Cooperative Energy Harvesting Networks with Spatially Random Users

Zhiguo Ding, Member, IEEE and H. Vincent Poor, Fellow, IEEE

Abstract—This paper considers a cooperative network with multiple source-destination pairs and one energy harvesting relay. The outage probability experienced by users in this network is characterized by taking the spatial randomness of user locations into consideration. In addition, the cooperation among users is modeled as a canonical coalitional game and the grand coalition is shown to be stable in the addressed scenario. Simulation results are provided to demonstrate the accuracy of the developed analytical results.

I. INTRODUCTION

Simultaneous power and signal transfer has been recognized as a promising energy harvesting technique, and it is particularly important to various energy constrained wireless networks without access to natural light or wind sources [1]– [3]. Wireless power and signal transfer has been studied in multiple antenna systems in [4], and its extension to the case with imperfect channel state information (CSI) at the transmitter has been considered in [5]. This new concept of energy harvesting is ideal for cooperative communication networks, in which the relay transmissions can be powered by the energy harvested from the incoming signals [6].

In this paper, we consider a cooperative network with multiple source-destination pairs communicating with each other via an energy harvesting relay. The contribution of this paper is two-fold. *Firstly* the spatial randomness of user locations is taken into consideration when the outage probability experienced by users is characterized. In the context of wireless power transfer, this randomness is particularly important since it determines the distance between the transceivers and hence describes the energy attenuation of the transmitted signals. *Secondly* the cooperation among users is modeled as a canonical coalitional game, and we show that in the high SNR regime, a grand coalition is always preferred, which means when forming a larger cooperative group the users cannot do worse than by acting alone. Simulation results are provided to demonstrate the accuracy of the developed analytical results.

II. COOPERATIVE ENERGY HARVESTING TRANSMISSIONS

Consider a scenario with N pairs of sources and destinations communicating with each other via an *energy harvesting* relay. Particularly the locations of 2N nodes are distributed uniformly in a disc, denoted by \mathcal{D} , with the relay located at its origin and D as its radius. Let h_i denote the channel gain between the *i*-th source and the relay, which we assume to be complex Gaussian (i.e., we assume Rayleigh fading), d_i denote the corresponding distance, and α denote the path loss factor. Similarly g_i and c_i are defined for the channels of the destinations. Prior to transmissions, no CSI, including channel gains and distances, is available.

The set of all pairs of nodes is denoted by \mathcal{N} . Prior to transmissions, user pairs who are willing to cooperate with each other form coalitions. Let $\mathcal{S}_k \in \mathcal{N}$ denote a coalition consisting of $|\mathcal{S}_k|$ source-destination pairs. $\mathcal{S} \triangleq \{\mathcal{S}_1, \dots, \mathcal{S}_K\}$ is a partition of \mathcal{N} , if $\mathcal{N} = \bigcup_{k=1}^K \mathcal{S}_k$ and $\mathcal{S}_i \cap \mathcal{S}_j = 0$, for any $i \neq j$. Denote the set of all possible partitions by \mathcal{B} . Define P_i^{ow} as the transmission power of the *i*-th source, and \tilde{P}_i^{ow} as the relay transmission power allocated to the *i*-th destination. The cooperative transmission strategy is described as follows:

- Phase I: Each source sends its message to the relay, and the sources of the same coalition cooperate in the sense that their power will be shared under the total power constraint, i.e. $\sum_{i \in S_k} P_i^{ow} = P^{total} \triangleq |S_k| P^{ow}$. Note that the sources communicate with the relay via orthogonal channels, and there is no direct source-destination link.
- Phase II: For each received transmission, the relay first tries to decode the message, and then carries out energy harvesting if there is any power left after decoding. Then the relay forwards the correctly decoded messages to the destinations. Let \hat{P}_i^{ow} denote the power that is harvested from the *i*-th source message. The user pairs in the same coalition are cooperating in the sense that their relaying power will be shared under the total harvested energy constraint, i.e. $\sum_{i \in S_k} \tilde{P}_i^{ow} = \sum_{i \in S_k} \hat{P}_i^{ow}$.

