Comments and Corrections

Correction to "Massive MIMO With Optimal Power and Training Duration Allocation"

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***Abstract*—We correct the proofs of Propositions 1 and 2 in [1].**

I. SUMMARY

In [1], the statements of Propositions 1 and 2 are correct, but the proofs are not. In what follows, we provide corrections to the proofs.

II. CORRECTIONS TO PROOFS

A. Corrections to Proof of Proposition 1

In Proposition 1, the choices $\bar{\tau} = K$, $\bar{p}_{\rm p} = \tau^* p_{\rm p}^*/K$, and $\bar{p}_{\rm u} = p_{\rm u}^*$ do not satisfy the total energy constraint:

$$\bar{\tau}\bar{p}_{\rm p} + (T - \bar{\tau})\bar{p}_{\rm u} = P. \tag{1}$$

The correct choice should be $\bar{\tau} = K$, $\bar{p}_{\rm p} = \tau^* p_{\rm p}^*/K$, and $\bar{p}_{\rm u} = \frac{P-\tau^* p_{\rm p}^*}{T-K}$.

Note that, with the above corrected choice of $\bar{p}_{\rm u}$, showing that $\mathcal{S}(\bar{\tau}, \bar{p}_{\rm p}, \bar{p}_{\rm u}) > \mathcal{S}(\tau^*, p_{\rm p}^*, p_{\rm u}^*)$ is not straightforward. In the following, we will provide a detailed proof of the inequality $\mathcal{S}(\bar{\tau}, \bar{p}_{\rm p}, \bar{p}_{\rm u}) > \mathcal{S}(\tau^*, p_{\rm p}^*, p_{\rm u}^*)$.

From [1, Eq. (6)], we have

$$\mathcal{S}(\bar{\tau}, \bar{p}_{\rm p}, \bar{p}_{\rm u}) = \sum_{k=1}^{K} g_k(\bar{\tau}, \bar{p}_{\rm p}, \bar{p}_{\rm u}) \tag{2}$$

where

$$g_k(\tau, p_{\rm p}, p_{\rm u}) \triangleq \left(1 - \frac{\tau}{T}\right)\log_2\left(1 + \frac{a_k \tau p_{\rm p} p_{\rm u}}{b_k \tau p_{\rm p} p_{\rm u} + c_k p_{\rm u} + d_k \tau p_{\rm p} + 1}\right). \tag{3}$$

Substitution of $\bar{\tau}$, $\bar{p}_{\rm p}$, and $\bar{p}_{\rm u}$ into (3) yields

$$g_k(\bar{\tau}, \bar{p}_{\rm p}, \bar{p}_{\rm u}) = \left(1 - \frac{K}{T}\right)\log_2\left(1 + \frac{\alpha_k}{\varsigma_k + \mu_k\left(1 - \frac{K}{T}\right)}\right) \tag{4}$$

where $\alpha_k \triangleq a_k \tau^* p_{\rm p}^* \frac{P-\tau^* p_{\rm p}^*}{T}$, $\varsigma_k \triangleq b_k \tau^* p_{\rm p}^* \frac{P-\tau^* p_{\rm p}^*}{T} + c_k \frac{P-\tau^* p_{\rm p}^*}{T}$, and $\mu_k \triangleq d_k \tau^* p_{\rm p}^* + 1$. Using Lemma 1 (given at the end of this subsection) and the fact that $\tau^* > K$, we obtain

$$g_k(\bar{\tau}, \bar{p}_{\rm p}, \bar{p}_{\rm u}) > \left(1 - \frac{\tau^*}{T}\right)\log_2\left(1 + \frac{\alpha_k}{\varsigma_k + \mu_k\left(1 - \frac{\tau^*}{T}\right)}\right) = g_k(\tau^*, p_{\rm p}^*, p_{\rm u}^*). \tag{5}$$

Therefore, we have $\mathcal{S}(\bar{\tau}, \bar{p}_{\rm p}, \bar{p}_{\rm u}) > \mathcal{S}(\tau^*, p_{\rm p}^*, p_{\rm u}^*)$ which proves that $\tau^* = K$.

Lemma 1: Assume that $a > 0$, $b > 0$, and $c > 0$. Then, the function

$$g(x) \triangleq x \log_2\left(1 + \frac{a}{b + cx}\right) \tag{6}$$

is a strictly increasing function in $x \in (0, \infty)$.

Proof: The first and the second derivatives of $g(x)$ are respectively given by

$$g'(x) = \frac{-1}{\ln 2}\frac{acx}{(b+cx)(a+b+cx)} + \frac{1}{\ln 2}\ln\left(1 + \frac{a}{b+cx}\right), \tag{7}$$

$$g''(x) = \frac{-ac^2(a+2b)x - 2abc(a+b)}{(b+cx)^2(a+b+cx)^2 \ln 2} < 0. \tag{8}$$

Since $g''(x) < 0$, $g'(x)$ is a strictly decreasing function in x. Thus,

$$g'(x) > g'(\infty) = 0. \tag{9}$$

The aforementioned result implies that $g(x)$ is a strictly increasing function in $x \in (0, \infty)$. ■

B. Corrections to Proof of Proposition 2

In the proof of [1, Prop. 2], some factors in the second derivative of $f_k(p_{\rm u})$ were missing. The proof of the inequality $\frac{\partial^2 f_k(p_{\rm u})}{\partial p_{\rm u}^2} \leq 0$ should be corrected as follows:

$$\omega_k \frac{\partial^2 f_k(p_{\rm u})}{\partial p_{\rm u}^2} = -b_k \hat{T}^2 (c_k - d_k \hat{T}) p_{\rm u}^3 - 3 b_k \hat{T}^2 (d_k P + 1) p_{\rm u}^2 + 3 b_k \hat{T} P (d_k P + 1) p_{\rm u} - (d_k P + 1)(b_k P^2 + c_k P + \hat{T}), \tag{10}$$

where $\omega_k \triangleq \frac{\left(b_k (P - \hat{T} p_{\rm u}) p_{\rm u} + c_k p_{\rm u} + d_k (P - \hat{T} p_{\rm u}) + 1\right)^3}{2 a_k}$, and $\hat{T} \triangleq T - K$. Since $P \geq \hat{T} p_{\rm u}$, we have

$$\omega_k \frac{\partial^2 f_k(p_{\rm u})}{\partial p_{\rm u}^2} = -b_k c_k \hat{T}^2 p_{\rm u}^3 - (d_k P + 1)(c_k P + \hat{T}) - \frac{3}{4} b_k \hat{T}^2 p_{\rm u}^2 - b_k \left(P - \frac{3}{2}\hat{T} p_{\rm u}\right)^2 - b_k d_k (P - \hat{T} p_{\rm u})^3 \leq 0. \tag{11}$$

Since $\omega_k > 0$, $\frac{\partial^2 f_k(p_{\rm u})}{\partial p_{\rm u}^2} \leq 0$.

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REFERENCES

[1] H. Q. Ngo, M. Matthaiou, and E. G. Larsson, "Massive MIMO with optimal power and training duration allocation," *IEEE Wireless Commun. Lett.*, vol. 3, no. 6, pp. 605–608, Dec. 2014.

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