{1857}=t_{918}+t_{1852}, t_{1514}=t_{1097}+t_{1298}, t_{522}=t_{391}+t_{406}, t_{1695}=$ $t_{522}+t_{1222}, t_{2157}=t_{1695}+t_{2150}, t_{2159}=t_{2149}+t_{2157}, t_{2160}=t_{655}+t_{2159}, t_{2166}=t_{710}+t_{2160}, t_{867}=t_{522}+t_{559}, t_{980}=$ $t_{583}+t_{867}, t_{2116}=t_{436}+t_{980}, t_{2117}=t_{606}+t_{2116}, t_{1988}=t_{839}+t_{980}, t_{636}=t_{522}+t_{577}, t_{1986}=t_{494}+t_{636}, t_{1989}=$ $t_{882}+t_{1986}, t_{1990}=t_{928}+t_{1989}, t_{1663}=t_{425}+t_{636}, t_{855}=t_{493}+t_{636}, t_{1992}=t_{855}+t_{1017}, t_{1618}=t_{773}+t_{855}, t_{390}=$ $r_{220}^{\prime}+r_{224}^{\prime}, t_{1073}=r_{196}^{\prime}+t_{390}, t_{1598}=t_{1073}+t_{1244}, t_{2233}=t_{1193}+t_{1598}, t_{2234}=t_{2228}+t_{2233}, t_{2089}=t_{1598}+t_{2083}, t_{639}=$ $t_{390}+t_{410}, t_{2368}=t_{639}+t_{1168}, t_{2371}=t_{2363}+t_{2368}, t_{1839}=t_{639}+t_{1835}, t_{1843}=t_{1837}+t_{1839}, t_{1399}=t_{639}+t_{747}, t_{1683}=$ $t_{945}+t_{1399}, t_{960}=t_{541}+t_{639}, t_{2306}=t_{596}+t_{960}, t_{1625}=t_{877}+t_{960}, t_{389}=r_{126}^{\prime}+r_{130}^{\prime}, t_{1759}=t_{389}+t_{859}, t_{1761}=$ $r_{208}^{\prime}+t_{1759}, t_{1762}=t_{1760}+t_{1761}, t_{1768}=t_{1758}+t_{1762}, t_{1771}=t_{1768}+t_{1769}, t_{1773}=t_{503}+t_{1771}, t_{1776}=t_{1773}+t_{1775}, t_{1778}=$ $t_{582}+t_{1776}, t_{1010}=t_{389}+t_{656}, t_{2071}=t_{792}+t_{1010}, t_{2074}=t_{709}+t_{2071}, t_{2081}=t_{2074}+t_{2079}, t_{2084}=t_{2081}+t_{2082}, t_{2088}=$ $r_{94}^{\prime}+t_{2084}, t_{990}=t_{389}+t_{517}, t_{2388}=t_{990}+t_{2385}, t_{500}=t_{389}+t_{396}, t_{723}=t_{489}+t_{500}, t_{2296}=t_{723}+t_{867}, t_{1398}=$ $t_{549}+t_{723}, t_{968}=r_{238}^{\prime}+t_{723}, t_{1242}=t_{659}+t_{968}, t_{2403}=t_{627}+t_{1242}, t_{640}=t_{500}+t_{534}, t_{1733}=t_{640}+t_{1608}, t_{1465}=$ $t_{620}+t_{640}, t_{388}=r_{142}^{\prime}+r_{146}^{\prime}, t_{2401}=r_{39}^{\prime}+t_{388}, t_{1780}=t_{388}+t_{1508}, t_{997}=t_{388}+t_{396}, t_{1320}=t_{861}+t_{997}, t_{1583}=$ $r_{126}^{\prime}+t_{1320}, t_{756}=t_{388}+t_{569}, t_{1480}=t_{756}+t_{1341}, t_{1303}=t_{540}+t_{756}, t_{1649}=t_{566}+t_{1303}, t_{1090}=t_{662}+t_{756}, t_{1619}=$ $t_{918}+t_{1090}, t_{722}=t_{388}+t_{553}, t_{2075}=r_{166}^{\prime}+t_{722}, t_{2080}=t_{1654}+t_{2075}, t_{2087}=t_{2078}+t_{2080}, t_{2094}=t_{2087}+t_{2088}, t_{2097}=$ $t_{2094}+t_{2095}, t_{1329}=t_{425}+t_{722}, t_{1563}=t_{564}+t_{1329}, t_{924}=t_{584}+t_{722}, t_{1670}=t_{561}+t_{924}, t_{505}=t_{388}+t_{423}, t_{2427}=$ $t_{505}+t_{1467}, t_{2428}=t_{1714}+t_{2427}, t_{1045}=t_{505}+t_{628}, t_{631}=t_{422}+t_{505}, t_{387}=r_{25}^{\prime}+r_{29}^{\prime}, t_{1838}=t_{387}+t_{481}, t_{1840}=$ $t_{479}+t_{1838}, t_{1141}=t_{387}+t_{610}, t_{1460}=t_{473}+t_{1141}, t_{820}=t_{387}+t_{703}, t_{1324}=r_{229}^{\prime}+t_{820}, t_{1456}=t_{1324}+t_{1362}, t_{1231}=$ $r_{102}^{\prime}+t_{820}, t_{1346}=t_{471}+t_{1231}, t_{708}=t_{387}+t_{520}, t_{1730}=t_{708}+t_{1663}, t_{827}=t_{708}+t_{728}, t_{1644}=t_{724}+t_{827}, t_{2104}=$ $t_{425}+t_{1644}, t_{2106}=t_{2103}+t_{2104}, t_{2112}=t_{416}+t_{2106}, t_{386}=r_{112}^{\prime}+r_{116}^{\prime}, t_{870}=t_{386}+t_{546}, t_{1592}=t_{736}+t_{870}, t_{1252}=$ $r_{22}^{\prime}+t_{870}, t_{1220}=t_{402}+t_{870}, t_{1687}=t_{1220}+t_{1267}, t_{753}=t_{386}+t_{398}, t_{2194}=t_{738}+t_{753}, t_{1018}=t_{666}+t_{753}, t_{385}=$ $r_{196}^{\prime}+r_{200}^{\prime}, t_{1736}=t_{385}+t_{553}, t_{1427}=t_{385}+t_{778}, t_{1209}=t_{385}+t_{824}, t_{895}=t_{385}+t_{639}, t_{1302}=t_{700}+t_{895}, t_{2420}=$ $t_{687}+t_{1302}, t_{1210}=t_{814}+t_{895}, t_{786}=t_{385}+t_{655}, t_{1278}=t_{742}+t_{786}, t_{1394}=t_{427}+t_{1278}, t_{525}=t_{385}+t_{415}, t_{1524}=$ $t_{525}+t_{606}, t_{2210}=t_{413}+t_{1524}, t_{1196}=t_{525}+t_{929}, t_{932}=t_{525}+t_{689}, t_{1689}=t_{932}+t_{990}, t_{2034}=t_{531}+t_{1689}, t_{2035}=$ $t_{902}+t_{2034}, t_{686}=t_{406}+t_{525}, t_{1782}=t_{686}+t_{1475}, t_{1783}=t_{477}+t_{1782}, t_{1258}=t_{586}+t_{686}, t_{1269}=t_{422}+t_{1258}, t_{2127}=$ $t_{454}+t_{1269}, t_{2132}=t_{1071}+t_{2127}, t_{2140}=t_{2128}+t_{2132}, t_{2146}=t_{1189}+t_{2140}, t_{2147}=t_{2145}+t_{2146}, t_{2148}=t_{2143}+t_{2147}, t_{384}=$ $r_{120}^{\prime}+r_{124}^{\prime}, t_{1750}=t_{384}+t_{1683}, t_{1752}=t_{1045}+t_{1750}, t_{844}=t_{384}+t_{634}, t_{1684}=t_{844}+t_{1608}, t_{752}=t_{384}+t_{580}, t_{1735}=$ $t_{752}+t_{1733}, t_{1737}=t_{728}+t_{1735}, t_{1743}=t_{575}+t_{1737}, t_{1188}=t_{453}+t_{752}, t_{512}=t_{384}+t_{386}, t_{1600}=t_{512}+t_{787}, t_{1195}=$ $t_{512}+t_{681}, t_{798}=t_{441}+t_{512}, t_{1390}=t_{662}+t_{798}, t_{1379}=t_{391}+t_{798}, t_{2208}=t_{1379}+t_{1555}, t_{2151}=t_{838}+t_{1379}, t_{2155}=$ $r_{155}^{\prime}+t_{2151}, t_{2163}=t_{2155}+t_{2162}, t_{2165}=t_{1125}+t_{2163}, t_{2167}=t_{2165}+t_{2166}, t_{784}=t_{512}+t_{556}, t_{1389}=t_{784}+t_{1170}, t_{2248}=$ $t_{1389}+t_{2239}, t_{1157}=t_{546}+t_{784}, t_{1505}=t_{915}+t_{1157}, t_{625}=t_{512}+t_{536}, t_{2169}=t_{625}+t_{818}, t_{2174}=t_{2169}+t_{2172}, t_{2181}=$ $t_{2170}+t_{2174}, t_{2182}=t_{2177}+t_{2181}, t_{2183}=t_{2180}+t_{2182}, t_{2184}=t_{2168}+t_{2183}, t_{1628}=t_{525}+t_{625}, t_{1312}=t_{625}+t_{628}, t_{1606}=$ $t_{680}+t_{1312}, t_{61}=t_{573}+t_{669}+t_{824}+t_{839}+t_{1051}+t_{1255}+t_{1290}+t_{1512}+t_{1522}+t_{1606}, t_{1415}=t_{61}+t_{1098}, t_{2193}=$ $t_{407}+t_{1415}, t_{1055}=t_{515}+t_{625}, t_{2187}=t_{998}+t_{1055}, t_{1337}=t_{397}+t_{1055}, t_{2321}=t_{1337}+t_{2318}, t_{2327}=t_{2261}+t_{2321}, t_{2329}=$ $t_{2323}+t_{2327}, t_{2332}=t_{2329}+t_{2331}, t_{383}=r_{88}^{\prime}+r_{92}^{\prime}, t_{2389}=t_{383}+t_{409}, t_{1764}=t_{383}+t_{579}, t_{1770}=t_{940}+t_{1764}, t_{1777}=$ $t_{1770}+t_{1774}, t_{1779}=t_{1777}+t_{1778}, t_{1159}=t_{383}+t_{875}, t_{1825}=t_{1159}+t_{1287}, t_{1827}=t_{1729}+t_{1825}, t_{635}=t_{383}+t_{392}, t_{1369}=$ $t_{627}+t_{635}, t_{1711}=r_{16}^{\prime}+t_{1369}, t_{849}=t_{467}+t_{635}, t_{1046}=t_{752}+t_{849}, t_{801}=t_{389}+t_{635}, t_{916}=t_{419}+t_{801}, t_{496}=$ $t_{383}+t_{398}, t_{1410}=t_{496}+t_{798}, t_{1916}=t_{546}+t_{1410}, t_{1918}=t_{1908}+t_{1916}, t_{1925}=t_{1914}+t_{1918}, t_{1928}=t_{1925}+t_{1926}, t_{1738}=$ $t_{1410}+t_{1736}, t_{1264}=t_{496}+t_{560}, t_{1666}=t_{537}+t_{1264}, t_{894}=t_{496}+t_{652}, t_{563}=t_{470}+t_{496}, t_{1064}=t_{534}+t_{563}, t_{1781}=$ $t_{779}+t_{1064}, t_{1787}=t_{951}+t_{1781}, t_{1791}=t_{1148}+t_{1787}, t_{1794}=t_{1789}+t_{1791}, t_{1798}=t_{775}+t_{1794}, t_{1801}=t_{1797}+t_{1798}, t_{1802}=$ $t_{459}+t_{1801}, t_{1015}=t_{563}+t_{640}, t_{1472}=t_{518}+t_{1015}, t_{1474}=t_{637}+t_{1472}, t_{692}=t_{403}+t_{563}, t_{1507}=t_{637}+t_{692}, t_{2298}=$ $t_{537}+t_{1507}, t_{1811}=t_{396}+t_{1507}, t_{991}=t_{692}+t_{926}, t_{382}=r_{2}^{\prime}+r_{6}^{\prime}, t_{1440}=r_{167}^{\prime}+t_{382}, t_{1132}=t_{382}+t_{631}, t_{1205}=$ $t_{629}+t_{1132}, t_{578}=t_{382}+t_{456}, t_{1705}=t_{578}+t_{1390}, t_{1096}=t_{578}+t_{826}, t_{942}=t_{533}+t_{578}, t_{1228}=t_{489}+t_{942}, t_{1959}=$ $t_{1228}+t_{1958}, t_{1961}=t_{1554}+t_{1959}, t_{513}=t_{382}+t_{402}, t_{1377}=t_{513}+t_{753}, t_{1285}=t_{513}+t_{708}, t_{1201}=t_{513}+t_{710}, t_{1578}=$ $t_{665}+t_{1201}, t_{958}=t_{486}+t_{513}, t_{58}=r_{56}^{\prime}+t_{564}+t_{673}+t_{682}+t_{716}+t_{730}+t_{746}+t_{778}+t_{807}+t_{907}+t_{958}+t_{973}+t_{997}+t_{1076}+$ $t_{1317}+t_{1457}+t_{1459}+t_{1550}+t_{1551}, t_{2008}=t_{58}+t_{451}, t_{2013}=t_{1580}+t_{2008}, t_{1072}=t_{58}+t_{871}, t_{1873}=t_{1072}+t_{1872}, t_{1876}=$
$t_{437}+t_{1873}, t_{1878}=t_{1870}+t_{1876}, t_{1884}=t_{1697}+t_{1878}, t_{1887}=t_{1884}+t_{1886}, t_{1890}=t_{717}+t_{1887}, t_{1891}=t_{1889}+t_{1890}, t_{1892}=$ $t_{1881}+t_{1891}, t_{1893}=t_{1707}+t_{1892}, t_{1674}=t_{1072}+t_{1662}, t_{1998}=t_{1674}+t_{1885}, t_{1500}=t_{697}+t_{958}, t_{2196}=t_{1401}+t_{1500}, t_{2198}=$ $t_{2193}+t_{2196}, t_{2200}=t_{2198}+t_{2199}, t_{2204}=t_{2200}+t_{2203}, t_{676}=t_{482}+t_{513}, t_{2111}=t_{676}+t_{747}, t_{2113}=t_{611}+t_{2111}, t_{1822}=$ $t_{441}+t_{676}, t_{1824}=t_{845}+t_{1822}, t_{1826}=t_{1141}+t_{1824}, t_{1828}=t_{1826}+t_{1827}, t_{1543}=t_{522}+t_{676}, t_{1327}=t_{573}+t_{676}, t_{917}=$ $t_{505}+t_{676}, t_{1462}=t_{917}+t_{1079}, t_{1165}=t_{534}+t_{917}, t_{1590}=t_{891}+t_{1165}, t_{381}=r_{11}^{\prime}+r_{15}^{\prime}, t_{691}=t_{381}+t_{477}, t_{1355}=$ $t_{622}+t_{691}, t_{1256}=t_{691}+t_{822}, t_{1461}=t_{636}+t_{1256}, t_{1250}=r_{124}^{\prime}+t_{691}, t_{1311}=t_{1000}+t_{1250}, t_{1586}=t_{1311}+t_{1585}, t_{850}=$ $t_{607}+t_{691}, t_{1948}=t_{850}+t_{1947}, t_{1950}=t_{462}+t_{1948}, t_{1952}=t_{1946}+t_{1950}, t_{1953}=t_{1951}+t_{1952}, t_{1955}=t_{974}+t_{1953}, t_{1001}=$ $t_{421}+t_{850}, t_{2030}=t_{1001}+t_{1377}, t_{2036}=t_{1644}+t_{2030}, t_{2037}=t_{875}+t_{2036}, t_{2038}=t_{2035}+t_{2037}, t_{2039}=t_{924}+t_{2038}, t_{2040}=$ $t_{1600}+t_{2039}, t_{1722}=t_{806}+t_{1001}, t_{495}=t_{381}+t_{390}, t_{1518}=t_{495}+t_{1155}, t_{1393}=t_{430}+t_{495}, t_{1808}=t_{629}+t_{1393}, t_{1814}=$ $t_{418}+t_{1808}, t_{1818}=t_{1811}+t_{1814}, t_{1296}=t_{495}+t_{536}, t_{1668}=t_{686}+t_{1296}, t_{777}=t_{433}+t_{495}, t_{1358}=t_{624}+t_{777}, t_{1934}=$ $t_{692}+t_{1358}, t_{1936}=t_{559}+t_{1934}, t_{1754}=t_{1358}+t_{1752}, t_{823}=t_{446}+t_{777}, t_{1949}=t_{823}+t_{1462}, t_{1956}=t_{590}+t_{1949}, t_{1962}=$ $t_{1956}+t_{1957}, t_{1829}=t_{823}+t_{1828}, t_{1831}=t_{569}+t_{1829}, t_{1544}=t_{823}+t_{896}, t_{568}=t_{410}+t_{495}, t_{1006}=t_{490}+t_{568}, t_{1727}=$ $t_{1006}+t_{1703}, t_{1253}=t_{620}+t_{1006}, t_{834}=t_{492}+t_{568}, t_{2393}=t_{579}+t_{834}, t_{2402}=t_{2393}+t_{2401}, t_{2408}=r_{104}^{\prime}+t_{2402}, t_{2415}=$ $t_{2408}+t_{2410}, t_{1077}=t_{629}+t_{834}, t_{1396}=t_{466}+t_{1077}, t_{1728}=t_{1396}+t_{1590}, t_{380}=r_{85}^{\prime}+r_{89}^{\prime}, t_{1012}=t_{380}+t_{925}, t_{1786}=$ $t_{1012}+t_{1785}, t_{1792}=t_{1786}+t_{1790}, t_{1793}=t_{1100}+t_{1792}, t_{1795}=t_{1343}+t_{1793}, t_{1799}=t_{1788}+t_{1795}, t_{1800}=t_{1330}+t_{1799}, t_{1803}=$ $t_{1783}+t_{1800}, t_{1804}=t_{781}+t_{1803}, t_{1805}=t_{1796}+t_{1804}, t_{1806}=t_{1802}+t_{1805}, t_{690}=t_{380}+t_{405}, t_{1657}=r_{36}^{\prime}+t_{690}, t_{1854}=$ $t_{1657}+t_{1851}, t_{1855}=t_{592}+t_{1854}, t_{1858}=t_{1054}+t_{1855}, t_{1861}=t_{1857}+t_{1858}, t_{1864}=t_{1853}+t_{1861}, t_{1866}=t_{1088}+t_{1864}, t_{1867}=$ $t_{1859}+t_{1866}, t_{1407}=t_{586}+t_{690}, t_{1173}=t_{690}+t_{699}, t_{1126}=t_{690}+t_{840}, t_{829}=t_{513}+t_{690}, t_{1696}=t_{829}+t_{929}, t_{1207}=$ $t_{778}+t_{829}, t_{1748}=t_{636}+t_{1207}, t_{1749}=t_{517}+t_{1748}, t_{501}=t_{380}+t_{387}, t_{1109}=t_{501}+t_{532}, t_{1671}=t_{507}+t_{1109}, t_{1536}=$ $t_{473}+t_{1109}, t_{1932}=t_{1536}+t_{1780}, t_{734}=t_{486}+t_{501}, t_{1435}=t_{734}+t_{1023}, t_{2154}=t_{1088}+t_{1435}, t_{2367}=t_{2154}+t_{2361}, t_{2372}=$ $t_{1413}+t_{2367}, t_{1142}=t_{568}+t_{734}, t_{63}=t_{61}+t_{513}+t_{746}+t_{1142}+t_{1427}+t_{1474}+t_{1511}+t_{1567}+t_{1618}, t_{2429}=t_{63}+t_{2428}, t_{2431}=$ $t_{452}+t_{2429}, t_{2432}=t_{2430}+t_{2431}, t_{2433}=t_{1537}+t_{2432}, t_{23}=t_{596}+t_{600}+t_{668}+t_{741}+t_{760}+t_{778}+t_{844}+t_{916}+t_{966}+$ $t_{1009}+t_{1116}+t_{1428}+t_{1461}+t_{2433}, t_{1751}=t_{23}+t_{1749}, t_{1755}=t_{697}+t_{1751}, t_{1756}=t_{1754}+t_{1755}, t_{1238}=t_{23}+t_{1019}, t_{122}=$ $t_{404}+t_{422}+t_{436}+t_{447}+t_{635}+t_{666}+t_{689}+t_{844}+t_{878}+t_{895}+t_{910}+t_{930}+t_{1182}+t_{1238}+t_{1649}+t_{1670}+t_{1730}+t_{1895}, t_{0}=$ $t_{608}+t_{2433}, t_{1106}=t_{0}+t_{63}, t_{1175}=t_{906}+t_{1106}, t_{626}=t_{0}+t_{23}, t_{744}=t_{608}+t_{626}, t_{54}=t_{381}+t_{504}+t_{624}+t_{744}+t_{761}+$ $t_{849}+t_{899}+t_{1012}+t_{1029}+t_{1228}+t_{1335}+t_{1515}+t_{1592}+t_{1631}+t_{1670}, t_{2032}=t_{479}+t_{744}, t_{2033}=t_{2031}+t_{2032}, t_{2041}=$ $t_{2033}+t_{2040}, t_{5}=t_{1143}+t_{2041}, t_{1594}=t_{5}+t_{946}, t_{1691}=t_{1436}+t_{1594}, t_{1138}=t_{400}+t_{744}, t_{53}=t_{449}+t_{468}+t_{487}+t_{518}+$ $t_{530}+t_{607}+t_{668}+t_{699}+t_{763}+t_{902}+t_{921}+t_{959}+t_{1019}+t_{1138}+t_{1146}+t_{1258}+t_{1473}+t_{1631}, t_{1208}=t_{53}+t_{686}, t_{1665}=$ $t_{1035}+t_{1208}, t_{8}=t_{390}+t_{408}+t_{415}+t_{427}+t_{484}+t_{555}+t_{583}+t_{690}+t_{818}+t_{941}+t_{1138}+t_{1397}+t_{1649}+t_{1686}+t_{1687}, t_{1603}=$ $t_{63}+t_{1138}, t_{99}=t_{434}+t_{435}+t_{464}+t_{476}+t_{479}+t_{507}+t_{525}+t_{620}+t_{739}+t_{1393}+t_{1603}+t_{1619}+t_{1730}, t_{1960}=t_{99}+t_{743}, t_{1963}=$ $t_{1960}+t_{1962}, t_{1965}=t_{1963}+t_{1964}, t_{1966}=t_{1961}+t_{1965}, t_{1967}=t_{1505}+t_{1966}, t_{55}=t_{1327}+t_{1967}, t_{1314}=t_{55}+t_{555}, t_{1504}=$ $t_{1018}+t_{1314}, t_{91}=t_{517}+t_{530}+t_{541}+t_{600}+t_{628}+t_{838}+t_{883}+t_{1012}+t_{1196}+t_{1452}+t_{1465}+t_{1504}+t_{1992}, t_{1202}=r_{247}^{\prime}+t_{55}, t_{1823}=$ $t_{628}+t_{1202}, t_{1830}=t_{1531}+t_{1823}, t_{1832}=t_{1830}+t_{1831}, t_{1833}=t_{1473}+t_{1832}, t_{2310}=t_{394}+t_{1967}, t_{1483}=t_{99}+t_{445}, t_{30}=$ $t_{53}+t_{380}+t_{426}+t_{474}+t_{652}+t_{915}+t_{963}+t_{1209}+t_{1366}+t_{1397}+t_{1483}+t_{1603}+t_{1988}+t_{2194}, t_{1810}=t_{30}+t_{1642}, t_{1812}=$ $t_{1705}+t_{1810}, t_{1813}=t_{99}+t_{1812}, t_{1815}=t_{1238}+t_{1813}, t_{1816}=t_{1753}+t_{1815}, t_{1817}=t_{498}+t_{1816}, t_{1819}=t_{896}+t_{1817}, t_{1757}=$ $t_{1483}+t_{1756}, t_{24}=t_{419}+t_{719}+t_{1757}, t_{1809}=t_{1757}+t_{1807}, t_{1820}=t_{1809}+t_{1819}, t_{1821}=t_{1818}+t_{1820}, t_{113}=t_{960}+t_{1821}, t_{32}=$ $t_{575}+t_{662}+t_{790}+t_{840}+t_{1061}+t_{1064}+t_{1258}+t_{1696}+t_{1821}+t_{1949}+t_{2169}+t_{2187}, t_{605}=t_{61}+t_{63}, t_{1016}=t_{605}+t_{741}, t_{873}=$ $t_{578}+t_{605}, t_{1367}=t_{873}+t_{916}, t_{2108}=t_{446}+t_{1367}, t_{1279}=t_{620}+t_{873}, t_{1261}=t_{394}+t_{1142}, t_{1896}=t_{531}+t_{1261}, t_{1902}=t_{556}+$ $t_{1896}, t_{685}=t_{501}+t_{516}, t_{1180}=t_{551}+t_{685}, t_{1271}=r_{30}^{\prime}+t_{1180}, t_{1151}=t_{526}+t_{685}, t_{816}=t_{631}+t_{685}, t_{1262}=t_{816}+t_{928}, t_{121}=$ $t_{113}+t_{382}+t_{480}+t_{500}+t_{543}+t_{782}+t_{975}+t_{1111}+t_{1157}+t_{1262}+t_{1447}+t_{1714}, t_{653}=t_{113}+t_{121}, t_{1289}=t_{382}+t_{653}, t_{2044}=$ $t_{877}+t_{1289}, t_{879}=t_{531}+t_{653}, t_{92}=t_{32}+t_{456}+t_{525}+t_{526}+t_{734}+t_{766}+t_{821}+t_{879}+t_{992}+t_{1086}+t_{1302}+t_{1554}+t_{2385}, t_{1552}=$ $t_{92}+t_{121}, t_{66}=t_{738}+t_{1552}+t_{2204}, t_{554}=t_{32}+t_{92}, t_{108}=t_{489}+t_{554}+t_{1955}, t_{2400}=t_{108}+t_{680}, t_{1679}=t_{108}+t_{538}, t_{1615}=$ $t_{554}+t_{1151}, t_{2382}=t_{1615}+t_{1696}, t_{2384}=t_{1004}+t_{2382}, t_{2386}=t_{1413}+t_{2384}, t_{2308}=t_{1615}+t_{2306}, t_{2312}=t_{2308}+t_{2310}, t_{2313}=$ $t_{2309}+t_{2312}, t_{765}=t_{546}+t_{554}, t_{114}=t_{122}+t_{513}+t_{532}+t_{685}+t_{765}+t_{928}+t_{1339}+t_{1628}+t_{1683}, t_{617}=t_{114}+t_{122}, t_{2049}=$ $t_{523}+t_{617}, t_{2307}=t_{692}+t_{2049}, t_{726}=t_{557}+t_{617}, t_{2422}=t_{765}+t_{2421}, t_{2423}=t_{553}+t_{2422}, t_{2424}=t_{2420}+t_{2423}, t_{1135}=$ $t_{765}+t_{868}, t_{1203}=t_{521}+t_{1135}, t_{6}=t_{438}+t_{498}+t_{518}+t_{691}+t_{796}+t_{900}+t_{1126}+t_{1203}+t_{1269}+t_{1293}+t_{1328}+t_{1453}+t_{1554}+$ $t_{1665}+t_{1679}, t_{1476}=t_{6}+t_{1030}, t_{72}=r_{180}^{\prime}+t_{1476}+t_{1893}, t_{1745}=t_{1203}+t_{1743}, t_{1734}=t_{410}+t_{879}, t_{1739}=t_{1411}+t_{1734}, t_{1741}=$ $t_{542}+t_{1739}, t_{1363}=t_{879}+t_{1016}, t_{2383}=t_{1363}+t_{1609}, t_{2387}=t_{2383}+t_{2386}, t_{2390}=t_{2387}+t_{2388}, t_{2391}=t_{2389}+t_{2390}, t_{97}=$ $t_{390}+t_{2391}, t_{969}=t_{97}+t_{746}, t_{56}=t_{385}+t_{486}+t_{536}+t_{590}+t_{653}+t_{969}+t_{1046}+t_{1099}+t_{1141}+t_{1181}+t_{1279}+t_{1288}+t_{1678}+$ $t_{1679}+t_{1686}+t_{2041}, t_{1509}=t_{56}+t_{649}, t_{1613}=t_{969}+t_{1106}, t_{22}=t_{465}+t_{520}+t_{560}+t_{608}+t_{1029}+t_{1474}+t_{1613}+t_{1619}, t_{1744}=$ $t_{22}+t_{1741}, t_{1154}=t_{22}+t_{97}, t_{64}=t_{466}+t_{697}+t_{790}+t_{822}+t_{838}+t_{918}+t_{939}+t_{1083}+t_{1141}+t_{1154}+t_{1285}+t_{1312}+t_{1954}, t_{1466}=$ $t_{64}+t_{1254}, t_{1448}=t_{64}+t_{1407}, t_{1740}=t_{467}+t_{1448}, t_{1742}=t_{1738}+t_{1740}, t_{1746}=t_{1742}+t_{1745}, t_{1747}=t_{1744}+t_{1746}, t_{100}=$ $t_{605}+t_{1747}, t_{1442}=t_{451}+t_{1154}, t_{2212}=t_{1442}+t_{1543}, t_{2295}=t_{1262}+t_{1427}, t_{2297}=t_{1017}+t_{2295}, t_{1136}=t_{816}+t_{942}, t_{2206}=$ $t_{1027}+t_{1136}, t_{2209}=t_{1535}+t_{2206}, t_{1431}=t_{484}+t_{1136}, t_{2418}=t_{838}+t_{1431}, t_{2419}=t_{1509}+t_{2418}, t_{2425}=t_{2419}+t_{2424}, t_{2426}=$ $t_{537}+t_{2425}, t_{74}=t_{1625}+t_{2426}, t_{880}=t_{74}+t_{726}, t_{2046}=t_{880}+t_{1196}, t_{1091}=t_{554}+t_{880}, t_{98}=t_{64}+t_{389}+t_{415}+t_{426}+t_{482}+$ $t_{752}+t_{917}+t_{921}+t_{975}+t_{1091}+t_{1264}+t_{1327}+t_{1335}+t_{1428}+t_{1509}+t_{2389}, t_{1931}=t_{98}+t_{1215}, t_{1939}=t_{1931}+t_{1936}, t_{1940}=$ $t_{1935}+t_{1939}, t_{1937}=t_{1091}+t_{1930}, t_{1938}=t_{1933}+t_{1937}, t_{1941}=t_{1938}+t_{1940}, t_{1942}=t_{1932}+t_{1941}, t_{1943}=t_{1579}+t_{1942}, t_{1944}=$ $t_{1466}+t_{1943}, t_{89}=t_{488}+t_{1944}, t_{1048}=t_{89}+t_{779}, t_{833}=t_{56}+t_{89}, t_{37}=t_{419}+t_{427}+t_{471}+t_{491}+t_{533}+t_{540}+t_{833}+t_{835}+$
$t_{882}+t_{921}+t_{956}+t_{997}+t_{1009}+t_{1018}+t_{1453}+t_{1515}+t_{1518}, t_{29}=t_{747}+t_{833}+t_{991}+t_{1045}+t_{1465}+t_{1510}+t_{1606}+t_{2426}, t_{1499}=$ $t_{29}+t_{833}, t_{76}=t_{54}+t_{518}+t_{531}+t_{551}+t_{928}+t_{979}+t_{1064}+t_{1091}+t_{1132}+t_{1413}+t_{1499}, t_{749}=t_{54}+t_{76}, t_{1834}=t_{607}+t_{749}, t_{1836}=$ $t_{1135}+t_{1834}, t_{1842}=t_{91}+t_{1836}, t_{1844}=t_{1840}+t_{1842}, t_{1845}=t_{726}+t_{1844}, t_{1846}=t_{1841}+t_{1845}, t_{1847}=t_{1843}+t_{1846}, t_{1848}=$ $t_{1504}+t_{1847}, t_{39}=t_{1205}+t_{1848}, t_{1589}=t_{640}+t_{749}, t_{90}=t_{30}+t_{526}+t_{680}+t_{806}+t_{995}+t_{1431}+t_{1589}+t_{1624}, t_{757}=$ $t_{30}+t_{90}, t_{2249}=t_{717}+t_{757}, t_{106}=t_{477}+t_{629}+t_{729}+t_{736}+t_{790}+t_{1034}+t_{1254}+t_{1531}+t_{2249}, t_{1545}=r_{247}^{\prime}+t_{106}, t_{1688}=$ $r_{79}^{\prime}+t_{1545}, t_{12}=r_{37}^{\prime}+r_{171}^{\prime}+r_{226}^{\prime}+t_{529}+t_{705}+t_{834}+t_{842}+t_{888}+t_{913}+t_{923}+t_{1081}+t_{1209}+t_{1215}+t_{1336}+t_{1385}+t_{1450}+t_{1566}+$ $t_{1638}+t_{1652}+t_{1685}+t_{1688}, t_{1593}=t_{12}+t_{537}, t_{102}=r_{118}^{\prime}+r_{140}^{\prime}+r_{157}^{\prime}+r_{187}^{\prime}+r_{241}^{\prime}+t_{411}+t_{568}+t_{907}+t_{967}+t_{1103}+t_{1139}+t_{1274}+$ $t_{1310}+t_{1332}+t_{1395}+t_{1398}+t_{1403}+t_{1412}+t_{1492}+t_{1516}+t_{1533}+t_{1535}+t_{1581}+t_{1593}+t_{1688}+t_{1720}, t_{1322}=t_{106}+t_{956}, t_{1723}=$ $r_{20}^{\prime}+t_{1322}, t_{2245}=t_{1723}+t_{2244}, t_{2246}=t_{2241}+t_{2245}, t_{2255}=t_{2243}+t_{2246}, t_{2250}=t_{2248}+t_{2249}, t_{2257}=t_{2250}+t_{2256}, t_{2259}=$ $t_{2257}+t_{2258}, t_{2260}=r_{148}^{\prime}+t_{2259}, t_{2262}=t_{2255}+t_{2260}, t_{2263}=t_{1233}+t_{2262}, t_{2311}=t_{1589}+t_{2307}, t_{2314}=t_{2311}+t_{2313}, t_{75}=$ $t_{494}+t_{2314}, t_{641}=t_{55}+t_{75}, t_{86}=r_{26}^{\prime}+r_{28}^{\prime}+r_{67}^{\prime}+r_{71}^{\prime}+r_{91}^{\prime}+r_{116}^{\prime}+r_{120}^{\prime}+r_{132}^{\prime}+r_{183}^{\prime}+r_{229}^{\prime}+t_{102}+t_{434}+t_{641}+t_{668}+t_{768}+t_{791}+$ $t_{806}+t_{1007}+t_{1043}+t_{1096}+t_{1111}+t_{1260}+t_{1282}+t_{1547}+t_{1575}+t_{1611}+t_{1620}, t_{1113}=t_{557}+t_{641}, t_{31}=t_{91}+t_{700}+t_{734}+t_{1113}+$ $t_{1511}+t_{1624}+t_{1628}+t_{1666}, t_{661}=t_{31}+t_{91}, t_{107}=t_{471}+t_{497}+t_{504}+t_{565}+t_{610}+t_{661}+t_{746}+t_{883}+t_{1174}+t_{1241}+t_{1692}, t_{2369}=$ $r_{37}^{\prime}+t_{107}, t_{2373}=t_{2369}+t_{2370}, t_{2374}=t_{2372}+t_{2373}, t_{1894}=t_{107}+t_{540}, t_{1245}=t_{107}+t_{981}, t_{2006}=t_{1245}+t_{2005}, t_{2012}=$ $r_{105}^{\prime}+t_{2006}, t_{2014}=t_{2012}+t_{2013}, t_{2015}=t_{2011}+t_{2014}, t_{78}=t_{1728}+t_{1885}+t_{2015}, t_{1160}=r_{42}^{\prime}+t_{78}, t_{1616}=t_{1160}+t_{1372}, t_{1706}=$ $r_{191}^{\prime}+t_{1616}, t_{1306}=r_{34}^{\prime}+t_{1245}, t_{80}=r_{30}^{\prime}+r_{139}^{\prime}+t_{407}+t_{432}+t_{442}+t_{464}+t_{607}+t_{663}+t_{724}+t_{742}+t_{808}+t_{809}+t_{923}+t_{938}+$ $t_{1096}+t_{1171}+t_{1200}+t_{1306}+t_{1321}+t_{1327}+t_{1523}+t_{1607}+t_{1653}+t_{1720}, t_{2186}=t_{1306}+t_{2184}, t_{2188}=t_{2179}+t_{2186}, t_{898}=$ $t_{107}+t_{551}, t_{1364}=r_{182}^{\prime}+t_{898}, t_{1643}=t_{514}+t_{1364}, t_{36}=t_{901}+t_{1435}+t_{1643}+t_{1706}+t_{2167}, t_{2164}=t_{1501}+t_{1643}, t_{2375}=$ $t_{2164}+t_{2374}, t_{2377}=t_{1047}+t_{2375}, t_{750}=t_{107}+t_{473}, t_{2185}=t_{750}+t_{2176}, t_{2190}=t_{2185}+t_{2188}, t_{1325}=t_{414}+t_{750}, t_{123}=$ $t_{99}+t_{439}+t_{446}+t_{522}+t_{640}+t_{809}+t_{930}+t_{952}+t_{1173}+t_{1174}+t_{1178}+t_{1232}+t_{1293}+t_{1325}+t_{1339}+t_{1694}+t_{2314}, t_{1093}=$ $t_{64}+t_{750}, t_{2205}=t_{1093}+t_{1822}, t_{1463}=t_{486}+t_{1093}, t_{904}=t_{641}+t_{661}, t_{1549}=t_{904}+t_{1463}, t_{62}=r_{247}^{\prime}+t_{649}+t_{711}+t_{725}+t_{829}+$ $t_{904}+t_{992}+t_{1090}+t_{1632}+t_{1666}+t_{1684}+t_{1747}+t_{1841}+t_{2391}, t_{82}=r_{108}^{\prime}+r_{121}^{\prime}+r_{135}^{\prime}+r_{170}^{\prime}+r_{239}^{\prime}+t_{62}+t_{571}+t_{667}+t_{764}+t_{1019}+$ $t_{1032}+t_{1200}+t_{1227}+t_{1433}+t_{1522}+t_{1718}+t_{1998}+t_{2015}+t_{2164}, t_{562}=t_{62}+t_{82}, t_{71}=t_{562}+t_{2148}, t_{1897}=t_{562}+t_{1894}, t_{21}=$ $t_{62}+t_{521}+t_{584}+t_{773}+t_{801}+t_{900}+t_{939}+t_{1016}+t_{1461}+t_{1500}, t_{1429}=t_{21}+t_{883}, t_{1712}=t_{62}+t_{1429}, t_{2109}=t_{1712}+t_{2108}, t_{2110}=$ $t_{2107}+t_{2109}, t_{2114}=t_{2110}+t_{2112}, t_{2115}=t_{2113}+t_{2114}, t_{40}=t_{108}+t_{391}+t_{433}+t_{569}+t_{596}+t_{631}+t_{684}+t_{734}+t_{746}+$ $t_{827}+t_{868}+t_{896}+t_{929}+t_{1104}+t_{1182}+t_{1557}+t_{1722}+t_{2115}, t_{876}=r_{20}^{\prime}+t_{21}, t_{1639}=t_{841}+t_{876}, t_{96}=t_{1639}+t_{1779}, t_{1075}=$ $t_{96}+t_{643}, t_{18}=r_{102}^{\prime}+r_{119}^{\prime}+r_{121}^{\prime}+r_{194}^{\prime}+r_{195}^{\prime}+r_{228}^{\prime}+r_{234}^{\prime}+t_{21}+t_{466}+t_{591}+t_{721}+t_{727}+t_{776}+t_{785}+t_{795}+t_{902}+t_{954}+t_{1052}+$ $t_{1053}+t_{1075}+t_{1118}+t_{1133}+t_{1286}+t_{1480}+t_{1494}+t_{1556}+t_{1573}+t_{1634}+t_{1729}, t_{1224}=t_{973}+t_{1075}, t_{971}=t_{860}+t_{876}, t_{1417}=$ $t_{470}+t_{971}, t_{59}=r_{162}^{\prime}+t_{501}+t_{649}+t_{794}+t_{808}+t_{893}+t_{895}+t_{931}+t_{965}+t_{1224}+t_{1252}+t_{1286}+t_{1392}+t_{1417}+t_{1529}, t_{20}=$ $r_{15}^{\prime}+r_{98}^{\prime}+r_{101}^{\prime}+r_{225}^{\prime}+t_{59}+t_{441}+t_{520}+t_{606}+t_{657}+t_{665}+t_{699}+t_{722}+t_{889}+t_{892}+t_{899}+t_{1122}+t_{1331}+t_{1449}+t_{1471}+t_{1491}+$ $t_{1566}+t_{1601}+t_{1647}+t_{1672}+t_{1983}, t_{1700}=t_{59}+t_{517}, t_{1058}=t_{59}+t_{808}, t_{1292}=t_{391}+t_{1058}, t_{2396}=t_{933}+t_{1292}, t_{2407}=$ $t_{2396}+t_{2404}, t_{2409}=t_{1037}+t_{2407}, t_{2411}=t_{1523}+t_{2409}, t_{2413}=t_{2400}+t_{2411}, t_{2414}=t_{2405}+t_{2413}, t_{2416}=t_{2414}+t_{2415}, t_{2417}=$ $t_{2403}+t_{2416}, t_{10}=t_{650}+t_{772}+t_{2417}, t_{2221}=r_{32}^{\prime}+t_{10}, t_{2223}=t_{1587}+t_{2221}, t_{2232}=t_{1591}+t_{2223}, t_{1185}=t_{10}+t_{627}, t_{1708}=$ $t_{564}+t_{1185}, t_{2017}=t_{632}+t_{1708}, t_{2025}=t_{1565}+t_{2017}, t_{2026}=t_{2020}+t_{2025}, t_{2029}=t_{2026}+t_{2028}, t_{77}=t_{1539}+t_{2029}, t_{2057}=$ $t_{565}+t_{2029}, t_{2059}=t_{891}+t_{2057}, t_{2064}=t_{2056}+t_{2059}, t_{2068}=t_{2064}+t_{2067}, t_{2069}=t_{1108}+t_{2068}, t_{2070}=t_{2060}+t_{2069}, t_{103}=$ $t_{633}+t_{2070}, t_{87}=r_{1}^{\prime}+r_{29}^{\prime}+r_{112}^{\prime}+r_{131}^{\prime}+t_{536}+t_{587}+t_{1150}+t_{1152}+t_{1159}+t_{1179}+t_{1229}+t_{1289}+t_{1351}+t_{1423}+t_{1432}+t_{1564}+$ $t_{1568}+t_{1604}+t_{1611}+t_{1656}+t_{2070}, t_{1375}=t_{381}+t_{1292}, t_{120}=r_{0}^{\prime}+r_{10}^{\prime}+r_{74}^{\prime}+r_{109}^{\prime}+r_{112}^{\prime}+r_{149}^{\prime}+r_{212}^{\prime}+r_{236}^{\prime}+t_{86}+t_{543}+t_{577}+$ $t_{614}+t_{625}+t_{688}+t_{807}+t_{846}+t_{1010}+t_{1045}+t_{1049}+t_{1126}+t_{1252}+t_{1308}+t_{1348}+t_{1375}+t_{1434}+t_{1454}+t_{1479}+t_{1491}+t_{1779}, t_{789}=$ $t_{86}+t_{120}, t_{1710}=t_{658}+t_{789}, t_{2315}=t_{1355}+t_{1710}, t_{2317}=t_{1099}+t_{2315}, t_{2319}=t_{976}+t_{2317}, t_{2330}=t_{547}+t_{2319}, t_{2333}=$ $t_{2328}+t_{2330}, t_{2334}=t_{2332}+t_{2333}, t_{85}=r_{4}^{\prime}+r_{44}^{\prime}+r_{252}^{\prime}+t_{391}+t_{483}+t_{655}+t_{788}+t_{797}+t_{821}+t_{836}+t_{892}+t_{1144}+t_{1190}+$ $t_{1257}+t_{1381}+t_{1385}+t_{1421}+t_{1438}+t_{1440}+t_{1477}+t_{1496}+t_{1532}+t_{1544}+t_{1583}+t_{1659}+t_{1693}+t_{1780}+t_{1806}+t_{2334}+t_{2412}, t_{101}=$ $r_{31}^{\prime}+r_{68}^{\prime}+r_{128}^{\prime}+r_{210}^{\prime}+t_{85}+t_{380}+t_{556}+t_{591}+t_{593}+t_{598}+t_{709}+t_{742}+t_{749}+t_{757}+t_{920}+t_{1046}+t_{1186}+t_{1223}+t_{1235}+$ $t_{1275}+t_{1359}+t_{1402}+t_{1447}+t_{1512}+t_{1525}+t_{1571}+t_{1621}+t_{1690}, t_{1163}=t_{651}+t_{789}, t_{1451}=r_{190}^{\prime}+t_{1163}, t_{67}=r_{205}^{\prime}+t_{1451}+$ $t_{1626}+t_{2190}, t_{104}=r_{38}^{\prime}+r_{117}^{\prime}+r_{133}^{\prime}+r_{158}^{\prime}+t_{67}+t_{536}+t_{557}+t_{585}+t_{646}+t_{696}+t_{886}+t_{915}+t_{1029}+t_{1100}+t_{1251}+t_{1395}+$ $t_{1464}+t_{1578}+t_{1586}+t_{1659}+t_{1708}+t_{1710}+t_{1986}+t_{1992}, t_{1541}=t_{104}+t_{1059}, t_{1651}=r_{148}^{\prime}+t_{1541}, t_{2214}=r_{125}^{\prime}+t_{1651}, t_{2229}=$ $t_{2214}+t_{2227}, t_{2230}=t_{1657}+t_{2229}, t_{2231}=t_{2219}+t_{2230}, t_{2235}=t_{2231}+t_{2234}, t_{2236}=t_{2232}+t_{2235}, t_{2237}=t_{2225}+t_{2236}, t_{2238}=$ $t_{2222}+t_{2237}, t_{2}=t_{92}+t_{1635}+t_{2238}, t_{1542}=t_{62}+t_{386}, t_{2267}=t_{555}+t_{1542}, t_{2277}=t_{1732}+t_{2267}, t_{2285}=t_{2277}+t_{2283}, t_{2288}=$ $t_{1725}+t_{2285}, t_{2289}=t_{743}+t_{2288}, t_{1074}=t_{62}+t_{100}, t_{95}=r_{146}^{\prime}+r_{152}^{\prime}+r_{196}^{\prime}+r_{206}^{\prime}+t_{472}+t_{605}+t_{654}+t_{683}+t_{727}+t_{920}+t_{967}+$ $t_{1022}+t_{1026}+t_{1036}+t_{1074}+t_{1092}+t_{1323}+t_{1346}+t_{1416}+t_{1430}+t_{1457}+t_{1613}+t_{1655}+t_{1698}+t_{1724}+t_{2082}+t_{2261}, t_{1527}=$ $t_{95}+t_{1175}, t_{69}=t_{1527}+t_{2263}, t_{1856}=t_{1074}+t_{1259}, t_{1868}=t_{1856}+t_{1865}, t_{1869}=t_{1867}+t_{1868}, t_{17}=t_{1700}+t_{1869}, t_{3}=$ $r_{115}^{\prime}+r_{128}^{\prime}+r_{144}^{\prime}+r_{146}^{\prime}+r_{150}^{\prime}+r_{173}^{\prime}+r_{204}^{\prime}+r_{211}^{\prime}+t_{491}+t_{504}+t_{745}+t_{759}+t_{786}+t_{827}+t_{894}+t_{972}+t_{1044}+t_{1048}+t_{1053}+$ $t_{1085}+t_{1193}+t_{1261}+t_{1277}+t_{1279}+t_{1313}+t_{1409}+t_{1574}+t_{1577}+t_{1584}+t_{1599}+t_{1617}+t_{1682}+t_{1869}, t_{996}=r_{212}^{\prime}+t_{62}, t_{57}=$ $r_{48}^{\prime}+r_{113}^{\prime}+r_{232}^{\prime}+t_{386}+t_{545}+t_{602}+t_{758}+t_{865}+t_{907}+t_{971}+t_{988}+t_{996}+t_{1133}+t_{1158}+t_{1223}+t_{1224}+t_{1227}+t_{1375}+t_{1401}+t_{1468}+$ $t_{1498}+t_{1715}, t_{1633}=t_{667}+t_{996}, t_{1645}=t_{57}+t_{1633}, t_{119}=r_{3}^{\prime}+r_{76}^{\prime}+r_{79}^{\prime}+r_{91}^{\prime}+r_{134}^{\prime}+r_{173}^{\prime}+r_{208}^{\prime}+r_{248}^{\prime}+t_{555}+t_{739}+t_{935}+$ $t_{1011}+t_{1077}+t_{1148}+t_{1152}+t_{1235}+t_{1324}+t_{1338}+t_{1384}+t_{1478}+t_{1602}+t_{1645}+t_{1655}+t_{1667}+t_{1697}+t_{1713}+t_{2217}+t_{2238}, t_{19}=r_{9}^{\prime}+$ $r_{253}^{\prime}+t_{558}+t_{825}+t_{936}+t_{1313}+t_{1355}+t_{1366}+t_{1420}+t_{1475}+t_{1479}+t_{1495}+t_{1565}+t_{1618}+t_{1645}+t_{1656}+t_{1658}+t_{1672}+t_{1693}, t_{885}=$ $t_{19}+t_{57}, t_{93}=r_{6}^{\prime}+r_{45}^{\prime}+r_{174}^{\prime}+t_{485}+t_{488}+t_{560}+t_{857}+t_{885}+t_{906}+t_{945}+t_{1014}+t_{1022}+t_{1117}+t_{1456}+t_{1471}+t_{1532}+t_{1542}+$ $t_{1609}+t_{1617}+t_{1641}+t_{1677}+t_{1719}, t_{1391}=r_{1}^{\prime}+t_{885}, t_{1623}=t_{1066}+t_{1391}, t_{94}=r_{52}^{\prime}+r_{59}^{\prime}+r_{138}^{\prime}+r_{183}^{\prime}+r_{190}^{\prime}+r_{194}^{\prime}+r_{241}^{\prime}+r_{250}^{\prime}+$
$t_{383}+t_{401}+t_{408}+t_{453}+t_{502}+t_{598}+t_{693}+t_{817}+t_{896}+t_{948}+t_{951}+t_{964}+t_{1010}+t_{1016}+t_{1150}+t_{1168}+t_{1309}+t_{1417}+t_{1584}+t_{1623}+$ $t_{1640}+t_{1702}, t_{2282}=t_{1033}+t_{1623}, t_{2287}=t_{2282}+t_{2286}, t_{2290}=t_{2287}+t_{2289}, t_{2292}=t_{2284}+t_{2290}, t_{2293}=t_{2276}+t_{2292}, t_{2294}=$ $t_{2291}+t_{2293}, t_{26}=r_{32}^{\prime}+r_{181}^{\prime}+r_{233}^{\prime}+t_{392}+t_{428}+t_{449}+t_{512}+t_{549}+t_{560}+t_{657}+t_{670}+t_{786}+t_{825}+t_{874}+t_{974}+t_{1009}+t_{1101}+$ $t_{1173}+t_{1231}+t_{1272}+t_{1371}+t_{1403}+t_{1404}+t_{1493}+t_{1540}+t_{1614}+t_{1691}+t_{2270}+t_{2294}, t_{112}=t_{26}+t_{2334}, t_{1353}=t_{976}+t_{1113}, t_{88}=$ $r_{96}^{\prime}+r_{181}^{\prime}+t_{453}+t_{671}+t_{681}+t_{1177}+t_{1353}+t_{1452}+t_{1469}+t_{1489}+t_{1503}+t_{1553}+t_{1604}+t_{1620}+t_{1630}+t_{1651}+t_{1705}, t_{110}=$ $r_{58}^{\prime}+r_{70}^{\prime}+r_{85}^{\prime}+r_{164}^{\prime}+r_{253}^{\prime}+t_{88}+t_{519}+t_{520}+t_{773}+t_{812}+t_{878}+t_{968}+t_{1060}+t_{1070}+t_{1172}+t_{1377}+t_{1405}+t_{1481}+t_{1502}+$ $t_{1704}+t_{1724}+t_{1725}+t_{1954}+t_{1955}+t_{2189}+t_{2190}+t_{2412}+t_{2417}, t_{25}=r_{101}^{\prime}+r_{106}^{\prime}+r_{187}^{\prime}+r_{231}^{\prime}+t_{382}+t_{506}+t_{597}+t_{689}+t_{693}+$ $t_{777}+t_{789}+t_{898}+t_{925}+t_{987}+t_{1005}+t_{1020}+t_{1082}+t_{1190}+t_{1233}+t_{1288}+t_{1349}+t_{1353}+t_{1393}+t_{1394}+t_{1478}+t_{1563}+t_{1574}+$ $t_{1575}+t_{1636}+t_{2148}+t_{2208}+t_{2270}, t_{1439}=t_{540}+t_{749}, t_{2042}=t_{422}+t_{1439}, t_{115}=t_{123}+t_{413}+t_{435}+t_{661}+t_{782}+t_{822}+t_{917}+$ $t_{1074}+t_{1363}+t_{1367}+t_{1536}+t_{1684}+t_{2042}+t_{2049}, t_{2045}=t_{2042}+t_{2044}, t_{2047}=t_{2045}+t_{2046}, t_{2050}=t_{534}+t_{2047}, t_{1319}=$ $t_{29}+t_{878}, t_{2118}=t_{1692}+t_{1944}, t_{2119}=t_{2117}+t_{2118}, t_{2120}=t_{1237}+t_{2119}, t_{2121}=t_{1694}+t_{2120}, t_{105}=t_{1319}+t_{2121}, t_{1025}=$ $t_{105}+t_{562}, t_{16}=t_{385}+t_{394}+t_{470}+t_{531}+t_{840}+t_{959}+t_{1025}+t_{1157}+t_{1174}+t_{1537}+t_{1650}, t_{989}=t_{16}+t_{496}, t_{1217}=$ $t_{544}+t_{989}, t_{1985}=t_{1217}+t_{1984}, t_{1991}=t_{1985}+t_{1987}, t_{1993}=t_{1990}+t_{1991}, t_{83}=t_{451}+t_{493}+t_{512}+t_{527}+t_{989}+t_{1253}+t_{1287}+$ $t_{1398}+t_{1567}+t_{1848}+t_{1988}+t_{1993}, t_{1899}=t_{1099}+t_{1217}, t_{1900}=t_{904}+t_{1899}, t_{1901}=t_{1897}+t_{1900}, t_{1903}=t_{1901}+t_{1902}, t_{1904}=$ $t_{507}+t_{1903}, t_{1905}=t_{1898}+t_{1904}, t_{14}=t_{478}+t_{1513}+t_{1905}, t_{1408}=t_{14}+t_{629}, t_{84}=t_{16}+t_{403}+t_{458}+t_{507}+t_{689}+t_{775}+t_{784}+$ $t_{839}+t_{855}+t_{979}+t_{1408}+t_{1549}+t_{1717}+t_{2187}, t_{51}=r_{132}^{\prime}+r_{172}^{\prime}+r_{220}^{\prime}+t_{84}+t_{456}+t_{470}+t_{580}+t_{600}+t_{601}+t_{671}+t_{674}+t_{860}+$ $t_{937}+t_{957}+t_{964}+t_{1195}+t_{1206}+t_{1274}+t_{1346}+t_{1434}+t_{1448}+t_{1456}+t_{1580}+t_{1586}+t_{1621}+t_{1631}+t_{1648}+t_{1692}+t_{1704}+t_{2154}, t_{805}=$ $t_{84}+t_{562}, t_{1490}=t_{805}+t_{1156}, t_{1166}=t_{405}+t_{805}, t_{2299}=t_{1166}+t_{2296}, t_{2300}=t_{561}+t_{2299}, t_{2301}=t_{2298}+t_{2300}, t_{2302}=$ $t_{796}+t_{2301}, t_{2303}=t_{1287}+t_{2302}, t_{2304}=t_{402}+t_{2303}, t_{2305}=t_{2297}+t_{2304}, t_{38}=t_{487}+t_{2305}, t_{1506}=t_{38}+t_{1490}, t_{7}=$ $t_{1506}+t_{1833}, t_{68}=t_{7}+t_{584}+t_{717}+t_{870}+t_{894}+t_{896}+t_{928}+t_{1002}+t_{1201}+t_{1202}+t_{1285}+t_{1288}+t_{1337}+t_{1398}+t_{1447}+t_{1449}+$ $t_{1510}+t_{1612}+t_{1627}+t_{2296}, t_{2207}=t_{1166}+t_{2205}, t_{2211}=t_{2207}+t_{2209}, t_{2213}=t_{2210}+t_{2211}, t_{47}=t_{29}+t_{89}+t_{105}+t_{428}+t_{438}+$ $t_{493}+t_{501}+t_{1015}+t_{1155}+t_{1460}+t_{1505}+t_{1513}+t_{1592}+t_{1722}+t_{2208}+t_{2212}+t_{2213}, t_{15}=t_{575}+t_{1543}+t_{1555}+t_{2213}, t_{81}=$ $t_{15}+t_{824}+t_{878}+t_{898}+t_{975}+t_{991}+t_{1253}+t_{1467}+t_{1717}+t_{2305}, t_{11}=r_{0}^{\prime}+r_{86}^{\prime}+r_{200}^{\prime}+t_{81}+t_{417}+t_{515}+t_{667}+t_{702}+t_{706}+$ $t_{730}+t_{881}+t_{888}+t_{909}+t_{926}+t_{983}+t_{1059}+t_{1071}+t_{1101}+t_{1121}+t_{1139}+t_{1195}+t_{1271}+t_{1376}+t_{1466}+t_{1577}+t_{1727}+t_{1732}, t_{34}=$ $r_{193}^{\prime}+t_{15}+t_{405}+t_{498}+t_{567}+t_{622}+t_{655}+t_{908}+t_{922}+t_{1039}+t_{1068}+t_{1096}+t_{1129}+t_{1151}+t_{1242}+t_{1280}+t_{1338}+t_{1422}+$ $t_{1488}+t_{1530}+t_{1578}+t_{1658}+t_{2189}, t_{52}=r_{49}^{\prime}+r_{98}^{\prime}+r_{100}^{\prime}+t_{34}+t_{387}+t_{434}+t_{597}+t_{661}+t_{721}+t_{750}+t_{856}+t_{955}+t_{1047}+$ $t_{1162}+t_{1212}+t_{1220}+t_{1273}+t_{1330}+t_{1368}+t_{1370}+t_{1405}+t_{1438}+t_{1464}+t_{1600}+t_{1637}+t_{1731}+t_{1893}+t_{1998}, t_{1907}=$ $t_{52}+t_{1043}, t_{1921}=t_{916}+t_{1907}, t_{1923}=t_{1909}+t_{1921}, t_{1924}=t_{1920}+t_{1923}, t_{1927}=t_{1922}+t_{1924}, t_{1929}=t_{1927}+t_{1928}, t_{70}=$ $r_{8}^{\prime}+r_{21}^{\prime}+r_{40}^{\prime}+r_{118}^{\prime}+r_{127}^{\prime}+r_{172}^{\prime}+t_{525}+t_{675}+t_{687}+t_{752}+t_{866}+t_{977}+t_{1118}+t_{1159}+t_{1240}+t_{1301}+t_{1309}+t_{1563}+t_{1583}+$ $t_{1599}+t_{1698}+t_{1929}, t_{9}=t_{1716}+t_{1929}, t_{1191}=t_{9}+t_{1153}, t_{35}=r_{16}^{\prime}+r_{30}^{\prime}+r_{41}^{\prime}+r_{86}^{\prime}+r_{102}^{\prime}+r_{110}^{\prime}+t_{457}+t_{648}+t_{677}+t_{723}+$ $t_{889}+t_{1100}+t_{1179}+t_{1191}+t_{1425}+t_{1484}+t_{1548}+t_{1622}+t_{1638}+t_{1660}+t_{1671}+t_{1695}+t_{2217}, t_{1538}=t_{35}+t_{1378}, t_{109}=$ $r_{7}^{\prime}+r_{152}^{\prime}+r_{159}^{\prime}+r_{202}^{\prime}+t_{462}+t_{642}+t_{679}+t_{704}+t_{754}+t_{759}+t_{811}+t_{947}+t_{1083}+t_{1207}+t_{1221}+t_{1230}+t_{1240}+t_{1265}+t_{1275}+t_{1352}+$ $t_{1412}+t_{1423}+t_{1488}+t_{1510}+t_{1538}+t_{1576}+t_{1579}+t_{1664}+t_{1667}+t_{1685}, t_{985}=r_{69}^{\prime}+t_{35}, t_{44}=r_{24}^{\prime}+r_{27}^{\prime}+r_{72}^{\prime}+r_{98}^{\prime}+r_{117}^{\prime}+r_{176}^{\prime}+$ $r_{190}^{\prime}+t_{109}+t_{422}+t_{738}+t_{849}+t_{894}+t_{908}+t_{937}+t_{985}+t_{1032}+t_{1033}+t_{1041}+t_{1128}+t_{1137}+t_{1152}+t_{1278}+t_{1342}+t_{1411}+t_{1430}+$ $t_{1544}+t_{1556}+t_{1581}+t_{1636}+t_{1727}, t_{2362}=t_{865}+t_{985}, t_{2365}=t_{2362}+t_{2364}, t_{2366}=t_{627}+t_{2365}, t_{2376}=t_{2167}+t_{2366}, t_{2378}=$ $t_{2376}+t_{2377}, t_{2380}=t_{2378}+t_{2379}, t_{2381}=t_{2371}+t_{2380}, t_{79}=t_{1057}+t_{2381}, t_{1291}=t_{79}+t_{1210}, t_{1709}=t_{402}+t_{1291}, t_{117}=$ $r_{222}^{\prime}+t_{437}+t_{483}+t_{571}+t_{644}+t_{684}+t_{726}+t_{759}+t_{761}+t_{819}+t_{877}+t_{970}+t_{1023}+t_{1145}+t_{1162}+t_{1178}+t_{1250}+t_{1268}+t_{1305}+$ $t_{1424}+t_{1494}+t_{1553}+t_{1675}+t_{1680}+t_{1699}+t_{1709}+t_{1711}, t_{43}=r_{47}^{\prime}+r_{90}^{\prime}+r_{105}^{\prime}+r_{123}^{\prime}+r_{188}^{\prime}+r_{253}^{\prime}+t_{440}+t_{453}+t_{499}+t_{533}+t_{623}+$ $t_{901}+t_{1062}+t_{1115}+t_{1174}+t_{1225}+t_{1263}+t_{1336}+t_{1425}+t_{1480}+t_{1497}+t_{1538}+t_{1561}+t_{1647}+t_{1690}+t_{1709}+t_{1731}+t_{2263}, t_{42}=$ $r_{69}^{\prime}+r_{135}^{\prime}+r_{141}^{\prime}+r_{147}^{\prime}+r_{228}^{\prime}+r_{243}^{\prime}+t_{384}+t_{492}+t_{559}+t_{595}+t_{740}+t_{757}+t_{832}+t_{866}+t_{894}+t_{924}+t_{938}+t_{974}+t_{1127}+t_{1134}+t_{1189}+$ $t_{1234}+t_{1299}+t_{1304}+t_{1349}+t_{1440}+t_{1454}+t_{1502}+t_{1673}+t_{1675}+t_{1676}+t_{2381}, t_{2077}=t_{853}+t_{985}, t_{2076}=t_{1191}+t_{1669}, t_{2092}=$ $t_{2076}+t_{2086}, t_{2096}=t_{2090}+t_{2092}, t_{2098}=t_{2073}+t_{2096}, t_{2099}=t_{1283}+t_{2098}, t_{2100}=t_{2077}+t_{2099}, t_{2101}=t_{2097}+t_{2100}, t_{2102}=$ $t_{2089}+t_{2101}, t_{41}=r_{113}^{\prime}+r_{240}^{\prime}+t_{403}+t_{480}+t_{487}+t_{521}+t_{626}+t_{651}+t_{944}+t_{1011}+t_{1030}+t_{1061}+t_{1129}+t_{1160}+t_{1277}+t_{1304}+$ $t_{1325}+t_{1371}+t_{1415}+t_{1420}+t_{1482}+t_{1520}+t_{1525}+t_{1553}+t_{1665}+t_{1732}+t_{2102}, t_{1}=r_{111}^{\prime}+t_{2102}, t_{45}=t_{427}+t_{439}+t_{541}+$ $t_{873}+t_{925}+t_{1045}+t_{1046}+t_{1156}+t_{1408}+t_{1468}+t_{1524}+t_{1627}+t_{1634}+t_{1712}, t_{118}=r_{23}^{\prime}+r_{68}^{\prime}+r_{127}^{\prime}+r_{167}^{\prime}+r_{217}^{\prime}+r_{229}^{\prime}+t_{45}+$ $t_{94}+t_{381}+t_{400}+t_{713}+t_{811}+t_{932}+t_{1021}+t_{1037}+t_{1071}+t_{1094}+t_{1188}+t_{1232}+t_{1270}+t_{1370}+t_{1381}+t_{1384}+t_{1389}+t_{1469}+$ $t_{1533}+t_{1546}+t_{1612}+t_{1641}+t_{1711}+t_{1721}+t_{1895}+t_{1905}, t_{1284}=t_{118}+t_{570}, t_{2341}=t_{1284}+t_{1588}, t_{2343}=t_{1648}+t_{2341}, t_{2357}=$ $t_{2343}+t_{2356}, t_{2358}=t_{2353}+t_{2357}, t_{2360}=t_{2358}+t_{2359}, t_{33}=t_{615}+t_{2360}, t_{1294}=r_{204}^{\prime}+t_{33}, t_{1646}=t_{1294}+t_{1514}, t_{50}=$ $r_{66}^{\prime}+r_{82}^{\prime}+r_{124}^{\prime}+r_{186}^{\prime}+t_{37}+t_{82}+t_{390}+t_{493}+t_{588}+t_{596}+t_{710}+t_{713}+t_{885}+t_{988}+t_{1003}+t_{1049}+t_{1057}+t_{1067}+t_{1084}+t_{1095}+$ $t_{1177}+t_{1260}+t_{1284}+t_{1368}+t_{1388}+t_{1444}+t_{1477}+t_{1548}+t_{1646}, t_{27}=r_{46}^{\prime}+r_{85}^{\prime}+r_{115}^{\prime}+r_{138}^{\prime}+r_{209}^{\prime}+t_{702}+t_{741}+t_{761}+t_{782}+t_{979}+$ $t_{1020}+t_{1032}+t_{1073}+t_{1102}+t_{1148}+t_{1274}+t_{1314}+t_{1374}+t_{1443}+t_{1487}+t_{1562}+t_{1576}+t_{1646}+t_{1652}+t_{1677}+t_{1715}+t_{1833}, t_{28}=$ $r_{5}^{\prime}+r_{104}^{\prime}+r_{110}^{\prime}+r_{210}^{\prime}+r_{244}^{\prime}+t_{461}+t_{464}+t_{494}+t_{565}+t_{626}+t_{748}+t_{856}+t_{921}+t_{961}+t_{962}+t_{1033}+t_{1263}+t_{1301}+t_{1308}+t_{1386}+$ $t_{1459}+t_{1460}+t_{1496}+t_{1654}+t_{1689}+t_{1691}+t_{1702}+t_{2360}, t_{1437}=t_{108}+t_{1025}, t_{1701}=t_{440}+t_{1437}, t_{13}=t_{980}+t_{1701}+t_{1993}, t_{48}=$ $t_{13}+t_{2115}, t_{73}=t_{48}+t_{411}+t_{473}+t_{500}+t_{541}+t_{766}+t_{959}+t_{991}+t_{999}+t_{1079}+t_{1282}+t_{1668}+t_{1753}+t_{2212}, t_{1726}=t_{73}+t_{500}, t_{4}=$ $t_{1204}+t_{1726}+t_{2294}, t_{124}=t_{24}+t_{448}+t_{490}+t_{543}+t_{553}+t_{607}+t_{1027}+t_{1061}+t_{1285}+t_{1518}+t_{1596}+t_{1597}+t_{1687}+t_{2121}, t_{111}=$ $r_{1}^{\prime}+r_{199}^{\prime}+t_{124}+t_{410}+t_{629}+t_{652}+t_{767}+t_{829}+t_{847}+t_{944}+t_{1048}+t_{1134}+t_{1188}+t_{1206}+t_{1265}+t_{1271}+t_{1394}+t_{1433}+t_{1455}+t_{1543}+$ $t_{1587}+t_{1593}+t_{1605}+t_{1669}+t_{1723}+t_{2152}, t_{943}=t_{124}+t_{494}, t_{2043}=t_{943}+t_{1499}, t_{2048}=t_{1111}+t_{2043}, t_{2051}=t_{2048}+t_{2050}, t_{46}=$ $t_{15}+t_{124}+t_{600}+t_{625}+t_{685}+t_{805}+t_{822}+t_{871}+t_{995}+t_{1208}+t_{1328}+t_{1426}+t_{1549}+t_{1753}+t_{2051}, t_{116}=t_{501}+t_{719}+t_{757}+$
$t_{2051}, t_{65}=r_{66}^{\prime}+t_{465}+t_{475}+t_{511}+t_{573}+t_{607}+t_{700}+t_{854}+t_{887}+t_{943}+t_{1048}+t_{1126}+t_{1668}+t_{1671}+t_{2194}+t_{2204}, t_{1211}=$ $r_{215}^{\prime}+t_{943}, t_{1345}=t_{116}+t_{1211}, t_{1629}=t_{27}+t_{1345}, t_{49}=t_{388}+t_{1629}+t_{1806}, p_{186}=t_{119}+t_{121}, p_{185}=t_{120}, p_{184}=$ $t_{121}+t_{123}, p_{183}=t_{124}, p_{182}=t_{117}+t_{119}, p_{181}=t_{118}+t_{120}, p_{180}=t_{123}, p_{179}=t_{122}+t_{124}, p_{178}=t_{109}+t_{113}, p_{177}=$ $t_{110}, p_{176}=t_{113}, p_{175}=t_{114}, p_{174}=t_{109}+t_{111}, p_{173}=t_{110}+t_{112}, p_{172}=t_{115}, p_{171}=t_{114}+t_{116}, p_{170}=t_{103}+t_{105}, p_{169}=$ $t_{104}, p_{168}=t_{104}, p_{167}=t_{105}+t_{107}+t_{108}, p_{166}=t_{108}, p_{164}=t_{102}+t_{104}, p_{165}=t_{101}+t_{103}+p_{164}, p_{162}=t_{106}+t_{108}, p_{163}=$ $t_{107}+p_{162}, p_{161}=t_{93}+t_{97}, p_{160}=t_{94}, p_{159}=t_{97}, p_{158}=t_{98}, p_{157}=t_{93}+t_{95}, p_{156}=t_{94}+t_{96}, p_{155}=t_{99}, p_{154}=$ $t_{98}+t_{100}, p_{153}=t_{85}+t_{89}, p_{152}=t_{86}, p_{151}=t_{86}, p_{150}=t_{89}+t_{90}, p_{149}=t_{90}, p_{147}=t_{86}+t_{88}, p_{148}=t_{85}+t_{87}+p_{147}, p_{145}=$ $t_{90}+t_{92}, p_{146}=t_{91}+p_{145}, p_{144}=t_{79}+t_{81}, p_{143}=t_{80}, p_{142}=t_{80}, p_{141}=t_{81}+t_{83}+t_{84}, p_{140}=t_{84}, p_{138}=t_{78}+t_{80}, p_{139}=$ $t_{77}+t_{79}+p_{138}, p_{136}=t_{82}+t_{84}, p_{137}=t_{83}+p_{136}, p_{135}=t_{69}+t_{73}, p_{134}=t_{70}, p_{133}=t_{70}, p_{132}=t_{73}+t_{74}, p_{131}=$ $t_{74}, p_{129}=t_{70}+t_{72}, p_{130}=t_{69}+t_{71}+p_{129}, p_{127}=t_{74}+t_{76}, p_{128}=t_{75}+p_{127}, p_{126}=t_{65}, p_{125}=t_{68}, p_{124}=t_{66}, p_{123}=$ $t_{66}, p_{122}=t_{67}, p_{121}=t_{57}+t_{61}, p_{120}=t_{58}, p_{119}=t_{58}, p_{118}=t_{61}, p_{117}=t_{61}+t_{62}, p_{116}=t_{62}, p_{115}=t_{57}+t_{59}, p_{113}=$ $t_{58}+t_{60}, p_{114}=p_{113}+p_{115}, p_{112}=t_{63}, p_{110}=t_{62}+t_{64}, p_{111}=t_{63}+p_{110}, p_{109}=t_{49}+t_{53}, p_{108}=t_{50}, p_{107}=t_{50}, p_{106}=$ $t_{53}, p_{105}=t_{53}+t_{54}, p_{104}=t_{54}, p_{103}=t_{49}+t_{51}, p_{101}=t_{50}+t_{52}, p_{102}=p_{101}+p_{103}, p_{100}=t_{55}, p_{98}=t_{54}+t_{56}, p_{99}=$ $t_{55}+p_{98}, p_{97}=t_{43}+t_{45}, p_{96}=t_{44}, p_{95}=t_{44}, p_{94}=t_{45}+t_{47}, p_{93}=t_{48}+p_{94}, p_{92}=t_{48}, p_{91}=t_{41}+t_{43}, p_{89}=$ $t_{42}+t_{44}, p_{90}=p_{89}+p_{91}, p_{88}=t_{47}, p_{86}=t_{46}+t_{48}, p_{87}=t_{47}+p_{86}, p_{85}=t_{33}+t_{37}, p_{84}=t_{34}, p_{83}=t_{34}, p_{82}=t_{37}, p_{81}=$ $t_{37}+t_{38}, p_{80}=t_{38}, p_{79}=t_{33}+t_{35}, p_{77}=t_{34}+t_{36}, p_{78}=p_{77}+p_{79}, p_{76}=t_{39}, p_{74}=t_{38}+t_{40}, p_{75}=t_{39}+p_{74}, p_{73}=$ $t_{25}+t_{29}, p_{72}=t_{26}, p_{71}=t_{26}, p_{70}=t_{27}+t_{29}, p_{69}=t_{28}, p_{68}=t_{28}, p_{67}=t_{29}+t_{31}+t_{32}, p_{66}=t_{32}, p_{65}=t_{29}, p_{64}=$ $t_{29}+t_{30}, p_{63}=t_{30}, p_{62}=t_{25}+t_{27}, p_{60}=t_{26}+t_{28}, p_{61}=p_{60}+p_{62}, p_{59}=t_{31}, p_{57}=t_{30}+t_{32}, p_{58}=t_{31}+p_{57}, p_{56}=$ $t_{17}+t_{21}, p_{55}=t_{18}, p_{54}=t_{19}+t_{21}, p_{53}=t_{20}, p_{52}=t_{20}, p_{51}=t_{21}+t_{23}, p_{50}=t_{24}+p_{51}, p_{49}=t_{24}, p_{48}=t_{21}, p_{47}=t_{22}, p_{46}=$ $t_{17}+t_{19}, p_{44}=t_{18}+t_{20}, p_{45}=p_{44}+p_{46}, p_{43}=t_{23}, p_{41}=t_{22}+t_{24}, p_{42}=t_{23}+p_{41}, p_{40}=t_{9}+t_{13}, p_{39}=t_{10}, p_{38}=$ $t_{10}, p_{37}=t_{11}+t_{13}, p_{36}=t_{12}, p_{35}=t_{12}, p_{34}=t_{13}+t_{15}, p_{33}=t_{16}+p_{34}, p_{32}=t_{16}, p_{31}=t_{13}, p_{30}=t_{13}+t_{14}, p_{29}=$ $t_{14}, p_{28}=t_{9}+t_{11}, p_{26}=t_{10}+t_{12}, p_{27}=p_{26}+p_{28}, p_{25}=t_{15}, p_{23}=t_{14}+t_{16}, p_{24}=t_{15}+p_{23}, p_{22}=t_{3}+t_{7}, p_{21}=$ $t_{4}+t_{8}, p_{20}=t_{1}+t_{5}, p_{19}=t_{2}, p_{18}=t_{2}, p_{17}=t_{3}+t_{5}, p_{16}=t_{4}, p_{15}=t_{4}, p_{14}=t_{5}+t_{7}, p_{13}=t_{8}+p_{14}, p_{12}=t_{8}, p_{11}=$ $t_{5}, p_{10}=t_{5}+t_{6}, p_{9}=t_{6}, p_{8}=t_{7}, p_{7}=t_{6}+t_{8}, p_{2}=t_{7}+p_{7}, p_{1}=p_{7}, p_{6}=t_{1}+t_{3}, p_{4}=t_{2}+t_{4}, p_{5}=p_{4}+p_{6}, p_{3}=t_{7}, p_{0}=t_{0}$.