 \hat{P}_i^{ow} is calculated based on the energy harvested from the *i*-th source message, denoted by E_i , i.e. $\hat{P}_i^{ow} = \frac{E_i}{\frac{T}{2}}$, where *T* is the duration of one cooperative frame. For each observation, the signal-to-noise ratio (SNR) threshold for correct decoding is $\epsilon \triangleq (2^{2R} - 1)$ to ensure $\frac{1}{2} \log(1 + SNR) > R$, where *R* denotes the targeted data rate. Any power left after decoding will be used for energy harvesting, i.e. $E_i = \frac{T\eta}{2} \left(\frac{|h_i|^2}{1 + d_i^{\alpha}} P_i^{ow} - \epsilon \right)$, where η denotes the energy harvesting efficiency.

Therefore, the relationship between \hat{P}_i^{ow} and P_i^{ow} depends on the channel gain and distance between the *i*-th source and the relay, as described in the following:

$$\hat{P}_{i}^{ow} = \begin{cases} \eta \left(\frac{|h_{i}|^{2}}{1+d_{i}^{\alpha}} P_{i}^{ow} - \epsilon \right), & \text{if } \frac{|h_{i}|^{2}}{1+d_{i}^{\alpha}} P_{i} \ge \epsilon \\ 0, & \text{if } \frac{|h_{i}|^{2}}{1+d_{i}^{\alpha}} P_{i} < \epsilon \end{cases}$$
(1)

Let P_i denote the outage probability experienced by the *i*th user pair, and therefore the achieved data rate between this pair is $R(1 - P_i)$. A natural choice for the coalition value of S_k is $v(S_k) = \sum_{i \in S_k} R \cdot (1 - P_i)$. The questions to be studied

The authors are with Department of Electrical Engineering, Princeton University, Princeton, NJ, 08544, USA. Z. Ding is also with School of Electrical, Electronic, and Computer Engineering Newcastle University, UK.

in this paper are how many coalitions should be formed, and how a user should choose a coalition to join. Such questions can be formulated in the following optimization problem:

$$\max_{P_i, \mathcal{S} \in \mathcal{B}} \sum_{\mathcal{S}_k \in \mathcal{S}} \left(\sum_{i \in \mathcal{S}_k} R \cdot (1 - P_i) \right)$$
s.t.
$$\sum_{i \in \mathcal{S}_k} P_i^{ow} = |\mathcal{S}_k| P^{ow}, \sum_{i \in \mathcal{S}_k} \tilde{P}_i^{ow} = \sum_{i \in \mathcal{S}_k} \hat{P}_i^{ow}.$$
(2)

III. CANONICAL COALITIONAL GAME

The optimization problem shown in (2) can be simplified by taking the following steps.

- Since no CSI is known prior to transmissions, all user pairs are equally important, which means that equal power allocation is optimal, i.e. $P_i^{ow} = P^{ow}$.
- Let S
 _k denote the subset of S_k containing all the sources whose information is successfully delivered to the relay. When no CSI is available at the relay, all users are equally important, so equal power allocation is optimal, i.e. P
 _i^{ow} = η|S_k|⁻¹ Σ_{j∈S_k} (^{|h_j|²}/_{1+d_j^o} P_j^{ow} ε).

By applying these steps, an equivalent form of (2) can be expressed as follows:

$$\max_{P_i, S \in \mathcal{B}} \sum_{S_k \in S} \left(\sum_{i \in S_k} R \cdot (1 - P_i) \right)$$
(3)
s.t.
$$P_i^{ow} = P^{ow}, \tilde{P}_i^{ow} = \frac{\eta}{|\tilde{S}_k|} \sum_{j \in \tilde{S}_k} \left(\frac{|h_j|^2}{1 + d_j^{\alpha}} P_j^{ow} - \epsilon \right).$$

Evaluating the outage probability is the key to finding the solution of (3), as discussed in the next section. The path loss factor α is set as 2, in order to obtain closed-form expressions.

A. Characterizing P_i

The following theorem provides the high SNR approximations for the outage probability.

