3-D Spatial Spread Quantifiers for Multipath Fading Wireless Channels

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Abstract

This letter proposes a novel generalized method to quantify the angular dispersion of radiated energy in realistic three dimensional space. Based on the first three coefficients of the 2-D Fourier transform, simple mathematical expressions are derived to independently or jointly quantify the angular dispersion of energy in a reduced 2-D or realistic 3-D space, respectively. The proposed quantifiers include mean direction of arrival, circular variance (CV), true standard deviation (TSD), angular spreads (AS), angular constriction (AC), and direction of maximum fading (DoMF). The proposed quantifiers are helpful in establishing an appropriate mechanism to measure the angular dispersion in 3-D space. These quantifiers provide results in radians with true physical meaning unlike their counterparts in the literature. Moreover, an analysis on the impact of distribution truncation/distortion on the degree of accuracy in measuring the angular dispersion is presented which signifies the importance of true angular spread quantification. The proposed quantifiers can thus be considered as an appropriate goodness-of-fit criterion in the characterization of spatial statistics of multipath fading channels.

Keywords: Multipath channels, fading, Angle-of-arrival, Gaussian, shape factors, and standard deviation.

1 Introduction

Various definitions to quantify angular dispersion of multipath components are available in the literature. These definitions include total angular span, beamwidth, the root mean square (RMS) value of the angular data, standard deviation (SD), and shape factors. Use of beamwidth and RMS value as the definition of angle spread are often ill suited for general application to periodic functions such as angle-of-arrival (AoA) distributions [1]. SD of the Gaussian angular energy distribution heavily depends on the total angular span and the degree of truncation of the distribution (if there is any truncation in the distribution). So the utilization of SD of an exact bell-shaped Gaussian function as the accurate SD of the angular data will certainly lead to wrong results. In [1], a theory of spatial shape factors to quantify the dispersion of multipath waves in azimuth (2-D) plane is proposed. Three shape factors include angular spread (AS), angular constriction (AC), and azimuthal direction of maximum fading (DoMF); which are calculated based on 0^{th}, 1^{st}, and 2^{nd} complex Fourier coefficients of the azimuth AoA distribution. The AS shape factor defined in [1] used to denote the spread in the range from 0 to 1, can be considered as the most useful one, since it is invariant under changes in transmitted power and under any series of rotational or reflective transformation of the distribution of AoA. However, this definition has the disadvantage of not providing direct physical information about the angle spread, i.e., information either in degrees or radians. Another approach to quantify the multipath dispersion in 2-D space is proposed in [2], which quantifies the azimuthal angular dispersion in terms of the circular variance (CV) and SD by using trigonometric moments. This approach proposes SD as an appropriate measure to quantify the AS, as it provides physical meanings by computing AS in radians. In [3], various three-dimensional shape factors are proposed based on six special spherical harmonics. However, this approach lacks the physical meaning of AS on the same grounds, as discussed above.

This letter proposes a novel generalized method of quantifying the spread of multipath energy in marginal (2-D) and joint (3-D) angular domains. Various outdoor measurement campaigns (e.g., [4]) show that the scattering structures in urban environments usually give rise to Gaussian distributed AoA profile. However, many outdoor propagation scenarios emerge when the spatial distribution of scatterers does not produce an exact Gaussian distributed AoA profile, but instead they form some truncated or distorted versions of Gaussian distributions. Therefore, a discussion on the factors which cause truncation in AoA profile is provided along with an analysis on the impact of degree-of-truncation on the proposed angle spread quantifier named true standard deviation (TSD). The proposed quantifier is helpful in finding the accurate angular spread of truncated or distorted angular distributions as well as of the angular data acquired in measurement campaigns.

