A Closed-Form Solution for RSS/AoA Target Localization by Spherical Coordinates Conversion

Slavisa Tomic, Marko Beko, Rui Dinis, and Paulo Montezuma

Abstract—**This letter addresses the problem of target localization in a 3-D space, utilizing combined measurements of received signal strength and angle of arrival (AoA). By using the spherical coordinate conversion and available AoA observations to establish new relationships between the measurements and the unknown target location, we derive a simple closed-form solution method. We then show that the proposed method has straightforward adaptation to the case where the target's transmit power is also not known. Simulation results validate the outstanding performance of the proposed method.**

Index Terms—**Wireless localization, received signal strength (RSS), angle of arrival (AoA), weighted least squares (WLS), wireless sensor network (WSN).**

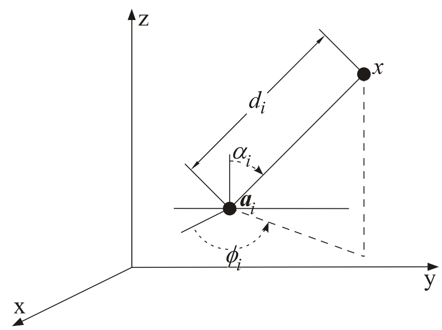

Fig. 1. Illustration of anchor and target locations in a 3-D space.

I. Introduction

TARGET localization has gained much attention recently due to its significance in both military and industrial applications [1]–[5]. Wireless localization schemes usually rely on range measurements [3], drawn from time of arrival, received signal strength (RSS), angle of arrival (AoA), or their combination.

Notable advance has been made in developing range/angle localization algorithms recently [6]–[15]. In [8], linear least squares (LS) and optimization based estimators were studied. An LS and a maximum likelihood (ML) estimators for merged RSS difference (RSSD) and AoA measurements were derived in [9]. In [10], a selective weighted LS (WLS) estimator for RSS/AoA localization problem was proposed. A WLS estimator for RSSD/AoA localization problem was presented in [11]. An estimator based on semidefinite relaxation technique was proposed in [12]. In [13], a second order cone programming estimator was derived. A bisection procedure was employed in [14] to find an *exact* solution on a computed trust region. In [15], a weighted linear LS (WLLS) estimator that uses an unbiased constant and the noise covariance was proposed.

All of the above methods are either designed for 2-D scenarios only [9], [10], [12], or extremely low noise power [11], or employ more sophisticated mathematical tools that lead to a significant increase in the computational complexity [12]–[14].

In this letter, we propose a novel target localization algorithm for 3-D space that merges RSS and AoA observations. By shifting from Cartesian to spherical coordinates and taking advantage of the acquired AoA measurements, we build new relationships between the measurements and the unknown target location, which results in a simple and closed-form solution. In contrast to the existing methods, the proposed one does not require further relaxations (which enlarge the set of possible solutions). The new method is computationally light and our numerical results show that it provides excellent accuracy, surpassing the state-of-the-art methods in general.

II. Problem Formulation

Let $\boldsymbol{x} \in \mathbb{R}^3$ be the unknown location of the target and $\boldsymbol{a}_i \in \mathbb{R}^3$, for $i = 1, \ldots, N$, be the known location of the i-th anchor. In order to determine the target's location, a hybrid system that combines range and angle measurements is employed, see Fig. 1. In Fig. 1, $\boldsymbol{x} = [x_x, x_y, x_z]^T$ and $\boldsymbol{a}_i = [a_{ix}, a_{iy}, a_{iz}]^T$ represent the coordinates of the target and the i-th anchor, respectively, while d_i, ϕ_i and α_i denote respectively the distance, azimuth angle and elevation angle between the target and the i-th anchor.

Here, we assume that the distance is drawn from the RSS information exclusively, since ranging based on RSS does not require additional hardware [16]. The RSS between the target and the i-th anchor, P_i, is defined as [17, Ch. 3]

$$P_i = P_0 - 10\gamma \log_{10} \frac{d_i}{d_0} + n_i, \text{ for } i = 1, \ldots, N \tag{1}$$

where P_0 (dBm) is the reference power at a distance d_0 ($d_0 \leq d_i$), γ is the path loss exponent (PLE), d_i is the distance

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