Theorem 1: In the high SNR regime, the outage probability experienced by the *i*-th user pair, $i \in S_k$, can be approximated as follows:

$$P_{i} \approx F_{x_{i}}\left(\frac{\epsilon}{P^{ow}}\right) + \sum_{n=1}^{|\mathcal{S}_{k}|} \frac{n}{|\mathcal{S}_{k}|} \frac{a^{n}}{(n-1)!} \left((n-1)!a^{-n}\right)$$
(4)
$$- \frac{2}{D^{2}\varpi} \left(\frac{\varpi}{a}\right)^{\frac{n+1}{2}} K_{n+1} \left(2\sqrt{\varpi a}\right) + \frac{2}{D^{2}\varpi} \times \left(\frac{(1+D^{2})\varpi}{a}\right)^{\frac{n+1}{2}} K_{n+1} \left(2\sqrt{(1+D^{2})\varpi a}\right)$$
(4)
$$\times \frac{|\mathcal{S}_{k}|!}{(|\mathcal{S}_{k}|-n)!n!} \left(F_{x_{i}}\left(\frac{\epsilon}{P^{ow}}\right)\right)^{|\mathcal{S}_{k}|-n} \left(1-F_{x_{i}}\left(\frac{\epsilon}{P^{ow}}\right)\right)^{n}$$

where $F_{x_i}(z) \triangleq 1 - \frac{e^{-z}}{D^2 z} + \frac{e^{-(1+D^2)z}}{D^2 z}$, $a = \frac{1}{2}(D^2 + 2)$, $\varpi = \frac{n\epsilon}{\eta P^{ow}}$, and $K_n(\cdot)$ denotes the modified Bessel function of the second kind.

Proof: Please refer to the appendix.

As demonstrated by the simulations in the next section, the analytical results in Theorem 1 are accurate even in a moderate SNR regime. However, these results are still too involved to be directly used for the analysis of coalition formation, which motivates the following corollary with more explicit expressions.

Corollary 1: When the SNR approaches infinity, i.e. $P^{ow} \rightarrow \infty$, an asymptotic expression for P_i , $i \in S_k$, is

$$\mathbf{P}_{i} \to \left(\frac{\epsilon D^{2}}{2} + \frac{\left(D^{2} + 2\right)^{2} \epsilon}{4\eta} \frac{|\mathcal{S}_{k}|}{(|\mathcal{S}_{k}| - 1)}\right) \frac{1}{P^{ow}} \tag{5}$$

if $|S_k| \ge 2$. And the outage probability for the users in singleton sets, i.e. $|S_k| = 1$, is given by

$$P_{i} \rightarrow \left(\frac{\epsilon D^{2}}{2} + \frac{\epsilon (D^{2} + 2)}{2\eta D^{2}} \left[\ln \sqrt{\frac{a\epsilon}{\eta P^{ow}}} + c_{0} \right] - (1 + D^{2})^{2} \left(\ln \sqrt{\frac{(1 + D^{2})\epsilon a}{\eta P^{ow}}} + c_{0} \right) \right] \frac{1}{P^{ow}}.$$
(6)

Proof: Please refer to the appendix.

B. Characterizing the addressed game

Based on the above theorem and corollary, we can show that a grand coalition is always preferred.

Corollary 2: When the SNR approaches infinity, the optimization problem in (2) becomes a canonical coalitional game with transferable utility (TU), and a grand coalition is stable.

Proof: The asymptotic result in (5) reveals the superadditivity property of the addressed game, i.e. $v(S_i) + v(S_j) < v(S_i \cup S_j)$, for any $|S_i| \ge 2$ and $|S_j| \ge 2$. The result in (6) reveals that the outage probability of a user forming a singleton set decays at a rate of $\frac{\log SNR}{SNR}$. Since a faster decaying rate of $\frac{1}{SNR}$ can be achieved by any coalition with a size larger than 1, singleton sets are not preferred, and any users in singleton sets will try to join in other existing coalitions. Therefore a grand coalition is a stable point of the system.

Let $\mathbf{x}^* \triangleq \{x_n^* = \frac{1}{N}v(\mathcal{N}), n \in \{1, \dots, N\}\}\$ denote the player payoff vector. Following Corollaries 1 and 2, the core of the addressed game can be characterized as follows.

Corollary 3: When the SNR approaches infinity, the core of the considered canonical coalitional game with TU, denoted by C, becomes non-empty and $\mathbf{x}^* \in C$.

