

Decision Fusion with Unknown Sensor Detection Probability

D. Ciuonzo, *Student Member, IEEE*, and P. Salvo Rossi, *Senior Member, IEEE*

Abstract—In this correspondence we study the problem of channel-aware decision fusion when the sensor detection probability is not known at the decision fusion center. Several alternatives proposed in the literature are compared and new fusion rules (namely “ideal sensors” and “locally-optimum detection”) are proposed, showing attractive performance and linear complexity. Simulations are provided to compare the performance of the aforementioned rules.

Index Terms—Decentralized detection, decision fusion, locally-optimum detection (LOD), wireless sensor networks (WSNs).

I. INTRODUCTION

DECISION fusion (DF) in wireless sensor networks (WSNs) attracted huge interest by the scientific community [1]. In some particular cases, assuming that the sensor probability of detection is higher than the corresponding false-alarm, the uniformly most powerful test is independent on the local sensor probabilities [2] and thus their knowledge is not needed. However, it is typically assumed that the sensor performance is known at the DF center (DFC) [3], [4], [5]. Indeed in the general case sensor performance is required in order to implement the optimal fusion rule, namely the likelihood ratio test (LRT). Unluckily, while the sensor false-alarm can be obtained (since it depends on the local threshold value and the sensing noise distribution), the detection probability is generally difficult to acquire, as it depends on the features of the (unknown) event being observed.

There are two common approaches tackling the aforementioned problem: (i) employing (sub-optimal) rules which *neglect the whole sensor performance*, such as the “diversity” statistics proposed in [3], [4], [6]; (ii) assuming the knowledge of the local false alarm probabilities and considering the detection probability as an unknown (deterministic) parameter, thus determining a *composite hypothesis test*¹. A first remarkable study in the latter direction is found in [7] where a fusion rule, obtained along the same lines of a generalized LRT (GLRT) derivation, has been proposed and shown to have promising results, i.e. being an affine statistic and outperforming the GLRT itself in the considered scenarios.

Unluckily, to the best of our knowledge the two approaches have not been compared yet, and thus it is not immediate whether the sole knowledge of the sensors false alarm probabilities is a potential benefit in the design of efficient fusion rules. Also, another (possibly) useful information is that the sensor detection probability is typically higher than the corresponding false alarm probability (since each “informative” receiver operating characteristic always outperforms an unbiased coin). We will show that jointly exploiting both information can produce performance gains.

In this letter we study channel-aware DF when the *false-alarm* probability of the generic sensor is *known*, while the *detection* probability is *unknown*. First, we perform (to best of our knowledge, for the first time) a detailed comparison of existing fusion alternatives,

not requiring knowledge of *sensor detection probability*, based on the approaches (i) (i.e. the *counting rule* [1]) and (ii) (i.e. the rule proposed in [7], denoted here as “*Wu rule*”). The comparison is strengthened by a theoretical analysis in the case of a large number of sensors, based on deflection measures [8]. Also, we derive two novel rules, based on “ideal sensors” assumption (approach (i)) [3], [4], [9] and locally-optimum detection (approach (ii)) [10]. For all the considered rules high/low signal-to-noise ratio (SNR) optimality properties are established in a scenario with identical sensors and a discussion on complexity and required system knowledge is reported. Finally, the case of non-identical sensors is considered.

The paper is organized as follows: Sec. II introduces the model; in Sec. III we derive and study the fusion rules, while in Sec. IV we generalize the analysis to the case of non-identical sensors; in Sec. V we compare the presented rules and confirm the theoretical findings through simulations; finally in Sec. VI we draw some conclusions; proofs are confined to the Appendix.

II. SYSTEM MODEL

The model is described as follows². We consider a decentralized binary hypothesis test, where K sensors are used to discriminate between the hypotheses of the set $\mathcal{H} = \{\mathcal{H}_0, \mathcal{H}_1\}$, representing the absence (\mathcal{H}_0) or the presence (\mathcal{H}_1) of a specific phenomenon of interest. The a priori probability of $\mathcal{H}_i \in \mathcal{H}$ is denoted $P(\mathcal{H}_i)$. The k th sensor, $k \in \mathcal{K} \triangleq \{1, 2, \dots, K\}$, takes a binary decision $d_k \in \mathcal{H}$ about the phenomenon on the basis of its own measurements, which is then mapped to a symbol $b_k \in \{0, 1\}$; without loss of generality (w.l.o.g.) we assume that $d_k = \mathcal{H}_i$ maps into $b_k = i$, $i \in \{0, 1\}$.

The quality of the k th sensor decisions is characterized by the conditional probabilities $P(b_k|\mathcal{H}_j)$: we denote $P_D \triangleq P(b_k = 1|\mathcal{H}_1)$ and $P_F \triangleq P(b_k = 1|\mathcal{H}_0)$ the probabilities of detection and false alarm of the k th sensor, respectively. Initially, we assume conditionally independent and identically distributed (i.i.d.) decisions; this restriction will be relaxed in Sec. IV. Also we assume $P_D > P_F$, because of the informativeness of the decision at each sensor. Differently from [4], we assume that P_F is *known* at the DFC, but on the other hand that the true P_D is *unknown*, as studied in [7].