2 3-D Spatial Spread Quantifiers

Let, $F_{n,m} = C_{n,m} + jS_{n,m}$, be defined as the n^{th} and m^{th} complex trigonometric moment of the angular energy distribution $p(\phi, \theta)$ along azimuth and elevation AoA, respectively.

$$F_{n,m} = \frac{1}{P_o} \int_{\phi_{\min}}^{\phi_{\max}} \int_{\theta_{\min}}^{\theta_{\max}} p(\phi, \theta) \mathrm{e}^{j(n\phi + m\theta)} \, \mathrm{d}\theta \, \mathrm{d}\phi. \tag{1}$$

The total energy distributed in angular domain is equal to $P_o = \int_{\phi_{\min}}^{\phi_{\max}} \int_{\theta_{\min}}^{\theta_{\max}} p(\phi, \theta) \mathrm{d}\theta \, \mathrm{d}\phi$. The azimuthal and elevational angular span can be defined as, $\phi_{\text{span}} = \phi_{\max} - \phi_{\min}$ and $\theta_{\text{span}} = \theta_{\max} - \theta_{\min}$, respectively. The complete 3D directional range is described in azimuth and elevation planes by a span of 2π and π, respectively. Since the use of trigonometric moments is advantageous in manipulating discrete data, we can extend our method from continuous distribution to the discrete data obtained in measurements. The trigonometric parameters, $C_{n,m}$ and $S_{n,m}$ for the angular energy distribution $p(\phi, \theta)$ are defined as,

$$C_{n,m} = \frac{1}{P_o} \int_{\phi_{\min}}^{\phi_{\max}} \int_{\theta_{\min}}^{\theta_{\max}} p(\phi, \theta) \cos(n\phi + m\theta) \mathrm{d}\theta \, \mathrm{d}\phi, \tag{2}$$

$$S_{n,m} = \frac{1}{P_o} \int_{\phi_{\min}}^{\phi_{\max}} \int_{\theta_{\min}}^{\theta_{\max}} p(\phi, \theta) \sin(n\phi + m\theta) \mathrm{d}\theta \, \mathrm{d}\phi. \tag{3}$$

In the case of discrete measured or observed data, the definitions for the trigonometric parameters can be modified as, $C_{n,m} = 1/\bar{P}_o \sum_{k=1}^{K} \sum_{\ell=1}^{L} f_{k,\ell} \cos(n\phi_k + m\theta_\ell)$, and $S_{n,m} =$

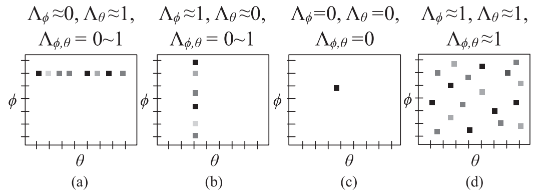

Figure 1: Different cases of joint and marginal AS quantification.

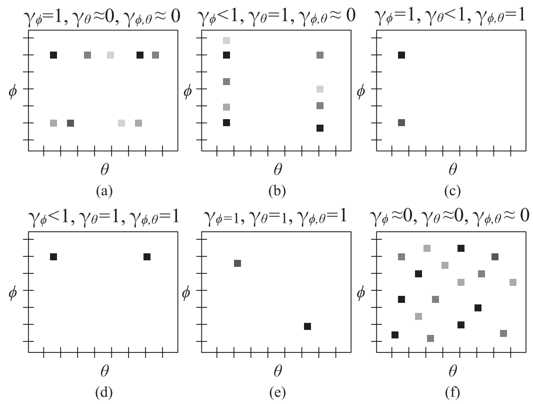

Figure 2: Different cases of joint and marginal AC quantification.

