# Ergodic Interference Alignment with Delayed Feedback

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#### Abstract

We propose new ergodic interference alignment techniques for K-user interference channels with delayed feedback. Two delayed feedback scenarios are considered – delayed channel information at transmitter (CIT) and delayed output feedback. It is proved that the proposed techniques achieve total 2K/(K+2) DoF which is higher than that by the retrospective interference alignment for for the delayed feedback scenarios.

#### **Index Terms**

Interference channel, degrees of freedom (DoF), ergodic interference alignment, delayed feedback.

## I. INTRODUCTION

In these days, interference management is one of the most important issues in wireless communication systems. In order to obtain high spectral efficiency, many interference management techniques have been proposed and studied. For the two-user interference channel, the capacity region is already known for weak and strong interference regions in [1] and [2]. For the moderate region, the capacity region is still unknown, but there are some works that the capacity regions can be achieved by rate-splitting within one bit [3]. The authors in [3] also proved that the optimal generalized degrees of freedom are achievable using the rate-splitting scheme.

M. G. Kang and W. Choi are with Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon 305-701, Korea (e-mail: casutar@kaist.ac.kr, wchoi@ee.kaist.ac.kr). Compared to the two-user interference channel, the general K-user interference channel have not been much known yet. Many researchers have studied degrees of freedom (DoF) to understand the asymptotic capacity because of the difficulty of finding the exact capacity region. For the K-user interference channel, the DoF was shown to be upper bounded by  $\frac{K}{2}$  in [4]. The authors in [5] showed that this upper bound can be achieved by the interference alignment (IA) scheme that all interfering signals from other transmitters are aligned in the same dimension to independently decode the desired signal at the receivers. This scheme operates in high SNR to guarantee independence between the desired signal dimension and the interference aligned dimension. In order to operate in any finite SNR, [6] proposed ergodic IA that all interfering signals are perfectly cancelled out by properly choosing two time indices. Using ergodic IA, each user can achieve half the interference-free ergodic capacity.

The IA schemes generally require perfect channel state information (CSI). In rapidly time varying channels, however, channel state information becomes outdated due to feedback delay. In other words, it is impractical to assume that transmitters have perfect knowledge of current channel state information. In order to solve this problem, recent studies [7]–[10] focus on exploiting imperfect channel state information – no channel state information at transmitter (CSIT) or delayed feedback information. It was shown in [9] that 9/8 DoF is achievable for the three-user interference channel with delayed CSIT and 6/5 DoF is achievable for the three-user interference channel with delayed without CSIT. More generally, [10] showed that  $K^2/(K^2-1)$  DoF is achievable for the K-user interference channel ( $K \ge 3$ ) with delayed CSIT and  $\lceil K/2 \rceil K/(\lceil K/2 \rceil (K-1) + 1)$  DoF is achievable for the K-user interference channel with delayed output feedback without CSIT.

In this paper, we assume two delayed feedback scenarios as follows. (i) *Delayed channel information at transmitter*: in this scenario, nothing but the past channel information is given at the transmitter. The channel information implies either channel state information or time index information. Output feedback is not assumed in this case. (ii) *Delayed output feedback without CSIT*: in this scenario, nothing but the past output feedback information is given at the transmitter and the channel information is not available at

the transmitter. We devise effective interference management strategies in the *K*-user interference channel for these two scenarios. The proposed schemes are developed in the framework of ergodic IA [6] and enables interference-free decoding of the desired message at the receiver. It is shown that the proposed strategies achieve  $\frac{2K}{K+2}$  DoF in the *K*-user interference channel for the scenarios of the delayed channel information and the delayed output feedback without CSIT. The proposed schemes achieve higher DoF than the retrospective IA [9], [10] in the *K* user interference channel with the same assumptions of delayed feedback.

#### II. SYSTEM MODEL AND PRELIMINARY

## A. Interference channel model

The received signal in the K-user interference channel is given by

$$\mathbf{Y}(t) = \mathbf{H}(t)\mathbf{X}(t) + \mathbf{Z}(t)$$
(1)

where  $\mathbf{Y}(t) = [Y_1(t) \ Y_2(t) \ \cdots \ Y_K(t)]^T$ , the transmitted signal vector  $\mathbf{X}(t) = [X_1(t) \ X_2(t) \ \cdots \ X_K(t)]^T \in \mathbb{C}^{K \times 1}$  with power constraint P,  $\mathbf{H}(t) \in \mathbb{C}^{K \times K}$  represents the time varying fading channel matrix and is given by