The k th sensor communicates to the DFC over a dedicated binary symmetric channel (BSC) and the DFC observes a noisy binary-valued signal y_k , that is $y_k = b_k$ with probability $(1 - P_{e,k})$ and $y_k = (1 - b_k)$ with probability $P_{e,k}$, which we collect as $\mathbf{y} \triangleq [y_1 \ \dots \ y_K]^T$. Here $P_{e,k}$ denotes the bit-error probability (BEP) of the k th link³. The BSC model arises when separation between sensing and communication layers is performed in the design phase (namely a “*decode-then-fuse*” approach [6]).

²*Notation* - Lower-case bold letters denote vectors, with a_n being the n th element of \mathbf{a} ; $\|\mathbf{a}\|_p$ denotes the ℓ_p -norm of \mathbf{a} ; upper-case calligraphic letters, e.g. \mathcal{A} , denote finite sets; $\mathbb{E}\{\cdot\}$, $\text{var}\{\cdot\}$ and $(\cdot)^t$ denote expectation, variance and transpose, respectively; $P(\cdot)$ and $p(\cdot)$ are used to denote probability mass functions (pmf) and probability density functions (pdf), respectively, while $P(\cdot|\cdot)$ and $p(\cdot|\cdot)$ their corresponding conditional counterparts; $\mathcal{N}_C(\mu, \sigma^2)$ denotes a proper complex-valued Gaussian pdf with mean μ and variance σ^2 , while $\mathcal{Q}(\cdot)$ is the complementary cumulative distribution function of a standard normal random variable; $\mathcal{U}(a, b)$ denotes a uniform pdf with support $[a, b]$; finally the symbol \sim means “distributed as”.

³Throughout this letter we make the reasonable assumption $P_{e,k} \leq \frac{1}{2}$.

Manuscript received Oct. 23, 2013; revised Nov. 31, 2013;

This work has been partially supported by the ERCIM within the Alain Bensoussan fellowship programme and by the Faculty of Information Technology, Mathematics and Electrical Engineering of the Norwegian University of Science and Technology, Trondheim, Norway, within the project CAMOS.

The authors are with the Dept. Industrial & Information Engineering, Second University of Naples, Aversa (CE), Italy (e-mail: {domenico.ciuonzo, salvorossi}@iee.org).

¹In the latter case it is assumed that the sensor detection probability is the same for all the sensors employed (i.e. a homogeneous scenario).

The pmf of \mathbf{y} is the same under both \mathcal{H}_0 and \mathcal{H}_1 , except that the value of the unknown parameter $P_1 \triangleq P(b_k = 1|\mathcal{H})$ is different. After denoting the pmf with $P(\mathbf{y}; P_1)$ the test is summarized as:

$$\mathcal{H}_0 : P_1 = P_F; \quad \mathcal{H}_1 : P_1 > P_F; \quad (1)$$

which is recognized as a *one-sided* (composite) test [11].

III. FUSION RULES

The final decision at the DFC is performed as a test comparing a signal-dependent fusion rule $\Lambda(\mathbf{y})$ and a fixed threshold γ :

$$\Lambda(\mathbf{y}) \underset{\hat{\mathcal{H}}=\mathcal{H}_0}{\overset{\hat{\mathcal{H}}=\mathcal{H}_1}{\geq}} \gamma \quad (2)$$

where $\hat{\mathcal{H}}$ denotes the estimated hypothesis. Hereinafter we propose different fusion rules for the considered problem.

(Clairvoyant) LRT - in this case we assume that also P_D is known at the DFC. The explicit expression of the LRT is given by

$$\begin{aligned} \Lambda_{\text{LRT}} &\triangleq \ln \left[\frac{P(\mathbf{y}; P_1 = P_D)}{P(\mathbf{y}; P_1 = P_F)} \right] = \sum_{k=1}^K \ln \left[\frac{P(y_k; P_1 = P_D)}{P(y_k; P_1 = P_F)} \right] \\ &= \sum_{k=1}^K \left\{ y_k \ln \left[\frac{\alpha_k(P_D)}{\alpha_k(P_F)} \right] + (1 - y_k) \ln \left[\frac{\beta_k(P_D)}{\beta_k(P_F)} \right] \right\} \end{aligned} \quad (3)$$

where $\alpha_k(P_1) \triangleq P(y_k = 1; P_1) = ((1 - 2P_{e,k}) \cdot P_1 + P_{e,k})$ and $\beta_k(P_1) \triangleq P(y_k = 0; P_1) = (1 - \alpha_k(P_1))$. It is apparent that Eq. (3) should not be intended as a realistic element of comparison, but rather as an optimistic upper bound on the achievable performance (since it makes use of both P_D and P_F). Differently, in this letter it is assumed that $P_{e,k}$ can be easily obtained, as in [12].

