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Decentralized TDOA Sensor Pairing in Multihop Wireless Sensor Networks

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***Abstract*—This letter is concerned with source localization based on time-difference-of-arrival (TDOA) measurements from spatially separated sensors in a wireless sensor network (WSN). Most of the existing works adopt a centralized sensor pairing strategy, where one sensor node is chosen as the common reference. However, due to the bandwidth and power constraints of multihop WSNs, it is well known that this kind of centralized methods is energy consuming due to the need of single and multihop transmissions of raw measurement data. In this letter, we propose a decentralized in-network sensor pairing method to acquire TDOA measurements for source localization. It is proved that the proposed decentralized in-network sensor pairing method can result in the same Cramer–Rao-Bound (CRB) as the centralized one at a far less communication cost.**

***Index Terms*—Dominating set, Fisher information matrix (FIM), sensor pairing, source localization, TDOA, wireless sensor network.**

I. Introduction

SOURCE localization based on measurements from spatially separated sensors is an important application of wireless sensor networks (WSNs) [1], [2]. The time-difference-of-arrival (TDOA) based source localization method can have a high accuracy and has been studied in many existing works, see e.g., [3]–[6]. For example, a non-iterative closed-form algorithm for 2-D geolocation is presented by Chan and Ho in [3]. A low-complexity weighted least-squares solution with a linear sensor array is proposed in [6]. While extensive research has been focused on algorithm development, limited attention has been paid to the problem of gathering TDOA measurements, especially in multihop WSNs, which is the main concern of this work.

Traditionally, TDOA measurements are collected in a centralized way which we call centralized sensor pairing, where one sensor node is chosen as the reference node and the other nodes broadcast their raw measurements to this reference node. However, due to the limited communication distance, most of the nodes can not communicate with the reference node directly, and much energy is consumed in multihop transmissions of the raw measurements. Hence, in practice, a new TDOA sensor pair information collection method with low requirements for both network bandwidth and power consumption is highly desirable.

It is well known that decentralized in-network signal processing and aggregation is more energy efficient than its centralized counterpart [7], [8]. In this letter, we propose a decentralized in-network sensor pairing method to collect TDOA measurements. The basic idea is to find a connected dominating set (CDS) for the graph of a muti-hop sensor network. A CDS of a sensor network is defined as a subset of nodes, called 'relay nodes' which form a connected network, such that any node in the original network is either a member of the CDS or is within the transmission range of at least one node in the CDS [9]. In this work, we apply the CDS to our TDOA sensor pairing problem. The nodes in the dominating set in our scheme work more like 'transceivers' than just 'relay nodes' in the literature. It is proved that the proposed decentralized method results in the same Cramer–Rao-Bound (CRB) as the centralized one at a far less communication cost. Furthermore, the network topology of the proposed sensor pairing method can be directly applied to the decentralized source localization and tracking problems.

The remainder of the paper is organized as follows. The problem formulation is stated in Section II. The proposed decentralized in-network sensor pairing method with performance analysis is presented in Section III. Section IV presents an application of the proposed sensor pairing method to decentralized source localization. Section V concludes the paper.

II. Problem Formulation

We consider a multihop WSN. Given a team of N nodes performing a source localization task. Each sensor pair can estimate the TDOA between them by using the generalized cross correlation method (GCC) [10]. A TDOA measurement by a sensor pair $\{i, j\}$ can be written as:

$$\hat{t}_{ij} = \frac{d_{ij}}{v} + e_{ij}, \tag{1}$$

where $d_{ij} = \|\mathbf{x}_i - \mathbf{x}_t\| - \|\mathbf{x}_j - \mathbf{x}_t\|$, $\mathbf{x}_i = [x_i, y_i]^T$ denotes the coordinates of sensor i, $\mathbf{x}_t = [x_t, y_t]^T$ is the source's coordinates, v is the propagation speed of the source signal. $e_{ij} \sim N(0, \sigma^2)$ is the TDOA estimation error where for simplicity we assume the variances of all TDOA measurements are equal.