Proof: The payoff vector \mathbf{x}^* is group-rational. And it is also individually rational since the outage probability of a user in a singleton set decays at a slow rate of $\frac{\log SNR}{SNR}$. The proof that $\mathbf{x}^* \in C$ can be shown by contradiction. Suppose that there is a coalition S that rejects the proposed payoff allocation, which means $\sum_{i\in S} x_i^* < v(S)$. Recall v(S) = $R\sum_{i\in S}(1-P_i)$. From Corollary 1, we first observe that the outage probability depends on only the size of the coalition, so the value of S can be simplified to $v(S) = R|S|(1-P_{|S|})$. Similarly for the grand set, we have $v(\mathcal{N}) = R|\mathcal{N}|(1-P_{|\mathcal{N}|})$, which means $x_i^* = R(1-P_{|\mathcal{N}|})$. The use of Corollary 1 yields $P_{|\mathcal{N}|} < P_{|S|}$, and therefore $\sum_{i\in S} x_i^* > v(S)$, which contradicts the initial claim. The proof is completed.

IV. NUMERICAL RESULTS

In this section, simulation results are provided to demonstrate the accuracy of the developed analytical results. In Fig. 1, the analytical results developed in Theorem 1 and the asymptotic results shown in Corollary 1 are compared to computer simulations. Recall that the approximation steps, such as the ones in (11) and (14), assume $\frac{D^2}{P^{ow}} \rightarrow 0$. As can be seen from the figure, for a small \mathcal{D} , the developed analytical results match exactly with simulations, and for a case with a larger \mathcal{D} there will be a gap which can be reduced by further increasing the SNR. In Fig. 2, the outage probability with different size coalitions is shown as a function of SNR. As can be seen from the figure, a larger coalition is always beneficial since users experience fewer errors and hence receive higher payoff. Particularly the case with a grand coalition achieves the best performance, which confirms Corollary 2.

V. CONCLUSION

In this paper, we have developed the outage probability for users in a cooperative network with an energy harvesting relay. In addition, the cooperation among users has been modeled as a canonical coalitional game and the grand coalition has been shown to be stable in the addressed scenario



Fig. 1. Simulation vs analytical results. R = 0.5 bit per channel use (BPCU).



Fig. 2. Outage probability for coalitions with different sizes.

Appendix

Proof of Theorem 1 : Define $x_i \triangleq \frac{|h_i|^2}{1+d_i^2}$ and $y_i \triangleq \frac{|g_i|^2}{1+c_i^2}$. Note that x_i and y_i are independent and identically distributed (i.i.d.). h_i and g_i are i.i.d. complex Gaussian variables, and the density function of the distance can be found by using the fact that the locations of the nodes are uniformly distributed in \mathcal{D} [7]. For any area A with a size of Δ and $A \in \mathcal{D}$, the distribution of each point W is $P(W \in A) = \frac{\Delta}{\pi D^2}$, and the corresponding probability density function (pdf) is $p_W(w) = \frac{1}{\pi D^2}$. In particular, it can be shown that the cumulative distribution function (CDF) of x_i is

$$F_{x_i}(z) \triangleq \mathcal{P}\left(x_i < z\right) = \int_{\mathcal{D}} \left(1 - e^{-(1+d_i^2)z}\right) p_W(w) dw, \quad (7)$$

where d_i is determined by the distance between the point W and the origin [8]. By applying polar coordinates, the CDF of x_i can be obtained as in Theorem 1, and its pdf is

$$f_{x_i}(z) = \frac{(1+z)e^{-z} - (1+z+zD^2)e^{-(1+D^2)z}}{D^2 z^2}.$$
 (8)

With equal power allocation, the outage probability of the *i*-th user pair, $i \in S_k$, is given by

$$P_{i} = F_{x_{i}}\left(\frac{\epsilon}{P^{ow}}\right) + \underbrace{\sum_{n=1}^{|\mathcal{S}_{k}|} P\left(i \in \tilde{\mathcal{S}}_{k}, y_{i} < \frac{\epsilon}{\tilde{P}_{i}^{ow}}, |\tilde{\mathcal{S}}_{k}| = n\right)}_{Q_{1}}.$$
 (9)