Ideal sensors (IS) rule - we obtain this rule by assuming that the sensing phase works ideally, that is $(P_D, P_F) = (1, 0)$. This simplifying assumption is exploited in Eq. (3), thus leading to:

$$\Lambda_{\text{IS}} \triangleq \sum_{k=1}^K (2y_k - 1) \ln \left[\frac{1 - P_{e,k}}{P_{e,k}} \right]. \quad (4)$$

The assumption behind Eq. (4) is not new: indeed it was considered in [3], [4], [9] to derive sub-optimal rules (i.e. the *maximum ratio* and the *equal gain combiners*) under different communication models.

Locally-optimum detection (LOD) rule - the one-sided nature of the test considered allows to pursue a LOD-based approach, whose implicit expression is given by [10], [11, chap. 6]

$$\Lambda_{\text{LOD}} \triangleq \frac{\partial \ln [P(\mathbf{y}; P_1)]}{\partial P_1} \Big|_{P_1=P_F} \times \left(\sqrt{I(P_F)} \right)^{-1}, \quad (5)$$

where $I(P_1)$ represents the *Fisher information* (FI), that is:

$$I(P_1) \triangleq \mathbb{E} \left\{ \left(\frac{\partial \ln [P(\mathbf{y}; P_1)]}{\partial P_1} \right)^2 \right\}. \quad (6)$$

The explicit form of Λ_{LOD} is shown in Eq. (7) at the top of the next page; the derivation is given in the Appendix.

Counting rule (CR) - this rule is widely used in DF (due to its simplicity and no requirements on system knowledge) and it is obtained by assuming that the communication channels are ideal, i.e.

$$\Lambda_{\text{CR}} \triangleq \sum_{k=1}^K y_k, \quad (8)$$

since $P_{e,k} = 0$ entails $\alpha_k(P_1) = P_1$ and irrelevant terms are incorporated in γ through Eq. (2).

Wu rule [7] - this rule was proposed by Wu *et al.* and it was shown to outperform a GLRT rule for all the scenarios considered. We

report only the final result and omit the details. First an *approximate*⁴ *maximum-likelihood* (ML) estimate of P_D is obtained as

$$\hat{P}_D \triangleq \frac{1}{K} \sum_{k=1}^K [(1 + 2P_{e,k})y_k - P_{e,k}], \quad (9)$$

then the following statistic is employed:

$$\Lambda_{\text{Wu}} \triangleq (\hat{P}_D - P_F). \quad (10)$$

Remark: when $P_{e,k} = P_e$ all the rules are *equivalent*⁵. Thus, when the SNR goes to infinity (i.e. $P_{e,k} \rightarrow 0$) all the rules undergo the same performance. The only exception is Λ_{IS} , since $\lim_{P_{e,k} \rightarrow 0} \Lambda_{\text{IS}} = +\infty$ (such a difference leads to a loss in performance, as shown in Sec. V). Differently, in the low SNR regime their behaviour is significantly different, as shown by the following proposition.

Proposition 1. *When the SNR is low at each link, Λ_{IS} and Λ_{LOD} approach Λ_{LRT} , while Λ_{Wu} does not.*

Proof: Λ_{IS} and Λ_{LOD} are *equivalent* to $\sum_{k=1}^K \psi(P_{e,k}) y_k$ and $\sum_{k=1}^K \phi(P_{e,k}) y_k$, respectively, where $\psi(P_{e,k}) \triangleq \ln \left[\frac{1 - P_{e,k}}{P_{e,k}} \right]$ and $\phi(P_{e,k}) \triangleq \frac{(1 - 2P_{e,k})}{\alpha_k(P_F) \beta_k(P_F)}$ (cf. Eqs. (4-7)). Also, $\Lambda_{\text{LRT}} = \sum_{k=1}^K (\chi(P_{e,k}) y_k + \vartheta(P_{e,k}) (1 - y_k))$, where we have denoted $\chi(P_{e,k}) \triangleq \ln \left[\frac{\alpha_k(P_D)}{\alpha_k(P_F)} \right]$ and $\vartheta(P_{e,k}) \triangleq \ln \left[\frac{\beta_k(P_D)}{\beta_k(P_F)} \right]$. When the SNR is small, we can approximate each $\psi(P_{e,k})$, $\phi(P_{e,k})$, $\chi(P_{e,k})$ and $\vartheta(P_{e,k})$ by a first-order Taylor series around $P_{e,k} = \frac{1}{2}$. Exploiting these expansions leads to $\sum_{k=1}^K \psi(P_{e,k}) y_k \approx 2 \sum_{k=1}^K (1 - 2P_{e,k}) y_k$, $\sum_{k=1}^K \phi(P_{e,k}) y_k \approx 4 \sum_{k=1}^K (1 - 2P_{e,k}) y_k$ and $\Lambda_{\text{LRT}} \approx 2(P_D - P_F) \sum_{k=1}^K [(1 - 2P_{e,k})(2y_k - 1)]$. Then, the Taylor-based approximations at low SNR are *all equivalent* and thus Λ_{IS} , Λ_{LOD} and Λ_{LRT} undergo the same performance. Finally, since Λ_{Wu} is *equivalent* to $\sum_{k=1}^K (1 + 2P_{e,k}) y_k$ (cf. Eqs. (9-10)), at low SNR it poorly approximates Λ_{LRT} , whose Taylor-based approximation is instead *equivalent* to $\sum_{k=1}^K (1 - 2P_{e,k}) y_k$. ■