$$\mathbf{H}(t) = \begin{bmatrix} H_{11}(t) & \cdots & H_{1K}(t) \\ \vdots & \ddots & \vdots \\ H_{K1}(t) & \cdots & H_{KK}(t) \end{bmatrix}.$$
(2)

where  $H_{ji}$  denotes the channel coefficient from transmitter *i* to receiver *j* and is an independent and identically distributed (i.i.d.) complex Gaussian random variable with distribution ~  $\mathcal{CN}(0,1)$ . At receivers, full channel state information is assumed to be available, i.e., CSIR. The element of the additive white Gaussian noise vector  $\mathbf{Z}(t) = [Z_1(t) \ Z_2(t) \ \cdots \ Z_K(t)]^T$  is assumed to follow complex Gaussian distribution ~  $\mathcal{CN}(0, N_0)$ .

## B. Preliminary: Ergodic IA with Full CSIT

The ergodic IA [6] requires perfect knowledge of channel state information at the transmitter (CSIT). For a K-user interference channel, K/2 total DoF can be achieved in an ergodic sense if the channel is time varying. Contrary to other IA techniques, the ergodic IA works for infinite SNR as well as any finite SNR since interfering signals are canceled out when the channel matrices at two different time instants satisfy a certain condition. Specifically, let  $t_1$  and  $t_2$  be the time instants (or time indices) at which the the channel matrices satisfy the following relationship:

$$\mathbf{H}(t_{1}) = \begin{bmatrix} H_{11}(t_{1}) & \cdots & H_{1K}(t_{1}) \\ \vdots & \ddots & \vdots \\ H_{K1}(t_{1}) & \cdots & H_{KK}(t_{1}) \end{bmatrix}$$
(3)  
$$\mathbf{H}(t_{2}) = c(t_{2}) \cdot \begin{bmatrix} H_{11}(t_{1}) & \cdots & -H_{1K}(t_{1}) \\ \vdots & \ddots & \vdots \\ -H_{K1}(t_{1}) & \cdots & H_{KK}(t_{1}) \end{bmatrix}$$
(4)

where  $c(t_2)$  is a complex valued constant and  $H_{kk}(t_2) = c(t_2)H_{kk}(t_1)$ ,  $H_{kj}(t_2) = -c(t_2)H_{kj}(t_1)$ ,  $k \neq j$ ,  $k, j \in 1, ..., K$ . At the time  $t_2$ , the message which was previously sent at the time  $t_1$  is again sent from the transmitter. In other words, the transmitted signal vector  $\mathbf{X}(t_1)$  is equal to  $\mathbf{X}(t_2)$ . To decode the message, receiver k adds the received signals at  $t_1$  and  $t_2$  and constructs a sufficient statistics for the message  $X_k(t_1)$  as

$$Y_k(t_1) + Y_k(t_2)/c(t_2) = 2H_{kk}(t_1)X_k(t_1) + Z_k(t_1) + Z_k(t_2)/c(t_2).$$
(5)

Then, the achievable rate is determined by

$$R_k = \frac{1}{2}\log(1 + \frac{2|H_{kk}|^2}{(1+1/c(t_2)^2)}SNR) - \epsilon$$
(6)

where  $SNR = \frac{P}{N_0}$ ,  $\epsilon > 0$ . Correspondingly, the total  $\frac{K}{2}$  DoF is achievable [6].

#### III. ERGODIC INTERFERENCE ALIGNMENT WITH DELAYED CIT

Contrary to the existing ergodic IA, we assume only full CSIR and imperfect or partial CSIT by feedback delay. Specifically, all receivers feed either channel state information or time indices back to the transmitters. Each transmitter cannot use the current channel information but can use the outdated channel information due to feedback delay.

## A. Three-user interference channel with delayed CSIT

In this subsection, we propose a new strategy to achieve high DoF when the receivers feed CSI back to the transmitters. To effectively establish the concept of the proposed scheme, we start from the three-user interference channel, i.e., K = 3.

*Theorem 1:* Three-user interference channel with delayed CSIT can achieve total  $\frac{6}{5}$  DoF by ergodic IA.