The first probability in (9) is $F_{x_i}\left(\frac{\epsilon}{P^{ow}}\right)$, and the second probability, Q_1 , can be expanded as follows:

$$Q_{1} = \sum_{n=1}^{|\mathcal{S}_{k}|} \frac{n}{|\mathcal{S}_{k}|} \underbrace{\mathbb{P}\left(y_{i} < \frac{\epsilon}{\tilde{P}_{i}^{ow}} \left| |\tilde{\mathcal{S}}_{k}| = n\right)}_{Q_{2,n}} \mathbb{P}(|\tilde{\mathcal{S}}_{k}| = n).$$

Since the source-relay channels are i.i.d., $P(|\tilde{S}_k| = n)$ is

$$\mathbb{P}(|\tilde{\mathcal{S}}_k|=n) = \frac{|\mathcal{S}_k|! \left(F_{x_i}\left(\frac{\epsilon}{P^{ow}}\right)\right)^{|\mathcal{S}_k|-n}}{(|\mathcal{S}_k|-n)!n!} \left(1 - F_{x_i}\left(\frac{\epsilon}{P^{ow}}\right)\right)^n.$$

On denoting $z = \sum_{j \in \tilde{S}_k} x_j$, $Q_{2,n}$ is given by

$$Q_{2,n} = P\left(y_i < \frac{\epsilon}{\tilde{P}_i^{ow}} \left| |\tilde{\mathcal{S}}_k| = n \right)$$

$$= \int_{n \frac{\epsilon}{P^{ow}}}^{\infty} F_{x_i} \left(\frac{\epsilon}{\frac{\eta P^{ow}}{|\tilde{\mathcal{S}}_k|} z - \epsilon \eta} \right) f_{z||\tilde{\mathcal{S}}_k| = n}(z) dz.$$
(10)

Directly finding the pdf of z conditioned on $|\tilde{\mathcal{S}}_k| = n$, $f_{z||\tilde{\mathcal{S}}_k|=n}(z)$, is difficult. To obtain an approximation of $Q_{2,n}$, first define $\gamma(z, P^{ow}) \triangleq \epsilon \left(\frac{\eta P^{ow}}{|\tilde{\mathcal{S}}_k|} z - \epsilon \eta\right)^{-1}$, and observe that $F_{x_i}(\gamma(z, P^{ow}))$ exhibits the following property:

$$F_{x_i}(\gamma(z, P^{ow})) \approx 1 - \frac{\left(1 - \gamma(z, P^{ow}) + \frac{1}{2}\gamma^2(z, P^{ow})\right)}{D^2\gamma(z, P^{ow})} + \frac{\left(1 - \gamma(z, P^{ow})(1 + D^2) + \frac{1}{2}\gamma^2(z, P^{ow})(1 + D^2)^2\right)}{D^2\gamma(z, P^{ow})} = \frac{1}{2}\gamma(z, P^{ow})(D^2 + 2) \to 0,$$
(11)

for any fixed z and a sufficiently large P^{ow} . Or in other words, when the transmission power is large, the probability quickly decreases to zero, i.e. $F_{x_i}(\gamma(z, P^{ow})) \to 0$, for z > e and e is small.

This means that for the integral in (10), a good approximation of $f_{z||\tilde{\mathcal{S}}_k|=n}(z)$, denoted by $\tilde{f}_{z||\tilde{\mathcal{S}}_k|=n}(z)$, needs to ensure $\tilde{f}_{z||\tilde{\mathcal{S}}_k|=n}(z) = f_{z||\tilde{\mathcal{S}}_k|=n}(z)$ for $z \to 0$, and a slight difference between two functions for a large z has an insignificant impact on the integral since $F_{x_i}(\gamma(z, P^{ow})) \to 0$ for a large z. Recall

that $z = \sum_{j \in \tilde{S}_k} x_j$, and the above rationale motivates us to find an approximation of $f_{x_j}(z)$ for $z \to 0$. Particularly when $z \to 0$, the pdf of x_i can be approximated as

$$\begin{split} f_{x_j}(z) &\approx \frac{1}{D^2 z^2} (1+z) \left(1-z+\frac{z^2}{2} \right) - \frac{1}{D^2 z^2} (1+z) \\ &+ z D^2) \left(1-(1+D^2)z+\frac{1}{2} (1+D^2)^2 z^2 \right) \\ &= \left(\frac{1}{2} D^2 + 1 \right) + o(z^2). \end{split}$$

Directly using the approximation $f_{x_j}(z) \approx \frac{1}{2}D^2 + 1$ means that the range of the variable is limited as $\left[0, \frac{2}{D^2+2}\right]$ to ensure the CDF normalized. Instead we use the following approximation:

$$f_{x_j}(z) \approx \left(\frac{1}{2}D^2 + 1\right)e^{-\left(\frac{1}{2}D^2 + 1\right)z},$$
 (12)

an exponential distribution with the parameter $(\frac{1}{2}D^2 + 1)$. As can be seen in Section IV, this approximation is tightly matched with the simulation results even in the moderate SNR regime.