It is worth noting that: (i) Prop. 1 does not require $P_{e,k}$ to be equal and that (ii) the low-SNR optimality of Λ_{IS} in Prop. 1 is coherent with the results shown in [4], [5], [6].

Wu rule vs CR deflection comparison: since all the considered rules are equivalent to scaled sums of independent Bernoulli random variables, the pmf $P(\Lambda|\mathcal{H}_i)$ is intractable [7]. Hence we rely on the so-called *deflection measures* [8] $D_i \triangleq \frac{(\mathbb{E}\{\Lambda|\mathcal{H}_1\} - \mathbb{E}\{\Lambda|\mathcal{H}_0\})^2}{\text{var}\{\Lambda|\mathcal{H}_i\}}$ to perform a theoretical comparison between Λ_{CR} and Λ_{Wu} . This choice is justified since, as K grows large, $P(\Lambda|\mathcal{H}_i)$ converges to a Gaussian pdf (in virtue of the *central limit theorem* [13]). It can be shown that for CR and Wu rule the deflections assume the following expressions:

$$D_{\text{CR},i} = \frac{\left(\sum_{k=1}^K m_k \right)^2}{\sum_{k=1}^K c_{i,k}}, \quad D_{\text{Wu},i} = \frac{\left(\sum_{k=1}^K n_k m_k \right)^2}{\sum_{k=1}^K n_k^2 c_{i,k}}, \quad (11)$$

where $m_k \triangleq (1 - 2P_{e,k})(P_D - P_F)$, $n_k \triangleq (1 + 2P_{e,k})$, $c_{0,k} \triangleq \alpha_k(P_F)(1 - \alpha_k(P_F))$ and $c_{1,k} \triangleq \alpha_k(P_D)(1 - \alpha_k(P_D))$. W.l.o.g., we assume $P_{e,k} \geq P_{e,k+1}$, which in turn gives $m_k \leq m_{k+1}$, $n_k \geq n_{k+1}$ and $c_{i,k} \geq c_{i,k+1}$ (since we assume $P_{e,k} \leq \frac{1}{2}$). Consequently, the *Chebyshev's sum inequalities* [14] $\sum_{k=1}^K n_k m_k \leq \frac{1}{K} \left(\sum_{k=1}^K m_k \right) \left(\sum_{k=1}^K n_k \right)$ and $\sum_{k=1}^K n_k^2 c_{i,k} \geq \frac{1}{K} \left(\sum_{k=1}^K c_{i,k} \right) \left(\sum_{k=1}^K n_k^2 \right)$ hold, which jointly give:

$$D_{\text{Wu},i} \leq D_{\text{Wu},i} \left(\frac{\sqrt{K} \|\mathbf{n}\|_2}{\|\mathbf{n}\|_1} \right)^2 \leq D_{\text{CR},i} \quad (12)$$

⁴This was derived under a high-SNR assumption [7].

⁵We use the term “equivalent” to refer to statistics which are equal up to a scaling factor and an additive term (both independent on \mathbf{y} and finite), thus leading to the same performance [11].

$$\Lambda_{\text{LOD}} = \left(\sum_{k=1}^K \frac{(1 - 2P_{e,k}) \cdot [(y_k - P_{e,k}) - (1 - 2P_{e,k})P_F]}{\alpha_k(P_F)\beta_k(P_F)} \right) \times \left(\sqrt{\sum_{k=1}^K \frac{(1 - 2P_{e,k})^2}{\alpha_k(P_F)\beta_k(P_F)}} \right)^{-1} \quad (7)$$

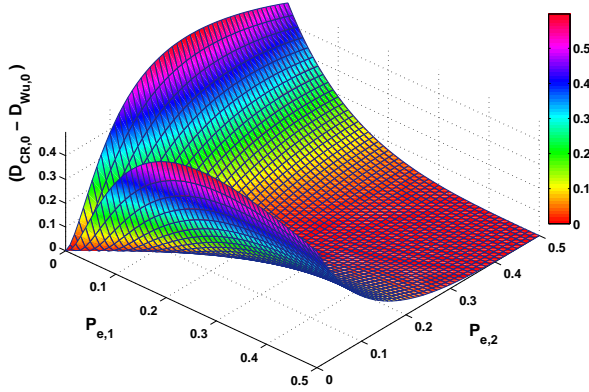


Figure 1. $(D_{\text{CR},0} - D_{\text{Wu},0})$ for $K = 2$ sensors as a function of $\{P_{e,1}, P_{e,2}\}$, conditionally i.i.d. decisions $(P_F, P_D) = (0.05, 0.5)$.