Proof: The proposed ergodic IA is carried out over two phases:

*Transmission phase 1*: Transmission phase 1 for data transmission is continued until  $t_2$ . At each time until  $t_2$ , new messages are continuously transmitted. For the time  $t_1$  and  $t_2$  at which the channel condition in (3) and (4) is satisfied, the received signals are given by

$$\mathbf{Y}(t_1) = \mathbf{H}(t_1)\mathbf{X}(t_1) + \mathbf{Z}(t_1), \tag{7}$$

$$\mathbf{Y}(t_2) = \mathbf{H}(t_2)\mathbf{X}(t_2) + \mathbf{Z}(t_2).$$
(8)

Due to delayed CSIT, the transmitters cannot recognize what the current channel states are so that they cannot send the same message of  $t_1$  at  $t_2$  as in the conventional ergodic IA. Thus, they just send independent messages at  $t_1$  and  $t_2$ . After the channel changes, the transmitters can figure out that the channel condition in (3) and (4) is satisfied at the previous time  $t_2$  due to delayed CSI feedback. Then, transmission phase 2 is entered.

Transmission phase 2: If the previous time was  $t_2$ , the transmitters send the following signals at time  $t_2 + 1, t_2 + 2, t_2 + 3$ , respectively:

Transmitter 1 at time 
$$t_2 + 1$$
:  $X_1(t_1) - X_1(t_2)$   
Transmitter 2 at time  $t_2 + 2$ :  $X_2(t_1) - X_2(t_2)$   
Transmitter 3 at time  $t_2 + 3$ :  $X_3(t_1) - X_3(t_2)$ .

After transmission phase 2 is completed, the transmission mode goes back to transmission phase 1. Each receiver adds its received signals at  $t_1$  and  $t_2$  and constructs the following variables.

$$Y_{1}(t_{1}) + Y_{1}(t_{2})/c(t_{2}) = H_{11}(t_{1})(X_{1}(t_{1}) + X_{1}(t_{2})) + H_{12}(t_{1})(X_{2}(t_{1}) - X_{2}(t_{2})) + H_{13}(t_{1})(X_{3}(t_{1}) - X_{3}(t_{2})) + Z_{1}(t_{1}) + Z_{1}(t_{2})/c(t_{2}),$$
(9)

$$Y_{2}(t_{1}) + Y_{2}(t_{2})/c(t_{2}) = H_{22}(t_{1})(X_{2}(t_{1}) + X_{2}(t_{2})) + H_{21}(t_{1})(X_{1}(t_{1}) - X_{1}(t_{2})) + H_{23}(t_{1})(X_{3}(t_{1}) - X_{3}(t_{2})) + Z_{2}(t_{1}) + Z_{2}(t_{2})/c(t_{2}),$$
(10)

$$Y_{3}(t_{1}) + Y_{3}(t_{2})/c(t_{2}) = H_{33}(t_{1})(X_{3}(t_{1}) + X_{3}(t_{2})) + H_{31}(t_{1})(X_{1}(t_{1}) - X_{1}(t_{2})) + H_{32}(t_{1})(X_{2}(t_{1}) - X_{2}(t_{2})) + Z_{3}(t_{1}) + Z_{3}(t_{2})/c(t_{2}).$$
(11)

Decoding at receiver 1: Using the received signal at  $t_2 + 2$  and  $t_2 + 3$ , receiver 1 removes the interfering signals from the other senders in (9). Then, we have an equation for  $X_1(t_1) + X_1(t_2)$ . Using another equation for  $X_1(t_1) - X_1(t_2)$  received at  $t_2 + 1$ , receiver 1 can decode both  $X_1(t_1)$  and  $X_1(t_2)$ .

Decoding at receiver 2: Similarly, receiver 2 removes the interfering signals in (10) using the received signal at  $t_2 + 1$  and  $t_2 + 3$  and decodes  $X_2(t_1)$  and  $X_2(t_2)$  using the received signal at  $t_2 + 2$  and (10).

Decoding at receiver 3: Similarly, receiver 2 removes the interfering signals in (11) using the received signal at  $t_2 + 1$  and  $t_2 + 2$  and decodes  $X_3(t_1)$  and  $X_3(t_2)$  using the received signal at  $t_2 + 3$  and (11).

According to the decoding procedure, the proposed scheme enables each receiver to decode its 2 messages in 5 symbol times. That is, total 6 messages are decodable over 5 symbol times and hence total 6/5 DoF is achievable.

*Theorem 2:* Total  $\frac{2K}{K+2}$  DoF is achievable in a *K*-user interference channel with delayed CSIT by ergodic IA.