The cross covariance between $\hat{t}_{ij}$ and $\hat{t}_{kl}$ denoted by $\mathrm{cov}(\hat{t}_{ij}, \hat{t}_{kl})$ is [3]

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$$\operatorname{cov}(\hat{t}_{ij}, \hat{t}_{kl}) = \begin{cases} \frac{1}{2}\sigma^2, & i = k \text{ and } j \neq l \text{ or } i \neq k \text{ and } j = l \\ -\frac{1}{2}\sigma^2, & i = l \text{ and } j \neq k \text{ or } i \neq l \text{ and } j = k \\ 0, & \text{otherwise} . \end{cases} \quad (2)$$

For N sensors, there is a total number of $N(N-1)/2$ possible TDOA sensor pairs. Let

$$\mathcal{I} = \{\{i, j\} | 1 \leq i < j \leq N\} \quad (3)$$

denote the set of all sensor pairs. $\mathcal{I}'$ is a subset of $\mathcal{I}$ and contains K ($K \leq N(N-1)/2$) sensor pairs whose TDOAs are used in source localization. By introducing the $K \times 1$ vectors

$$\hat{\mathbf{t}} = \begin{bmatrix} \hat{t}_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \mathbf{d} = \begin{bmatrix} d_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \mathbf{e} = \begin{bmatrix} e_{ij} \\ \vdots \end{bmatrix}_{\{i,j\} \in \mathcal{I}'}, \quad (4)$$

the measurement model in matrix form becomes

$$\hat{\mathbf{t}} = \frac{1}{v}\mathbf{d} + \mathbf{e}. \quad (5)$$

The problem of source localization is to estimate the source location $\mathbf{x}_t$ given the measurement vector $\hat{\mathbf{t}}$ and the sensor location $\mathbf{x}_i$, $i = 1, \ldots, N$.

Most of the existing research works focus on algorithm development for source localization. Differently, in this letter, we will study how to form TDOA sensor pairs or how to collect TDOA measurements in a bandwidth and power limited multihop sensor network for source localization, which is still an open problem.

III. TDOA Sensor Pairing in Multihop Networks

A. Network Model

Let us represent a multihop WSN as a graph defined by $\mathcal{G} := (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the node set, $\mathcal{N} := \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the edge set. If node k and node l can communicate with each other directly, we define the link by $(k, l) \in \mathcal{E}$.

Several definitions from graph theory are used in this paper.

Definition 1: Maximum Degree is the maximum count of edges emanating from a single node.

Definition 2: Dominating Set S is defined as a subset of $\mathcal{N}$ such that each node in $\mathcal{N} - S$ is adjacent to at least one node in S.

Definition 3: Connected Dominating Set (CDS): C is a dominating set of $\mathcal{G}$ which induces a connected subgraph of $\mathcal{G}$.

B. Proposed TDOA Sensor Pairing Method

In the literature, most of the works adopt a centralized sensor pairing strategy, as shown in Fig. 1(a), where one sensor node (black node) is chosen as the reference node. In the centralized sensor pairing method, all the TDOAs can be estimated without redundancy. However, it may not work well in a multihop sensor network. This is mainly because that raw measurements are involved in transmissions resulting in high communication overhead and high power consumption.

To reduce the requirements for both network bandwidth and power consumption, we propose an in-network sensor pairing method to collect TDOA measurements while guarantee the quality of source localization, as shown in Fig. 1(b). The solution involves finding a CDS for the network graph [11]. The nodes in the CDS work as a network backbone to collect the TDOA measurements of the whole network. The proposed in-network sensor pairing method consists of the following two steps:

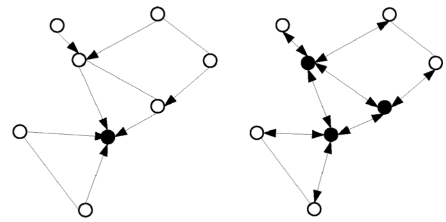

Fig. 1. Sensor Pairing. (a) centralized. (b) decentralized.