Table I

COMPARISON OF RULES W.R.T. SYSTEM KNOWLEDGE REQUIREMENTS.

Fusion rule	Required parameters
(Clairvoyant) LRT	$P_D, P_F, P_{e,k}$
LOD rule	$P_F, P_{e,k}$
IS rule	$P_{e,k}$
CR	none
Wu rule [7]	$P_F, P_{e,k}$

where $\mathbf{n} \triangleq [n_1 \ \dots \ n_K]^t$ and the first inequality arises from the application of *Cauchy-Schwartz inequality* [15] to $\|\mathbf{n}\|_1$.

In Fig. 1 we illustrate $(D_{\text{CR},0} - D_{\text{Wu},0})$ (in a WSN with $K = 2$) as a function of $(P_{e,1}, P_{e,2})$ in a scenario with $(P_F, P_D) = (0.05, 0.5)$. It is confirmed that $D_{\text{Wu},i}$ is always dominated by $D_{\text{CR},i}$ and that the effect is more pronounced when $P_{e,1}$ and $P_{e,2}$ differ significantly (indeed when $P_{e,1} = P_{e,2}$, Λ_{Wu} is equivalent to Λ_{CR}). The superiority of Λ_{CR} is also confirmed via the results in Sec. V.

Discussion on complexity and system knowledge: as discussed in [7], Λ_{Wu} being *affine* in \mathbf{y} (cf. Eqs. (9-10)) is one of the main advantages w.r.t. the GLRT. This feature reduces the complexity at the DFC and facilitate performance analysis. Since all the considered alternatives (i.e. Λ_{IS} , Λ_{LOD} and Λ_{CR}) are also affine functions of \mathbf{y} , they exhibit the same advantages. On the other hand, as summarized in Tab. I, the presented fusion rules have different requirements in terms of system knowledge. In fact, while Λ_{LOD} and Λ_{Wu} entail the same requirements (i.e. P_F and $P_{e,k}$), Λ_{IS} only needs $P_{e,k}$. Finally, Λ_{CR} does not require any parameter for its implementation.

IV. EXTENSION TO NON-IDENTICAL SENSORS SCENARIO

In this section we generalize the proposed rules to a scenario with non-identical sensors, i.e. $(P_{D,k}, P_{F,k})$, $k \in \mathcal{K}$, where $P_{F,k}$ is *known* but $P_{D,k}$ is *still unknown* at the DFC.

(Clairvoyant) LRT - Λ_{LRT} is readily obtained by replacing $\alpha_k(P_D)$ (resp. $\alpha_k(P_F)$) with $\alpha_k(P_{D,k})$ (resp. $\alpha_k(P_{F,k})$) in Eq. (3).

LOD fusion rule - the rule is naturally extended to conditionally independent and non-identically distributed (i.n.i.d.) decisions:

$$\check{\Lambda}_{\text{LOD}} \triangleq \sum_{k=1}^K \frac{\partial \ln[P(y_k; P_1)]}{\partial P_1} \bigg|_{P_1=P_{F,k}} \times \left(\sqrt{I_k(P_{F,k})} \right)^{-1} \quad (13)$$

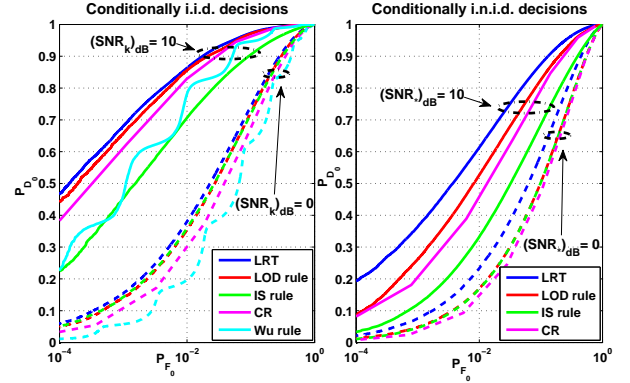


Figure 2. $P_{D,0}$ vs. $P_{F,0}$; WSN with $K = 10$ and $(\text{SNR}_k)_{\text{dB}} \in \{0, 10\}$ (resp. $(\text{SNR}_*)_{\text{dB}} \in \{0, 10\}$); $(P_F, P_D) = (0.05, 0.5)$ (resp. $(P_{FU}, P_{DE}) = (0.2, 0.6)$) for conditionally i.i.d. (resp. i.n.i.d.) decisions.

CR, IS and Wu fusion rules - in this scenario Λ_{IS} retains the same form as in Eq. (4), while it is apparent that $\Lambda_{\text{CR}} = \sum_{k=1}^K y_k$ does not arise from the assumption $P_{e,k} = 0$ in Λ_{LRT} . Nonetheless we will still keep Λ_{CR} in the comparison of Sec. V, since it represents a natural “ $P_{D,k}$ -unaware” alternative. Finally, we discard Eq. (10) from our comparison, since the (approximate) ML estimate in Eq. (9) is performed assuming $P_{D,k} = P_D$.