*Proof:* For a K-user interference channel, two independent messages sent at time  $t_1$  and  $t_2$  are decoded at each receiver over K + 2 symbol times. As in the three-user interference channel, the time  $t_1$  and  $t_2$  correspond to transmission phase 1. If the transmitters realize the channel matrix at time  $t_2$  satisfies the condition in (3) and (4), transmission phase 2 starts. Then, transmitter  $k, k \in \{1, ..., K\}$ , sends the signal  $X_k(t_1) - X_k(t_2)$  at time  $t_2 + k$ . Similarly to the three-user interference channel, each receiver can decode its two messages over K + 2 symbol times. Therefore, total  $\frac{2K}{K+2}$  DoF is achievable in a K-user interference channel by the proposed ergodic IA.

Fig. 1 shows the achievable DoF by the proposed ergodic IA (solid line) and the retrospective IA [10] (dashed line) with delay CSIT according to the number of users in a *K*-user interference channel. The total achievable DoF by the retrospective IA is  $\frac{K^2}{K^2-1}$ . It starts from  $\frac{9}{8}$  for a three-user case and converges to 1 as *K* goes to infinity. On the other hand, the total achievable DoF by the proposed ergodic IA starts from  $\frac{6}{5}$  and converges to 2.

# C. K-user interference channel with delayed time index feedback

If the receivers feed the time indices at which the condition in (3) and (4) is satisfied, total achievable DoF is the same as the case that delay CSIT is used.

*Theorem 3:* Total achievable DoF by the proposed ergodic IA with delay time index feedback in a K-user interference channel is  $\frac{2K}{K+2}$ .

*Proof:* The strategy in Section III-B can be applied. Instead of using the channel state information (i.e., channel matrix) to find the time  $t_1$  and  $t_2$  at which the condition in (3) and (4) is satisfied, the receivers send the time indices  $t_1$  and  $t_2$  since receivers can find them by the assumption of full CSIR. After receiving the time indices, the transmitters realize that the previous time was  $t_2$  and enter into

## IV. ERGODIC INTERFERENCE ALIGNMENT WITH DELAYED OUTPUT FEEDBACK WITHOUT CSIT

In this section, we assume full CSIR and delayed output feedback. The received signals themselves are fed back to the transmitters. Each transmitter cannot use the channel information but can use only the delayed received output feedback. It is also assumed that the receivers can reform the output signals and feed them back to the transmitters if necessary.

## A. Three-user interference channel

Our new proposed strategy is first applied to a three-user interference channel for better explanation of the proposed idea.

*Theorem 4:* Total  $\frac{6}{5}$  DoF is achievable by the proposed ergodic IA in a three-user interference channel when only delayed output feedback information is available.

*Proof:* The operation of the proposed ergodic IA is classified into two phases:

Transmission phase 1: The transmission phase 1 for data transmission is continued until  $t_2$ . At each time until  $t_2$ , new messages are continuously transmitted. For the time  $t_1$  and  $t_2$  at which the channel condition in (3) and (4) is satisfied, the received signals are given by (7) and (8). Similarly to the case of delayed CIT in Section III, the transmitters cannot send the same message at  $t_1$  and  $t_2$  because they cannot realize that  $t_2$  is the time instant at which the channel condition in (3) and (4) is satisfied due to the absence of CSIT. Therefore, the transmitters continue to send independent messages at  $t_2$ . However, the receivers know that the channel condition in (3) and (4) is satisfied at  $t_2$  owing to full CSIR so that they construct the following output feedback information and send them back to the transmitters after receiving the signals at  $t_2$ .

Receiver 1:  $(Y_1(t_1) + Y_1(t_2)/c(t_2))/H_{11}(t_1)$ Receiver 2:  $(Y_2(t_1) + Y_2(t_2)/c(t_2))/H_{22}(t_1)$ Receiver 3:  $(Y_3(t_1) + Y_3(t_2)/c(t_2))/H_{33}(t_1).$ 

Then, the transmitters can figure out that the previous time  $t_2$  was the time instant at which the channel condition in (3) and (4) is satisfied after receiving the delayed output feedback information. However, note that they do not know the time instant  $t_1$  as well as the message sent at  $t_1$ . Once after receiving the delayed output feedback information, transmission phase 2 is entered.