Conditioned on $x_j > \frac{\epsilon}{P^{ow}}$, the Laplace transform of x_j is

$$\mathcal{L}(f_{x_j|x_j > \frac{\epsilon}{P^{ow}}}(t)) = \frac{1}{1 - F_{x_j}(\frac{\epsilon}{P^{ow}})} \int_{\frac{\epsilon}{P^{ow}}}^{\infty} e^{-st} f_{x_j}(t) dt$$
$$\approx \left(\frac{1}{2}D^2 + 1\right) e^{\frac{D^2\epsilon}{2P^{ow}}} \frac{e^{-(s + \left(\frac{1}{2}D^2 + 1\right))\frac{\epsilon}{P^{ow}}}}{s + \left(\frac{1}{2}D^2 + 1\right)}$$

The Laplace transform for the pdf of the sum is

$$\mathcal{L}(f_{z||\tilde{\mathcal{S}}_k|=n}(z)) \approx \left(\frac{1}{2}D^2 + 1\right)^n \frac{e^{-\frac{n\epsilon s}{P^{\delta w}}}}{\left(s + \left(\frac{1}{2}D^2 + 1\right)\right)^n}$$

which yields the pdf of z conditioned on $|S_k| = n$ as follows:

$$f_{z||\tilde{\mathcal{S}}_k|=n}(z) \approx \frac{a^n \left(z - n\frac{\epsilon}{P^{ow}}\right)^{n-1} e^{-a\left(z - n\frac{\epsilon}{P^{ow}}\right)}}{(n-1)!}$$

for $\frac{n\epsilon}{Pow} \leq z \leq \infty$. On substituting it into the probability of $Q_{2,n}$ we obtain the following:

$$Q_{2,n} = \int_{n\frac{\epsilon}{Pow}}^{\infty} F_{x_i}\left(\frac{n\epsilon}{\eta P^{ow}}\frac{1}{z-n\frac{\epsilon}{Pow}}\right) f_{z||\tilde{\mathcal{S}}_k|=n}(z)dz$$
$$\approx \frac{a^n}{(n-1)!} \int_0^{\infty} F_{x_i}\left(\frac{n\epsilon}{\eta P^{ow}}\frac{1}{t}\right) e^{-at}t^{n-1}dt.$$

After some algebra manipulations, the probability $Q_{2,n}$ can be expressed as

$$Q_{2,n} \approx \frac{a^n}{(n-1)!} \left((n-1)! a^{-n} \right)$$

$$- \frac{2}{D^2 \varpi} \left(\frac{\varpi}{a} \right)^{\frac{n+1}{2}} K_{n+1} \left(2\sqrt{\varpi a} \right) + \frac{2}{D^2 \varpi}$$

$$\times \left(\frac{(1+D^2) \varpi}{a} \right)^{\frac{n+1}{2}} K_{n+1} \left(2\sqrt{(1+D^2) \varpi a} \right) .$$
(13)

By combining (9), (10) and (13), the theorem is proved.

Proof of Corollary 1: By applying the series representation of Bessel functions, $x^n \mathbf{K}_n(x)$ can be approximated as [9]

$$x^{2}\mathbf{K}_{2}(x) = \frac{1}{2}\left(4 - x^{2}\right) + \frac{x^{4}}{8}\left(-\ln\frac{x}{2} - c_{0}\right) + o(x^{6}\ln x),$$

$$x^{n}\mathbf{K}_{n}(x) = \frac{1}{2}\sum_{l=0}^{n-1} \frac{(-1)^{l}(n-l-1)!}{l!} \frac{x^{2l}}{2^{2l-n}} + o(x^{2n}\ln x),$$