1) Finding a CDS: Finding a CDS has been well studied in the literature [11], [12]. In this letter, a sequential greedy algorithm presented in [11] is adopted to find a CDS which is presented in Algorithm 1. For other algorithms, we refer readers to [12] and the references therein. Note that the CDS for a graph may not be unique.

Algorithm 1: Sequential Greedy Algorithm

1:Coloring all nodes white.

2:Selecting a node that causes the maximum reduction of the number of white nodes. Once a node is selected, it is marked black and its white neighbors are marked gray.

3:The algorithm iteratively scans the gray nodes and their white neighbors, and selects a gray node or a pair of nodes (a gray node and one of its white neighbors), whichever has the maximal number of white neighbors. The selected node or the selected pair of nodes are marked black, with their white neighbors marked gray.

4:Algorithm terminates when there is no white node left.

5:All the black nodes form a connected dominating set (CDS).

An example of finding a CDS is presented in Fig. 2. From the above algorithm, the nodes in a CDS will be marked black and their neighbors are gray. We assume that there are p sensors in the dominating set, then $N/(1 + \triangle) \leq p \leq N/2$, where $\triangle$ denotes the maximum degree of the graph [13].

In real applications, a CDS of a sensor network can be determined offline if the topology of the network remains fixed during the online sensing tasks.

Remark 1: It is important to keep the cardinality of the dominating set small. In this case, we can find a CDS with minimum cardinality, which is called minimum CDS (MCDS). Finding the MCDS in a connected network was proved to be NP-complete. Some heuristic algorithms can be found in [14] and the references therein.

2) Decentralized TDOA Sensor Pairing: After the CDS is determined, one node in the CDS will be selected as the **Team Leader**. The team leader is responsible for collecting the TDOA information from the other CDS nodes and computing

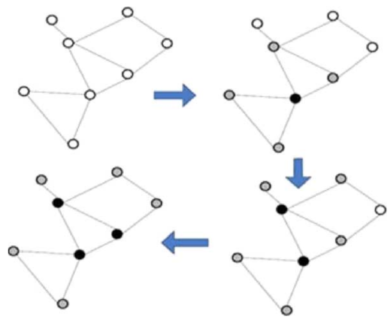

Fig. 2. An example of finding a CDS.

the source location estimate. The TDOA sensor pairing can be achieved by using the following decentralized in-network TDOA sensor pairing method as presented in Algorithm 2.

Algorithm 2: Sensor Pairing in Multi-hop Sensor Networks

1:Finding a connected dominating set using Algorithm 1.

2:The nodes in the dominating set (black nodes) broadcast their **raw measurement data** to their corresponding one-hop neighbors (including the gray and/or black nodes), respectively.

3:The gray/black nodes estimate the TDOAs accordingly using their **local** processors. For two or more black nodes which are neighbours, only one of them needs to compute TDOAs.

4:The gray nodes transmit the estimated TDOAs (which are **scalars**) back to their neighboring black nodes.

5:The black nodes forward their collected/estimated TDOAs to the **Team Leader**.

6:The **Team Leader** estimates the source location using the received TDOAs.

Remark 2: As stated in the Algorithm 2, the gray nodes will do the sensor pairing with the nearest black node in the CDS.

Remark 3: The proposed sensor pairing method may give rise to redundant TDOA information which will be eliminated by the**Team Leader** when calculating the source location.

In the next subsection, we give performance analysis for the proposed in-network sensor pairing method in terms of estimation accuracy and communication cost.

C. Performance Analysis

In this letter, we use the CRB as a metric to evaluate the estimation accuracy. CRB is a lower bound for the error covariance matrix of any unbiased estimator.