*Transmission phase 2*: After the output feedback signals are received, the transmitters send their signals at time  $t_2 + 1, t_2 + 2, t_2 + 3$ , respectively:

Transmitter 1 at time 
$$t_2 + 1$$
:  $(Y_1(t_1) + Y_1(t_2)/c(t_2))/H_{11}(t_1) - 2X_1(t_2) =$   
 $(X_1(t_1) - X_1(t_2)) + (H_{12}(t_1)(X_2(t_1) - X_2(t_2)) + H_{13}(t_1)(X_3(t_1) - X_3(t_2)) + Z_1(t_1) + Z_1(t_2)/c(t_2))/H_{11}(t_1)$   
Transmitter 2 at time  $t_2 + 2$ :  $(Y_2(t_1) + Y_2(t_2)/c(t_2))/H_{22}(t_1) - 2X_1(t_2) =$   
 $(X_2(t_1) - X_2(t_2)) + (H_{21}(t_1)(X_1(t_1) - X_1(t_2)) + H_{23}(t_1)(X_3(t_1) - X_3(t_2)) + Z_2(t_1) + Z_2(t_2)/c(t_2))/H_{22}(t_1)$   
Transmitter 3 at time  $t_2 + 3$ :  $(Y_3(t_1) + Y_3(t_2)/c(t_2))/H_{33}(t_1) - 2X_1(t_2) =$   
 $(X_3(t_1) - X_3(t_2)) + (H_{31}(t_1)(X_1(t_1) - X_1(t_2)) + H_{32}(t_1)(X_2(t_1) - X_2(t_2)) + Z_3(t_1) + Z_3(t_2)/c(t_2))/H_{33}(t_1)$ 

After transmission phase 2 is completed, the transmission mode goes back to transmission phase 1.

Decoding at receiver 1: Linearly combining the received signals at  $t_2 + 1$ ,  $t_2 + 2$  and  $t_2 + 3$ , receiver 1 can obtain the values of  $X_1(t_1) - X_1(t_2)$ ,  $X_2(t_1) - X_2(t_2)$  and  $X_3(t_1) - X_3(t_2)$ . By substituting the values of  $X_2(t_1) - X_2(t_2)$  and  $X_3(t_1) - X_3(t_2)$  to  $Y_1(t_1) + Y_1(t_2)/c(t_2)$ , the value of  $X_1(t_1) + X_1(t_2)$  can also be obtained. Then, receiver 1 can decode both  $X_1(t_1)$  and  $X_1(t_2)$  because it has two independent equations on  $X_1(t_1)$  and  $X_1(t_2)$  – one is given in terms of  $X_1(t_1) - X_1(t_2)$  and the other is given in terms of  $X_1(t_1) + X_1(t_2)$ . Decoding at receiver 2: Similarly, receiver 2 can decode  $X_2(t_1)$  and  $X_2(t_2)$  using linear combination and substitution.

Decoding at receiver 3: Similarly, receiver 3 can decode  $X_3(t_1)$  and  $X_3(t_2)$  using linear combination and substitution.

In this way, all 6 messages are decoded in 5 symbol times so that total 6/5 DoF is achievable by the proposed ergodic IA.

## B. K-user interference channel

*Theorem 5:* When only delayed output feedback information is available at the transmitters, total  $\frac{2K}{K+2}$ DoF is achievable in a *K*-user interference channel by the proposed ergodic IA

Proof: For a K-user interference channel, two independent messages sent at time  $t_1$  and  $t_2$  are decoded at each receiver over K + 2 symbol times, where the time time  $t_1$  and  $t_2$  correspond to transmission phase 1. After the receivers receive the signals at  $t_2$ , receiver  $k, k \in \{1, ..., K\}$ , feed the output  $(Y_k(t_1) - Y_k(t_2)/c(t_2))/H_{kk}(t_1)$  back to its own transmitter. After the output feedback signals are received, the transmitters realize that the previous time instant was  $t_2$  at which the channel condition in (3) and (4) is satisfied and enters into transmission phase 2. In transmission phase 2, transmitter k sends the signal  $(Y_k(t_1) - Y_k(t_2)/c(t_2))/H_{kk}(t_1) - 2X_k(t_2)$  at only time  $t_2 + k$ . As in the three-user interference channel, each receiver decode its two messages over K + 2 symbol times. Therefore, total  $\frac{2K}{K+2}$  DoF is achievable in a K-user interference channel by the proposed ergodic IA.

Fig. 2 shows total achievable DoF by the proposed ergodic IA (solid line) and the retrospective IA [10] (dashed line) with delayed output feedback without CSIT according to the number of users. The total achievable DoF of the retrospective IA is  $\frac{\lceil K/2 \rceil K}{\lceil K/2 \rceil (K-1)+1}$ . It starts from  $\frac{6}{5}$  for the three-user case and converges to 1 as *K* goes to infinity. On the other hand, the total achievable DoF by the proposed ergodic IA starts from  $\frac{6}{5}$  and approaches to 2 as *K* goes to infinity.