$1/\bar{P}_o \sum_{k=1}^{K} \sum_{\ell=1}^{L} f_{k,\ell} \sin(n\phi_k + m\theta_\ell)$; where, $\bar{P}_o = \sum_{k=1}^{K} \sum_{\ell=1}^{L} f_{k,\ell}$, and $f_{k,\ell}$ is the number of occurrences for the azimuth and elevation AoA ϕ_k and θ_ℓ in the (k,ℓ) bin of the 2-D histogram. The 2-D Fourier coefficients can also be obtained as, $F_{n,m} = \rho_{n,m}\, \mathrm{e}^{j\beta_{n,m}}$, where the mean resultant and the direction of trigonometric moments are $\rho_{n,m} = \left(C_{n,m}^2 + S_{n,m}^2\right)^{1/2}$ and $\beta_{n,m} = \arctan(S_{n,m}/C_{n,m})$, respectively. A complex 2-D Fourier coefficient matrix of first three harmonics can be expressed as,

$$\begin{array}{cc} & \begin{array}{ccc} \overbrace{\quad\quad}^{0^{\text{th}}} & \overbrace{\quad\quad}^{1^{\text{st}}} & \overbrace{\quad\quad}^{2^{\text{nd}}} \end{array} \\ \mathbb{F} = & \left[\begin{array}{ccc} F_{00} & F_{01} & F_{02} \\ F_{10} & F_{11} & \color{gray}{F_{12}} \\ F_{20} & \color{gray}{F_{21}} & F_{22} \end{array}\right] \end{array}. \tag{4}$$

The inter-connection of coefficients is pertinent to show, as it offers simplification operations in the following derivations, $F_{1,1} \approx F_{0,1} \times F_{1,0}$ and $F_{2,2} \approx F_{0,2} \times F_{2,0}$. This further suggests that, $\rho_{1,1} \approx \rho_{1,0} \times \rho_{0,1}$, $\rho_{2,2} \approx \rho_{2,0} \times \rho_{0,2}$, $\beta_{1,1} \approx \beta_{1,0} + \beta_{0,1}$, and $\beta_{2,2} \approx \beta_{2,0} + \beta_{0,2}$. The coefficients F_{12} and F_{21} are not used in the derivations, therefore shaded in gray in the matrix definition. The 0^{th} moment represents multipath angular power density, $P_o = F_{00}$.

2.1 3-D Shape Factors

This section presents an extension of multipath shape factors proposed in [1] from 2D (azimuthal) to 3D propagation. The conventional azimuthal AS shape factor Λ (using similar notations, as in [1]) defines the concentration of multipath about a single azimuthal direction. We define azimuthal, elevational, and joint AS shape factors to calculate the concentration of energy independently or jointly along azimuth and/or elevation axes. The azimuthal and elevational AS shape factors are defined as $\Lambda_\phi = \sqrt{1-\rho_{10}^2}$ and $\Lambda_\theta = \sqrt{1-\rho_{01}^2}$, respectively. The joint AS shape factor can be obtained as,

$$\Lambda_{\phi,\theta} = \sqrt{1-\rho_{11}^2}. \tag{5}$$

These AS shape factors range from 0 to 1, with 0 denoting a signal arriving from exactly one direction and 1 denoting no clear bias in angular distribution about a single direction. The relationship of independent azimuthal and elevational AS shape factors with the joint AS shape factor can be expressed as, $\Lambda_{\phi,\theta} \approx \sqrt{\Lambda_\phi^2 + \Lambda_\theta^2 - \Lambda_\phi^2\Lambda_\theta^2}$. The proposed AS shape factors provide flexibility to quantify the dispersion in a reduced (integrated) 2-D space or a realistic 3-D space; see four different cases illustrated in Fig. 1.

The AC shape factor defined in [1], is a measure of the concentration of AoA distribution about two physical directions in azimuth plane. We extend this shape factor to three AC shape factors, viz: azimuthal, elevational, and joint AC. The azimuthal and elevation AC can be defined as, $\gamma_\phi = \left(|F_{20} - F_{10}^2|\right) / \left(1 - |F_{10}|^2\right)$, $\gamma_\theta = \left(|F_{02} - F_{01}^2|\right) / \left(1 - |F_{01}|^2\right)$, respectively. The joint 3D AC can be defined as,