for $n \ge 3$. For $n \ge 3$, $Q_{2,n}$ can be approximated as follows:

$$Q_{2,n} \approx \frac{a^n}{(n-1)!} \left((n-1)!a^{-n} - \frac{1}{D^2 \varpi 2^{n+1}a^{n+1}} \right)$$
$$\times \sum_{j=0}^n \frac{(-1)^j (n-j)!}{j!} \frac{(2\sqrt{\varpi a})^{2j}}{2^{2j-n-1}} + \frac{1}{D^2 \varpi 2^{n+1}a^{n+1}}$$
$$\times \sum_{j=0}^n \frac{(-1)^j (n-j)!}{j!} \frac{\left(2\sqrt{(1+D^2)\varpi a}\right)^{2j}}{2^{2j-n-1}} \approx \frac{a\left(D^2+2\right)}{2(n-1)} \varpi$$

And for n = 2, we can obtain the following:

$$Q_{2,2} \approx \frac{\varpi (D^2 + 2)}{2D^2} \left(\left(\ln \sqrt{\varpi a} + c_0 \right) - (1 + D^2)^2 \right) \\ \left(\ln \sqrt{(1 + D^2)\varpi a} + c_0 \right) \right).$$

Note that $Q_{2,n}$ for $n \geq 3$ decays at a rate of $\frac{1}{P^{ow}}$ and $Q_{2,2}$ has a much slower decay rate of $\frac{\log P^{ow}}{P^{ow}}$.

By applying the above approximations, when $|S_k| \ge 2$, the outage probability experienced by a user in S_k is asymptotically equivalent to the following expression:

$$P_{i} = F_{x_{i}}\left(\frac{\epsilon}{P^{ow}}\right) + \sum_{n=1}^{|\mathcal{S}_{k}|} \frac{n}{|\mathcal{S}_{k}|}Q_{2,n} \cdot P(|\tilde{\mathcal{S}}_{k}| = n)$$

$$\rightarrow \frac{\epsilon D^{2}}{2P^{ow}} + \sum_{n=1}^{|\mathcal{S}_{k}|} \frac{n}{|\mathcal{S}_{k}|} \frac{a\left(D^{2}+2\right)}{2(n-1)}\left(\frac{n\epsilon}{\eta P^{ow}}\right)$$

$$\times \frac{|\mathcal{S}_{k}|!}{(|\mathcal{S}_{k}| - n)!n!} \left(\frac{\epsilon}{2P^{ow}}\right)^{|\mathcal{S}_{k}| - n}$$

$$= \frac{\epsilon D^{2}}{2P^{ow}} + \frac{a\left(D^{2}+2\right)}{2(|\mathcal{S}_{k}| - 1)} \frac{|\mathcal{S}_{k}|\epsilon}{\eta} \frac{1}{P^{ow}}, \quad (14)$$

and the first part of the corollary is proved. The second part of the corollary can be proved using similar steps.

REFERENCES

- [1] L. R. Varshney, "Transporting information and energy simultaneously," in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Toronto, Canada, Jul. 2008. P. Grover and A. Sahai, "Shannon meets Tesla: wireless information and
- [2] power transfer," in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Austin, TX, Jun. 2010
- X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power [3] transfer: Architecture design and rate-energy tradeoff," [online]. Available: http://arxiv.org/abs/1205.0618, 2012.
- [4] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," IEEE Trans. Wireless Commun., vol. 12, no. 5, pp. 1989-2001, May 2013.
- [5] Z. Xiang and M. Tao, "Robust beamforming for wireless information and power transmission," IEEE Wireless Commun. Letters, vol. 1, no. 4, pp. 372-375, Jan. 2012.
- [6] A. A. Nasir, X. Zhou, S. Durrani, and R. A. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," [online]. Available: http://arxiv.org/abs/1212.5406, 2012.
- [7] J. F. C. Kingman, Poisson Processes. Oxford University Press, Oxford, UK, 1993.
- [8] H. Wang, S. Ma, T.-S. Ng, and H. Poor, "A general analytical approach for opportunistic cooperative systems with spatially random relays," IEEE Trans. Wireless Commun., vol. 10, no. 12, pp. 4122-4129, 2011.
- [9] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series and Products, 6th ed. New York: Academic Press, 2000.