The CRB for TDOA based localization has been derived in [3]

$$\mathbf{J}^{-1} = v^2(\mathbf{G}\Sigma^{-1}\mathbf{G}^T)^{-1}, \tag{6}$$

where

$$\mathbf{G} = [\mathbf{g}_{ij}, \ldots]_{\{i,j\}\in\mathcal{I}},$$
$$\mathbf{g}_{ij} = \mathbf{g}_i - \mathbf{g}_j,$$

$$\mathbf{g}_i = \begin{bmatrix} \frac{x_t - x_i}{\sqrt{(x_t-x_i)^2+(y_t-y_i)^2}} \\ \frac{y_t - y_i}{\sqrt{(x_t-x_i)^2+(y_t-y_i)^2}} \end{bmatrix},$$

and Σ denotes the covariance matrix of TDOA measurement noises.

Theorem 1: If the network is connected, then the proposed decentralized in-network sensor pairing results in the same CRB with the centralized one.

Proof: Without loss of generality, we assume that the sensors with IDs $1, 2, \ldots$ are in a chosen CDS. TDOA measurements for the centralized pairing and the proposed decentralized pairing methods can be represented by

$$\hat{\mathbf{t}}_{ce} \triangleq \begin{bmatrix} \hat{t}_{21} \\ \hat{t}_{31} \\ \vdots \\ \hat{t}_{N1} \end{bmatrix} = \mathbf{T}_1 \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix}, \hat{\mathbf{t}}_{de} \triangleq \begin{bmatrix} \hat{t}_{i1} \\ \vdots \\ \hat{t}_{j2} \\ \vdots \end{bmatrix} = \mathbf{T}_2 \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix} \tag{7}$$

respectively, where $\hat{\mathbf{t}}_{ce}, \hat{\mathbf{t}}_{de} \in \mathbb{R}^{(N-1)\times 1}$ and t_i, $\forall i$ denotes the time of arrival of i-th sensor which is actually not known. $\mathbf{T}_1$ and $\mathbf{T}_2$ are transformation matrices and both of them are of dimension $(N-1)\times N$. $\mathbf{T}_1$ and $\mathbf{T}_2$ only have one '1' and one '-1' in each row. $\mathbf{T}_1$ can be represented by

$$\mathbf{T}_1 = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -1 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{8}$$

$\mathbf{T}_2$ is a little more complicated. Here we give an example to show how $\mathbf{T}_2$ looks like. We assume nodes 1 and 2 are the only two nodes in a CDS and they are connected. For simplicity, we further assume that node 3 is connected to 2 and the other nodes are all connected to node 1. Then $\mathbf{T}_2$ can be written as

$$\mathbf{T}_2 = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \ddots & \vdots \\ -1 & 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ -1 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{9}$$

From (9), we can see that through a simple elementary transformation, i.e., the first row is added to the second row, then $\mathbf{T}_2$ can be transformed to $\mathbf{T}_1$.

Since the network is connected, it can be easily verified by induction method that for a general $\mathbf{T}_2$, it can also be transformed to $\mathbf{T}_1$ through elementary transformations, i.e.,

$$\mathbf{T}_2 = \mathbf{U}_{cd}\mathbf{T}_1. \tag{10}$$

where $\mathbf{U}_{cd}$ is a $(N-1)\times(N-1)$ elementary transformation matrix.

The CRBs for the centralized pairing and the proposed decentralized in-network pairing methods are denoted by $\mathbf{CRB}_{ce} = v^2(\mathbf{G}_{ce}\Sigma_{ce}^{-1}\mathbf{G}_{ce}^T)^{-1}$ and $\mathbf{CRB}_{de} = v^2(\mathbf{G}_{de}\Sigma_{de}^{-1}\mathbf{G}_{de}^T)^{-1}$, respectively.