$$\gamma_{\phi,\theta} = \frac{|F_{22} - F_{11}^2|}{1 - |F_{11}|^2}. \tag{6}$$

The proposed AC shape factors range from 0 to 1, with 1 denoting the extreme case of signals arriving exactly from two directions and 0 denoting no clear bias in the two directions. The definition of these AC parameters is illustrated in Fig. 2 for six different scenarios. The azimuthal constriction, γ_ϕ, quantises the distribution of AoA about exactly two directions in azimuth plane integrated over the elevation plane. Similarly, the elevational constriction, γ_θ, quantises the distribution of AoA about exactly two angles in the elevation plane integrated over the azimuth plane. The joint AC, $\gamma_{\phi,\theta}$, quantises the joint azimuth and elevation AoA distribution about exactly any two directions in realistic 3-D space.

The DoMF is the orientation for which the wave-number spread is maximized. The azimuth and elevation DoMF can be obtained in radian by, $\chi_{\phi}^{\max} = 0.5 \arctan \{(S_{20} - 2C_{10}S_{10}) / (S_{10}^2 + C_{20} - C_{10}^2)\}$ and $\chi_{\theta}^{\max} = 0.5 \arctan \{(S_{02} - 2C_{01}S_{01}) / (S_{01}^2 + C_{02} - C_{01}^2)\}$, respectively. The composite DoMF can be obtained as, $\chi_{\phi,\theta}^{\max} = 0.5 \arctan ((S_{22} - 2C_{11}S_{11})/(S_{11}^2 + C_{22} - C_{11}^2))$, which represents $\chi_{\phi,\theta}^{\max} \approx \frac{1}{2} \left(\chi_{\phi}^{\max} + \chi_{\theta}^{\max} + \mu_{\phi} + \mu_{\theta}\right)$.

2.2 True Standard Deviation

The first moment gives the mean direction of AoA distribution, therefore, the mean AoA along azimuth and elevation directions can be obtained as $\mu_{\phi} = \beta_{10}$ and $\mu_{\theta} = \beta_{01}$, respectively. We define a basic measure of angular dispersion along azimuth and elevation AoA, the circular variance. It can be obtained in azimuth and elevation planes as, $\varsigma_{\phi} = 1 - \rho_{10}$ and $\varsigma_{\theta} = 1 - \rho_{01}$, respectively. The circular variance CV ranges between 0 and 1. The values closer to 0 represent high concentration of AoA distribution along the mean (μ) direction of AoA, whereas, a value closer to 1 represent widely dispersed AoA distribution. It is invariant under any changes in transmitted power and under any series of rotational or reflective transformation of $p(\phi, \theta)$. The joint CV parameter can be represented as, $\varsigma_{\phi,\theta} = 1 - \rho_{11}$. This joint CV is interlinked with marginal CVs as, $\varsigma_{\phi,\theta} \approx \varsigma_{\phi} + \varsigma_{\theta} - \varsigma_{\phi}\varsigma_{\theta}$. The CV is inter-related with AS shape factor as, $\Lambda_{\phi,\theta} = \sqrt{2\varsigma_{\phi,\theta} - \varsigma_{\phi,\theta}^2}$.

The definitions of CV can be extended to define TSD in azimuth and elevation planes as, $\sigma_{\phi} = \sqrt{-2\ln(1 - \varsigma_{\phi})} = \sqrt{-2\ln(\rho_{10})}$ and $\sigma_{\theta} = \sqrt{-2\ln(1 - \varsigma_{\theta})} = \sqrt{-2\ln(\rho_{01})}$, respectively. The joint TSD,

$$\sigma_{\phi,\theta} = \sqrt{-2\ln(1 - \varsigma_{\phi,\theta})} = \sqrt{-2\ln(\rho_{11})}. \tag{7}$$

The joint TSD $\sigma_{\phi,\theta}$ represents the Euclidean separation between σ_{ϕ} and σ_{θ}, which can also be obtained as, $\sigma_{\phi,\theta} \approx \sqrt{\sigma_{\phi}^2 + \sigma_{\theta}^2}$. These relationships for SD of angular energy distribution give the true physical information about the angular dispersion of the multipath waves in radians. Like the CV, TSD is also invariant under changes in transmitted power and under any series of rotational or reflective transformation of $p(\phi, \theta)$. Since it provides true physical information about the dispersion of the multipath signals in space, it can also stand as the major candidate for the unanimous definition of angle spread of multipath signals.