V. NUMERICAL RESULTS

In this section we compare the performance of the proposed rules in terms of system false alarm and detection probabilities, defined as

$$P_{F_0} \triangleq \Pr\{\Lambda > \gamma | \mathcal{H}_0\}, \quad P_{D_0} \triangleq \Pr\{\Lambda > \gamma | \mathcal{H}_1\}, \quad (14)$$

respectively, where Λ is the generic statistic employed at the DFC.

Similarly as in [7], we consider communication over a Rayleigh fading channel via on-off keying, i.e. $x_k = h_k b_k + w_k$, where $x_k \in \mathbb{C}$, $h_k \sim \mathcal{N}_{\mathbb{C}}(0, 1)$, $w_k \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_w^2)$; h_k is assumed known at the DFC and therefore coherent detection is employed. Given these assumptions, $P_{e,k} = \mathcal{Q}(\frac{|h_k|}{2\sigma_w})$ holds. We define the (individual) communication SNR as the (average individual) received energy divided by the noise power, that is in the i.i.d. case

$$\text{SNR}_k \triangleq \frac{\mathbb{E}\{|h_k b_k|^2\}}{\sigma_w^2} = \frac{P_{D,k}P(\mathcal{H}_1) + P_{F,k}P(\mathcal{H}_0)}{\sigma_w^2}, \quad (15)$$

while in the i.n.i.d. case $\text{SNR}_* \triangleq \mathbb{E}_{(P_{D,k}, P_{F,k})}\{\text{SNR}_k\}$. Here we assume $P(\mathcal{H}_i) = \frac{1}{2}$; the figures are based on 10^6 Monte Carlo runs.

In Fig. 2 we report $P_{D,0}$ vs. $P_{F,0}$ in a scenario with conditionally i.i.d. and i.n.i.d. decisions, respectively⁶. We study a WSN with $K = 10$ and local performance equal to $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ in the i.i.d case while $P_{F,k} \sim \mathcal{U}(0, P_{FU})$, $P_{D,k} = (P_{F,k} + \Delta P)$ and $\Delta P \sim \mathcal{U}(0, P_{DE})$ in the i.n.i.d. case, where $(P_{FU}, P_{DE}) = (0.2, 0.6)$. We report scenarios with $(\text{SNR}_k)_{\text{dB}} \in \{0, 10\}$ (resp. $(\text{SNR}_*)_{\text{dB}}$, where $\text{SNR}_* = \frac{P_{FU} + P_{DE}/2}{2\sigma_w^2}$ in the i.n.i.d. case). It is apparent that Λ_{LOD} and Λ_{IS} approach Λ_{LRT} at $(\text{SNR}_k)_{\text{dB}} = 0$ in the i.i.d. case (confirming Prop. 1), while there is a moderate loss

⁶Note that the concavity of the plots is not apparent, as instead suggested from the theory [11]; this is due to the use of a log-linear scale.

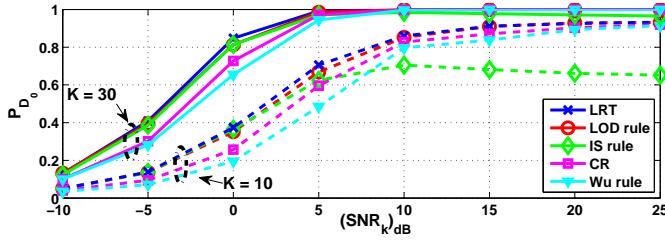


Figure 3. P_{D0} vs. $(\text{SNR}_k)_{\text{dB}}$; $P_{F0} = 0.01$. WSN with $K \in \{10, 30\}$ sensors; $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$.

in the i.i.d. case⁷. However, Λ_{IS} suffers from significant loss in performance in both cases $(\text{SNR}_k)_{\text{dB}} = 10$ and $(\text{SNR}_k)_{\text{dB}} = 10$. Also, in the i.i.d. case Λ_{Wu} is outperformed by both Λ_{CR} and Λ_{LOD} , the latter being the best choice. Finally, the *oscillating behaviour* of Λ_{Wu} is explained since the approximate ML estimate \hat{P}_D (cf. Eq. (9)) is *not reliable* when the WSN is not of large size. Moreover the performance of \hat{P}_D further degrades at low-medium SNR, since $\mathbb{E}\{\hat{P}_D|\mathcal{H}_1\} = \frac{1}{K} \sum_{k=1}^K ((1 - 4P_{e,k}^2) \cdot P_D + 2P_{e,k}^2)$, i.e. when $P_{e,k}^2$ is not negligible, the estimator is *biased* (even if K grows large), as opposed to the exact ML estimate [16].