#### V. CONCLUSION

In this paper, we proposed new ergodic IA techniques in K-user interference channels with delayed feedback. Total achievable DoF by the proposed ergodic IA is derived for two scenarios of delayed feedback – delayed channel information and delayed output feedback information. We showed that total 2K/(K + 2) DoF is achievable by the proposed schemes for both scenarios. The proposed ergodic IA schemes achieve higher DoF than the retrospective IA when the feedback information is outdated.

## REFERENCES

- V. S. Annapureddy and V. V. Veeravalli, "Gaussian interference networks: sum capacity in the low-interference regime and new outer bounds on the capacity region," *IEEE Trans. Inform. Theory*, vol. 55, no. 7, pp. 3032-3050, Jul. 2009.
- M. Costa and A. Gamal, "The capacity region of the discrete memoryless interference channel with strong interference," *IEEE Trans. Info. Theory*, vol. 33, no. 5, pp.710-711, Sept. 1987.
- [3] R. H. Etkin, D. N. C. Tse and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Info. Theory*, vol. 54, no. 12, pp. 5534-5562, Dec. 2008.
- [4] A. Host-Madsen and A. Nosratinia, "The multiplexing gain of wireless networks," in *Proc. IEEE Int. Symp. Info. Theory*, Adelaide, SA, Sep. 2005, pp. 2065-2069.
- [5] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Info. Theory*, vol. 54, no. 8, pp. 3425-3441, Aug. 2008.
- [6] B. Nazer, M. Gastpar, S. A. Jafar and S. Vishwanath, "Ergodic interference alignment," in *Proc. IEEE Int. Symp. Info. Theory*, Seoul, Korea, Jun. 2009.
- [7] C. Huang, S. A. Jafar, S. Shamai and S. Vishwanath, "On degrees of freedom region of MIMO networks without channel state information at transmitters," *IEEE Trans. Info. Theory*, vol. 58, no. 2, pp. 849-857, Feb. 2012.
- [8] S. A. Jafar, "Blind interference alignment," IEEE Trans. Info. Theory, vol. 6, no. 3, pp. 216-227, Jun. 2012.
- [9] H. Maleki, S. A. Jafar and S. Shamai, "Retrospective interference alignment," in *Proc. IEEE Int. Symp. Info. Theory*, Saint Petersburg, Russia, Jul. 2011.
- [10] L. Maggi and L. Cottatellucci, "Retrospective interference alignment for intereference channels with delayed feedback," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, Paris, France, Apr. 2012.

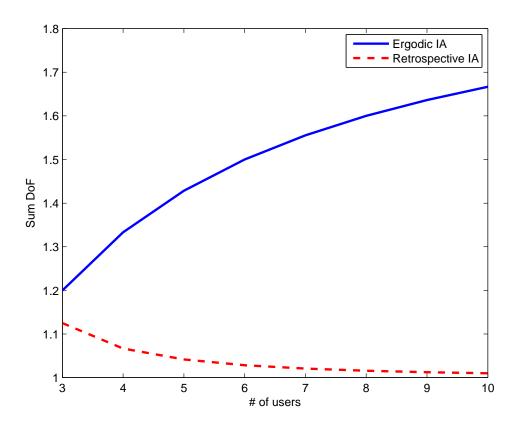


Fig. 1. Sum degrees of freedom with delayed CSIT

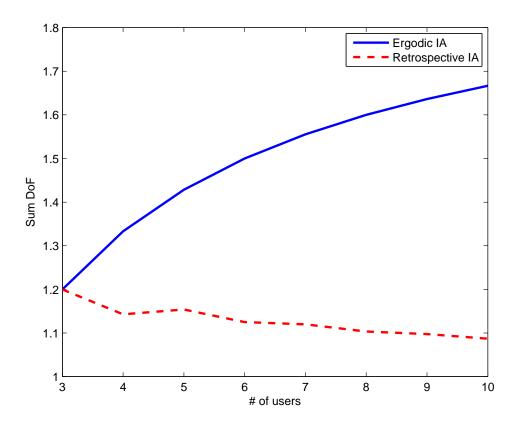


Fig. 2. Sum degrees of freedom with delayed output feedback without CSIT