The proposed TSD quantifiers are inter-related with the angular spread shape factor as, $\sigma_\phi = \sqrt{-\ln(1-\Lambda_\phi^2)}$ and $\sigma_\theta = \sqrt{-\ln(1-\Lambda_\theta^2)}$. Moreover, the circular variance can also be linked with the angular spread shape factor as, $\Lambda_\phi = \sqrt{2\varsigma_\phi - \varsigma_\phi^2}$ and $\Lambda_\theta = \sqrt{2\varsigma_\theta - \varsigma_\theta^2}$.

3 Effect of Distribution Truncation on True Standard Deviation

This section presents an analysis on the degree of accuracy of the proposed TSD quantifier for the cases of exact Gaussian and distorted Gaussian AoA distributions. While the Gaussian case is just an example, the proposed analysis can be extended to any distribution and measurement data. Various measurement campaigns in the literature (see e.g., [4]) show that the scattering structures in urban environments usually give rise to the Gaussian distribution of AoA observed at the base station (BS). However, many situations have been observed when the distribution of the AoA does not match exactly a true bell-shaped Gaussian. Such situations usually emerge in outdoor environments when the spatial distribution of scatterers does not produce exact Gaussian distributions in AoA, but instead they form some truncated, distorted, cut or constant-added versions of Gaussian distributions. Therefore, the pdf of azimuth AoA is usually modeled as wrapped Gaussian [5, 6]. The major factors which give rise to the situations where the angular distribution is subjected to truncation, can be categorized as follows,

- *Factor 1:* The use of directional antenna at the BS eliminates the scatterers falling out of the beam-range of antenna. This gives rise to a truncated distribution of the AoA at the BS [7]. This sort of situations usually belongs to the space division multiple access (SDMA) systems which rely on the use of adaptive narrow-beam antennas and the nonhomogeneous distribution of users in a cellular system to increase system capacity [8].

- *Factor 2:* The propagation environment of streets crowded with automobile traffic leads to

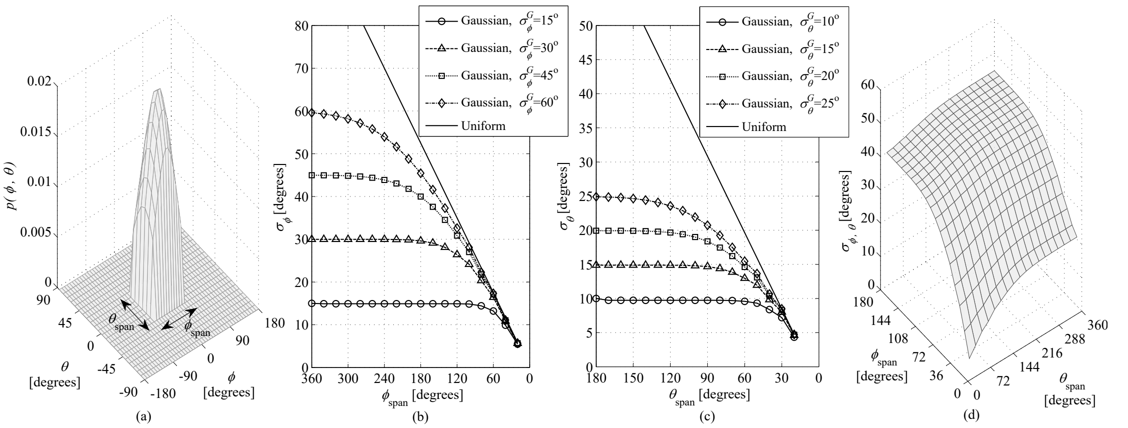

Figure 3: (a) Truncated 3-D Gaussian distributed AoA, ($\sigma_\phi^G = 40^o$ and $\sigma_\theta^G = 20^o$), (b) Effect of distribution truncation on azimuthal angular spread, (c) Effect of distribution truncation on elevational angular spread, (d) Effect of distribution truncation on joint angular spread, ($\sigma_\phi^G = 40^o$ and $\sigma_\theta^G = 20^o$).