Fig. 3 shows P_{D0} vs. $(\text{SNR}_k)_{\text{dB}}$, assuming⁸ $P_{F0} = 0.01$; we simulate a i.i.d. scenario, where $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ and we report the cases $K \in \{10, 30\}$. First, simulations confirm the theoretical findings in Sec. III: (i) only Λ_{IS} and Λ_{LOD} approach Λ_{LRT} at low SNR, while (ii) all the considered rules undergo the same performance as the SNR increases. The only exception is given by Λ_{IS} , which keeps close to Λ_{LRT} at low-to-moderate SNR values and exhibits a *unimodal behaviour*, which is consequence of $\lim_{P_{e,k} \rightarrow 0} \Lambda_{\text{IS}} = +\infty$, as discussed in Sec. III. In fact as $P_{e,k} \rightarrow 0$, the possible errors are mainly due to the sensing part; on the other hand Λ_{IS} assumes a perfect sensing phase (cf. Eq. (4)), thus misleadingly conjecturing that the whole process is error-free. Finally, Λ_{LOD} is close to Λ_{LRT} over the whole SNR range considered, while Λ_{Wu} has a significant loss in performance and it is always “counter-intuitively” outperformed by Λ_{CR} (with no requirements on system knowledge).

Finally, in Fig. 4 we show P_{D0} vs. K , assuming $P_{F0} = 0.01$. We study a i.i.d. setup in the cases $(\text{SNR}_k)_{\text{dB}} \in \{0, 10\}$ (dashed and solid lines, resp.). We analyze the scenarios $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ (scenario A, as in [4]) and $(P_{F,k}, P_{D,k}) = (0.4, 0.6)$ (scenario B, as in [7]). The simulations confirm the performance improvement given by Λ_{LOD} with respect to Λ_{CR} and Λ_{IS} (at the expenses of slightly higher requirements on system knowledge) and the significant improvement with respect to Λ_{Wu} (the latter being *always* outperformed by Λ_{CR} , even when K is large, as proved in Sec. III). For example, in scenario A with $(\text{SNR}_k)_{\text{dB}} = 0$, Λ_{LOD} achieves $P_{D0} \approx 0.8$ with $K \approx 30$ sensors as opposed to $K \approx 43$ when Λ_{Wu} is employed.

VI. CONCLUSIONS

In this letter we studied DF when the DFC knows the false-alarm probability of the generic sensor, but does not the detection probability. Wu rule is always (counter-intuitively, since it makes use of BEPs and false alarm probabilities) outperformed by the simpler counting rule, thus does not exploit effectively the required system parameters. This result is confirmed by a deflection-based analysis, with CR *always* dominating Wu rule, irrespective of the specific

⁷In fact, it can be verified that Prop. 1 does not hold in the latter scenario.

⁸In order to keep a fair comparison, we allow for *rule randomization* whenever its discrete nature does not allow to meet the desired P_{F0} exactly.

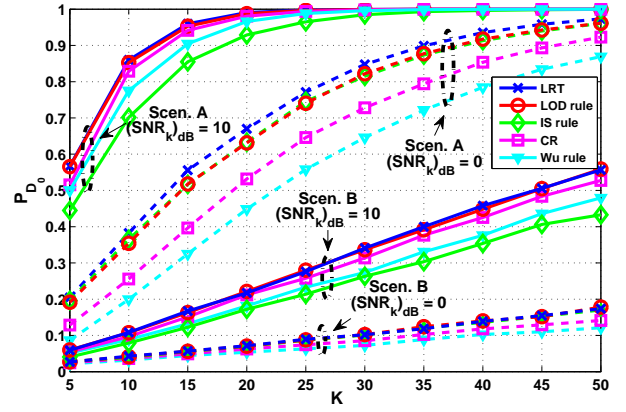


Figure 4. P_{D0} vs. K ; $P_{F0} = 0.01$. WSN with $(\text{SNR}_k)_{\text{dB}} \in \{0, 10\}$; $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ (scen. A) and $(P_{F,k}, P_{D,k}) = (0.4, 0.6)$ (scen. B).

BEPs and local performance (in the i.i.d case) considered. Differently, the proposed LOD and IS based rules are appealing in terms of complexity and performance. LOD rule was shown to be close to the clairvoyant LRT over a realistic SNR range (thus *effectively* exploiting knowledge of BEPs and false alarm probabilities), both for conditionally i.i.d. and i.n.i.d. decisions, as opposed to IS rule (only requiring the BEPs for its implementation) being close to the LRT only at low-medium SNR. Optimality of both rules was proved at low SNR in the i.i.d. case, thus motivating the knowledge of false-alarm probability only at medium SNR in a *homogeneous* scenario.