According to (7) and (10), it is easy to obtain that

$$\mathbf{G}_{de} = \mathbf{G}_{ce}\mathbf{U}_{cd}^T.$$

For the covariance matrix, we have

$$\begin{aligned}\frac{\Sigma_{de}}{v^2} &= E[(\mathbf{t}_{de} - E(\mathbf{t}_{de}))(\mathbf{t}_{de} - E(\mathbf{t}_{de}))^T] \\ &= \mathbf{U}_{cd}E[(\mathbf{t}_{ce} - E(\mathbf{t}_{ce}))(\mathbf{t}_{ce} - E(\mathbf{t}_{ce}))^T]\mathbf{U}_{cd}^T \\ &= \mathbf{U}_{cd}\frac{\Sigma_{ce}}{v^2}\mathbf{U}_{cd}^T \qquad (11)\end{aligned}$$

Hence $\Sigma_{de} = \mathbf{U}_{cd}^T\Sigma_{ce}\mathbf{U}_{cd}$. Then, we can obtain that

$$\begin{aligned}\mathbf{CRB}_{de} &= v^2(\mathbf{G}_{de}\Sigma_{de}^{-1}\mathbf{G}_{de}^T)^{-1} \\ &= v^2\big((\mathbf{G}_{ce}\mathbf{U}_{cd}^T)(\mathbf{U}_{cd}\Sigma_{ce}\mathbf{U}_{cd}^T)^{-1}(\mathbf{G}_{ce}\mathbf{U}_{cd})^T\big)^{-1} \\ &= v^2\big(\mathbf{G}_{ce}(\mathbf{U}_{cd}^T(\mathbf{U}_{cd}^T)^{-1})\Sigma_{ce}^{-1}((\mathbf{U}_{cd})^{-1}\mathbf{U}_{cd})\mathbf{G}_{ce}^T\big)^{-1} \\ &= v^2\big(\mathbf{G}_{ce}\Sigma_{ce}^{-1}\mathbf{G}_{ce}^T\big)^{-1} \\ &= \mathbf{CRB}_{ce}. \qquad (12)\end{aligned}$$

This completes the proof. ■

Remark 4: From the above theorem, we can see that if combining all the TDOA measurements collected by the nodes (redundant TDOAs are ignored) in a chosen CDS, then the corresponding CRB will be the same as the centralized one. One important issue for the above result is that the network should be connected, i.e., there is at least a spanning tree in the network. If not, (10) does not hold and then TDOA information can not be collected by the black nodes in the CDS.

Remark 5: In-network aggregation in sensor networks will reduce communication cost and power consumption [8]. In the proposed method, only one-hop communications are required to accomplish the sensor pairing task which is a better choice compared with the centralized sensor pairing where long-distance transmissions are involved. Hence the proposed decentralized sensor pairing method requires a less communication cost. Some remarks on the communication cost of the CDS can be found in [9]. Our analysis for the communication cost of the proposed decentralized sensor pairing method can be found in [15].

IV. Application to Decentralized Source Localization

One important application of the proposed sensor pairing method is the decentralized source localization. In the literature, most of the algorithms developed are centralized which is not realistic in general. In our method, each black node collects several TDOA measurements (for source localization in a 2-dimensional space, at least two TDOAs are required) and it is capable of estimating the source location by using the existing methods. Then an overall source location estimate can be obtained by fusing all the estimates from the black nodes in a centralized or a sequential mode.

In Fig. 3, we give an example of the decentralized source localization by using the structure of the proposed in-network sensor pairing method, where the unknown parameters of the source are estimated sequentially by the black nodes in the CDS. Many kinds of the estimators can be applied, such as extended Kalman filter (EKF), nonlinear least-squares (NLS) method, particle filter (PF), etc.

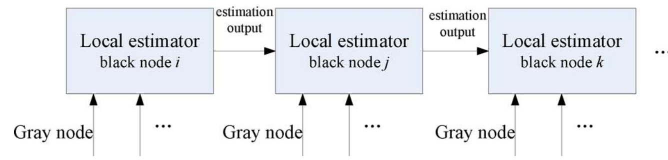

Fig. 3. Decentralized source localization architecture.

V. Conclusions

In this letter, we have proposed an in-network sensor pairing method to collect TDOA measurements. Compared to the centralized pairing method, it has lower requirements for both network bandwidth and energy consumptions in data transmissions. It has been proved that the proposed in-network sensor pairing can result in the same CRB as the centralized one at a far less communication cost. Finally, we pointed out that the proposed sensor pairing method can be applied to the decentralized source localization problem.

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