a truncated AoA distribution caused by the physical dimensions of the streets. 3-D angular investigations at BS [9] show street canyon dominated propagation.

- *Factor 3:* The measurement campaigns of both indoor [10] and outdoor [4] environments demonstrate that in addition to the double exponential Laplacian (indoor case) and bell-shaped Gaussian (outdoor case), there are always very uniform tails on both sides of the mean AoA in the angular domain. The formation of these uniform tails in the distribution of AoA is in fact the aftermath of the reflections/scattering of the radio signal from the scatterers that surround BS. The uniform tails cause deformation in the shape of Gaussian or Laplacian, and hence alter the measure of angle spread, i.e., the SD.

- *Factor 4:* Far scatterers like high-rise buildings (in urban environments) or mountains (in rural environments) significantly contribute to the multipath scattering phenomenon in addition to the scattering structures located near the mobile stations [9]. Hence, they give rise to non-symmetric and non-isotropic scattering. This kind of scattering induces a non-uniform and non-symmetric distribution in the AoA as seen at the BS.

Measurements in [4] show that in addition to bell-shaped Gaussian distribution in AoA, there is always some additional part, which distorts the Gaussian function. This additional part certainly disturbs the angle spread measure. That is why the definition of SD of the exact bell-shaped Gaussian function, (σ_ϕ^G and σ_θ^G), cannot be trusted to be used in calculating the spatial fading correlations among antenna array elements. The effect of distribution truncation on the proposed angular spread quantifier TSD is shown in Fig. 3. It is evident that the proposed definition of TSD (σ_ϕ, σ_θ, $\sigma_{\phi,\theta}$) of the angular energy distribution of the AoA, depends on the peak(s) of the distribution as well as on the angular span of the data. SD of the exact bell-shaped Gaussian function, σ_*^G, remains equal to the proposed TSD, σ_*, of the Gaussian distribution as long as the total angular span remains more than $8\sigma_*^G$ (or twice of $4\sigma_*^G$). The angular span of $8\sigma_*^G$ encompasses about 99% of the total energy, this refers to the case of no truncation. As soon as the angular span lowers $8\sigma_*^G$ (in case of a truncated Gaussian), the TSD of the angular energy starts decreasing, and it decreases sharply for the smaller values of the span. This definition of TSD in 3-D space can stand as the major candidate for the unanimous definition of angle spread of multipath signals to assist in calculating more accurate spatial correlations and other high order statistics of multipath fading channels for emerging wireless communication networks.

Various measurement campaigns and analytical models for characterization of wireless multipath fading channels have been proposed in the literature. No such measure of the angular dispersion has been developed so far, on which the proximity of these models could be tested. Usually, least squares error (LSE) is used to measure the fitness of analytical results over the field measurement results [11]. However, the shape of the distribution of energy in angular domain is not so important rather the variance of the angle spread is important. The proposed TSD offers a realistic measure of almost all necessary information about the angle spread in 3-D space to be used as a goodness-of-fit measure, no matter what functions and how much truncation are used.

4 Conclusions

New generalized quantifiers to measure the dispersion of energy in realistic 3-D space have been proposed. The proposed quantifiers measure the angular dispersion of energy in joint 3-D space (or reduced 2-D planes) and provides the results in radians with true physical sense unlike their counterparts in the literature. An analysis on the impact of distribution truncation has been presented to demonstrate the applicability of the proposed quantifiers as goodness-of-fit criterion in the characterization of spatial statistics of multipath fading channels.

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