APPENDIX

We start expressing the log-likelihood $\ln[P(\mathbf{y}; P_1)]$ explicitly:

$$\ln[P(\mathbf{y}; P_1)] = \sum_{k=1}^K \{y_k \ln[\alpha_k(P_1)] + (1 - y_k) \ln[\beta_k(P_1)]\} \quad (16)$$

where $\alpha_k(P_1)$ and $\beta_k(P_1)$ have the same meaning as in Eq. (3). Eq. (16) easily provides the numerator in Eq. (5):

$$\begin{aligned} \frac{\partial \ln[P(\mathbf{y}; P_1)]}{\partial P_1} &= \sum_{k=1}^K \frac{\partial \ln[P(y_k; P_1)]}{\partial P_1} \\ &= \sum_{k=1}^K \frac{(1 - 2P_{e,k}) \cdot [(y_k - P_{e,k}) - (1 - 2P_{e,k}) P_1]}{\alpha_k(P_1) \beta_k(P_1)}. \end{aligned} \quad (17)$$

On the other hand, we notice that $I(P_1) = \sum_{k=1}^K I_k(P_1)$, where $I_k(P_1) \triangleq \mathbb{E} \left\{ \left(\frac{\partial \ln[P(y_k; P_1)]}{\partial P_1} \right)^2 \right\}$, since y_k are (conditionally) independent. Hence, we can evaluate each $I_k(P_1)$ separately. Considering the explicit form of $\frac{\partial \ln[P(y_k; P_1)]}{\partial P_1}$ in Eq. (17), squaring and taking the expectation leads to:

$$I_k(P_1) = (1 - 2P_{e,k})^2 \frac{\mathbb{E} \{ ((1 - 2P_{e,k})P_1 - (y_k - P_{e,k}))^2 \}}{\alpha_k(P_1)^2 \cdot \beta_k(P_1)^2}. \quad (18)$$

The average in the r.h.s. of Eq. (18) is given explicitly as follows:

$$\mathbb{E} \{ ((1 - 2P_{e,k})P_1 - (y_k - P_{e,k}))^2 \} = \alpha_k(P_1) \beta_k(P_1), \quad (19)$$

which can be substituted in Eq. (18) to obtain $I_k(P_1)$ in closed form. Summing all the (independent) contributions $I_k(P_1)$ leads to:

$$I(P_1) = \sum_{k=1}^K I_k(P_1) = \sum_{k=1}^K \frac{(1 - 2P_{e,k})^2}{\alpha_k(P_1) \cdot \beta_k(P_1)}. \quad (20)$$

Finally substituting Eqs. (16) and (20) in Eq. (5) provides Eq. (7).

REFERENCES

- [1] P. K. Varshney, *Distributed Detection and Data Fusion*, 1st ed. Springer-Verlag New York, Inc., 1996.
- [2] D. Ciuonzo, G. Romano, and P. Salvo Rossi, "Optimality of received energy in decision fusion over Rayleigh fading diversity MAC with non-identical sensors," *IEEE Trans. Signal Process.*, vol. 61, no. 1, pp. 22–27, Jan. 2013.
- [3] A. Lei and R. Schober, "Coherent Max-Log decision fusion in wireless sensor networks," *IEEE Trans. Commun.*, vol. 58, no. 5, pp. 1327–1332, May 2010.
- [4] B. Chen, R. Jiang, T. Kasetkasem, and P. K. Varshney, "Channel aware decision fusion in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 52, no. 12, pp. 3454–3458, Dec. 2004.
- [5] R. Jiang and B. Chen, "Fusion of censored decisions in wireless sensor networks," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2668–2673, Nov. 2005.
- [6] D. Ciuonzo, G. Romano, and P. Salvo Rossi, "Channel-aware decision fusion in distributed MIMO wireless sensor networks: Decode-and-fuse vs. decode-then-fuse," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2976–2985, Aug. 2012.
- [7] J.-Y. Wu, C.-W. Wu, T.-Y. Wang, and T.-S. Lee, "Channel-aware decision fusion with unknown local sensor detection probability," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1457–1463, Mar. 2010.
- [8] B. Picinbono, "On deflection as a performance criterion in detection," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 31, no. 3, pp. 1072–1081, Jul. 1995.
- [9] D. Ciuonzo, G. Romano, and P. Salvo Rossi, "Performance analysis of maximum ratio combining in channel-aware MIMO decision fusion," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4716–4728, Sep. 2013.
- [10] S. A. Kassam and J. B. Thomas, *Signal detection in non-Gaussian noise*. Springer-Verlag New York, 1988.
- [11] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume 2: Detection Theory*. Prentice Hall PTR, Jan. 1998.
- [12] S. Chaudhari, J. Lundén, V. Koivunen, and H. V. Poor, "Cooperative sensing with imperfect reporting channels: Hard decisions or soft decisions?" *IEEE Trans. Signal Process.*, vol. 60, no. 1, pp. 18–28, Jan. 2012.
- [13] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, 3rd ed. McGraw-Hill Companies, Feb. 1991.
- [14] G. H. Hardy, J. E. Littlewood, and G. Pólya, *Inequalities, Cambridge Mathematical Library*. Cambridge University Press, 1988.
- [15] D. S. Bernstein, *Matrix mathematics: theory, facts, and formulas*. Princeton University Press, 2009.
- [16] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume 1: Estimation Theory*. Prentice Hall PTR, 1993.