Pointwise multiplication (149 multiplications): $\boldsymbol{g}=\boldsymbol{p} \cdot \boldsymbol{c}$, where $\boldsymbol{c}=\left(1, \alpha^{5}, \alpha^{143}, \alpha^{199}, \alpha^{204}, \alpha^{136}, \alpha^{68}, \alpha^{204}, \alpha^{68}, \alpha^{38}\right.$, $\alpha^{222}, \alpha^{111}, \alpha^{137}, \alpha^{183}, \alpha^{219}, \alpha^{170}, 1,1, \alpha^{170}, 1,1, \alpha^{170}, 1, \alpha^{5}, \alpha^{143}, \alpha^{199}, \alpha^{204}, \alpha^{136}, \alpha^{68}, \alpha^{38}, \alpha^{222}, \alpha^{111}, \alpha^{137}, \alpha^{183}$, $\alpha^{219}, \alpha^{170}, 1,1, \alpha^{170}, 1,1, \alpha^{5}, \alpha^{143}, \alpha^{199}, \alpha^{204}, \alpha^{136}, \alpha^{68}, \alpha^{38}, \alpha^{1}, \alpha^{11}, \alpha^{137}, \alpha^{183}, \alpha^{219}, \alpha^{170}, 1,1, \alpha^{170}, 1, \alpha^{5}, \alpha^{143}$, $\alpha^{199}, \alpha^{204}, \alpha^{136}, \alpha^{68}, \alpha^{38}, \alpha^{222}, \alpha^{111}, \alpha^{137}, \alpha^{183}, \alpha^{170}, 1,1, \alpha^{170}, 1,1, \alpha^{5}, \alpha^{143}, \alpha^{199}, \alpha^{204}, \alpha^{136}, \alpha^{68}, \alpha^{38}, \alpha^{222}, \alpha^{111}$, $\alpha^{170}, 1,1, \alpha^{5}, \alpha^{143}, \alpha^{199}, \alpha^{204}, \alpha^{136}, \alpha^{68}, \alpha^{137}, \alpha^{183}, \alpha^{219}, \alpha^{17}, 1,1,1, \alpha^{5}, \alpha^{143}, \alpha^{199}, \alpha^{204}, \alpha^{136}, \alpha^{68}, \alpha^{38}, \alpha^{222}, \alpha^{111}$, $\alpha^{170}, 1,1, \alpha^{5}, \alpha^{143}, \alpha^{199}, \alpha^{204}, \alpha^{136}, \alpha^{68}, \alpha^{38}, \alpha^{222}, \alpha^{111}, \alpha^{17}, 1,1,1, \alpha^{153}, \alpha^{170}, 1, \alpha^{68}, 1, \alpha^{5}, \alpha^{143}, \alpha^{204}, \alpha^{136}, \alpha^{38}$, $\alpha^{222}, \alpha^{170}, 1,1, \alpha^{5}, \alpha^{143}, \alpha^{204}, \alpha^{136}, \alpha^{137}, \alpha^{183}, \alpha^{170}, 1,1, \alpha^{5}, \alpha^{143}, \alpha^{204}, \alpha^{136}, \alpha^{38}, \alpha^{222}, \alpha^{170}, 1,1, \alpha^{5}, \alpha^{199}, \alpha^{204}$, $\alpha^{68}, \alpha^{38}, \alpha^{111}, \alpha^{170}, 1, \alpha^{5}, \alpha^{143}, \alpha^{204}, \alpha^{136}, \alpha^{137}, \alpha^{183}, \alpha^{170}, 1,1, \alpha^{5}, \alpha^{199}, \alpha^{204}, \alpha^{68}, \alpha^{38}, \alpha^{111}, \alpha^{170}, 1, \alpha^{5}, \alpha^{199}, \alpha^{204}$, $\left.\alpha^{68}, \alpha^{137}, \alpha^{219}, \alpha^{170}, 1\right)$.

Post-additions (177 additions): $\boldsymbol{S}^{\prime}=\boldsymbol{P}^{T} \boldsymbol{g}$. Note that in the following sequence we use $\boldsymbol{S}$ directly to avoid extra permutation.
$t_{273}=g_{18}+g_{20}, t_{269}=g_{110}+g_{116}, t_{270}=g_{113}+t_{269}, t_{271}=g_{119}+t_{270}, t_{272}=g_{121}+t_{271}, S_{30}=g_{111}+g_{114}+$ $g_{117}+g_{120}+t_{272}, t_{266}=g_{52}+g_{54}, t_{267}=g_{49}+t_{266}, t_{264}=g_{92}+g_{95}, t_{262}=g_{89}+g_{97}, t_{263}=g_{86}+t_{262}, t_{265}=$ $t_{263}+t_{264}, S_{22}=g_{87}+g_{90}+g_{93}+g_{96}+t_{265}, t_{260}=g_{32}+g_{37}, t_{256}=g_{77}+g_{80}, t_{255}=g_{74}+g_{83}, t_{257}=g_{85}+t_{255}, t_{258}=$ $t_{256}+t_{257}, S_{17}=g_{75}+g_{78}+g_{81}+g_{84}+t_{258}, t_{248}=g_{98}+g_{104}, t_{249}=g_{107}+t_{248}, t_{250}=g_{109}+t_{249}, t_{251}=g_{101}+$ $t_{250}, S_{26}=g_{99}+g_{102}+g_{105}+g_{108}+t_{251}, t_{246}=g_{63}+g_{73}, t_{247}=g_{71}+t_{246}, t_{244}=g_{2}+g_{5}, t_{241}=g_{41}+g_{44}, t_{268}=$ $t_{241}+t_{267}, S_{9}=g_{42}+g_{45}+g_{50}+g_{53}+t_{268}, t_{240}=g_{15}+g_{17}, t_{237}=g_{57}+g_{60}, t_{236}=g_{100}+g_{106}, t_{242}=g_{103}+t_{236}, S_{13}=$ $t_{242}+t_{251}, t_{233}=g_{1}+g_{4}, t_{276}=g_{12}+t_{233}, t_{277}=t_{240}+t_{276}, S_{2}=g_{13}+g_{16}+t_{244}+t_{277}, t_{274}=g_{9}+t_{233}, t_{275}=$ $t_{273}+t_{274}, S_{8}=g_{10}+g_{19}+t_{244}+t_{275}, t_{230}=g_{58}+g_{61}, t_{245}=t_{230}+t_{237}, S_{13}=g_{64}+g_{72}+t_{245}+t_{247}, S_{11}=$ $g_{66}+g_{67}+g_{68}+g_{69}+g_{70}+t_{245}, t_{226}=g_{91}+g_{94}, t_{232}=g_{88}+t_{226}, S_{11}=t_{232}+t_{265}, t_{225}=g_{59}+g_{62}, t_{235}=$ $g_{65}+t_{225}, S_{22}=t_{235}+t_{237}+t_{247}, t_{223}=g_{112}+g_{118}, t_{239}=g_{115}+t_{223}, S_{15}=t_{239}+t_{272}, t_{222}=g_{76}+g_{82}, t_{224}=$ $g_{79}+t_{222}, S_{26}=t_{224}+t_{258}, t_{221}=g_{43}+g_{46}, S_{18}=g_{47}+g_{48}+g_{55}+g_{56}+t_{221}+t_{241}, t_{229}=g_{51}+t_{221}, S_{5}=t_{229}+t_{268}, t_{220}=$ $g_{3}+g_{6}, t_{234}=g_{14}+t_{220}, S_{1}=t_{234}+t_{277}, S_{16}=g_{7}+g_{8}+g_{21}+g_{22}+S_{1}+t_{240}, t_{231}=g_{11}+t_{220}, S_{4}=t_{231}+t_{275}, t_{219}=$ $g_{25}+g_{28}, t_{228}=g_{27}+t_{219}, t_{238}=g_{24}+t_{228}, t_{227}=g_{23}+t_{219}, t_{243}=g_{26}+t_{227}, t_{259}=g_{35}+t_{243}, t_{261}=t_{259}+t_{260}, S_{6}=$ $g_{33}+g_{36}+t_{238}+t_{261}, S_{3}=g_{34}+t_{261}, t_{252}=g_{29}+t_{243}, t_{253}=g_{38}+t_{252}, t_{254}=g_{40}+t_{253}, S_{24}=g_{30}+g_{39}+t_{238}+t_{254}, S_{12}=$ $g_{31}+t_{254}, S_{31}=g_{179}+g_{180}+g_{181}+g_{182}+g_{183}+g_{184}+g_{185}+g_{186}, S_{29}=g_{171}+g_{172}+g_{173}+g_{174}+g_{175}+g_{176}+g_{177}+g_{178}, S_{27}=$ $g_{162}+g_{163}+g_{164}+g_{165}+g_{166}+g_{167}+g_{168}+g_{169}+g_{170}, S_{25}=g_{154}+g_{155}+g_{156}+g_{157}+g_{158}+g_{159}+g_{160}+g_{161}, S_{23}=$ $g_{145}+g_{146}+g_{147}+g_{148}+g_{149}+g_{150}+g_{151}+g_{152}+g_{153}, S_{21}=g_{136}+g_{137}+g_{138}+g_{139}+g_{140}+g_{141}+g_{142}+g_{143}+g_{144}, S_{19}=$ $g_{127}+g_{128}+g_{129}+g_{130}+g_{131}+g_{132}+g_{133}+g_{134}+g_{135}, S_{17}=g_{122}+g_{123}+g_{124}+g_{125}+g_{126}, S_{0}=g_{0}$.

Overall 149 multiplications and 3970 additions over $\operatorname{GF}\left(2^{8}\right)$ are needed.

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